Review Article

Scientific Review on the Ionospheric Absorption and Research Prospects of a Complex Eikonal Model for One-Layer Ionosphere

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The present paper conducts a scientific review on ionospheric absorption, extrapolating the research prospects of a complex eikonal model for one-layer ionosphere. As regards the scientific review, here a quasi-longitudinal (QL) approximation for nondeviative absorption is deduced which is more refined than the corresponding equation reported by Davies (1990). As regards the research prospects, a complex eikonal model for one-layer ionosphere is analyzed in depth here, already discussed by Settimi et al. (2013). A simple formula is deduced for a simplified problem. A flat, layered ionospheric medium is considered, without any horizontal gradient. The authors prove that the QL nondeviative amplitude absorption according to the complex eikonal model is more accurate than Rawer's theory (1976) in the range of middle critical frequencies.

1. Introductive Review

Absorption is the process by which ordered energy of the radio wave is transformed into heat and electromagnetic (e.m.) noise by electron collisions with neutral molecules and ionized particles [1].

1.1. Ionospheric Absorption. When the absorption is small and spatial diffraction is neglected, absorption is given by the imaginary part β of the complex propagation function k. The absorption loss L_a (in decibels) is given by $L_a = 8.68(-\int \beta ds)$. If W_r is the received power, W_u is the unabsorbed power that would have been received in the absence of absorption, and $\rho = (W_r/W_u)^{1/2}$ is the effective amplitude reflection coefficient:

$$L_a = -10\log_{10}\left(\frac{W_r}{W_u}\right) = -20\log_{10}\rho. \tag{1a}$$

In scientific work, absorption is sometimes expressed in Np (nepers). On the basis of the natural logarithm, $1\mathrm{Np} = 8.68\,\mathrm{dB}$.

The wave amplitude E decays exponentially with distance; that is, $E = E_0(-\int \beta ds)$. Hence, the absorption in Np is

$$L_a = \int \beta ds = -\ln\rho. \tag{1b}$$

In the absence of the geomagnetic field, the Appleton-Hartree formula gives the absorption β of a wave of angular frequency ω, per unit path length, in a medium containing N electrons per unit volume [1]

$$\beta = \frac{q_e^2/\varepsilon_0}{2m_e c} \frac{1}{n_R} \frac{N\nu}{\omega^2 + \nu^2} = 4.6 \cdot 10^{-2} \frac{1}{n_R} \frac{N\nu}{\omega^2 + \nu^2}, \tag{2}$$

where β is expressed in dB/km, N in cm^{-3}, and ν in s^{-1} and c is the speed light in vacuum, ε_0 is the constant permittivity of vacuum, m_e is electron mass, q_e is the charge of electron, n_R is the refractive index, and ν is the electron collisional frequency. Equation (2) enables us to define two types of absorption [1]:

(i) nondeviative absorption, which occurs in regions where n_R is approximately unity but where the product $N \cdot \nu$ is large. This is the type of absorption of high frequency (HF) and very high frequency (VHF) waves that occurs in the D region;

(ii) deviative absorption, which occurs near the top of the ray-trajectory or anywhere along the path where marked bending takes place, for example, when $n_R \rightarrow 0$. Deviative absorption is associated with group retardation.

In the presence of the geomagnetic field, the nondeviative absorption coefficient is [1]

$$\beta = 4.6 \cdot 10^{-2} \frac{N\nu}{\left(\omega \pm \omega_L\right)^2 + \nu^2}, \tag{3}$$

where $\omega_L = \omega_H \cos \theta_H$ is the magnetic angular gyrofrequency corresponding to the longitudinal (parallel) component of the geomagnetic field, θ_H is the angle between the direction of wave propagation and the geomagnetic field, $\omega_H = B_0(q_e/m_e)$, and B_0 is the amplitude of magnetic induction field and where the + and − signs refer to the ordinary and extraordinary waves, respectively. From this, we see that the nondeviative absorption of the extraordinary wave is greater than that of the ordinary wave. This is particularly important on frequencies near the gyrofrequency (i.e. 1 to 2 MHz) where the extraordinary wave is heavily absorbed [1].

1.2. Martyn's Absorption Theorem. The absorption $L_a^{(ob)}$ of a wave at angular frequency ω incident on a flat ionosphere with angle φ_0 is related to the absorption $L_a^{(v)}$ of the equivalent vertical wave, at an angular frequency $\omega \cos \varphi_0$, by

$$L_a^{(ob)}\Big|_{\omega_{ob}=\omega} = L_a^{(v)}\Big|_{\omega_v=\omega \cos \varphi_0} \cos \varphi_0. \tag{4}$$

This theorem shows that the additional absorption of the oblique wave, caused by the longer ray path, is more than compensated for by the absorption decrease because of the higher frequency [2].

1.3. Absorption in Some Model Layers. To provide some insight into the dependence of absorption on electron density profiles N, solar zenith angle χ, frequency f, and so forth, the total absorptions in some simple layers are given in Table 1 (reproduced from Davies [1]). The wave frequency is much greater than the collision frequency, and the geomagnetic field is ignored. Of particular interest are equations (b) and (d) in Table 1. Equation (b) has been used, as a profile of the E region, to separate D-layer and E-layer absorptions. Equation (d) shows that absorption varies as $\cos^{3/2} \chi$. In the cases that include both deviative and nondeviative absorption, that is, (b) and (e), the inverse square dependence on wave frequency does not hold.

2. The Quasi-Longitudinal (QL) Approximation for Non-Deviative Absorption

The complete treatise of the propagation for e.m. waves in any magnetoplasma is rather complex; here, we restrict ourselves to a relatively simple discussion, based on common assumptions reported by [3].

As well-known, the phase refractive index n can be calculated from the Appleton-Hartree equation [1]:

$$n^2 = \left(n_R - in_I\right)^2 = 1 - \frac{X}{1 - iZ - \left(Y_T^2/2 \cdot (1 - X - iZ)\right) \pm \sqrt{\left(Y_T^4/4 \cdot (1 - X - iZ)^2\right) + Y_L^2}}, \tag{5}$$

where $X = \omega_p^2/\omega^2$ (ω being the angular frequency of the radio wave, $\omega_p = \sqrt{Nq_e^2/m_e\varepsilon_0}$ the plasma frequency, and N the profile of electron density); $Y_L = Y \cos \theta_H$, $Y_T = Y \sin \theta_H$ (θ_H being the angle between the wave vector and the geomagnetic field), and $Y = \omega_H/\omega$ (being $\omega_H = B_0(q_e/m_e)$ the angular gyrofrequency, and B_0 the amplitude of magnetic induction field); $Z = \nu/\omega$ (ν being the collision frequency).

For the known birefringence of ionospheric plasma, this relationship allows to derive two refractive indices, for the ordinary ray $n^{[ORD]}$ and the extraordinary ray $n^{[EXT]}$, where the refractive indices $n^{[ORD],[EXT]}$ are complex quantities (being $n^{[ORD]} = n_R^{[ORD]} + in_I^{[ORD]}$ and $n^{[EXT]} = n_R^{[EXT]} + in_I^{[EXT]}$, with obvious meaning of symbols). The two refractive indices are obtained from (5) through the choice of positive or negative signs, which must be decided applying the so-called Booker's rule [1]. Once defined the critical frequency

$\omega_c = (\omega_B/2) \cdot \sin^2\theta_H/\cos \theta_H$, this rule states that, to achieve continuity of $n_R^{[ORD]}$ ($n_R^{[EXT]}$) and $n_I^{[ORD]}$ ($n_I^{[EXT]}$), if $|\omega_c/\nu| > 1$, the positive (negative) sign in (5) must be considered both for $X < 1$ and for $X > 1$, while, if $|\omega_c/\nu| < 1$, the positive (negative) sign for $X < 1$ and negative (positive) for $X > 1$ must be considered.

Referring to a specialized text [3] for the discussion of applicability of the quasi-transverse (QT) and quasi-longitudinal (QL) approximations, here we will limit ourselves to pointing out that, for values of X much smaller than 1 (the wave frequency is much larger than the plasma frequency), the QL approximation holds within wide limits (it is acceptable, for $Z^2 \ll 1$, up to values of θ_H tending to $\pi/2$). The application of QT-QL criterion leads to the conclusion that, in the D-layer, the QL propagation occurs if the waves are propagating at relatively high frequencies (usually, an ionospheric radio-link works at frequency $f \geq 2$ MHz).

TABLE 1: Total absorption in model layers (reproduced from Davies [1]).

