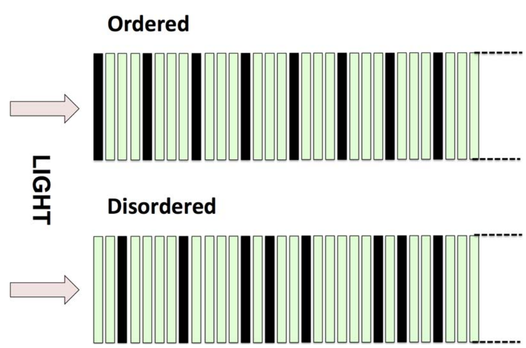
IEEE Photonics Journal
An IEEE Photonics Society Publication

Open Access

Light Transmission Properties and Shannon Index in One-Dimensional Photonic Media With Disorder Introduced by Permuting the Refractive Index Layers

Volume 5, Number 6, December 2013

M. Bellingeri
D. Cassi
L. Criante
F. Scotognella



DOI: 10.1109/JPHOT.2013.2289988
1943-0655 © 2013 IEEE

Light Transmission Properties and Shannon Index in One-Dimensional Photonic Media With Disorder Introduced by Permuting the Refractive Index Layers

M. Bellingeri,[1] D. Cassi,[1] L. Criante,[2] and F. Scotognella[3]

[1]Dipartimento di Fisica, Università di Parma, 43124 Parma, Italy
[2]Center for Nanoscience and Technology@PoliMi, Istituto Italiano di Tecnologia, 20133 Milano, Italy
[3]Dipartimento di Fisica, Politecnico di Milano, 20133 Milano, Italy

DOI: 10.1109/JPHOT.2013.2289988
1943-0655 ©2013 IEEE

Manuscript received September 12, 2013; revised October 25, 2013; accepted October 25, 2013. Date of publication November 7, 2013; date of current version November 20, 2013. F. Scotognella acknowledges financial support from Italian Ministry of University and Research (project PRIN 2010- 2011 "DSSCX" 20104XET32). Corresponding author: F. Scotognella (e-mail: francesco.scotognella@polimi.it).

Abstract: In this paper, we have studied, with a numerical method, how introducing disorder affects the light transmission properties of 1-D photonic structures over a wide range of wavelengths. A new type of disorder is introduced by permuting the refractive index layers in the optical medium. We compared the light transmission properties of ideal photonic crystals and of disordered media with the same kind and number of scattering elements for different sample lengths. We have calculated the transmission properties, by the transfer matrix method, of thousands of different disordered structures in order to perform a statistical analysis. We found that, below a certain sample length, disorder induces less average light reflection than ordered structures, whereas above the threshold length, disordered structures show more average reflection. Moreover, we have quantified the disorder of the structures with the Shannon index. We have found a decrease in the average light transmission as a function of the Shannon index. Furthermore, the sample length affects the trend of the average transmission as a function of the Shannon index.

Index Terms: Photonic crystals, disordered structures, structure-property relationship.

1. Introduction

Wave propagation in periodic and disordered materials still remains one of the most intriguing problems in physics. For example, in solid state physics, the introduction of disorder can transform a conducting material into an insulator, and can be described by the concept of Anderson localization [1]. Theoretical studies and experiments have shown that disorder hinders transport, leading to deviations from classical diffusion in systems containing disorder [2]–[4], as well as in fully random potentials [5]–[7].

The study of ordered and disordered topologies (at length scales of the wavelength of light) is a crucial topic for a better comprehension of light transport in photonic media. In photonic structures, two different materials give rise to spatial variations in refractive index of the order of the wavelength of light. When the different domains form periodic structures, these are called photonic crystals [8]–[11]. This is a relevant topic within physics and materials science, e.g., for the study of their optical properties and for the fabrication of optical filters, lasers and other optical devices [12]–[16].

When two domains have large-order rotational symmetry but no translational symmetry, they form a quasi-crystal [17] that can exhibit unusual optical properties [18]. Disordered photonic structures consist of a random mixing of refractive index domains. The optical properties of disordered structures have been studied in the pioneering works of Kuga *et al.* [19], Wolf *et al.* [20] and Van Albada *et al.* [21], in which the weak localization of light in these systems is experimentally demonstrated. Although widely used in applications such as random lasers [22], [23], recent observations (e.g., photon Levy flights [24], disorder enhanced light transport [25]) pay testament to the great effort underway to better understand the singular features of these photonic materials. Anderson localization has been experimentally demonstrated in one-dimensional random photonic structures, showing a narrow peak in the transmission spectrum and an average of the transmission coefficient logarithm which decreases linearly with the sample thickness [4], [26]. Furthermore, optical necklace states have been observed [26], as theoretically predicted by Pendry [27] and Tartakovskii [28].

An interesting way to correlate the disorder in a photonic structure with its optical properties is to describe the structural homogeneity of such a structure. The degree of disorder-homogeneity of the medium can be quantified by a diversity index (Shannon index) and it is possible to correlate the average light transmission over a large range of wavelengths to the structural disorder [29]–[34].

In this paper, we analyze the optical properties of one-dimensional photonic structures introducing topological disorder. The disorder, employed for the first time in these kind of structures, is introduced by permuting the refractive index layers in the optical medium. Since each disordered photonic structure can show different light transmission properties, we repeat a large number of experiments in order to produce a statistical ensemble of light transmission outputs. We study the average transmission as a function of the sample length and observe that, below a certain length, disordered structures transmit more light than ordered structures, whereas above such lengths the ordered structures transmit more than the disordered. This characteristic is independent of the kind of light localization in the structure. Moreover, we have quantified the disorder of the structures with the Shannon index. First, we underline that the calculations herein presented, made with the transfer matrix method, are in good agreement with previous simulations by finite element methods [32]. Second, in the disordered media we have found that the average transmission decrease as a function of the Shannon index for each structure and it depends on the sample length.

