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Critical phenomena in ferromagnetic antidot lattices

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In this paper a quantitative theoretical formulation of the critical behavior of soft mode frequencies as a function of an applied magnetic field in two-dimensional Permalloy square antidot lattices in the nanometric range is given according to micromagnetic simulations and simple analytical calculations. The degree of softening of the two lowest-frequency modes, namely the edge mode and the fundamental mode, corresponding to the field interval around the critical magnetic field, can be expressed via numerical exponents. For the antidot lattices studied we have found that: a) the ratio between the critical magnetic field and the in-plane geometric aspect ratio and (b) the ratio between the numerical exponents of the frequency power laws of the fundamental mode and of the edge mode do not depend on the geometry. The above definitions could be extended to other types of in-plane magnetized periodic magnetic systems exhibiting soft-mode dynamics and a fourfold anisotropy. [http://dx.doi.org/10.1063/1.4944666]

I. INTRODUCTION

Since the pioneering works of the 60s and 70s on the static scaling laws[1,2] and on their generalization to dynamical properties of critical phenomena,[3–5] the critical behavior of dynamic excitations in physical systems has been widely investigated. Among them remarkable results were obtained for magnetic systems studied, for instance, by using the dynamic Ising model and the dynamic Heisenberg model.[6–9]

However, all the above mentioned studies were carried out by investigating the critical behavior of dynamic excitations as a function of temperature. On the other hand, in these last years great attention has been devoted to the study of the static and dynamical properties of one-dimensional, two-dimensional (2D) and three-dimensional periodic magnetic systems, because of their challenging features.[10–14] In recent years, a series of works have focused the attention on the study of soft modes associated to critical phase transitions, for instance, in magnetic media exhibiting topological defects and in 2D antidot lattices (ADLs).[15,16]

To the best of our knowledge, there are not yet in the literature investigations dealing quantitatively with the critical properties of soft modes in low-dimensional magnetic systems in correspondence of critical phase transitions driven by an external magnetic field. In this paper, we propose a quantitative study of the critical behavior as a function of the external magnetic field of the lowest-frequency modes, the so called soft modes, namely in order of increasing frequency the edge mode (EM) and the resonant mode of the system, the fundamental (F) mode, in 2D square ADLs having periodicity and holes size in the nanometric range.[16] In this analysis the external magnetic field takes the role played, in general, by the temperature.

The theoretical investigation is carried out via the Dynamical Matrix Method (DMM), an Hamiltonian-based and finite-difference micromagnetic approach developed within the conservative dynamics regime and extended to 2D periodic magnetic systems.[17] Micromagnetic simulations are

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supported by a simple analytical treatment based on the formulation of some quantities independent of the geometry. In particular, for the ADLs studied in the nanometric range, it has been found that: 1) the ratio between the critical field and the in-plane aspect ratio $r = d/s$ with d the hole diameter and s the hole separation and 2) the ratio between the numerical exponents of the EM and F mode frequency power laws are constant quantities.

II. MICROMAGNETIC FRAMEWORK

The simulation was performed on a 2D square array of circular holes of different diameters embedded into a ferromagnetic Permalloy (Py) material. The samples investigated have periodicity $a = 420$ nm and thickness $L = 30$ nm, whereas the diameter d of the holes ranges between 100 nm and 260 nm. The equilibrium magnetization state was calculated by using typical Py magnetic parameters: $A = 1.3 \times 10^{-6}$ erg/cm, $\gamma/2\pi = 2.8$ GHz/kOe, $M_s = 800$ emu/cm^3 with A the exchange stiffness constant, γ the gyromagnetic ratio and M_s the saturation magnetization. According to the above values of the parameters the exchange length $l_{exch} = (A/2\pi M_s^2)$ is about 6 nm. The external magnetic field **H** was applied along the y axis symmetry direction of the ADLs (see Fig. 1) and the ground-state magnetization **M** was determined by using OOMMF with periodic boundary conditions[18] and prismatic 5 nm × 5 nm × 30 nm cells. The choice of prismatic cells having 5 nm in-plane size is justified by the fact that it is a value comparable to the exchange length of the Py material. To simulate an infinitely extended system (along the x and y directions) the DMM with implemented 2D boundary conditions[17] has been employed so that the unit cell extracted contains all the physical effects due to the presence of the other ADs belonging to the other unit cells and the dynamic magnetization of collective modes is expressed in Bloch form.