Electron-density profile	Collision profile	Type of absorption	Absorption	Comments
$N = N_0$	$\nu = \nu_0$	Nondeviative	$\int \beta ds = 1.2 \cdot 10^{-7} f^{-2} N_0 \nu_0 T$ (a)	T is slab thickness
Parabolic: $N = N_0 \left[1 - \left(\dfrac{h - h_0}{2H} \right)^2 \right]$	$\nu = \nu_0$	Deviative and nondeviative	$\int \beta ds = \dfrac{\nu_0 H}{c} \left(\dfrac{1 + x^2}{2x} \ln \dfrac{1 + x}{1 - x} - 1 \right)$ (b)	c is light speed in vacuum; H is scale height; $x = f / f_c$, f_c is critical frequency
$N = N_0$	$\nu = \nu_0 \exp(-z)$	Nondeviative	$\int \beta ds = 1.2 \cdot 10^{-7} f^{-2} N_0 \nu_0 H \left[1 - \exp\left(-\dfrac{T}{H} \right) \right]$ (c) $z = \dfrac{h - h_0}{H}$	
Chapman: $N = N_0 \exp[1 - \sec \chi \exp(-z)]$	$\nu = \nu_0 \exp(-z)$	Nondeviative	$\int \beta ds = 4.13 \dfrac{\nu_0 H}{c} \dfrac{\cos^{3/2} \chi}{x^2}$ (d)	Double traverse
$N = a(h - h_0)$	$\nu = \nu_0$	Deviative and nondeviative	$\int \beta ds = \dfrac{2}{3} f^2 \dfrac{\nu_0}{c} \dfrac{1}{ak}$ (e)	Wave reflected, $k = 80.5$

If the QL approximation is applied for $Z^2 \ll 1$, then $\theta_H \ll 1 \Rightarrow Y_L \cong Y$ and (5) reduces to [1]

$$n^2 = (n_R - i n_I)^2 \cong 1 - \frac{X}{1 - iZ \pm Y}. \tag{6}$$

After some manipulation, (6) is divided into two equations, one for the real part and one for the imaginary part:

$$n_R^2 - n_I^2 \cong 1 - \frac{X(1 \pm Y)}{(1 \pm Y)^2 + Z^2}, \tag{7a}$$

$$2 n_R n_I \cong \frac{XZ}{(1 \pm Y)^2 + Z^2} \cong \frac{XZ}{(1 \pm Y)^2}, \tag{7b}$$

and under the simplifying condition $n_I \ll n_R$, the imaginary part of the refractive index is derived by coupling the previous two equations (7a)-(7b):

$$
\begin{aligned}
n_I &\cong \frac{1}{2} \frac{1 - n_R^2}{n_R} \frac{Z}{1 \pm Y} \left[1 + \left(\frac{Z}{1 \pm Y} \right)^2 \right] \\
&= \frac{1}{2} \frac{1 - n_R^2}{n_R} \frac{\nu/\omega}{1 \pm \omega_H/\omega} \left[1 + \left(\frac{\nu/\omega}{1 \pm \omega_H/\omega} \right)^2 \right].
\end{aligned} \tag{8}
$$

In the case of nondeviative absorption, occurring far from the reflection level, that is, with real refractive index near unity, $n_R \to 1$, the local absorption coefficient can be expressed as

$$
\begin{aligned}
\beta &= n_I \frac{\omega}{c} \\
&\approx \frac{\omega}{c} \frac{1}{2} (1 - n_R^2) \frac{\nu/\omega}{1 \pm \omega_H/\omega} \left[1 + \left(\frac{\nu/\omega}{1 \pm \omega_H/\omega} \right)^2 \right] \\
&= \frac{1}{2} (1 - n_R^2) \frac{\nu}{c} \frac{\omega}{\omega \pm \omega_H} \left[1 + \left(\frac{\nu}{\omega \pm \omega_H} \right)^2 \right].
\end{aligned} \tag{9}
$$

Note that the present paper has deduced a QL approximation for nondeviative absorption (9) which is more refined than the corresponding equation reported by Davies [1]. In fact, Davies' equation is deduced, in the right limit $n_R \to 1$, only from (7b), and without accounting also (7a).

3. The Variation of Collision Frequency with the Altitude

The average number of collisions ν which an electron makes per unit time with the atmosphere molecules depends upon the number density of the molecules and, therefore, on the density and composition of the atmosphere. Then, a decreasing exponential law holds in an atmosphere which is constant in composition [4]:

$$\nu(h) = \nu_{max} \exp\left(-\frac{h - h_{max}}{H} \right), \tag{10}$$

where H is the atmospheric scale height, ν_{max} is a constant, that is, $\nu_{max} = \nu(h_{max})$, and h_{max} is the height corresponding to the maximum electron density N_{max}; that is, $N_{max} = N(h_{max})$. The constant ν_{max} is not the maximum collision frequency but the collision frequency at the "maximum height" h_{max}. On equal terms, this maximum occurs for a null solar zenith angle χ; that is, $\chi = 0$. In practice, H takes different values at different levels, and the law can only be expected to hold over ranges of h so small that H may be treated as constant. A useful summary of the factors which affect the value of ν have been made by [5].

It is found that changes of the value of ν affect the propagation of radio waves far less than changes of the electron density profile N. For many purposes, it is, therefore, permissible to treat ν as constant over a small range of height h. This is especially true at high frequencies (HFs) (greater than 2 MHz), where the wavelength is small compared with the scale height H, which is approximately 10 km. Here, more generally, considering a short range of heights h, that is, $|h - h_{max}| \ll H$, the collision frequency (10) can be expanded in a Taylor's series at the first order:

$$\nu(h) \cong \nu_{max} \left(1 - \frac{h - h_{max}}{H} \right) \longrightarrow \nu_{max}, \tag{11}$$

so that the collision frequency $\nu(h)$ is approximately a function decreasing linearly with the height h.

The scale height H is defined as [4]

$$H = \frac{k_B \langle T \rangle}{m \langle g \rangle}, \tag{12}$$

where k_B is the Boltzmann's constant and m is the mean molecular mass, varying with the atmospheric composition and, therefore, with the altitude (on the ground, m is assumed to be about 29 times the mass of a hydrogen atom, which is approximately equal to $4.7 \cdot 10^{-26}$ kg) [3].

The Earth's gravity acceleration g can be expressed as function of the geographic colatitude angle θ and height h, applying, if necessary, the free air correction (FAC) which accounts for altitudes above sea level, by the International Gravity Formula (IGF) 1967 [6]:

$$
\begin{aligned}
&g\left(h,\theta\right) \\
&= g_0\left[1 + K_1\sin^2\left(\frac{\pi}{2} - \theta\right) - K_2\sin^2 2\left(\frac{\pi}{2} - \theta\right)\right] - K_3 h,
\end{aligned}
\tag{13}
$$

where $g_0 = 9.780327 \, \text{m/s}^2$, $K_1 = 5.3024 \cdot 10^{-3}$, $K_2 = 5.8 \cdot 10^{-6}$, and $K_3 = 3.086 \cdot 10^{-6} \, \text{s}^{-2}$, so that, for a one-layer ionosphere between the heights h_1 and h_2, being $|h_1 - h_2| \ll h_{\max}$, the mean value of gravity acceleration is

$$
\begin{aligned}
\langle g \rangle &= \frac{1}{\pi}\frac{1}{h_2 - h_1}\int_0^\pi d\theta \int_{h_1}^{h_2} g\left(h,\theta\right) dh \\
&= g_0\left(1 + \frac{K_1 - K_2}{2}\right) - K_3\frac{h_1 + h_2}{2},
\end{aligned}
\tag{14}
$$

and the mean absolute temperature can be calculated as [Appendix A]

$$
\langle T \rangle = T_{\max}\left[1 - \gamma\frac{m\langle g \rangle}{2k_B T_{\max}}\left(h_1 + h_2 - 2h_{\max}\right)\right],
\tag{15}
$$

where $\gamma = 2/7$ and $T_{\max}$ is not the maximum temperature but the temperature at the "maximum height" $h_{\max}$; that is, $T_{\max} = T(h_{\max})$.

This paper has deduced (15) as mean value $\langle T \rangle$ of the absolute temperature profile $T = T(h)$ for a one-layer ionosphere between the heights h_1 and h_2. This average value can be used as an effective value representing the temperature profile of layer ionosphere. The formula (15) results in more accuracy than assuming the temperature to be constant on the whole layer; that is, $T(h) = T(h_1) = T(h_2) = \text{const}$.

4. A Dipole Model of Geomagnetic Field

The Earth, as a whole, is source of a magnetic field, the geomagnetic field, which as a first-order acceptable approximation can be assimilated to the field of a dipole located in the Earth's centre, with magnetic moment equal to $8.1 \cdot 10^{22} \, \text{A} \cdot \text{m}^2$ and tilted of approximately $\Delta\theta = 11°$ compared to the Earth's rotation.

An ionospheric model of the Earth's magnetic field consists of an eccentric dipole. The magnetic gyrofrequency ω_H is a function of the height h above the ground and the geomagnetic colatitude λ [7]:

$$
\omega_H\left(h,\lambda\right) = \omega_{H0}\left(\frac{R_T}{R_T + h}\right)^3\left(1 + 3\cos^2\lambda\right),
\tag{16}
$$

where ω_{H0} is the gyrofrequency at the equator on the ground and R_T the Earth's mean radius ($R_T = 6371$ km).

The magnetic dip angle $I(\lambda)$ is given by [7]

$$
\tan I = 2\cot\lambda.
\tag{17}
$$

The dipole model of the Earth's magnetic field uses the axis of a computational coordinate system as the axis for dipole field. When using this dipole model, the computational coordinate system is a geomagnetic coordinate system, and the Earth's magnetic field is defined in geomagnetic coordinates. Reference [8] describes the transformations between the geographic and geomagnetic coordinate systems, respectively, with colatitudes θ and $\lambda = \theta - (\pi/180°)\Delta\theta$.