2. Experimental Details

Herein we have studied two different photonic structures. The photonic media are stratified structures in which the thickness of each layer is 100 nm. The structures are composed of two materials with different refractive indices. In the first structure the refractive indices of the two materials are $n_{HI1} = 1.69$ and $n_{LO1} = 1.6$, chosen in order to obtain a low refractive index contrast. In the second structure, inspired by the work of Bertolotti *et al.* [26], the parameters were chosen such that $4d_{HI2}n_{HI2} = 4d_{LO2}n_{LO2} = 1500$ nm using $n_{HI2} = 2.13$ and $n_{LO2} = 1.46$ as refractive indices of the two materials and $d_{HI2} = 176$ nm and $d_{LO2} = 258.6$ nm as their thicknesses. For both structures we have considered three sizes of the crystal unit cell: a unit cell composed of a layer of high refractive index material and a layer of low refractive index material (henceforth labeled **2**), a unit cell composed of a layer of high refractive index material and two layers of low refractive index material (**3**) and a unit cell composed of a layer of high refractive index material and three layers of low refractive index material (**4**). To distinguish between the photonic structures with high and low refractive index contrast, we name them **2LC**, **3LC** and **4LC** when the materials with n_{HI1} and n_{LO1} constitute the structure (LC stands for low contrast), **2HC**, **3HC** and **4HC** when the materials with n_{HI2} and n_{LO2} constitute the structure (HC stands for high contrast).

We have studied the average light transmission of the structures as a function of the number of unit cells constituting the crystal, starting from 10 unit cells, up to 100 unit cells.

To design the random structures we have made permutations of the refractive layers. In this manner a certain degree of disorder is introduced in the photonic media and the periodicity of the structure is lost. This implies that the distance between high refractive elements is not uniform and the unit cells have different proportions of high and low refractive layers.

For a 1D random photonic sequence with n possible positions and k_i elements with the same refractive index (indistinguishable elements) we have a total number of possible different disordered photonic crystals given by the multinomial coefficient [33]

$$\binom{n}{k_1, k_2, \ldots, k_m} = \frac{n!}{k_1! k_2! \ldots k_m!} \tag{1}$$

where n is the total number of elements (layers) and k_i the number of repeated elements (high and low refractive layers).

For example in the case of $n = 120$ (30 unit cells **4**), $k_1 = 90$ (low refractive layers) and $k_2 = 30$ (high refractive layers) the total number of distinguishable permutations are 1.697454×10^{28}. The representation of ordered and disordered crystals with $n = 120$ layers are shown in the first page of the Supplemental Material.

Finally, we have correlated the disorder of the random one-dimensional structures to the Shannon–Wiener index [34]. The Shannon–Wiener (H') index is a diversity index widely used in statistics and in information theory, and it is defined as

$$H' = -\sum_{j=1}^{s} p_j \log p_j \tag{2}$$

where p_j is the proportion of the j-fold species and s is the number of the species. In our study p_j indicates the proportion of high scattering elements in each unit cell. With a certain number of species, the quantity represented by $\log(s)$ is the maximum value of the Shannon index, i.e., when all species show the same proportion of individuals. Dividing H' by $\log(s)$ we can normalize the index constraining it within the range (0, 1). The more the structure is homogeneous, the more the Shannon index is. The periodic and ideal photonic crystal, in which each unit cell contains the same proportion of high scattering elements, the Shannon index is 1. We have realized 1000 permutations of each crystal length (10 000 for each type of structure). For each replica we calculated the Shannon index and the transmission properties.

To calculate the transmission spectra we have employed the transfer matrix method; a general technique that is widely used in optics for the description of stacked layers and it is extensively described in ref. [35]–[37]. We have considered isotropic, nonmagnetic materials shaping the system glass/multilayer/air (in which glass is the sample substrate) and an incidence of the light normal to the stacked layer surface. n_0 and n_S are the refractive indexes of air and glass, respectively, while E_m and H_m are the electric and magnetic fields in the glass substrate. To determine the electric and magnetic fields in air, E_0 and H_0, we have solved the following system:

$$\begin{bmatrix} E_0 \\ H_0 \end{bmatrix} = M_1 \cdot M_2 \cdot \ldots \cdot M_m \quad \begin{bmatrix} E_m \\ H_m \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_m \\ H_m \end{bmatrix} \tag{3}$$

where

$$M_j = \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix}$$

with $j = (1, 2, \ldots, m)$, is the characteristic matrix of each layers. The elements of the transmission matrix $ABCD$ are

$$A_j = D_j = \cos(\phi_j), \quad B_j = -\frac{i}{p_j}\sin(\phi_j), \quad C_j = -i\, p_j \sin(\phi_j) \tag{4}$$

where n_j and d_j, hidden in the angle ϕ_j, are, respectively the effective refractive index and the thickness of the layer j. In the case of normal incidence of the probe beam, the phase variation of the

