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Procedia Engineering 199 (2017) 2531–2536

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

On the moving multi-loads problem in discontinuous beam structures with interlayer slip

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Abstract

This contribution proposes an efficient approach to the moving multi-loads problem on two-layer beams with interlayer slip and elastic translational supports. The Euler-Bernoulli hypothesis is assumed to hold for each layer separately, and a linear constitutive relation between the horizontal slip and the interlaminar shear force is considered. It is shown that, using the theory of generalized functions to treat the discontinuous response variables, exact eigenfunctions can be derived from a characteristic equation built as determinant of a 6 x 6 matrix. Building pertinent orthogonality conditions for the deflection eigenfunctions, a closed-form analytical response is established in the time domain. The proposed procedure is illustrated for a two-layer beam with interlayer slip, elastically supported at the center and acted upon by moving multi-loads.

Keywords: Euler-Bernoulli beam, interlayer slip, translational support, moving loads.

1. Introduction

In most cases, static and dynamic analysis of composite beams is based on the hypothesis of rigid inter-connection between the layers, applying either equivalent-single-layer theories or layerwise laminate theories [1]. However, in layered wood systems and in composite steel-concrete structures, the assumption of rigid bond between the layers cannot be used because the layer interfaces are generally subject to relative horizontal displacement. Interlayer slip affects both strength and deformation of the layered beam, as recognized in various studies. For instance, linear static analysis of layered beams with flexible bond is presented by Schnabl et al. [2], while the dynamic problem is treated by Girhammar and Pan [3] and Adam et al. [4]. In this context, the present contribution proposes an efficient approach to the moving multi-loads problem on *discontinuous* layered beams with interlayer slip, with discontinuities imposed by in-span elastic translational supports. Existing studies on discontinuous beams under moving loads are numerous but, in general, concern homogeneous beams. In general, they are based on modal superposition procedures

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Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.436

[5–10], using exact [5–7] or approximate modes [8–10]. Finite element (FE) approaches were also adopted [11] as an alternative to modal superposition procedures for discontinuous beams. Although modal superposition and FE methods provide effective solutions in all studies described above, their application is still limited to some extent. The efficiency of both methods depends on the number of discontinuity locations, suffering from increasing computational effort as the number of discontinuity locations increases. Additionally, the FE approach requires re-meshing and re-building of the stiffness matrix for each potential set of discontinuity parameters. For this reason, recently, an innovative modal superposition approach, independent of the number of discontinuity locations, was presented for discontinuous homogeneous beams [12–14], deriving exact modes by the theory of generalized function [15–17] with pertinent orthogonality conditions. In the present paper, the theory of generalized functions is used to reformulate and solve the vibration problem of two-layer beams with elastic interlayer slip subjected to multi-moving loads and carrying in-span elastic translational supports.

2. Governing equations for discontinuous two-layer beam with interlayer slip

Consider a two-layer beam of length L with interlayer slip between the layers, carrying an arbitrary number N of translational supports at abscissas x_j along the axis (Fig. 1). The equation of flexural beam motion is derived neglecting the effect of rotatory and longitudinal inertia, in absence of external axial forces and considering that the Bernoulli-Euler hypothesis is not applicable to the entire cross-section of the beam, due to the interlayer slip, but remains valid for each individual layer. Recalling the mathematical steps carried out by Adam et al. [4] for continuous layered beams with interlayer slip, and considering the recent studies conducted by Di Lorenzo et al. [13] and Adam et al. [14] on the moving load problem of discontinuous homogeneous beams, the equation of motion of the considered beam subjected to a series of NL concentrated loads F_i $(i = 1, ..., NL)$ with constant velocity V_0 and initial location s_i (see Fig. 1) is derived as

$$\frac{\bar{\partial}^6 w(x,t)}{\partial x^6} - \alpha^2 \frac{\bar{\partial}^4 w(x,t)}{\partial x^4} + \frac{\mu}{EI_0}\frac{\bar{\partial}^4 w(x,t)}{\partial x^2 \partial t^2} - \frac{\mu\,\alpha^2}{EI_\infty}\frac{\partial^2 w(x,t)}{\partial t^2} + \frac{\alpha^2 P_j\,\delta(x-x_j)}{EI_\infty} - \frac{P_j\,\delta^{(2)}(x-x_j)}{EI_0} \tag{1}$$
$$= -\frac{\alpha^2}{EI_\infty}\sum_{i=1}^{NL} F_i\,\delta\left(x - V_0 t + s_i\right)\left[H(t-t_i^0) - H(t-t_i^E)\right] + \frac{1}{EI_0}\sum_{i=1}^{NL} F_i\,\delta^{(2)}\left(x - V_0 t + s_i\right)\left[H(t-t_i^0) - H(t-t_i^E)\right]$$