For a one-layer ionosphere between the heights h_1 and h_2, the mean value of magnetic gyrofrequency is

$$
\begin{aligned}
&\langle \omega_H \rangle \\
&= \frac{1}{\pi}\frac{1}{h_2 - h_1}\int_0^\pi d\lambda \int_{h_1}^{h_2} \omega_H\left(h,\lambda\right) dh \\
&= \omega_{H0}\frac{1}{\pi}\int_0^\pi\left(1 + 3\cos^2\lambda\right) d\lambda\frac{1}{h_2 - h_1}\int_{h_1}^{h_2}\left(\frac{R_T}{R_T + h}\right)^3 dh \\
&= K_H\omega_{H0}\frac{R_T^3\left(h_1 + h_2 + 2R_T\right)}{\left(h_1 + R_T\right)^2\left(h_2 + R_T\right)^2},
\end{aligned}
\tag{18}
$$

where $K_H = 0.770982$. Thus, the Earth's magnetic field is represented by a dipole model, which is eccentric (centred) in the geographic (geomagnetic) coordinate system, and the mean value of magnetic gyrofrequency is calculated as an integral of gyrofrequency referred to the geomagnetic coordinates. At ionospheric heights, that is, $h \ll R_T$, the gyrofrequency ω_H is almost independent from the height h and at middle geographic latitudes $\pi/2 - \theta = \pi/2 - \lambda - (\pi/180°)\Delta\theta$ such that $-\pi/4 < \pi/2 - \theta < \pi/4$, it is almost constant: $\omega_H(h,\lambda) \cong \langle\omega_H\rangle$ ($\langle f_H \rangle = \langle\omega_H\rangle/2\pi \cong 1.2$ MHz) [9].

The paper has deduced (18) as mean value $\langle\omega_H\rangle$ of the magnetic gyrofrequency profile $\omega_H = \omega_H(h,\lambda)$ for a one-layer ionosphere between the heights h_1 and h_2. This average value can be used as an effective value representing the gyrofrequency profile of layer ionosphere. The formula (18) results in more accuracy than assuming the gyrofrequency to be constant for the whole layer; that is, $\omega_H(h,\lambda) \cong \omega_{H0}$.

5. Chapman's One-Layer Ionosphere and QL Nondeviative Absorption

Reference [10] elaborated a theory on the solar photoionization in the Earth's atmosphere, which still retains a fundamental importance in the field of ionospheric physics. As basis of this theory there are common assumptions reported by [3].

A remarkable simplifying hypothesis is to consider stationary conditions, which approximately occur around the true solar noon, when the zenith angle χ of Sun, and hence

the electron density N, varies slowly in time. In dealing with various issues, stationary condition is approximately extended to all the daylight hours. Under this hypothesis, it results in the following [1]:

$$N(z) = N_{\text{ref}} \exp \frac{1 - z - \sec\chi \exp(-z)}{2}$$

$$\cong N_{\text{ref}} \exp \frac{1 - z - \exp(-z)}{2}, \qquad \chi \longrightarrow 0. \tag{19}$$

The curve trend of normalized electron density N/N_{ref} is a Chapman's function of the reduced relative height $z = (h - h_{\text{max}})/H$ and the zenith angle χ.

The nondeviative absorption occurs, for all ray-paths, in the D-layer and only for those paths with reflection in the F-layers, also in the E-layer; the Chapman's theory is a good approximation for nondeviative absorption on all these ray-paths, insomuch that the trend of the electron density N with height h satisfies a well-known implicit relationship [1] and, in the limit of quasi-stationarity, the explicit relationship (19).

Considering a vertical radio sounding with just one ionospheric reflection, the amplitude absorption $L_a^{(v)}$ (1a)-(1b) can be expressed as a decreasing exponential function $L_{12}^{(v)} = \exp[-\beta_{12}^{(v)}]$ of the integral absorption coefficient $\beta_{12}^{(v)}$ across the vertical propagation path $\gamma : h_1 = 0 \rightarrow h_2 = h$, which is defined as the definite integral $\beta_{12}^{(v)} = \int_{h_1}^{h_2} \beta(h)dh$ of the local absorption coefficient $\beta(h)$ between the heights $h_1 = 0$ and $h_2 = h$ (the reader who would like proving such issue could find an in-depth analysis in Section 4 of Settimi et al. paper [11]). Applying the QL approximation for nondeviative absorption as reported by Davies [1], the variation of collision frequency with the altitude (10), the dipole model of geomagnetic field (18), and Chapman's one-layer ionosphere (19), after some manipulation [9], this results in the following:

$$\beta_{12}^{(v)} = -\ln L_{12}^{(v)} = \int_{h_1}^{h_2} \beta(h)\,dh$$

$$= \frac{\sqrt{2\pi e}}{2} \frac{q_e^2/\varepsilon_0}{m_e c} N_{\text{max}} \nu_{\text{max}} H \frac{\cos^{3/2}\chi}{(\omega \pm \langle \omega_H \rangle)^2 + \nu_{\text{max}}^2}$$

$$\cong \frac{\sqrt{2\pi e}}{2} \frac{q_e^2/\varepsilon_0}{m_e c} \frac{N_{\text{max}} \nu_{\text{max}} H}{(\omega \pm \langle \omega_H \rangle)^2 + \nu_{\text{max}}^2}, \qquad \chi \longrightarrow 0. \tag{20a}$$

Instead, considering an oblique radio sounding with one ionospheric reflection, the Martyn's absorption theorem [2] assures that the integral absorption coefficient $\beta_{12}^{(ob)}$ of a wave at frequency f incident on a flat ionosphere with angle φ_0 is

further dependent on the secant of φ_0 (magnitude order and units correctly revised with respect to Rawer [12]):

$$\beta_{12}^{(ob)}$$

$$\cong 1.344 \cdot 10^{-4} \left[\text{m}^2/\text{s}\right]$$

$$\times \frac{10^6 \cdot N_{\text{max}} \left[\text{cm}^{-3}\right] \cdot \nu_{\text{max}} \left[\text{s}^{-1}\right] \cdot 10^3 \cdot H \, [\text{km}]}{\left\{10^6 \left(f \pm \langle f_H \rangle\right) [\text{MHz}]\right\}^2 + \left\{(1/2\pi) \nu_{\text{max}} \left[\text{s}^{-1}\right]\right\}^2} \sec\varphi_0. \tag{20b}$$

Equations (20a)-(20b) shows that, on equal terms, the QL nondeviative amplitude absorption $[L]_{\text{dB}}$ is inversely proportional to the square of frequency f^2 for the e.m. waves and increases as the angle of incidence φ_0; that is, it increases with the decrease of the elevation angle. Inserting in (20a)-(20b) the typical values for the D and E layers, it follows that, as to be expected, the absorption occurs mainly in the D layer and the amplitude absorption order is a few tens of dB.

6. The Complex Eikonal Model for One-Layer Ionosphere

6.1. The Complex Eikonal Model.
A previous paper of Settimi et al. [11] conducted a scientific review on the complex eikonal, extrapolating the research prospects on the ionospheric ray-tracing and absorption.

As regards the scientific review, the eikonal equation is expressed, and some complex-valued solutions are defined corresponding to complex rays and caustics. Moreover, the geometrical optics is compared to the beam tracing method, introducing the limit of the quasi-isotropic and paraxial complex optics approximations. Finally, the quasi-optical beam tracing is defined as the complex eikonal method applied to ray-tracing, discussing the beam propagation in cold magnetized plasma.

As regards the research prospects, the cited paper has proposed to address the following scientific problem: in absence of electromagnetic (e.m.) sources, consider a material medium which is time invariant, linear, optically isotropic, generally dispersive in frequency, and inhomogeneous in space, with the additional condition that the refractive index is assumed varying even strongly in space. The paper continues the topics discussed by [13], proposing a novelty with respect to the other referenced bibliography; indeed, the absorption is assumed nonnegligible, so the medium is dissipative. In mathematical terms, the refractive index belongs to the field of complex numbers. The dissipation plays a significant role, and even the eikonal function belongs to the complex numbers field. Under these conditions, suitable generalized complex eikonal and transport equations are derived.

In fact, if the dissipative absorption is supposed to be not negligible in the 3D space $\vec{r} = (x, y, z)$, which is filled by a material medium with complex refractive index, that is $n(\vec{r}) = n_R(\vec{r}) + i n_I(\vec{r})$, then the approximation of quasi-optics allows to introduce an e.m. field $V(\vec{r}) = A(\vec{r}) \exp[i k_0 S(\vec{r})]$ whereby

the eikonal is a complex function, that is $S(\vec{r}) = S_R(\vec{r}) + iS_I(\vec{r})$, satisfying to the pair of eikonal equations [11]

$$\left|\nabla S_R\right|^2 - n_R^2(\vec{r}) = \left|\nabla S_I\right|^2 - n_I^2(\vec{r}) = C = \text{const}, \qquad (21a)$$

$$\nabla S_R \cdot \nabla S_I = n_R(\vec{r})\, n_I(\vec{r}). \qquad (21b)$$

Once assumed the null value to be allowable for the constant $C = 0$ in (21a), and defining the versor $\hat{s} = d\vec{r}/ds$ as tangent to the curvilinear coordinate s, then the two real scalar equations (21a)-(21b) can be collected in just one complex vector equation [11]:

$$\nabla S = n(\vec{r})\,\hat{s} = n(\vec{r})\,\frac{d\vec{r}}{ds}. \qquad (22)$$

Under the hypothesis $C = 0$, (21a)-(21b) for the complex eikonal $S(\vec{r})$ are reduced into two independent equations for the real and imaginary part of eikonal function, respectively, $S_R(\vec{r})$ and $S_I(\vec{r})$, the first $\nabla S_R = n_R(\vec{r})\hat{s}$ solving the ray-tracing and the second $\nabla S_I = n_I(\vec{r})\hat{s}$ to derive the amplitude absorption; in these conditions, the ray-tracing and absorption problems become uncoupled, and the eikonal equation (22) belonging in the complex numbers field $[n(\vec{r}), S(\vec{r}) \in \mathbb{C}]$ is formally equal to the corresponding one in the real numbers field $[n(\vec{r}), S(\vec{r}) \in \mathbb{R}]$ [13].