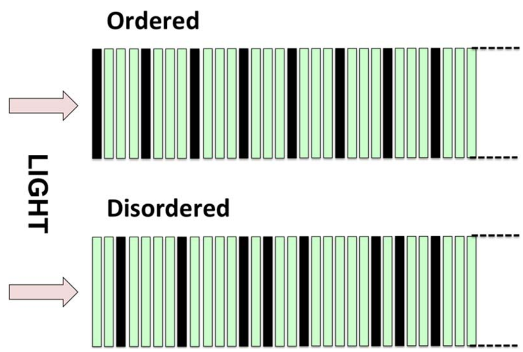


Fig. 1. Schematic of the structure **4** (first 32 layers): (top) ordered and periodic ideal photonic crystals; (bottom) disordered and aperiodic photonic structures.

wave passing the j-fold layer is $\phi_j = (2\pi/\lambda)n_j d_j$, while the coefficient $p_j = \sqrt{\varepsilon_j/\mu_j}$ in TE wave and $q_j = 1/p_j$ replace p_j in TM wave. Inserting Equation (4) into Equation (3) and using the definition of transmission coefficient

$$t = \frac{2p_s}{(m_{11} + m_{12}p_0)p_s + (m_{21} + m_{22}p_0)} \tag{5}$$

it is possible to write the light transmission as

$$T = \frac{p_0}{p_s}\,|t|^2. \tag{6}$$

The bandwidth in which we calculate the average light transmission is 900 nm broad, centered at the photonic band gap of the corresponding ordered ideal photonic crystal (for example, the corresponding ideal photonic crystal of the **2LC** structure has a photonic band gap centered at 658 nm; therefore, for this structure the range of wavelengths is 208–1108 nm). We have normalized the average light transmission of the disordered media to the average light transmission of the ideal photonic crystal.

3. Results

In Fig. 1(a) we show a schematic of the ideal one-dimensional photonic crystal and in Fig. 1(b) that of a disordered one-dimensional photonic structure. The black layers correspond to the high refractive index material while the green layers correspond to the low refractive index material.

Fig. 2(a) depicts the transmission spectra for the cell size 4 ideal photonic crystals and Fig. 2(b) for disordered structure of the same size (**4LC**) over the entire wavelength range (844–1744 nm). Introducing light along the ideal photonic crystal we observed the fundamental photonic band gap at 1294 nm. In the disordered structure, we do not observe an intense photonic band gap, but rather the presence of several minima along all the considered spectral range. Fig. 2(c) and (d) displays transmission spectra for ordered and disordered **4HC** structures, in the range 2603–3503 nm.

It is interesting to compare transmission of disordered structures herein studied with that of the disordered structure proposed by Faist *et al.* [38]. In that structure, disorder is referred to by a Gaussian distribution of layer thicknesses, with a standard deviation σ. Owing to the randomness in layer thickness, the calculated reflectivity spectrum of this structure was different to the ordered structure; although, both ordered and disordered structures show a very similar photonic band gaps in the region 850–950 nm (Figure 11 and 12 in Ref. [38]). This could be ascribed to the fact that thickness Gaussian distribution generates mild disorder. Instead, for the structures studied in our analysis, the transmission spectrum is affected by disorder in the entire spectral region investigated, including the photonic band region, and is more similar to the one showed in Ref. [26].

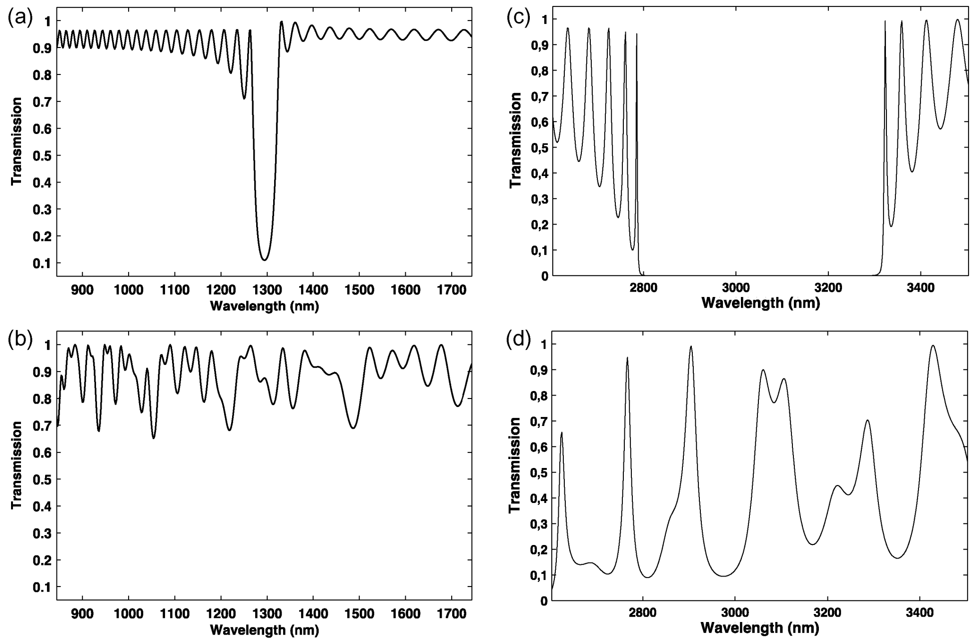


Fig. 2. Transmission spectra for: (a) **4LC** ideal photonic crystal, (b) **4LC** disordered structures (in the range 844–1744 nm), (c) **4HC** ideal photonic crystal and (d) **4HC** disordered structures (2603–3503 nm). All these structures contain 40 unit cells.