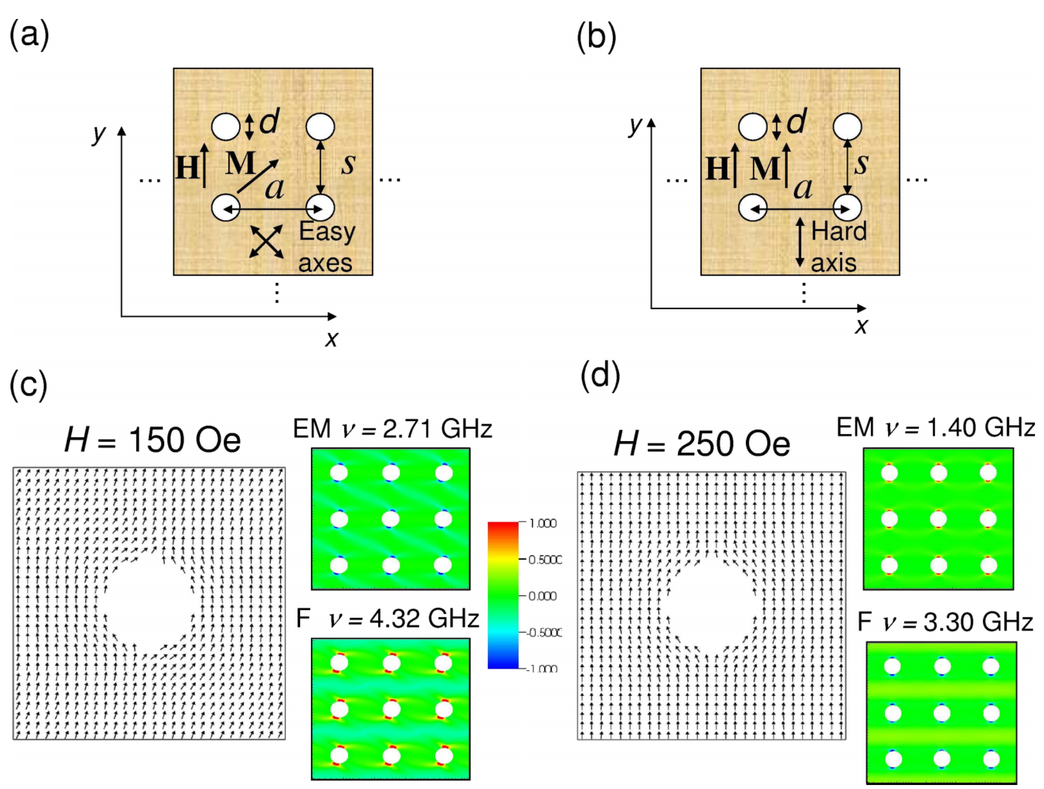

FIG. 1. (a) Sketch of the ADL. The diameter d and the periodicity a are indicated together with **M**, **H** and of the easy axes for $H < H_c$. (b) As in panel (a) but for $H > H_c$. The hard axis parallel to the y direction is also indicated. (c) Calculated ground-state magnetization in the unit cell for $d = 140$ nm at $H = 150$ Oe ($H < H_c$) . The corresponding spatial profiles (real part of the out-of-plane dynamic magnetization) and frequencies of the EM and F modes are depicted. (d) As in panel (c) but at $H = 250$ Oe ($H > H_c$). The intensity of the critical field is $H_c = 200$ Oe.