The present paper does not include the transfer equation for the field amplitude (see Settimi et al. [11]), from which one can derive a relationship for the refractive attenuation of radio wave [14], valid along a ray tube: $I(\vec{r})\Delta S = \text{const}$, where ΔS is the cross-section square of a ray tube, and $I(\vec{r}) = n_R(\vec{r})A^2(\vec{r})$ the field intensity which is proportional to the square amplitude of e.m. field $A^2(\vec{r})$ times the real part of refractive index $n_R(\vec{r})$. The transfer equation for the field amplitude states the intensity law of geometrical optics [13], which is an evolution of the expression for the intensity in terms of the flow tubes. The e.m. energy propagates within the flow tube and the intensity varies in inverse proportion to the section of tube. The relationship for the refractive attenuation of radio wave involves just the geometric attenuation due to the enlargement of wave front with the propagation [15]. The intensity carried by each ray may decreases along the distance, even if the medium is loss-free, since, as the wave propagates, the intensity is distributed over an ever-widening surface.

6.2. One-Layer Ionosphere.

In order to solve the ionospheric ray-tracing and absorption problems, Settimi et al. [11] have prospected a novel point of view. Equations (21a)-(21b) or (22) for complex eikonal are derived assuming the material medium as optically isotropic. However, there exist suitable conditions in which (21a)-(21b) or (22) can be referred to the Appleton-Hartree equations (5) or (6) for ionospheric magnetoplasma, which becomes anisotropic at the presence of geomagnetic field. Indeed, in agreement with [16–18], the quasi-isotropic approximation (QIA) of geometrical optics can be applied for weakly anisotropic inhomogeneous media, so that the eikonal equations hold alternatively for both the ordinary and extraordinary rays, which propagate independently in the magnetoplasma by experiencing each a different refractive index.
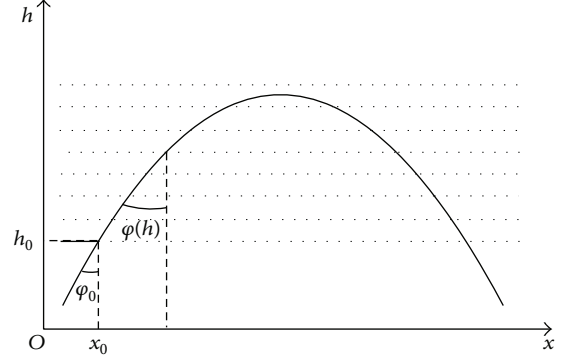

FIGURE 1: Flat, layered ionospheric medium described by a linearized profile (25a)-(25b) and (26a)-(26b) (reproduced from Settimi et al. [11]).

Let us consider a flat, layered ionospheric medium (Figure 1, reproduced from Settimi et al. [11]), without any horizontal gradient, characterized by an electron density profile dependent only on the altitude, as for the complex refractive index,

$$n = n(h) = n_R(h) + in_I(h). \qquad (23)$$

Fix the axis of abscissa x, orthogonal to the axis of heights h, which produce the space plane xh. Initially, a generic optical ray is passing through a point (x_0, h_0), forming an angle φ_0 with the heights axis h. Along the optical path, the ray changes its angle φ with respect to the axis h. This angle $\varphi(h)$ depends on the initial conditions (x_0, h_0) and it is a function of the height h. In fact, the refraction law of Snell-Descartes states for the real part of refractive index:

$$n_R(h)\sin\varphi(h) = n_R(h_0)\sin\varphi_0 = R. \qquad (24)$$

The ionosphere, in presence of collisions, is assumed to be weakly interacting with the static geomagnetic field. A linearized analytic profile can be adopted for the complex refractive index [11] (Figure 1):

$$n(h) = 1, \qquad h < h_0, \qquad (25a)$$

$$n(h) = n_R(h) + in_I(h), \qquad h \geq h_0,$$

$$n_R(h) = n_0 + \alpha_R(h - h_0), \qquad (25b)$$

$$n_I(h) = \alpha_I,$$

where the coefficients α_R and α_I are functions of the angular frequency ω, the collision frequency $\nu_{\max}$ (11), the atmospheric scale height H (12), the mean value of magnetic gyrofrequency $\langle\omega_H\rangle$ (18), and the height of ionosphere bottom h_0, the refractive index across the ionosphere-neutral atmosphere boundary $n_0 = n(h_0)$. In a first-order approximation, the boundary refractive index n_0 could be assumed as a real number slightly different from 1, that is, in any case $n_0 \neq 1$, so that the refractive index $n(h)$ is a discontinuous function of height h, that is, crossing $h = h_0$.

The linear refractive index (25a), (25b) is sufficient to highlight that the geometrical attenuation is modelled just

by the transport equation, and, therefore, the dissipative absorption just by the complex eikonal equations (21a)-(21b); in fact if $n(h) \cong \alpha_R h$, then $I(h_2)/I(h_1) \cong n(h_1)/n(h_2) = h_1/h_2$ [11].

Inserting the refractive index (25a) to solve the real part $\nabla S_R = n_R(\vec{r})\hat{s}$ of complex eikonal equation (22), then the optical path $h = h(x)$ is obtained [11] (Figure 1):

$$\frac{n_R(h)}{R} = \cosh\left[\frac{\alpha_R}{R}(x - \overline{x})\right], \tag{26a}$$

$$\overline{x} = x_0 - \frac{R}{\alpha_R}\ln\left(\frac{1 + \cos\varphi_0}{\sin\varphi_0}\right). \tag{26b}$$

In this case, the optical rays show, unless vertical shifts, the trend of hyperbolic cosines, known as catenaries, which can be approximated to parabolas.

Inserting the refractive index (25b) to solve the imaginary part $\nabla S_I = n_I(\vec{r})\hat{s}$ of complex eikonal equation (22), if the height h is not so low, that is, $h \gg h_0$, then the imaginary part $S_I(h)$ of eikonal function is approximately independent from the Snell-Descartes' constant R [11]:

$$S_I(h) \cong \frac{\alpha_I}{\alpha_R}n_R(h), \qquad h \gg h_0. \tag{27}$$

Regardless of geometrical attenuation, the amplitude absorption due to dissipation effects can be calculated from the imaginary part of complex eikonal function. In fact, in the ionospheric plasma, recalling that the collision frequency is a function $\nu(h)$ of height h, as the complex eikonal $S(h) = S_R(h) + iS_I(h)$, the local absorption coefficient $\beta(h)$ can be defined as

$$\beta(h) = \frac{\omega}{c}S_I(h) \cong \frac{\omega}{c}\frac{\alpha_I}{\alpha_R}n_R(h). \tag{28}$$

If $n_I(h) = \alpha_I \to 0$, then $n(h) \to n_R(h) \in \mathbb{R}$; moreover $S_I(h) \to 0$, so $S(h) \to S_R(h) \in \mathbb{R}$; and, finally, $\beta(h) \to 0$.

Considering a vertical radio sounding with just one ionospheric reflection, once applied (28), the integral absorption coefficient $\beta_{12}^{(v)} = \int_{h_1}^{h_2}\beta(h)dh$ across any vertical propagation path $\gamma: h_1 \to h_2$ is proportional to the optical path $\Delta l_{12}^{(v)} = \int_{h_1}^{h_2}n_R(h)dh$; that is [11],

$$\beta_{12}^{(v)} \cong \frac{\omega}{c}\frac{\alpha_I}{\alpha_R}\Delta l_{12}^{(v)}$$

$$= \frac{\omega}{c}\frac{\alpha_I}{\alpha_R}(h_2 - h_1)\left[n_0 + \frac{\alpha_R}{2}(h_1 + h_2 - 2h_0)\right], \tag{29}$$

$$|h_2 - h_1| \ll h_0 < h_{\max}.$$

Instead, considering an oblique radio sounding with one ionospheric reflection, the Martyn's absorption theorem [2] assures that the absorption coefficient $\beta_{12}^{(ob)}$ of a wave at angular frequency ω incident on a flat ionosphere with angle φ_0 is related to the absorption coefficient $\beta_{12}^{(v)}$ of the equivalent vertical wave, at an angular frequency $\omega\cos\varphi_0$, by $\beta_{12}^{(ob)}\big|_{\omega_{ob}=\omega} = \beta_{12}^{(v)}\big|_{\omega_v=\omega\cos\varphi_0}\cos\varphi_0$.

6.3. A Simple Formula for a Simplified Problem.

Settimi et al. [11] proposed (29), useful to calculate the absorption due to the propagation across the ionospheric D-layer, which can be approximately modelled by a linearized complex refractive index (25a), (25b), covering a short range of heights between $h_1 = 50$ km and $h_2 = 80$–90 km approximately. According to [4], rocket techniques have evidenced that, in the daytime, the D-layer shows, almost as a rule, its maximum (and minimum) of electron density in the vicinity of 80 km (and 85 km). In authors opinion, this evidence is not so strong, and even if the refractive index $n_R(h)$ is not a monotonically decreasing function of height h along the D-E layers valley, anyway this is not a substantial correction; $n_R(h)$ can be linearized up to $h_2 = 80$–90 km approximately.

