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## New iterative method to obtain the softening curve in concrete.

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### Abstract

An original procedure to determine the softening curve in concrete has been proposed by the authors. This inverse method combines experimental results, finite element simulations and an iterative algorithm to adjust the experimental data. The end product of the process is a softening curve that allows us to very accurately reproduce the experimental curves. The proposed method calculates the fracture energy from the cohesive softening curve model, which in turn is iteratively determined by adjusting experimental load-displacement data of three-point bending tests. The procedure has been successfully applied to two conventional concretes.

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**Keywords:** Cohesive zone model, softening curve, concrete, fiber reinforced concrete

### 1. Introduction

The cohesive zone model is one of the most extended techniques to simulate the entire fracture process in concrete. It was initially introduced in the 60's by Dugdale (Dugdale 1960) and Barenblatt (Barenblatt 1962), to explain the stress singularity at the tip of a crack, and a decade later was developed and generalized by Hillerborg (Hilleborg et al 1976). The model has been successfully applied to describe the fracture of quasi-brittle materials (Guinea et al 1994, Bazant and Planas 1998, Elices et al 2002, Planas et al 1999, 2003, 2005 and 2006), ceramics, polymers and even metals (Gómez et al 2012).

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The cohesive zone model simulates the damage mechanisms preceding failure as a crack that transmits loads across its lips. The relationship between the transferred stress and the opening displacement is a material property called softening curve. The direct measurement of this function is complex and consequently, indirect procedures are used. These methods approximate the material curve by analytical functions depending on several parameters that are experimentally determined (Guinea et al 1994, Planas et al 1999).

The bilinear curve is one of the simplified proposed models to represent the softening curve. This function is formed by two straight lines, and depends on four parameters: the cohesive strength, the fracture energy and the coordinates of the vertex that separates the two linear parts. This curve can predict concrete behavior in a relatively satisfactory way (Guinea et al 1994).

In this paper, a new procedure to determine the softening curve based on the application of an iterative algorithm is proposed. The results show that the degree of approximation of the model predictions to the experimental results is considerably improved over with respect to previous works. The proposal does not postulate any analytical function, and the shape of the softening of the curve is a result of the algorithm. The methodology combines experimental data and numerical simulations in an iterative process with the aim of fitting the experimental values.

The algorithm has been successfully applied to two conventional concretes. The experimental program, taken from the literature, is discussed in the next section. Numerical modeling and the proposed algorithm is fully described in the third and fourth sections. Finally, the results are shown in section fifth, where a softening curve that almost perfectly reproduces the experimental data is reported.

#### Nomenclature

CMOD	Crack mouth opening displacement
CMOD <sub>exp</sub>	Experimental crack mouth opening displacement
CMOD <sub>i</sub>	Numerical crack mouth opening displacement at iteration i
E	Young's modulus
f	Softening curve
f <sub>t</sub>	Cohesive stress
P	Applied load
P <sub>max</sub>	Maximum load
P <sub>max,exp</sub>	Experimental maximum load
P <sub>max,num</sub>	Maximum load in the numerical calculation
w	Cohesive displacement
w <sub>c</sub>	Critical cohesive displacement
w <sub>i</sub>	Cohesive displacement at iteration i
w <sub>k</sub>	Cohesive displacement at the vertex of the bilinear softening curve
β	Iterative algorithm parameter
σ	Cohesive stress
σ <sub>i</sub>	Cohesive stress at iteration i
σ <sub>j</sub>	Cohesive stress at iteration j
σ <sub>k</sub>	Cohesive stress at the vertex of the bilinear softening curve

## 2. Experimental data

Experimental results of three point bending fracture tests have been taken from the literature in order to validate the proposed methodology (Fathy et al 2008). The experiments include two different concretes with design resistance of 25 and 40 MPa. The load vs. displacement data are shown in Fig 1 and the experimental device in Fig 2.