We found that introducing disorder produces a significant increase in the average light transmission compared to the ideal photonic crystal below a certain length of the sample. Beyond this length, the disordered crystal showed less average light transmission than ideal periodic photonic crystal. In Fig. 3 we show the distribution of the normalized average transmissions (T) for the crystal **4LC** for each sample length (L). In this figure the average transmission of the disordered medium is normalized to the ideal photonic crystal average transmission. For short sample length– in this case less than 50 unit cells–the majority of the transmissions of the disordered media is over the value of 1. Beyond this length, the mean value of the normalized average transmission increases and the majority of disordered media transmissions is lower than the value of the ideal photonic crystal. When the sample length is more than 70 unit cells all of the normalized average transmission for the disordered media is less than 1.

We found two patterns of light localization in disordered media as a function of the refractive index contrast. Disordered crystals with low refractive index contrast showed a linear decrease of the average light transmission as a function of the sample length (Fig. 4, panels LC). This linear law indicates a weak localization of light in the medium [26]. When the refractive index contrast is high we found a strong light localization pattern and increasing the sample length we observe a sharp decay of the average light transmission (Fig. 4, panels HC). This decay is linear when plotting the natural logarithm of the measured average transmission versus the sample length and indicates the presence of Anderson localization of light in the medium [26], [30]. This strong localization of light in the medium for the HC disordered structure, as also observed in ref. [26], is due to the higher refractive index contrast that results in a more intense reflection from each interface in the crystal.

Different refractive contrasts induce different types of light localization. In fact, it is worth noting that, while in HC structures we observe an Anderson localization regime (in accordance with ref. [26]), for low contrast (LC) structures it is evident that the mean average transmission (and not its logarithm) is linear as a function of sample length. However, for all crystals, disordered structures transmit more light than ordered structures below a certain length, whereas above such a length the trend is reversed. For this reason the observed pattern is independent from the type of light localization.

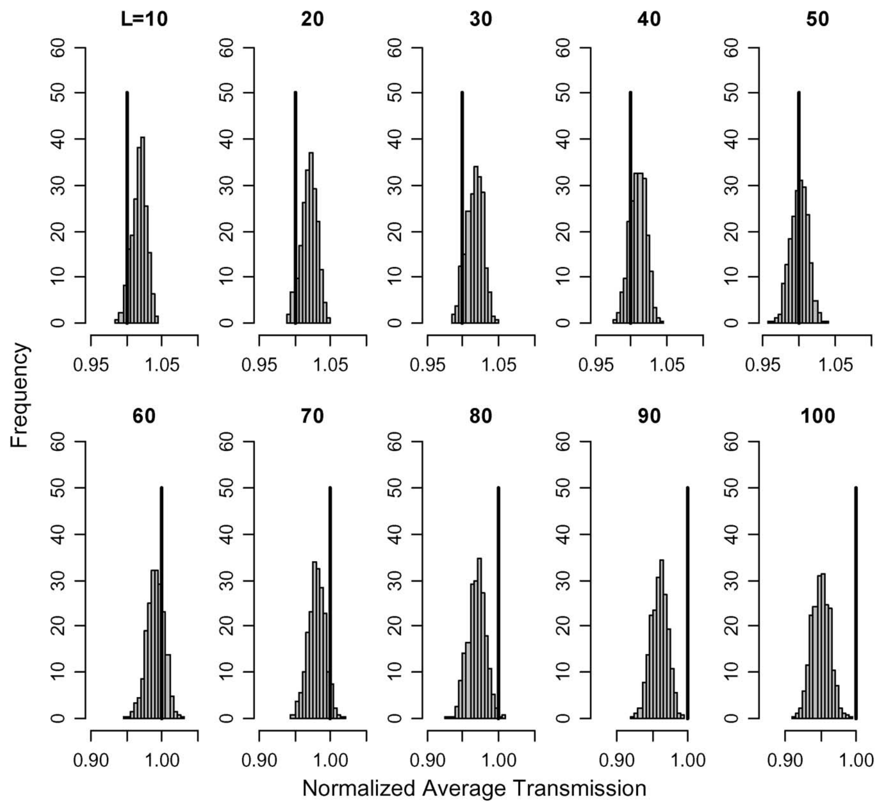


Fig. 3. Histograms of the crystal **4LC** normalized average transmission (T) for each sample length (L). The main legend in each panel is the sample length of the analyzed crystal. The ideal photonic crystal light transmission is 1 and is depicted by the vertical line. Below a certain sample length the largest amount of the average transmission results for the disordered media is higher than the light transmission of the ideal photonic crystal (e.g., in this case before $L = 50$), while over this length the pattern is preserved and disordered structures transmit less than periodic and ordered crystals.

The observed mechanism can be described from a phenomenological point of view. As shown in Fig. 2, the periodic structure can reflect a certain narrow range of wavelengths efficiently, thereby resulting in a photonic band gap. The disordered structure, without periodicity, reflects a broad range of wavelengths producing several transmission minima. Below a certain sample length (i.e., number of layers), the strong reflectance in the photonic band gap region lowers the average transmission of the ideal photonic crystal with respect to the one of the disordered structure. In the ideal periodic crystal, when the sample has a length such that the photonic band gap reaches $T = 0\%$, there is no further decrease in the average light transmission. In other word, when light of a certain range of wavelength is completely reflected from the ideal periodic crystal (band gap), no additional light reflection is possible from this medium, even increasing the sample length. This reflection saturation is clearly visible in Fig. 4 where, for both LC and HC structures, the ideal photonic crystal average transmission trend reaches a plateau and does not decrease upon further increase in the sample length. Conversely, in the disordered structure, increasing the sample length always produces (in the length range considered in this study) an increase in the reflectance. The average light transmission trend for disordered structures does not show a plateau. For this reason, beyond a certain length the average light transmission of disordered media becomes lower than the periodic crystal.