Thus, the theoretical bases of present paper were laid, where the further expansion of (29) will lead to a formula for the ionospheric absorption more accurate than some theoretical models (20a)-(20b), using the Chapman's profile reported by Rawer [12].

Indeed, Appendix B, supposing the analytical continuity of complex eikonal model (28) with the QL approximation for nondeviative absorption (9), demonstrates the necessary and sufficient condition to equate the collision frequency deriving from the refractive index (25a), (25b) to the variation of collision frequency with the altitude (11). The QL nondeviative absorption (9), deduced in this paper, is more refined than the corresponding equation reported by Davies [1]; and, linearizing the involved equations, here are obtained the coefficients α_R and α_I as functions of the angular frequency ω, the collision frequency $\nu_{\max}$, the scale height H, the mean magnetic gyrofrequency $\langle\omega_H\rangle$, and the height of ionosphere bottom h_0, the refractive index across the ionosphere-neutral atmosphere boundary $n_0 = n(h_0)$; that is [Appendix B],

$$\alpha_R = -\frac{1}{2}\frac{\left(1 - n_0^2\right)/H}{1 + ((h_{\max} - h_0)/H)}, \tag{30a}$$

$$\alpha_I = -\frac{(1 - n_0)^2}{H}\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle}\left[1 + \left(\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle}\right)^2\right]. \tag{30b}$$

A reasonable hypothesis should be assumed for (30a)-(30b); the boundary refractive index n_0 is a real number slightly less than 1; that is, $n_0 < 1$, so that both coefficients α_R and α_I are negative; that is, $\alpha_R < 0$ and $\alpha_I < 0$.

Therefore, the linearized analytic profile for complex refractive index (25b) can be rearranged as

$$n_R(h) \cong n_0 - \frac{1 - n_0^2}{2}\frac{(h - h_0)/H}{1 + (h_{\max} - h_0)/H}, \qquad h \geq h_0, \tag{31a}$$

$$n_I = -\frac{(1 - n_0)^2}{H}\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle}\left[1 + \left(\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle}\right)^2\right]. \tag{31b}$$

Just the reasonable hypothesis assumed below equations (30a)-(30b) could imply the expected conclusions for (30a)-(30b); the real refractive index $n_R(h)$ is a decreasing function of height h, while the imaginary refractive index n_I is negative, as substantially correct for any ionospheric profile of the D-layer [4].

Moreover, considering a vertical radio sounding with just one ionospheric reflection, once the optical path is calculated

$$\Delta l_{12}^{(v)} = \int_{h_1}^{h_2} n_R(h)\, dh$$

$$\cong (h_2 - h_1)\left[n_0 - \frac{1 - n_0^2}{4} \frac{(h_1 + h_2 - 2h_0)/H}{1 + (h_{max} - h_0)/H}\right], \quad (32)$$

$$|h_2 - h_1| \ll h_0 < h_{max},$$

it is proportional to the integral absorption coefficient (29), re-arranged as

$$\beta_{12}^{(v)} \cong 2\frac{1 - n_0}{1 + n_0}\Delta l_{12}^{(v)}\left(1 + \frac{h_{max} - h_0}{H}\right)\frac{\nu_{max}}{c}$$

$$\times \frac{\omega}{\omega + \langle \omega_H \rangle}\left[1 + \left(\frac{\nu_{max}}{\omega + \langle \omega_H \rangle}\right)^2\right]. \quad (33)$$

Note that the refractive index $n_0 = n(h_0)$ can be computationally assumed as $n_0 = 1 - \varepsilon_{max}$ for any ray-tracing program, where ε_{max} is defined as the maximum allowable relative error in single step length for any of the equations being integrated [7].

Instead, considering an oblique radio sounding with one ionospheric reflection, the Martyn's absorption theorem [2] assures that the integral absorption coefficient $\beta_{12}^{(ob)}$ of a wave at angular frequency ω incident on a flat ionosphere with angle φ_0 is further dependent on the secant of φ_0. A simple formula for a simplified problem results:

$$\beta_{12}^{(ob)}\Big|_{\omega_{ob}=\omega} = \beta_{12}^{(v)}\Big|_{\omega_v = \omega \cos\varphi_0}\cos\varphi_0$$

$$\cong 2\frac{1 - n_0}{1 + n_0}\Delta l_{12}^{(v)}\left(1 + \frac{h_{max} - h_0}{H}\right)\frac{\nu_{max}}{c}\frac{\omega}{\omega + \langle \omega_H \rangle}$$

$$\times \frac{\cos\varphi_0}{\cos\varphi_0}\left[1 + \left(\frac{\nu_{max}}{\omega + \langle \omega_H \rangle}\frac{1}{\cos\varphi_0}\right)^2\right]\cos\varphi_0$$

$$= 2\frac{1 - n_0}{1 + n_0}\Delta l_{12}^{(v)}\left(1 + \frac{h_{max} - h_0}{H}\right)\frac{\nu_{max}}{c}\frac{\omega}{\omega + \langle \omega_H \rangle}$$

$$\times \left[\frac{1}{\sec\varphi_0} + \left(\frac{\nu_{max}}{\omega + \langle \omega_H \rangle}\right)^2\sec\varphi_0\right]. \quad (34)$$

7. Examples

Figure 2 compares two profiles of electron density modelling the ionospheric D-layer between the heights h_1 and $h_2 > h_1$; the first profile agrees with the linearized complex refractive index (31a)-(31b), and is defined by a lower limit (h_0, N_0), such that $h_0 < h_1$, while the second profile responds to Chapman's (19) and is specified by a relative maximum (h_{max}, N_{max}), such that $h_{max} > h_2$.

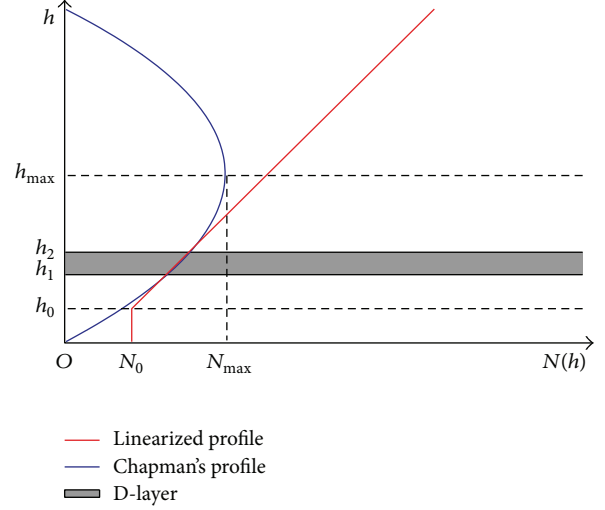Figures 3 and 4 consider an oblique radio sounding, with just one ionospheric reflection, between the transmitter

FIGURE 2: Comparison between two profiles of electron density modelling the ionospheric D-layer between the heights h_1 and $h_2 > h_1$.

Rome, Italy (41.89°, 12.49°) and the receiver Chania, Crete (35.52°, 24.02°) stations. The D-layer, represented by a complex eikonal model, covers a short range of heights between $h_1 = 50$ km and $h_2 = 80$–90 km, the absolute temperature decreasing, respectively, from $T_1 = 273$ K to $T_2 = 187$ K [3]. The whole ionosphere, represented by a Chapman's profile, is characterized by a maximum of electron density N_{max} which occurs at height $h_{max} = 300$ km, corresponding to a collision frequency $\nu_{max} = \nu(h_{max}) = 1.60512\,\text{s}^{-1}$. The linearized profile of complex refractive index (31a)-(31b) defined by a height of the ionosphere bottom $h_0 = h_1 = 50$ km and a refractive index across the ionosphere-neutral atmosphere boundary $n_0 = n(h_0) = 1 - \varepsilon_{max}$ (ε_{max} being the maximum allowable single step error, that is, $\varepsilon_{max} \leq 10^{-6}$) is related to the collision frequency ν_{max} (11), the atmospheric scale height H (12), and the mean value of magnetic angular gyrofrequency $\langle \omega_H \rangle$ (18). The Chapman's profile (19), specified by a solar zenith angle approximately null $\chi = 0$ and a scale height $H = 62$ km [7], is correlated to the mean magnetic gyrofrequency $f_H = 1.2$ MHz. Suppose that the critical frequency at the Earth's equator f_{c0}, calculated as $f_{c0}^2 = K \cdot N_{max}$ [being $K = (q_e^2/\varepsilon_0)/(4\pi^2 m_e) = 8.061382 \cdot 10^{-5}\,\text{MHz}^2\cdot\text{cm}^3$], can assume the following values: $f_{c0} = 3.65$ MHz, $f_{c0} = 5.65$ MHz, $f_{c0} = 6.85$ MHz, $f_{c0} = 7.60$ MHz, $f_{c0} = 7.80$ MHz, $f_{c0} = 8.60$ MHz, $f_{c0} = 9.15$ MHz, and $f_{c0} = 10.45$ MHz.