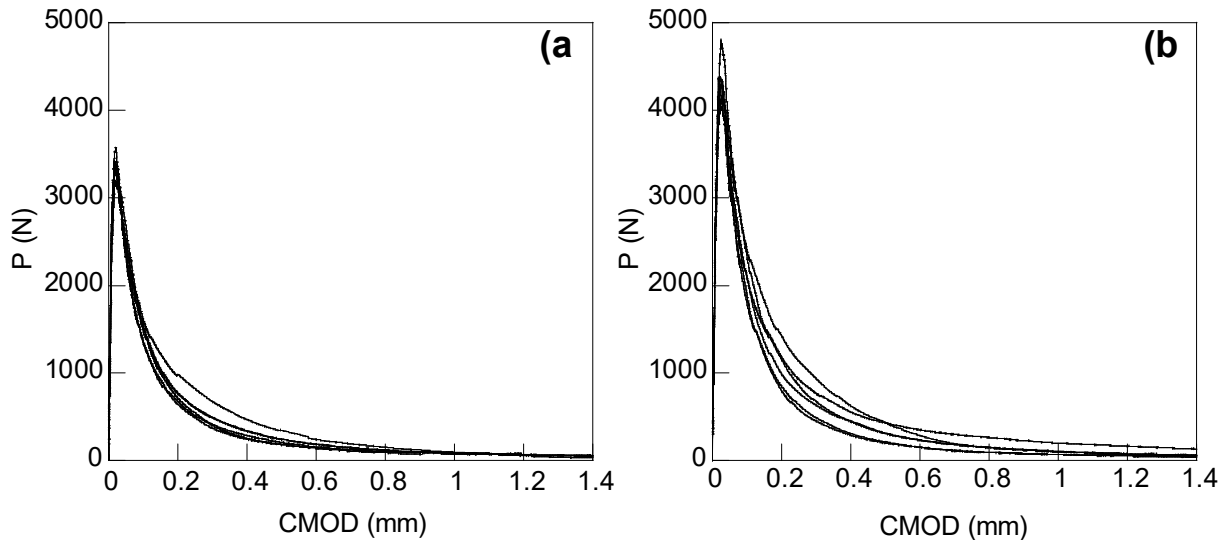


Fig. 1. Experimental load-CMOD curves: (a) Concrete 1; (b) Concrete 2.

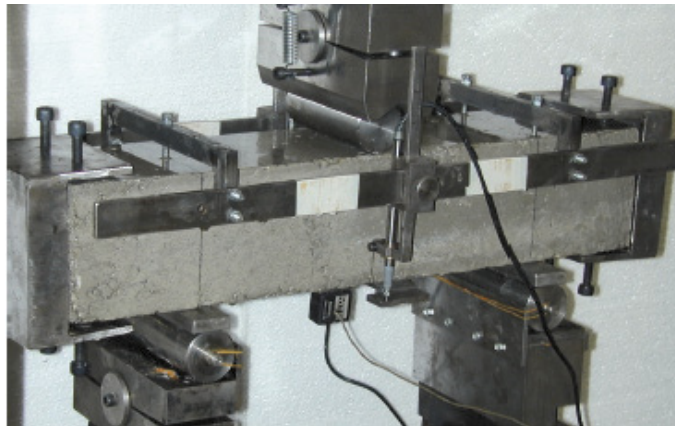


Fig. 2. Three point bending experimental device

The three-point bending tests were stable until the end of the test. There exists an initial linear part followed by a maximum load and a load decay until almost zero value. This is produced by a stable crack propagation in the symmetry plane of the specimen. From the experimental results the bilinear softening curve parameters were obtained following the procedure described in (Fathy et al 2008). The results appear in Table 1.

Table 1. Bilinear softening curve parameters and Young's modulus.

	Concrete 1	Concrete 2
$f_t$ (MPa)	2.24	2.84
E (GPa)	31.6	32.5
$w_c$ (mm)	0.272	0.293
$\sigma_k$ (MPa)	0.375	0.378
$w_k$ ( $\mu\text{m}$ )	15.6	21.8

### 3. Numerical simulation

The three-point bending tests have been modeled using the finite element method with the commercial code ABAQUS v6.13.4. 2D meshes formed by four node elements were used under plane stress hypothesis. The number of elements in the specimen ligament were 400. The material outside the fracture zone is considered linear elastic. The Young's modulus and Poisson's ratio are taken from the literature (Fathy et al 2008). The cohesive region has been introduced as a non-linear spring band whose load-displacement behavior is given by the softening curve. In Fig. 3, experimental and numerical load-CMOD curves are compared. It can be noticed that the fitting is relatively good for both concrete types.