Interestingly, in a study on the average light transmission in 2D photonic structures, in which high refractive index pillars are aggregated forming disordered structures, it is observed that average light reflection increases faster in the disordered structures compared to the ordered ones [28]. The pattern observed in this study for 1D structures, in which above a certain sample length disorder induces more reflection, is in agreement with this result [30].

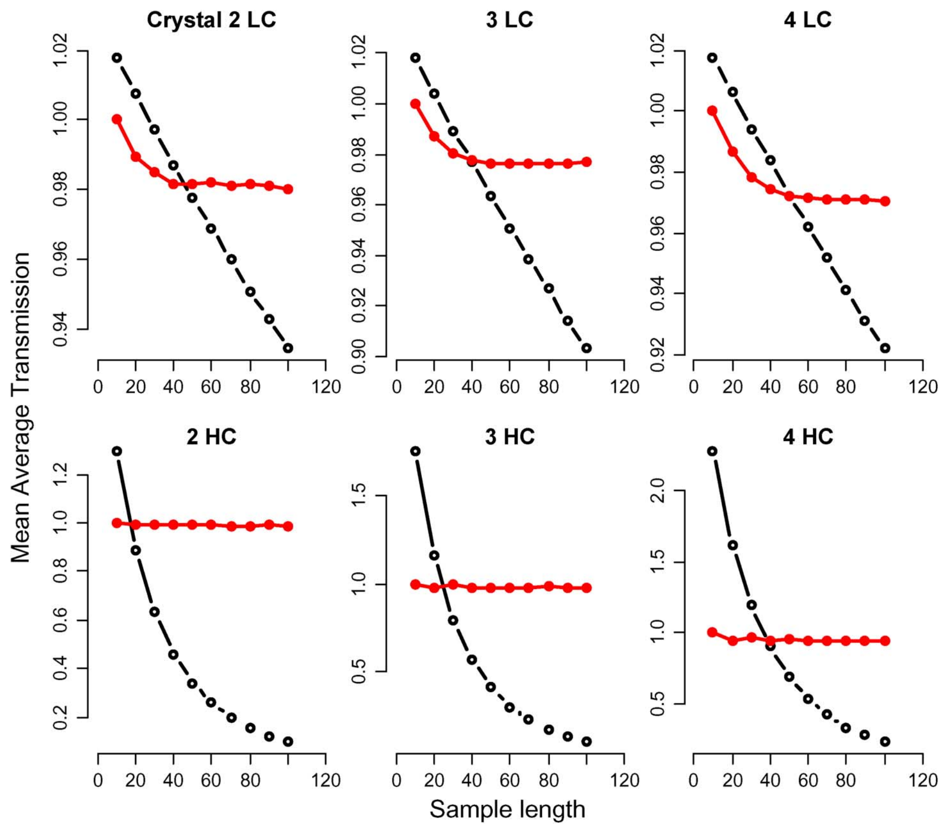


Fig. 4. Mean average transmission for each crystal as a function of the sample length (defined as numbers of unit cells). Red line: ideal periodic photonic crystal; black line: disordered crystal. The main legend in each panel indicates the unit cell size features of the analyzed crystal. The observed pattern is independent of the kind of localization showed in the structure. For each plot, data are normalized to the average transmission of the photonic crystal with the shorter sample length (i.e., 10 unit cells).

For the structures herein studied, we have analyzed the localization length l^*, defined as $l^* = -L/\langle lnT \rangle$ (where L is the sample length) [39]. We observed that, only for the **HC** structures, $\langle lnT \rangle$ shows a linear dependence with respect to the sample length. For such structures, we have thus extrapolated from the fit the localization lengths, which are (11.2 ± 2.4) μm for **2HC**, (16.4 ± 3.3) μm for 3**HC**, (28.7 ± 6.8) μm for **4HC**. Differently from Bertolotti *et al.* [26], where l^* is found to be about (14.9 ± 2.4) μm, we found a slightly stronger localization for the medium. This is due to the different type of disorder. In [26] disorder is introduced by giving each layer a 50% probability to be of type high or low refractive index (resulting in a variable ratio of high/low refractive layers), while in our case disorder is due to permutations of constant ratio of high/low refractive index layers.

Moreover, in order to understand how disorder affects the transmission properties of each replica, we have calculated the average light transmission as a function of the Shannon index for each sample length, with the corresponding plots reported in the Supplemental Material (Figures A1-A6). We have fitted linear regressions observing a wide variability but a clear decrease in the average light transmission as a function of the Shannon index ($p < 0.01$). The decreasing rate is different with respect to the sample length. In Fig. 5 we show the slope coefficients (of the linear regressions) as a function of the sample length. For LC structures we have noticed a linear decrease of the slope versus sample length, while a non-monotonic trend occurs for the HC structures: the slope decreases up to a minimum (corresponding to a sample length of 30–40 unit cells) and then increases for longer sample lengths.