The paper has proven that the amplitude absorption according to the Settimi et al. [11] complex eikonal model (32), (33), and (34) is generally more accurate than Rawer's [12] theory (20a)-(20b), since it arises in continuity with the QL approximation for nondeviative absorption (9), deduced here, which is more refined than the corresponding equation reported by Davies [1].

As regards the ordinary ray, the complex eikonal absorption curve becomes overestimated with respect to the Rawer's one, that is, the absorption relative deviation is larger than
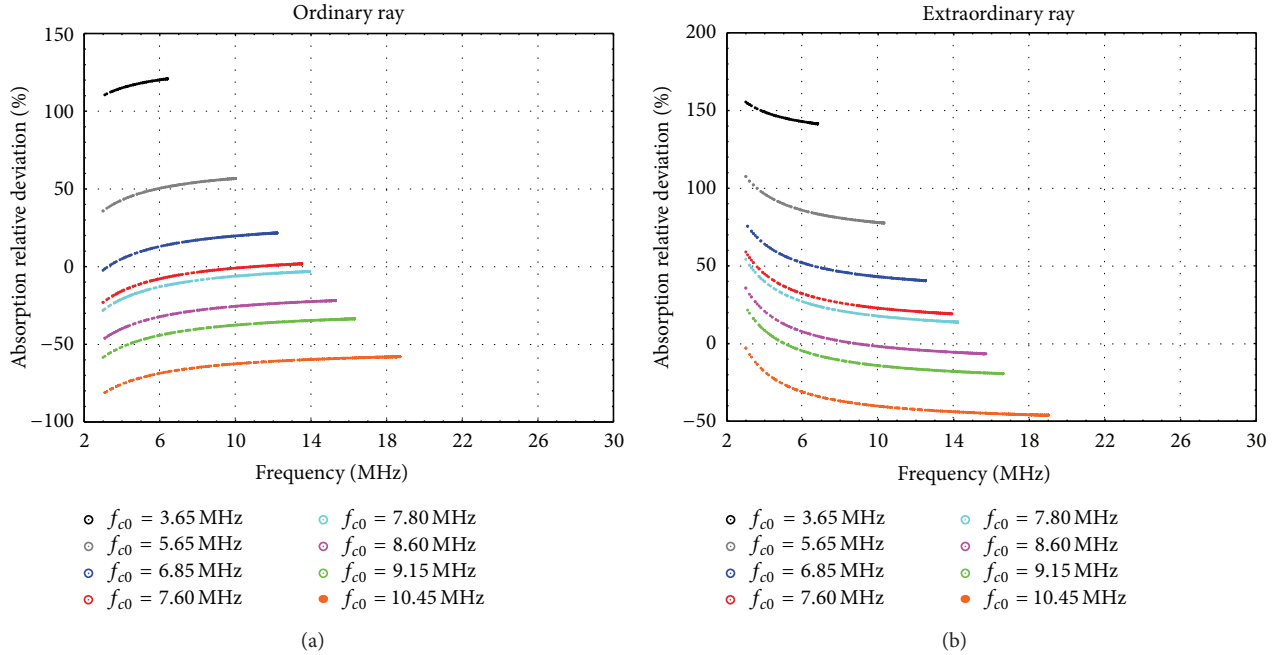
FIGURE 3: Plots of the QL nondeviative amplitude absorption [dB], according to the Settimi et al. [11] complex eikonal model (32), (33), and (34) and Rawer's [12] theory (20a)-(20b), as a function of the frequency f (MHz), for both the ordinary and extraordinary rays. The critical frequency f_{c0} at equator assumes the following values: (a) f_{c0} = 3.65 MHz, (b) f_{c0} = 5.65 MHz, (c) f_{c0} = 6.85 MHz, (d) f_{c0} = 7.60 MHz, (e) f_{c0} = 7.80 MHz, (f) f_{c0} = 8.60 MHz, (g) f_{c0} = 9.15 MHz, and (h) f_{c0} = 10.45 MHz.

50%, when referring to the low critical frequencies ($f_{c0} \leq$ 5.65 MHz). Indeed, the complex eikonal model (32), (33), and (34) is not numerically reliable for the low critical frequencies, as the integral absorption coefficient (28) implies tacitly that not so low altitudes are involved [11]. Moreover, the complex eikonal absorption curve becomes underestimated with respect to the Rawer's one; that is, the absorption relative deviation is less than −50%, even when referring to the highest critical frequencies ($f_{c0} \geq$ 10.45 MHz). Indeed, the complex eikonal model (32), (33), and (34) is not numerically reliable even for the highest critical frequencies, when the linearized complex refractive index (25a)-(25b) fails, having to be replaced by a parabolic or even cubic profile [11]. Finally, applying the exclusion principle, necessarily the complex eikonal absorption curve is more accurate than the Rawer's one just when referring to the middle critical frequencies (6.85 MHz $\leq$ f_{c0} $\leq$ 7.80 MHz). The fitting between the absorption curves becomes optimal; that is, the absorption relative deviation tends to 0%, when referring to the high critical frequencies (8.60 MHz $\leq$ f_{c0} $\leq$ 9.15 MHz). Indeed, as proved, the D-layer is generally more carefully defined by the Settimi et al. [11] complex eikonal model than the Rawer's [12] theory. The D-layer is opaque to the middle critical frequencies, when the optical ray is reflected at altitudes next to the D-layer (h < 100 km), being so strongly absorbed ($[L]_{dB} \geq$ 10 dB). In this case, the D-layer plays a primary role; it follows that a more careful description of the D-layer concurs to promote the complex eikonal model as more accurate compared to Rawer's theory. Instead, the D-layer is transparent to the high critical frequencies, when the optical

ray is reflected at altitudes well away from the D-layer ($h \gg$ 100 km), being so weakly absorbed ($[L]_{dB} \leq$ 1 dB). In this other case, the D-layer plays only a secondary role; it follows that, even if the D-layer description is more carefully defined by the complex eikonal model than Rawer's theory, anyway the fitting between the corresponding absorption curves is optimal.

As regards the extraordinary ray, the complex eikonal absorption curve remains overestimated with respect to the Rawer's one for the majority of critical frequencies (3.65 MHz $\leq$ f_{c0} $\leq$ 9.15 MHz). The fitting between the complex eikonal and Rawer's absorption curve becomes optimal, that is, the absorption relative deviation tends to 0%, when referring to the highest critical frequencies ($f_{c0} \geq$ 10.45 MHz). Indeed, even if the extraordinary ray is reflected at lower altitudes than the ordinary ray [3], anyway it is more absorbed [12] and then is characterized by a higher maximum usable frequency (MUF). It follows that, with respect to the ordinary ray, the fitting between the complex eikonal and Rawer's absorption curves for the extraordinary ray occurs at higher critical frequencies.

8. Conclusions and Future Work

The present paper conducted a scientific review on ionospheric absorption, extrapolating the research prospects of a complex eikonal model for one-layer ionosphere.