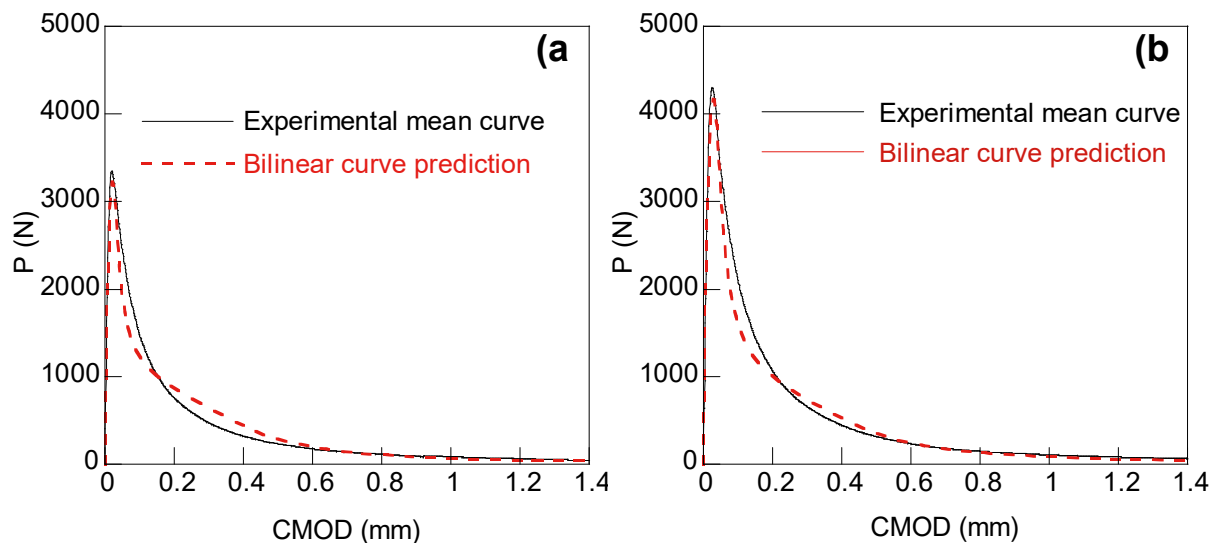


Fig. 3. Experimental data versus bilinear softening curve numerical predictions: (a) Concrete 1; (b) Concrete 2.

### 4. Iterative algorithm

To improve the fitting shown in Fig. 3, the following algorithm is proposed, where the softening curve is modified through two successive transformations. The first transformation is applied to the cohesive displacement,  $w$ , by defining a new softening curve  $\sigma_{i+1}=f(w_{i+1})$  calculated as

$$w_{i+1}(\sigma) = w_i(\sigma) \frac{CMOD_{exp}(P)}{CMOD_i(P)} \quad (1)$$

The cohesive stress,  $\sigma$ , and the applied load,  $P$ , are related through the following expression

$$\frac{\sigma}{f_t} = \left( \frac{P}{P_{\max}} \right)^\beta \tag{2}$$

where  $\beta$  is an algorithm parameter that varies among 0.5 and 3 and  $P_{\max}$  is the maximum load. Fig. 4 shows the evolution of the softening curve and the P-CMOD predicted curve during the application of the algorithm. Fig. 5 compares the transformed  $i+1$  P-CMOD curve versus the curve in state  $i$ .