III. RESULTS AND DISCUSSION

The analysis is restricted to the behavior of the EM and F mode frequencies vs. the intensity H of the magnetic field in the vicinity of the critical field H_c where the frequencies of these two collective modes soften. The spin dynamics is studied at the centre of the Brillouin zone but similar results could be obtained for non-vanishing Bloch wave vector. A sketch of the studied system is shown in Fig. 1(a) for $H < H_c$ where **M** is parallel to the easy axis (45° away from the y direction) and in Fig. 1(b) for $H > H_c$ where **M** is parallel to the hard axis (y direction). In Fig. 1(c)-1(d) the corresponding ground-state magnetization in the unit cell determined according to OOMMF and the spatial profiles of the EM and F mode are depicted. While the EM amplitude is mainly confined outside the holes and is mainly negligible in the horizontal channels, the amplitude of the F mode is rather appreciable also in the horizontal channels comprised between the holes (note the spatial profiles rotation for $H < H_c$ consequence of the magnetization rotation).

The critical phase transition takes place at H_c whose value depends on the studied geometry. It is important to note that the occurrence of the phase transition is not only due to the crucial role played by the applied magnetic field, but by the concomitant action played by the size of the holes of the periodic system to determine the ground-state magnetization. Indeed, the presence of holes gives a significant contribution to the demagnetizing energy. For the phenomenological model developed to explain the critical and reorientational phase transition, for the discussion of the behavior of all collective modes and for the comparison with experimental frequencies see Ref. 16.

The critical phase transition can be characterized by two phases having different symmetries. The phase corresponding to $H > H_c$ is marked by a static magnetization **M** aligned with the external magnetic field **H** along the y axis (hard axis), while in the phase corresponding to $H < H_c$ **M** is not anymore collinear with **H** and progressively rotates away from the y axis aligning at a given external magnetic field intensity along the easy axis forming a 45° angle with **H**. Hence, for $H > H_c$ a magnetic field of strong magnitude orders the ground-state magnetization forcing it to align along the hard axis, while for $H < H_c$ the ordering mechanism driven by the external magnetic field weakens. As a result, for $H > H_c$ the ground-state magnetization is characterized by a rotational symmetry under the global symmetry group of rotations that is lower with respect to that of the phase occurring for $H < H_c$. A strong external magnetic field lowers the ground-state rotational symmetry leading to a preferential and fixed spatial direction of the static magnetization.

The critical phase transition studied here can be described quantitatively by an order parameter written in scalar form as:

$$\Psi = 2\left(\frac{< M_y >}{< M >}\right)^2 - 1. \tag{1}$$

The order parameter is proportional to the square ratio between the y-component of the static magnetization normalized to the total magnetization (in modulus) both evaluated in each prismatic cell and averaged over the number of prismatic cells belonging to the unit cell. Therefore, its definition expresses the correlation of the static magnetization over the short length scale given by the exchange length comparable to the size of a prismatic cell.

In our case:

$$\begin{aligned} &\Psi = 0 \text{ for } \mathbf{M} \text{ parallel to the easy axis } H < H_c \\ &\Psi = 1 \text{ for } \mathbf{M} \text{ parallel to the hard axis } H \geq H_c, \end{aligned} \tag{2}$$

namely Ψ is equal to one when the static magnetization is aligned along the hard axis (y-axis) and continuously reduces its value till to zero when **M** is aligned along the easy axis for a given $H < H_c$.

The aforementioned definition of Ψ was introduced to recall the behavior of the order parameter describing quantitatively the typical critical phase transitions taking place as a function of temperature where the order parameter continuously varies from one to zero and the two phases have different symmetries (e.g. the ferromagnetic-paramagnetic phase transition). Moreover, according to Eq. (1), the order parameter does not change its sign for a change of sign of the averaged y component of the static magnetization.

Let's $r = d/s$ be the geometric in-plane ratio with d the diameter of the holes and $s = a - d$ the separation between adjacent horizontal rows (or adjacent vertical rows) of ADLs. The dependence of H_c on r was studied for $0.3 \le r \le 1.6$. Indeed, it was found by means of OOMMF simulations that, for $a = 420$ nm, the critical phase transition does not occur anymore at $d < 100$ nm corresponding to $r < 0.3$. As shown in Fig. 2, the magnitude of the critical field increases linearly with r passing from about 120 Oe for $r = 0.3$ to about 750 Oe for $r = 1.6$ and the slope found by means of a linear fitting procedure turns out to be about 481 Oe.