As regards the scientific review, a quasi-longitudinal (QL) approximation was deduced for nondeviative absorption which is more refined than the corresponding equation
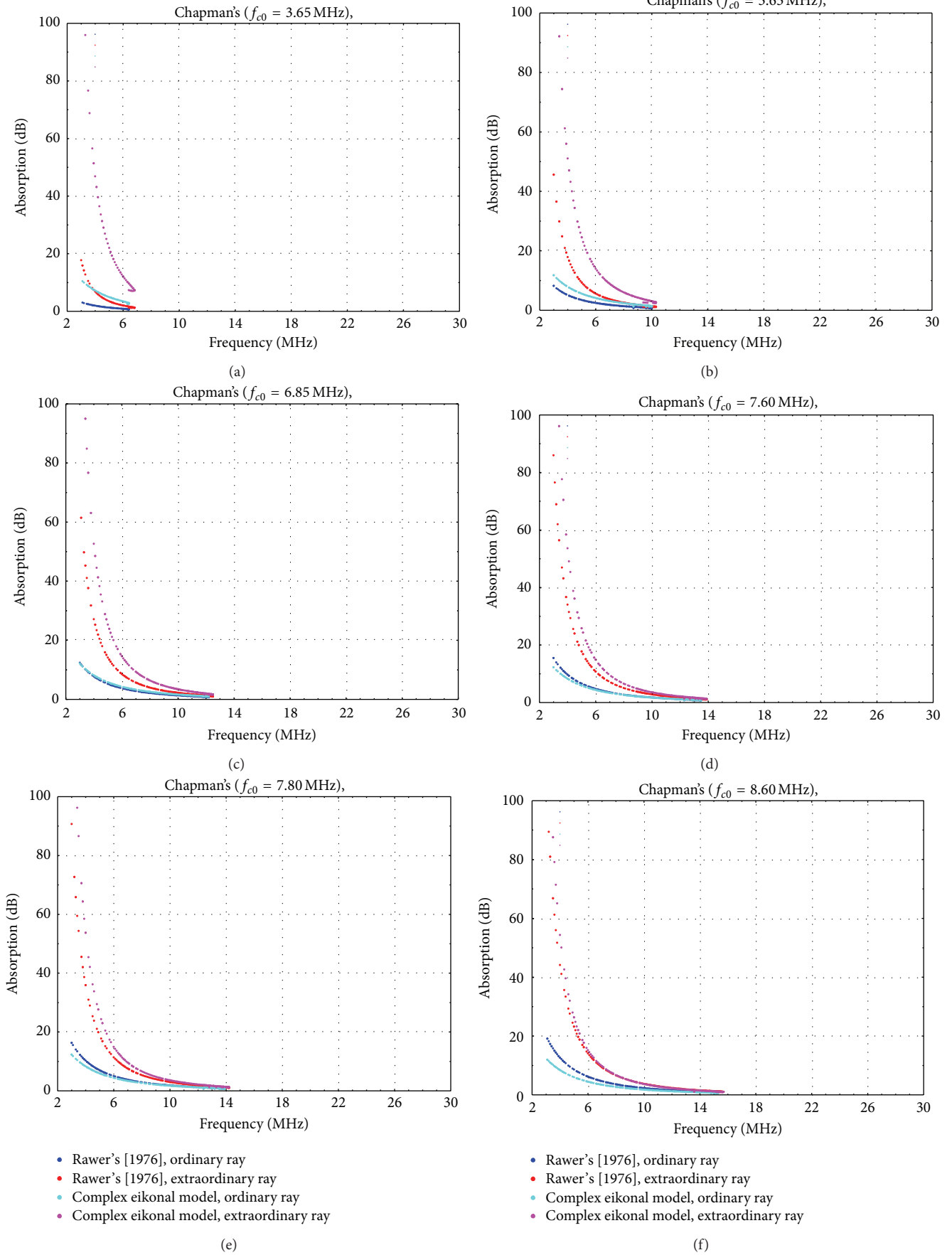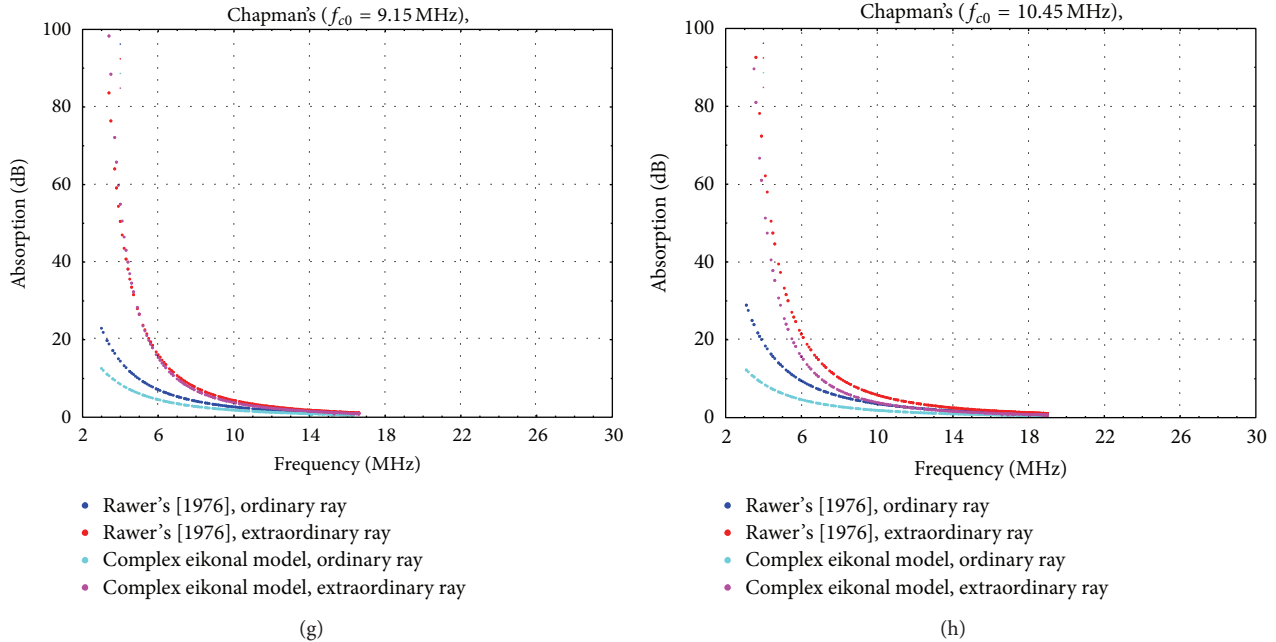
(a)

(b)

(c)

(d)

(e)

- Rawer's [1976], ordinary ray
- Rawer's [1976], extraordinary ray
- Complex eikonal model, ordinary ray
- Complex eikonal model, extraordinary ray

(f)

- Rawer's [1976], ordinary ray
- Rawer's [1976], extraordinary ray
- Complex eikonal model, ordinary ray
- Complex eikonal model, extraordinary ray

FIGURE 4: Continued.

(g)

(h)

FIGURE 4: Plots of the percentage relative deviation [%], between the quasi-longitudinal (QL) approximation for nondeviative amplitude absorption according to the Settimi et al. [11] complex eikonal model (32), (33), and (34) and Rawer's [12] theory (20a)-(20b), as a function of the frequency f (MHz), for both the ordinary (a) and extraordinary (b) rays [The critical frequency at the Earth's equator f_{c0} assumes the following values: $f_{c0} = 3.65$ MHz, $f_{c0} = 5.65$ MHz, $f_{c0} = 6.85$ MHz, $f_{c0} = 7.60$ MHz, $f_{c0} = 7.80$ MHz, $f_{c0} = 8.60$ MHz, $f_{c0} = 9.15$ MHz, $f_{c0} = 10.45$ MHz].

reported by Davies [1]. Similarly, further improvements were proposed for calculating more accurately the atmospheric scale height involved into the variation of collision frequency with the altitude and the mean value of magnetic angular gyrofrequency relative to the dipole model of geomagnetic field. Moreover, the authors reviewed Chapman's one-layer ionosphere, referring to the QL approximation for nondeviative absorption reported by Rawer [12].

As regards the research prospects, a complex eikonal model for one layer ionosphere was analyzed in depth here, already discussed by Settimi et al. [11]. In order to solve the ionospheric ray-tracing and absorption problems, a novel point of view was prospected. The complex eikonal equations were derived assuming a material medium as optically isotropic. However, in agreement with the quasi-isotropic approximation of geometrical optics, these equations can be referred to as the Appleton-Hartree refractive index for an ionospheric magnetoplasma, which becomes only weakly anisotropic in the presence of geomagnetic field.

Finally, a simple formula was deduced for a simplified problem. A flat, layered ionospheric medium is considered, without any horizontal gradient. This paper proposed a new formula, useful to calculate the amplitude absorption due to the propagation across the ionospheric D-layer, which can be approximately modelled by a linearized analytic profile of complex refractive index, covering a short range of heights between $h_1 = 50$ km and $h_2 = 80$–90 km approximately.

The authors proved that the QL nondeviative amplitude absorption according to the complex eikonal model is more accurate than Rawer's theory [12] in the range of middle critical frequencies.

The theoretical paper explained the bases for an applicative study [19]. The simple complex eikonal equations for calculating the QL approximation of nondeviative amplitude absorption due to the propagation across the D-layer were encoded as subroutine of an ionospheric ray-tracing (IONORT) program [20]. The IONORT program, which simulates the three-dimensional (3D) ray-tracing for high frequency (HF) waves in the ionosphere, runs on the assimilative IRI-SIRMUP-P (ISP) discrete model over the Mediterranean area [21, 22]. The IONORT-ISP results were compared to a more elaborate semiempirical formula, that is, the ICEPAC [23], which refers to various phenomenological parameters such as the critical frequency of E-layer. The complex eikonal model for QL nondeviative amplitude absorption is as reliable as the ICEPAC formula, with the advantage of being implemented more easily, since the proposed model depends just on parameters of the electron density profile, which are numerically determinable, such as the maximum height.

Appendices

A. Appendix A

Let us model the plasma as an electron gas. The thermodynamics of adiabatic (energy conserving) gas parcel displacements describes expansion cooling or compression

heating associated with adiabatic processes taking place in a compressible fluid. A diatomic ideal gas, defining the Rydberg's constant R, is characterized by a specific heat at constant pressure $c_p = (7/2) \cdot R$. Its absolute temperature T and pressure p are linked by the equation of state [24]

$$T = Ap^{\gamma}, \qquad (A.1)$$

where A is a constant and $\gamma = R/c_p = 2/7$.

Once measured, at the "maximum height" $h_{\max}$ [see Section 3], the temperature $T_{\max}$ and the pressure $p_{\max}$, the constant A is calculated; that is, $A = T_{\max}/(p_{\max})^{\gamma}$, so that (A.1) can be recast as

$$T = Ap^{\gamma} = \frac{T_{\max}}{p_{\max}^{\gamma}} p^{\gamma} = T_{\max} \left(\frac{p}{p_{\max}} \right)^{\gamma}. \qquad (A.2)$$

The hydrostatic approximation states that, in the vertical direction, the most important forces acting on a parcel of gas are the vertical pressure gradient and gravity. This simplification is common in dynamical models, although it neglects some phenomena such as sound waves, and is not appropriate for high resolution models. Introducing the atmospheric scale height $H = k_B \langle T \rangle / m \langle g \rangle$, where k_B is the Boltzmann's constant, m the mean molecular mass of atmosphere, and $\langle g \rangle$ the mean value of gravity acceleration, if H (and the absolute temperature T) varies little with height h, the pressure vertical distribution p of the ionosphere can be represented in a convenient form [3]:

$$p(h) = p_{\max} \exp \left(-\frac{h - h_{\max}}{H} \right). \qquad (A.3)$$

The scale height H represents the altitude which corresponds to a reduction by $1/e$ in the pressure. It can be shown that H represents twice the distance through which electrons having the equipartition of translational energy, $(1/2)k_B \langle T \rangle$, can rise in the vertical direction against the force of gravity.