This is a complex and interesting trend and could be explained noting that in the LC structures the variability of the average transmission increases with the sample length (Fig. 5). Higher average

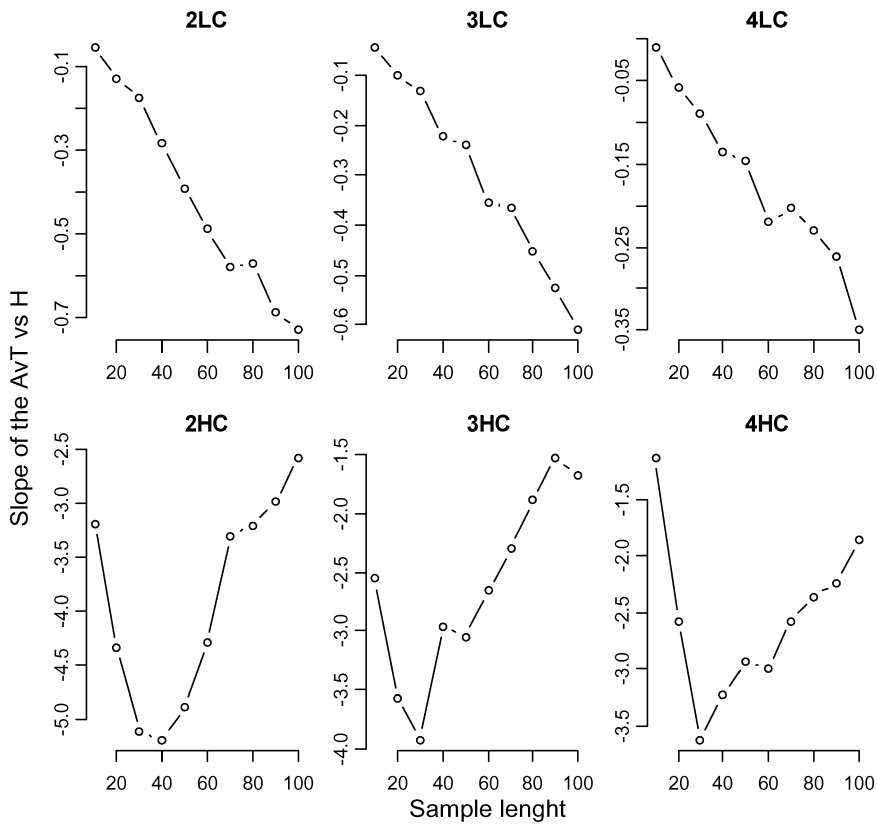


Fig. 5. Slope of the average light transmission as function of the Shannon index for each crystal length (plots with average light transmission as a function of the Shannon index are reported in the Supplemental Material).

transmission values are possible even with longer sample lengths. Instead, in the longer HC structures (e.g., $>$ 30 unit cells) high average transmission values become unlikely and most of the average transmission values are close to the minimum, such that the decreasing variability (e.g., lower higher values and the plateau over the minimum value of the average transmission) induces a lower slope.

In ref. [32], with a finite element method, the average light transmission decreases by increasing the Shannon index up to a certain value, while it is growing with a Shannon index above this value. This trend is confirmed in this analysis by using the transfer matrix method (Figure A7 in the Supplemental Material). With such approach, the trend is corroborated and it is worth noting that the structures, obtained by a random permutation of the high refractive layers, show values that lay at the minimum of the trend. This result shows how delivering the elements at random produces, in average, photonic structures with very low transmission properties.

Finally, to investigate how disorder affects the distribution of the scattering elements in the crystal and to topologically describe the structures, we analyzed the distance between the high refractive index layers in the disordered media. This distance corresponds to the number of low refractive index layers between a pair of high refractive index layers. We call this length as the high refractive distance (HRD).

The ideal photonic crystal has a periodic structure and the HRD is the same in the entire medium. For example, an ideal photonic crystal with 120 layers and cell size of 4 has a uniform HRD of 3 (i.e., 3 low refractive layers between a couple of high refractive elements); an ideal crystal of size 6 has a HRD equal to 5. For this reason, the distribution of the HRD in the ideal photonic crystal is a delta function with a characteristic value. The HRD in disordered crystals is not uniform in the

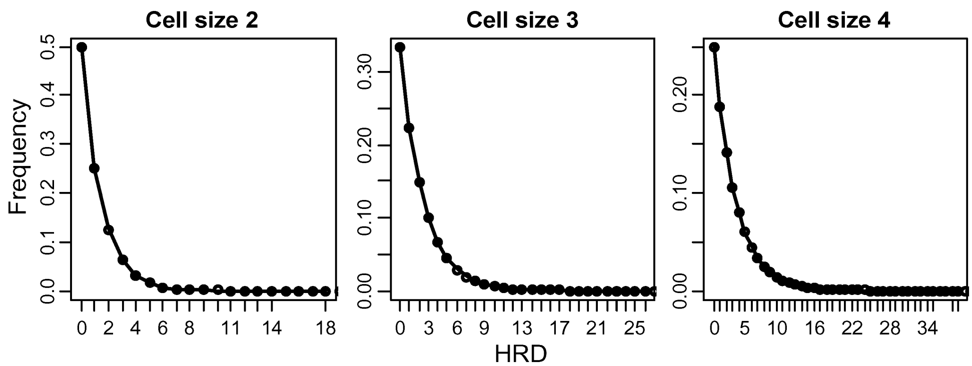


Fig. 6. HRD distribution for each crystal size. The line indicates the empirical results and is calculated by averaging over 10000 crystal simulations. Circles indicate theoretical prediction by Eq. (7).

medium. It is possible to analytically derive the probability $P(n)$ of having a HRD of length n using combinatorics

$$P(n) = \frac{(k_2 - 1) \cdot \binom{k_1 + k_2 - n - 1}{k_2 - 1}}{\binom{k_1 + k_2}{k_1}} \tag{7}$$

where k_1 and k_2 are the number of low and high refractive index layers in the medium, respectively.

Equation (7) describes a sharp decay in the probability of having a HRD of length n and the distribution of the HRD in a disordered medium exhibits a large variance. The HRD distributions of the disordered crystals studied are shown in Fig. 6 and they fit well to the Eq. (7).