Straightforwardly, it can be proved that the scaling relation $I = h_c/r$ with $h_c = \frac{H_c}{4\pi M_s}$ does not depend on the ADL geometry. As a matter of fact, for all the geometries studied, we have found from micromagnetic simulations that $I = 0.04$. This scaling relation has a considerable advantage: by using it one can immediately get H_c , viz.

$$H_c = 4\pi M_s I\, r, \tag{3}$$

whatever array periodicity and diameter is studied under the limiting condition $d < a$ that should be fulfilled in order to reproduce an ADL. For instance, for $r = 2.5$ ($d = 300$ nm and $a = 420$ nm not simulated) we get $H_c = 1000$ Oe according to Eq. (3) and so on. Note that the value of H_c is in accordance with the one obtained in the framework of the phenomenological model developed in Ref. 16 where the critical field turned out to be approximately the opposite of the first-order demagnetizing field averaged over the volume of the unit cell.

Since the analysis is focused on collective spin precession around the equilibrium magnetization, the critical phase transition can be investigated also by studying the behavior of soft mode (EM and F mode) frequencies as a function of the external magnetic field in the proximity of the critical field. In this respect, numerical exponents expressing the degree of softening can be introduced.

To study in a quantitative way the behavior of soft modes as a function of the external magnetic field and in the vicinity of the critical field it is useful to define the reduced field $h = \frac{H-H_c}{H_c}$ with $h \to 0^{\pm}$. The behavior of soft mode frequencies close to the critical point (H_c) can be written in the form of a power law, viz:

$$\nu_i = A_i |h|^{\lambda_i}, \tag{4}$$

where λ_i is the exponent expressing the degree of softening with $i =$ EM, F and Ai is a coefficient expressed in GHz.

In Fig. 3 the calculated frequency curves of the EM and of the F mode (black) as a function of the external magnetic field for H close to H_c according to DMM are compared to the resulting fitting curves (red) obtained via Eq. (4) for three different diameters.

Interestingly, for both modes the curvature of the frequency is the same for $h \to 0^{\pm}$ and this behavior justifies the use of the modulus in Eq. (4). In particular, for $d = 140$ nm $\lambda_{EM} = 0.40$ and

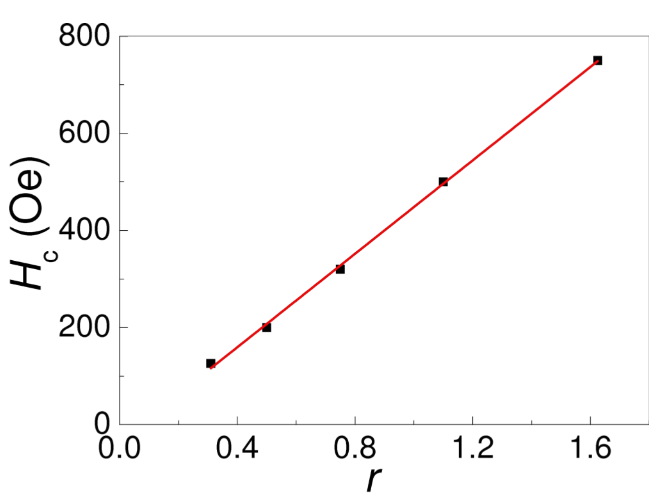

FIG. 2. Critical field H_c vs. in-plane aspect ratio r. Squares: calculated values of the critical field. Line: fitted data.