Once a short range of heights h is considered, that is, $|h - h_{\max}| \ll H$ [see Section 3], the pressure (A.3) is expanded in a Taylor's series at the first-order:

$$p(h) \cong p_{\max} \left(1 - \frac{h - h_{\max}}{H} \right)$$
$$= p_{\max} \left[1 - \frac{m \langle g \rangle}{k_B T} (h - h_{\max}) \right]. \qquad (A.4)$$

Reducing the system of two equations (A.2) and (A.4),

$$T = T_{\max} \left(\frac{p}{p_{\max}} \right)^{\gamma} \cong T_{\max} \left[1 - \frac{m \langle g \rangle}{k_B T} (h - h_{\max}) \right]^{\gamma}$$
$$\cong T_{\max} \left[1 - \gamma \frac{m \langle g \rangle}{k_B T} (h - h_{\max}) \right], \qquad (A.5)$$

into the solving equation,

$$\frac{T^2}{T_{\max}} - T + \gamma \frac{m \langle g \rangle}{k_B} (h - h_{\max}) = 0, \qquad (A.6)$$

the temperature distribution of the ionosphere is derived:

$$T(h) = \frac{1 \pm \sqrt{1 - 4 (1/T_{\max}) \gamma (m \langle g \rangle / k_B) (h - h_{\max})}}{2/T_{\max}}$$
$$\cong \frac{T_{\max}}{2} \left[1 + 1 - 2 \frac{1}{T_{\max}} \gamma \frac{m \langle g \rangle}{k_B} (h - h_{\max}) \right]$$
$$= T_{\max} \left[1 - \gamma \frac{m \langle g \rangle}{k_B T_{\max}} (h - h_{\max}) \right] > 0. \qquad (A.7)$$

Finally, for a one-layer ionosphere between the heights h_1 and h_2, being $|h_1 - h_2| \ll h_{\max}$, the mean value of temperature can be calculated:

$$\langle T \rangle = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} T(h) \, dh$$
$$\cong \frac{T_{\max}}{h_2 - h_1} \int_{h_1}^{h_2} \left[1 - \gamma \frac{m \langle g \rangle}{k_B T_{\max}} (h - h_{\max}) \right] dh$$
$$= \frac{T_{\max}}{h_2 - h_1} \left[(h - h_{\max}) - \gamma \frac{m \langle g \rangle}{k_B T_{\max}} \frac{(h - h_{\max})^2}{2} \right]_{h_1}^{h_2}$$
$$= T_{\max} \left[1 - \gamma \frac{m \langle g \rangle}{k_B T_{\max}} (h_1 + h_2 - 2h_{\max}) \right]. \qquad (A.8)$$

B. Appendix B

Assuming the reader to be familiar with the mathematical symbols and their physical meaning, let us report the QL approximation for nondeviative absorption $\beta(h)$ (9), where the magnetic angular gyrofrequency $\omega_H(h, \theta)$ (16) is assumed coincident with its mean value $\langle \omega_H \rangle$ (18), being approximately a constant:

$$\beta(h) \approx \frac{1}{2} \left[1 - n_R^2(h) \right] \frac{\nu(h)}{c}$$
$$\times \frac{\omega}{\omega \pm \langle \omega_H \rangle} \left\{ 1 + \left[\frac{\nu(h)}{\omega \pm \langle \omega_H \rangle} \right]^2 \right\}. \qquad (B.1)$$

Considering a short range of heights h, that is, $|h - h_{\max}| \ll H$, the variation of collision frequency with the altitude $\nu(h)$ (10) can be expanded in a Taylor's series at the first-order in $(h - h_{\max})$ (11):

$$\nu(h) \cong \nu_{\max} \left(1 - \frac{h - h_{\max}}{H} \right) \longrightarrow \nu_{\max}, \qquad (B.2)$$

such that the QL nondeviative absorption (B.1) is conveniently simplified as

$$
\beta(h)
$$

$$
\approx \frac{1}{2}\left[1-n_R^2(h)\right]\frac{\nu(h)}{c}\frac{\omega}{\omega\pm\langle\omega_H\rangle}\left\{1+\left[\frac{\nu(h)}{\omega\pm\langle\omega_H\rangle}\right]^2\right\}
$$

$$
\approx \frac{1}{2}\left[1-n_R^2(h)\right]\frac{\nu(h)}{c}\frac{\omega}{\omega\pm\langle\omega_H\rangle}\left[1+\left(\frac{\nu_{max}}{\omega\pm\langle\omega_H\rangle}\right)^2\right].
$$
(B.3)

Similarly for the additional mathematical symbols and their physical meaning, consider the complex eikonal model for local absorption coefficient $\beta(h)$ (28):

$$
\beta(h)\cong \frac{\omega}{c}\frac{\alpha_I}{\alpha_R}n_R(h), \qquad h\geq h_0.
$$
(B.4)

Comparing the QL approximation (B.3) and the complex eikonal model (B.4), an alternative expression is obtained for the collision frequency as a function of height $\nu(h)$; that is,

$$
\nu(h)\cong 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}\frac{n_R(h)}{1-n_R^2(h)},
$$
(B.5)

which can be expanded in a Laurent's series close to the neighbourhood of pole $n_R(h)\rightarrow 1$, according to the nondeviative absorption condition:

$$
\nu(h)\cong 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}\frac{n_R(h)}{1-n_R^2(h)}
$$

$$
\approx 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left\{-\frac{1}{2[n_R(h)-1]}-\frac{1}{4}+\frac{1}{8}[n_R(h)-1]\right.
$$

$$
\left.-\frac{1}{16}[n_R(h)-1]^2+\cdots\right\}
$$
(B.6)

$$
\approx 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left\{-\frac{1}{2[n_R(h)-1]}-\frac{1}{4}\right\}.
$$

Within the complex eikonal model, consider the linearized analytic profile which is adopted for the complex refractive index $n(h)$ (25a)-(25b):

$$
n(h)=n_R(h)+in_I(h), \qquad h\geq h_0,
$$

$$
n_R(h)=n_0+\alpha_R(h-h_0),
$$
(B.7)

$$
n_I(h)=\alpha_I.
$$

The collision frequency (B.6), in explicit form

$$
\nu(h)\cong 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left\{-\frac{1}{2[n_0-1+\alpha_R(h-h_0)]}-\frac{1}{4}\right\},
$$
(B.8)

can be expanded in a Taylor's series approximately limited at the first-order in $(h-h_0)$, for a narrow one-layer ionosphere covering the short range of heights between h_1 and h_2, being $|h_2-h_1|\ll h_0<h_{max}$, so that

$$
\nu(h)\cong 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left\{-\frac{1}{2[n_0-1+\alpha_R(h-h_0)]}-\frac{1}{4}\right\}
$$

$$
\approx 2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left[-\frac{1}{4}+\frac{1}{2}\frac{1}{1-n_0}+\frac{1}{2}\frac{\alpha_R(h-h_0)}{(1-n_0)^2}\right]
$$
(B.9)

$$
=2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}
$$

$$
\times\left[\frac{1}{4}\frac{1+n_0}{1-n_0}+\frac{1}{2}\frac{\alpha_R(h-h_0)}{(1-n_0)^2}\right].
$$

Equating terms in (B.2) and (B.9) gives a system of two linear equations,

$$
2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}\left[\frac{1}{4}\frac{1+n_0}{1-n_0}-\frac{1}{2}\frac{\alpha_R h_0}{(1-n_0)^2}\right]
$$

$$
=\nu_{max}\left(1+\frac{z_{max}}{H}\right),
$$

$$
2\frac{\alpha_I}{\alpha_R}\frac{\omega\pm\langle\omega_H\rangle}{1+(\nu_{max}/(\omega\pm\langle\omega_H\rangle))^2}\frac{1}{2}\frac{\alpha_R}{(1-n_0)^2}
$$

$$
=-\frac{\nu_{max}}{H},
$$
(B.10)

which can be solved for the two variables α_R and α_I; that is,

$$
\alpha_R=-\frac{1}{2}\frac{(1-n_0^2)/H}{1+((h_{max}-h_0)/H)},
$$
(B.11)

$$
\alpha_I=-\frac{(1-n_0)^2}{H}\frac{\nu_{max}}{\omega\pm\langle\omega_H\rangle}\left[1+\left(\frac{\nu_{max}}{\omega\pm\langle\omega_H\rangle}\right)^2\right].
$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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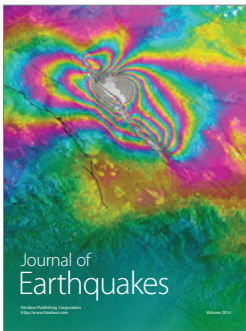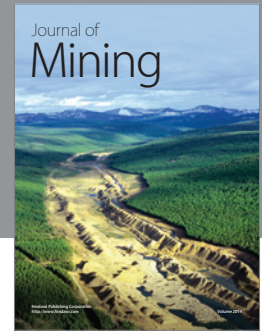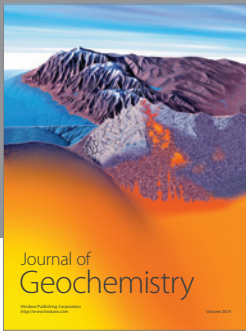
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