4. Conclusion

We have studied the optical properties of disordered one-dimensional photonic structures introducing a new type of disorder, with the aim to compare the differences between ordered and disordered media. We have observed an interesting pattern of the light transmission as a function of the sample length: below a certain length, disordered structures transmit more light than ordered structures, whereas above this length the ordered structures transmit more than the disordered. This characteristic is independent of the light localization pattern observed in the structure. Our results point out interesting technological implications: i) techniques using disordered media with the aim of maximizing light trapping (or light transport) should consider the specific length of the sample; ii) over a certain sample length, the ideal periodic crystal becomes unable to increase light trapping, but remains very effective for light transport, compared to the disordered medium in spectral regions outside of the photonic band gaps. When the disorder of the structure is quantified by the Shannon index, we found that the average light transmission decreases as a function of the Shannon index for each type of crystal. The slope of the decrease of the average light transmission versus the Shannon index depends on the sample length. Furthermore, we have observed that, quantifying the disorder by the Shannon index, the studied random structures show an average light transmission that is in agreement with a trend, for 1D structure, reported in ref. [32]. For this reason, the finite element method and the transfer matrix method produce similar results but with a less computational time of algorithms. The presented results can be useful for the study of light scattering in diffusive media for diagnostic imaging [38] and for engineering of highly scattering layers for light coupling and trapping enhancement in photovoltaic devices [41]. For the latter, we envisage the fabrication of photovoltaic cells covered with cheap plastic random multilayers with a thickness such that light transmission of the random structure is higher than in an ordered structure of the same thickness.

Acknowledgment

We thank Marcelo J. P. Alcocer, Dr. Francesco Tassone and Prof. Guglielmo Lanzani for useful discussion and support.

References

[1] P. W. Anderson, "Absence of diffusion in certain random lattices," *Phys. Rev.*, vol. 109, no. 5, pp. 1492–1505, Mar. 1958.
[2] P. A. Lee and T. V. Ramakrishnan, "Disordered electronic systems," *Rev. Mod. Phys.*, vol. 57, no. 2, pp. 287–337, Apr.–Jun. 1985.
[3] T. Schwartz, G. Bartal, S. Fishman, and M. Segev, "Transport and Anderson localization in disordered two-dimensional photonic lattices," *Nature*, vol. 446, no. 7131, pp. 52–55, Mar. 2007.
[4] Y. Lahini, A. Avidan, F. Pozzi, M. Sorel, R. Morandotti, D. N. Christodoulides, and Y. Siberberg, "Anderson localization and nonlinearity in one-dimensional disordered photonic lattices," *Phys. Rev. Lett.*, vol. 100, no. 1, pp. 013906-1–013906-4, Jan. 2008.
[5] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, "Localization of light in a disordered medium," *Nature*, vol. 390, no. 6661, pp. 671–673, Dec. 1997.
[6] A. A. Chabanov, M. Stoytchev, and A. Z. Genack, "Statistical signatures of photon localization," *Nature*, vol. 404, no. 6780, pp. 850–853, Apr. 2000.
[7] M. Störzer, P. Gross, C. M. Aegerter, and G. Maret, "Observation of the critical regime near Anderson localization of light," *Phys. Rev. Lett.*, vol. 96, no. 6, pp. 063904-1–063904-4, Feb. 2006.
[8] K. Sakoda, *Optical Properties of Photonic Crystals*. Berlin, Germany: Springer-Verlag, 2005.
[9] E. Yablonovitch, "Inhibited spontaneous emission in solid-state physics and electronics," *Phys. Rev. Lett.*, vol. 58, no. 20, pp. 2059–2062, May 1987.
[10] S. John, "Strong localization of photons in certain disordered dielectric superlattices," *Phys. Rev. Lett.*, vol. 58, no. 23, pp. 2486–2489, Jun. 1987.
[11] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light*. Princeton, NJ, USA: Princeton Univ. Press, 1995.
[12] C. Lopez, "Materials aspects of photonic crystals," *Adv. Mater.*, vol. 15, no. 20, pp. 1679–1704, Oct. 2003.
[13] F. Scotognella, D. P. Puzzo, M. Zavelani-Rossi, J. Clark, M. Sebastian, G. A. Ozin, and G. Lanzani, "Two-photon poly (phenylenevinylene) DFB laser," *Chem. Mater.*, vol. 23, no. 3, pp. 805–809, Feb. 2011.
[14] F. Scotognella, A. Monguzzi, F. Meinardi, and R. Tubino, "DFB laser action in a flexible fully plastic multilayer," *Phys. Chem. Chem. Phys.*, vol. 12, no. 2, pp. 337–340, Jan. 2010.
[15] S. Valligatla, A. Chiasera, S. Varas, N. Bazzanella, D. Narayana Rao, G. C. Righini, and M. Ferrari, "High quality factor 1-D Er3+-activated dielectric microcavity fabricated by rf-sputtering," *Opt. Exp.*, vol. 20, no. 19, pp. 21 214–21 222, Sep. 2012.
[16] Y. Li, L. M. Fortes, A. Chiappini, M. Ferrari, and R. M. Almeida, "High quality factor Er-doped Fabry–Perot microcavities by sol–gel processing," *J. Phys. D, Appl. Phys.*, vol. 42, no. 20, p. 205 104, Oct. 2009.
[17] D. Shechtman, I. Blech, D. Gratias, and J. W. Cahn, "Metallic phase with long-range orientational order and no translational symmetry," *Phys. Rev. Lett.*, vol. 53, no. 20, pp. 1951–1953, Nov. 1984.
[18] A. Ledermann, L. Cademartiri, M. Hermatschweiler, C. Toninelli, G. A. Ozin, D. S. Wiersma, M. Wegener, and G. Von Freymann, "Three-dimensional silicon inverse photonic quasicrystals for infrared wavelengths," *Nat. Mater.*, vol. 5, no. 12, pp. 942–945, Dec. 2006.
[19] Y. Kuga and A. Ishimaru, "Backscattering enhancement of random discrete scatterers," *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 1, no. 8, pp. 831–835, Aug. 1984.
[20] P.-E. Wolf and G. Maret, "Weak localization and coherent backscattering of photons in disordered media," *Phys. Rev. Lett.*, vol. 55, no. 24, pp. 2696–2699, Dec. 1985.
[21] M. P. Van Albada and A. Lagendijk, "Observation of weak localization of light in a random medium," *Phys. Rev. Lett.*, vol. 55, no. 24, pp. 2962–2965, Dec. 1985.
[22] H. Cao, Y. G. Zhao, S. T. Ho, E. W. Seelig, Q. H. Wang, and R. P. H. Chang, "Random laser action in semiconductor powder," *Phys. Rev. Lett.*, vol. 82, no. 11, pp. 2278–2281, Mar. 1999.
[23] D. S. Wiersma, "The physics and applications of random lasers," *Nat. Phys.*, vol. 4, no. 5, pp. 359–367, May 2008.
[24] P. Barthelemy, J. Bertolotti, and D. S. Wiersma, "A Levy flight for light," *Nature*, vol. 453, no. 7194, pp. 495–498, May 2008.
[25] L. Levi, M. Rechtsman, B. Freedman, T. Schwartz, O. Manela, and M. Segev, "Disorder-enhanced transport in photonic quasicrystals," *Science*, vol. 332, no. 6037, pp. 1541–1544, Jun. 2011.
[26] J. Bertolotti, S. Gottardo, D. S. Wiersma, M. Ghulinyan, and L. Pavesi, "Optical necklace states in Anderson localized 1D systems," *Phys. Rev. Lett.*, vol. 94, no. 11, pp. 113903-1–113903-11, Mar. 2005.
[27] J. B. Pendry, "Quasi-extended electron-states in strongly disordered-systems," *J. Phys. C, Solid State Phys.*, vol. 20, no. 5, pp. 733–742, Feb. 1987.
[28] A. V. Tartakovskii, M. V. Fistul, M. E. Raikh, and I. M. Ruzin, "Hopping conductivity of metal–semiconductor–metal contacts," *Sov. Phys. Semicond.*, vol. 21, pp. 370–373, 1987.
[29] M. Bellingeri, S. Longhi, and F. Scotognella, "Transmission of light in crystals with different homogeneity: Using Shannon index in photonic media," *J. Eur. Opt. Soc.-Rapid Pub.*, vol. 5, p. 10 041, 2010.
[30] M. Bellingeri and F. Scotognella, "Anomalous behaviour of light reflection in crystals with different homogeneity," *Opt. Mater.*, vol. 33, no. 8, pp. 1258–1261, Jun. 2011.
[31] M. Bellingeri and F. Scotognella, "Threshold behaviour in light reflection tuning the disorder in photonic media," *IEEE Photon. J.*, vol. 3, no. 6, pp. 1157–1163, Dec. 2011.