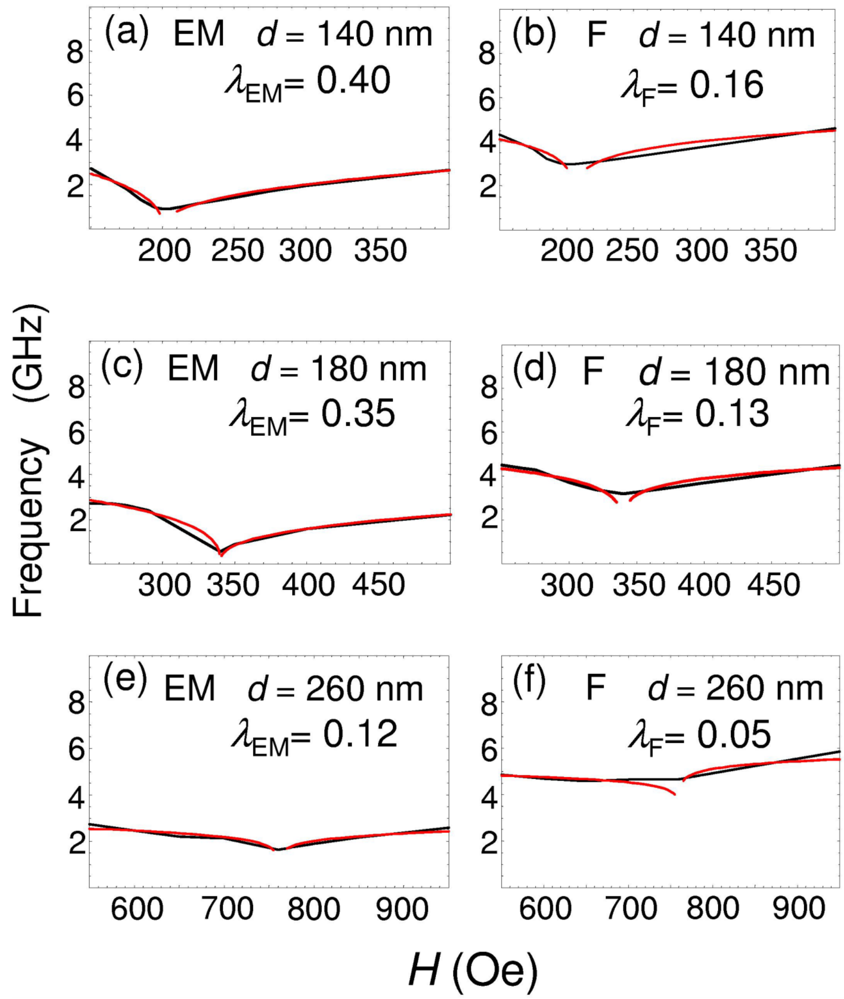

FIG. 3. (a) Frequency of the EM vs. H in the interval around H_c for $d = 140$ nm. Black curve: micromagnetic simulation. Red curve: fitting curve obtained according to Eq. (4). (b) As in (a) but for the F mode. (c) As in (a) but for $d = 180$ nm. (d) As in (b) but for $d = 180$ nm. (e) As in (a) but for $d = 260$ nm. (f) As in (b) but for $d = 260$ nm. The values of the numerical exponents are also indicated.

$\lambda_F = 0.16$, while for $d = 180$ nm $\lambda_{EM} = 0.35$ and $\lambda_F = 0.13$. Instead, for $d = 260$ nm the result of the fit was $\lambda_{EM} = 0.12$ and $\lambda_F = 0.05$. The overall agreement is very good apart from some discrepancies close to H_c for the F mode frequencies and for $H < H_c$ corresponding to the continuous change of the ground-state magnetization by decreasing H. In Fig. 3 the ranges of H for $H < H_c$ are different for each d and the minimum values of H correspond to the alignment of the ground-state $\mathbf{M}$ with the easy axis, while the chosen ranges of H for $H > H_c$ are approximately the same (about 200 Oe) for each d.