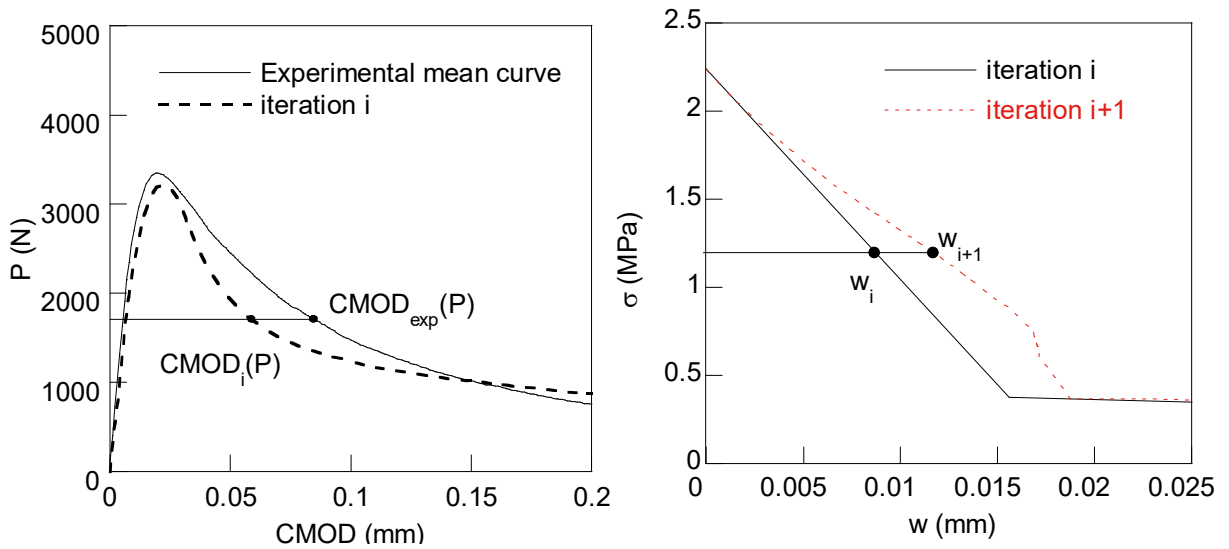


Fig. 4. Evolution of the numerical load-displacement curve and the softening curve with the iterative algorithm: a) Load-CMOD curve, b) softening curve.

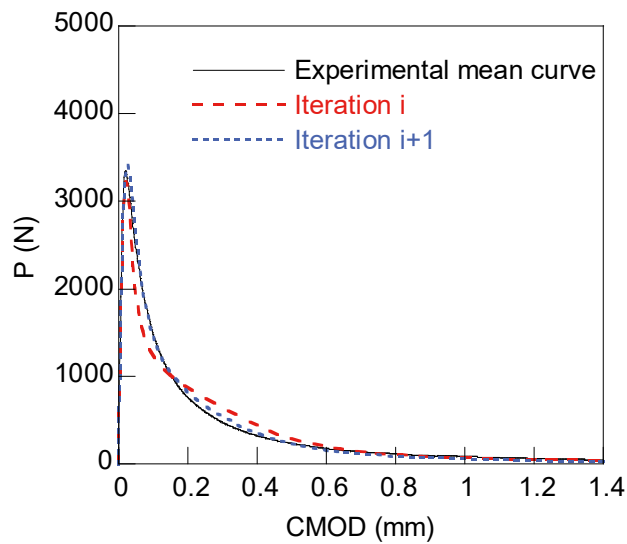


Fig. 5. Experimental mean curve versus numerical ones corresponding to iterations  $i$  and  $i + 1$ .

The procedure is repeated several times and the softening curve is modified until the equations (1) and (2) provide the best results.

After that, the second transformation, defined by the next expression, is applied.

$$\sigma_{j+1} = \sigma_j \frac{P_{\max, \text{exp}}}{P_{\max, \text{num}}} \quad (3)$$

The equation (3) is applied once and the loop defined by (1) and (2) is repeated. The procedure finishes when the transformed curve does not improve the fit.

The numerical implementation of the algorithm has been done in Python, the auxiliary language used by ABAQUS finite element code to write the output results. The Python application allows to launch calculations, analyze results and automatically modify the softening curve.

## 5. Results

The above algorithm has been applied to the experimental results described in paragraph 2. In both concrete samples, a new softening curve has been obtained which reproduces the experimental data much better than before.

The following figures show the quality of the model. In both cases there is an excellent fit between the numerical results and the experimental values.