[32] M. Bellingeri and F. Scotognella, "Light transmission behaviour as a function of the homogeneity in one dimensional photonic crystals," *Photon. Nanostruct.*, vol. 10, no. 1, pp. 126–130, Jan. 2012.

[33] V. Bryant, *Aspects of Combinatorics*. Cambridge, U.K.: Cambridge Univ. Press, 1993.

[34] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, no. 4, pp. 379–423, Jul. 1948.

[35] M. Born and E. Wolf, *Principles of Optics*. Cambridge, U.K.: Cambridge Univ. Press, 1999.

[36] A. Petcu and L. Preda, "The optical transmission of one-dimensional photonic crystal," *Rom. J. Phys.*, vol. 54, no. 5/6, pp. 539–546, 2009.

[37] A. Crisanti, G. Paladin, and A. Vulpiani, *Products of Random Matrices in Statistical Physics*. Berlin, Germany: Springer-Verlag, 1993.

[38] J. Faist, J.-D. Ganière, P. Buffat, S. Sampson, and F. K. Reinhart, "Characterization of GaAs/(GaAs)n(AlAs)m surface-emitting laser structures through reflectivity and high resolution electron microscopy measurements," *J. Appl. Phys.*, vol. 66, no. 3, pp. 1023–1032, Aug. 1989.

[39] A. A. Asatryan, P. A. Robinson, L. C. Botten, R. C. McPhedran, N. A. Nicorovici, and C. M. de Sterke, "Effects of geometric and refractive index disorder on wave propagation in two-dimensional photonic crystals," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 62, no. 4, pp. 5711–5720, Oct. 2000.

[40] R. C. Polson and Z. V. Vardeny, "Random lasing in human tissues," *Appl. Phys. Lett.*, vol. 85, no. 7, pp. 1289–1291, Aug. 2004.

[41] C. J. Barbe, F. Arendse, P. Comte, M. Jirousek, F. Lenzmann, V. Shklover, and M. Gratzel, "Nanocrystalline titanium oxide electrodes for photovoltaic applications," *J. Amer. Ceram. Soc.*, vol. 80, no. 12, pp. 3157–3171, Dec. 1997.