It is not surprising that the two collective mode frequency curves are characterized by different curvatures. The different curvatures are ascribed to the localization features of the two collective modes (see Fig. 1) which experience different demagnetizing effects. Moreover, by increasing the diameter, the curvatures of both EM and F mode frequencies reduce resulting in lower values of the corresponding numerical exponents. However, we have found that the ratio β_λ of the two exponents is constant, independently of the diameter investigated, viz. $\beta_\lambda = \frac{\lambda_F}{\lambda_{EM}} = 0.4$. In other words, the ratio of the frequency curvature for the two soft modes close to H_c does not depend on the geometric in-plane ratio r so that $\beta_\lambda(r) = \beta_\lambda$ for each r investigated.

To study the dynamical critical behavior and to search for a dynamic critical point corresponding to the static one it would be necessary to calculate microscopic physical quantities like the spatial correlation function and the dynamic susceptibility, but this will be the subject of a further investigation. This analysis could confirm that the found constant quantities can be regarded as universal quantities over any length scale.

IV. CONCLUSIONS

In summary, in this work the critical phenomena linked to the dynamics of soft modes studied as a function of an external magnetic field in 2D Py ADLs have been investigated from a quantitative point of view. According to micromagnetic simulations and analytical calculations it has been found that the critical field can be determined by means of a scaling relation and that the ratio of the frequency curvature of the two soft modes close to the critical magnetic field does not depend on the geometry of the ADL studied. The investigation of critical properties exhibited as a function of an external magnetic field can be performed on the available experimental data[16] and can be extended both theoretically and experimentally to other kinds of 2D periodic magnetic systems composed by other ferromagnetic materials that exhibit fourfold anisotropy. These results could be of interest for the design and interpretation of microwave properties and could open the route to the quantitative description of critical phenomena in different classes of low-dimensional magnetic systems where the critical properties are investigated as a function of the external magnetic field.

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[1] L. P. Kadanoff, Physics **2**, 263 (1966).
[2] B. Widom, J. Chem. Phys. **43**, 3898 (1965).
[3] B.I. Halperin and P.C. Hohenberg, Phys. Rev. Lett. **19**, 700 (1967).
[4] B.I. Halperin and P.C. Hohenberg, Phys. Rev. **177**, 952 (1969).
[5] P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**, 436 (1977).
[6] R.J. Glauber, J. Math. Phys. **4**, 294 (1963).
[7] H.W. Huang, Phys. Rev. B **11**, 1144 (1975).
[8] H. W. Diehl, M. Krech, and H. Karl, Phys. Rev. B **66**, 024408 (2002).
[9] K. Chen and D.P. Landau, Phys. Rev. B **49**, 3266 (1994).
[10] M. Madami, D. Bisero, G. Gubbiotti, S. Tacchi, G. Carlotti, K. Nakano, and T. Ono, IEEE Trans. Magn. **48**, 1269 (2012).
[11] R. Zivieri, F. Montoncello, L. Giovannini, F. Nizzoli, S. Tacchi, M. Madami, G. Gubbiotti, G. Carlotti, and A.O. Adeyeye, IEEE Trans. Mag. **47**, 1563 (2011).
[12] M. Krawczyk and D. Grundler, J. Phys.: Condens. Matter **26**, 123202 (2014).
[13] S. Mamica, M. Krawczyk, M. L. Sokolovskyy, and J. Romero-Vivas, Phys. Rev. B **86**, 144402 (2012).
[14] R. Zivieri, Solid State Phys. **63**, 151 (2012).
[15] F. J. Buijnsters, A. Fasolino, and M. I. Katsnelson, Phys. Rev. B **89**, 174433 (2014).
[16] R. Zivieri, P. Malagò, L. Giovannini, S. Tacchi, G. Gubbiotti, and A.O. Adeyeye, J. Phys.: Condens. Matter **25**, 336002 (2013) and references therein..
[17] L. Giovannini, F. Montoncello, and F. Nizzoli, Phys. Rev. B **75**, 024416 (2007).
[18] *OOMMF User's Guide, Version 1.0*, edited by M. Donahue and D. Porter, interagency report NISTIR 6376 (National Institute of Standards and Technology, Gaithersburg, MD).