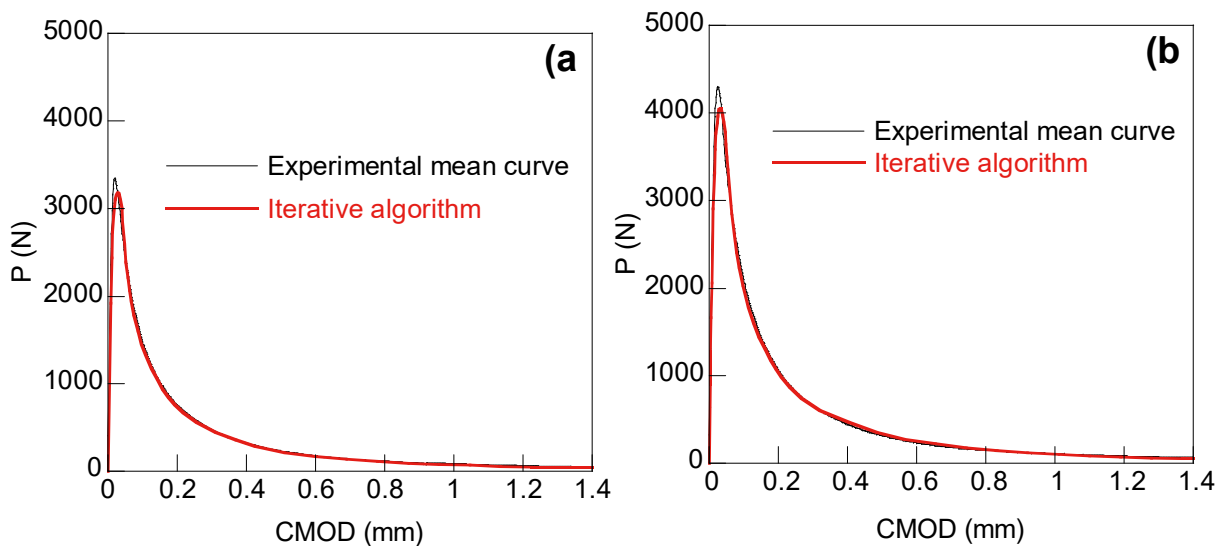


Fig. 6. Experimental and numerically predicted P-CMOD curves: a) Concrete 1 b) Concrete 2.

The final softening curves are plotted in Fig 7 and compared with the bilinear ones. In both cases the final curve has a similar initial slope to the bilinear one, and some smoothing is observed at the region near the internal vertex.

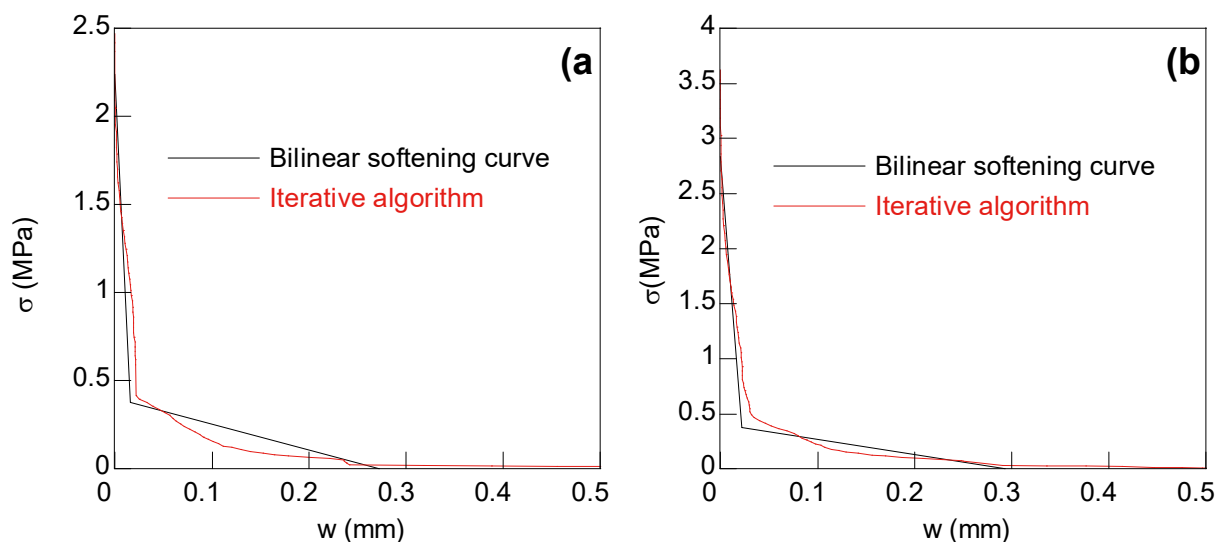


Fig. 7. Final softening curves comparing with the bilinear ones: a) Concrete 1 b) Concrete 2.

## 6. Conclusions

An original iterative method for determining the softening curve of concrete has been proposed. The procedure reduces the difference between the numerical and experimental results by modifying the softening curve.

The algorithm has been successfully applied to two conventional concretes.

The final softening functions allow the calculation of load-CMOD curves that almost perfectly fit the experimental data of three point bending tests.

The final softening curve does not substantially differ from the original bilinear softening curve, keeping the overall shape.

The new method is an alternative to indirect conventional procedures based on bilinear softening curve, but also could be seen as a complementary method for improving them.

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