In Eq. (1), $w(x,t)$ is the transversal displacement of the beam, and μ is the beam mass per unit length. Heaviside functions $H(t-t_i^0)$ and $H(t-t_i^E)$ indicate the arrival and the departure of F_i at time instants $t_i^0 = s_i/V_0$ and $t_i^E = s_i + L/V_0$, respectively. In coefficient α^2, given as [4] $\alpha^2 = k\left[\left(EA_0/EA_p\right) + \left(r^2/EI_0\right)\right]$, variable r represents the vertical distance between the elastic centers of gravity of the two layers (see Fig. 1), k is the elastic slip modulus, $EA_0 = E_1A_1 + E_2A_2$ is the longitudinal stiffness, E_1 (E_2) is Young's moduli and A_1 (A_2) the cross sectional area of the upper (lower) layer, $EI_0 = E_1I_1 + E_2I_2$ denotes the bending stiffness of the corresponding beam with non-composite action (no bond), while $EA_p = E_1A_1E_2A_2$.

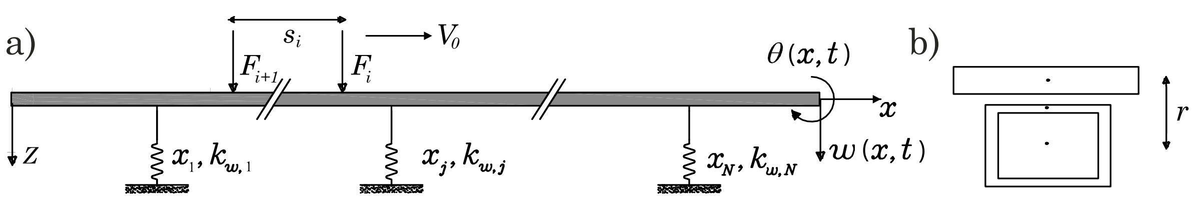

Fig. 1. a) Discontinuous composite beam with interlayer slip. b) Cross-section. Positive sign conventions are reported

Furthermore, in Eq. (1), EI_∞ is the bending stiffness of the corresponding beam with rigid interlayer bond [4],

$$EI_\infty = EI_0 + \frac{r^2 E_1A_1E_2A_2}{EA_0} \tag{2}$$

Because of the discontinuities at the elastic supports, in Eq. (1) the space-derivatives are generalized derivatives, as denoted by the over-bar. Variable $\delta()$ is Dirac's delta function, $\delta^{(2)}()$ denotes its second formal derivative with respect to x , and $P_j = -k_{w,j} w\left(x_j, t\right)$ is the reaction force at the jth elastic translation support of stiffness $k_{w,j}$. Note that in this formulation both limits $\alpha \to \infty$ (rigid bond) and $\alpha \to 0$ (no bond) can be realized without numerical difficulties. If no translation support is attached at $x = x_j$, i.e. $P_j = 0$, Eq. (1) reverts to the equation of motion of a *continuous* composite beam with interlayer slip [4].

3. Dynamic response analysis

In the first step, the eigenfunctions of the boundary problem represented by Eq. (1) are determined. Based on the standard separate variables approach, a displacement eigenfunction $\Phi(x)$ of the two-layer discontinuous beam with interlayer slip complies with the following ordinary differential equation

$$\frac{\bar{d}^6\Phi(x)}{dx^6} - \alpha^2\frac{\bar{d}^4\Phi(x)}{dx^4} - \frac{\mu\omega^2}{EI_0}\frac{\bar{d}^2\Phi(x)}{dx^2} + \frac{\mu\alpha^2\omega^2}{EI_\infty}\Phi(x) + \frac{\alpha^2}{EI_\infty}p_j\,\delta(x-x_j) - \frac{p_j}{EI_0}\,\delta^{(2)}(x-x_j) = 0 \tag{3}$$

Here, p_j is related to the unknown reaction force at the jth support for $j = 1, ..., N$, defined as $p_j = -k_{w,j}\Phi\left(x_j\right)$. Now, the eigenfunctions for displacement, rotation, bending moment, shear force, individual layer axial force and interlaminar shear force are collected in vector $\mathbf{Y}(x) = \left[\Phi(x)\ \Theta(x)\ \Upsilon(x)\ \Gamma(x)\ \Sigma(x)\ \Psi(x)\right]^T$. In analogy to Eq. (3), vector $\mathbf{Y}(x)$ can be rewritten in the following general form

$$\mathbf{Y}(x) = \mathbf{\Omega}(x)\,\mathbf{c} + \sum_{j=1}^{N} p_j\,\mathbf{J}\left(x, x_j\right) = \begin{bmatrix} \Omega_{\Phi_1} & \Omega_{\Phi_2} & \Omega_{\Phi_3} & \Omega_{\Phi_4} & \Omega_{\Phi_5} & \Omega_{\Phi_6} \\ \Omega_{\Theta_1} & \Omega_{\Theta_2} & \Omega_{\Theta_3} & \Omega_{\Theta_4} & \Omega_{\Theta_5} & \Omega_{\Theta_6} \\ \Omega_{\Upsilon_1} & \Omega_{\Upsilon_2} & \Omega_{\Upsilon_3} & \Omega_{\Upsilon_4} & \Omega_{\Upsilon_5} & \Omega_{\Upsilon_6} \\ \Omega_{\Gamma_1} & \Omega_{\Gamma_2} & \Omega_{\Gamma_3} & \Omega_{\Gamma_4} & \Omega_{\Gamma_5} & \Omega_{\Gamma_6} \\ \Omega_{\Sigma_1} & \Omega_{\Sigma_2} & \Omega_{\Sigma_3} & \Omega_{\Sigma_4} & \Omega_{\Sigma_5} & \Omega_{\Sigma_6} \\ \Omega_{\Psi_1} & \Omega_{\Psi_2} & \Omega_{\Psi_3} & \Omega_{\Psi_4} & \Omega_{\Psi_5} & \Omega_{\Psi_6} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} + \sum_{j=1}^{N} p_j \begin{bmatrix} J_\Phi^{(p)} \\ J_\Theta^{(p)} \\ J_\Upsilon^{(p)} \\ J_\Gamma^{(p)} \\ J_\Sigma^{(p)} \\ J_\Psi^{(p)} \end{bmatrix} \tag{4}$$

That is, $\mathbf{Y}(x)$ is built as the sum of the solutions $\mathbf{\Omega}(x)\,\mathbf{c}$ of the homogeneous equation, representing the eigenfunction of the bare beam and the particular solution $\sum_{j=1}^{N} p_j\ \mathbf{J}\left(x, x_j\right)$ associated with the unknowns p_j. In Eq. (4), $\mathbf{c} = \left[c_1\ c_2\ c_3\ c_4\ c_5\ c_6\right]^T$ is a 6×1 vector of integration constants, and $\mathbf{J}\left(x, x_j\right)$ contains the integrals associated with a unit transverse force, applied at $x = x_j$. All terms in Eq. (4) are available in a simple analytical form. Specifically, to build the vector of particular solutions $\mathbf{J}\left(x, x_j\right)$, equations of motion are used starting from the following novel expression of the particular integral for displacement eigenfunction $\Phi(x)$, obtained by Mathematica [18] using pertinent generalized functions

$$\begin{aligned} J_\Phi^{(p)} = &\left[EI_\infty\ EI_0{}^2/b\right](R_1 - R_2)(R_1 - R_3)(R_2 - R_3)\left\{\left[\left(\tfrac{R_2 - R_3}{\sqrt{R_1}}\right)\left(EI_\infty R_1 - EI_0\alpha^2\right)\right]\sinh\left(\sqrt{R_1}(x - x_j)\right)\right. \\ &\left. - \left[\left(\tfrac{R_1 - R_3}{\sqrt{R_2}}\right)\left(EI_\infty R_2 - EI_0\alpha^2\right)\right]\sinh\left(\sqrt{R_2}(x - x_j)\right) + \left[\left(\tfrac{R_1 - R_2}{\sqrt{R_3}}\right)\left(EI_\infty R_3 - EI_0\alpha^2\right)\right]\sinh\left(\sqrt{R_3}(x - x_j)\right)\right\} H(x - x_j) \end{aligned} \tag{5}$$

where symbol b is a constant defined as

$$b = \mu\ \omega^2\left[4EI_0{}^3 EI_\infty\ \alpha^8 + EI_0\left(-27EI_0{}^2 + 18EI_0\ EI_\infty + EI_\infty^2\right)\alpha^4\mu\ \omega^2 + 4EI_\infty^2\ \mu^2\omega^4\right] \tag{6}$$

with R_1, R_2, R_3 denoting the roots of the characteristic polynomial of the homogeneous differential Eq. (3). The unknowns p_j can be expressed in terms of vector $\mathbf{c}$, and thus, also $\mathbf{Y}(x)$ can be expressed in compact form through vector $\mathbf{c}$ only. At this stage, the eigenvalue problem can be formulated using the boundary conditions of the beam, obtaining six equations, expressed in general form $\mathbf{Bc} = \mathbf{0}$, where each equation corresponds to a component of vector $\mathbf{Y}(x)$ evaluated at the beam ends. It is worth underscoring that matrix $\mathbf{B}$ is always a 6×6 matrix, independent of the number of supports along the beam. The characteristic equation of the eigenvalue problem is the determinant of matrix $\mathbf{B}$, i.e. $\det \mathbf{B} = 0$, whose roots ${\omega_n}^2$ are the squared natural circular frequencies of the beam (subscript n is added to indicate the infinite number of the eigensolutions). Once vector $\mathbf{c}$ has been obtained from $\mathbf{Bc} = \mathbf{0}$, an exact closed-form expression can be derived for the vectors of eigenfunctions $\mathbf{Y}_n(x)$ through the compact form of Eq. (4). This is one advantage over the exact classical approach, where in each beam segment between two consecutive application points of supports, six integration constants need to be specified, resulting in $6 \times (N+1)$ constants for N application points, to be computed on enforcing the boundary conditions and matching conditions over continuous beam segments. After the eigenfunctions have been derived, the vibration problem of the beam shown in Fig. 1 is solved through modal analysis, $w(x,t) = \sum_{n=1}^{\infty} R_n(t)\,\Phi_n(x)$, with $\Phi_n(x)$ denoting the nth displacement eigenfunction, and $R_n(t)$ is the corresponding modal coordinate. Substituting the modal series into Eq. (1), multiplying this expression by the mth eigenfunction $\Phi_m(x)$, integrating over the beam length L, and considering the pertinent orthogonality relations for the eigenfunctions [4], allows to write an ordinary differential equation for the nth modal coordinate $R_n(t)$ in analogy to a single-degree-of-freedom oscillator,

$$\frac{d^2 R_n(t)}{dt^2} + \omega_n^2 R_n(t) = \frac{1}{m_n} V_n(t) \tag{7}$$

where m_n denotes the nth modal mass

$$m_n = -\frac{\mu\alpha^2}{EI_\infty}\int_0^L \Phi_n^2(x)\,dx + \frac{\mu}{EI_0}\int_0^L \frac{d^2\Phi_n(x)}{dx^2}\Phi_n(x)\,dx \tag{8}$$

and V_n is the nth modal load

$$\begin{aligned} V_n = &-\left(\alpha^2/EI_\infty\right)\int_0^L \Phi_n(x)\,F_i\,\delta(x - V_0\,t + s_i)\left[H\left(t - t_i^0\right) - H\left(t - t_i^E\right)\right]dx \\ &+ (1/EI_0)\int_0^L \Phi_n(x)\,F_i\,\delta^{(2)}(x - V_0\,t + s_i)\left[H\left(t - t_i^0\right) - H\left(t - t_i^E\right)\right]dx \end{aligned} \tag{9}$$

In the time domain, the solution of Eq. (7) is given, for instance, by means of Duhamel's convolution integral.

4. Numerical example

Consider the compound high-speed train bridge of length $L = 40\,m$, composed of a concrete deck (upper layer) and two symmetrically arranged steel girders (lower layer), simply supported at both ends and equipped with an elastic translational support of stiffness $k_{w,1} = 20\,10^7\,N/m$ at the center (see Fig. 2). The characteristic layer parameters are [19]: $A_1 = 2.08\,m^2$, $E_1 = 33\,10^9\,N/m^2$, $E_1 I_1 = 61.2\,10^7\,Nm^2$ for the upper layer; $A_2 = 0.195\,m^2$, $E_2 = 210\,10^9\,N/m^2$, $E_2 I_2 = 42.5\,10^9\,Nm^2$ for the lower layer. The bridge mass per unit length is $\mu = 15000\,kg/m^3$, and $r = 1.98\,m$ is the actual distance between the centroids of the individual layers. The effect of the steel anchor bolts, which couple flexibly both layers, is modeled as continuous elastic connection with slip modulus $k = 10^7\,N/m^2$. The bridge is subjected to a high-speed train used in [20], modeled as a series of concentrated forces, which represent the axle loads of a train composed of one rail car and seven passenger cars. Each car has at both ends a bogie with two axles each, with distances specified in Fig. 2: $h = c = 2.80\,m$, $b = e = 2.27\,m$, $g = 13\ m$ and $d = 24.34\,m$. The axle loads of the rail cars and passenger cars are respectively 200 kN and $116.5\,kN$. In the current application it is assumed that the loads cross the beam with the first critical speed of first order, i.e. $V_0 = \omega_1 d/2\pi$ [21], yielding $V_0 = 104.91\,m/s$ for the fundamental circular beam frequency $\omega_1 = 27.08\,rad/s$. The next three natural frequencies are: $\omega_2 = 42.6\,rad/s$, $\omega_3 = 98.5\,rad/s$, $\omega_4 = 168\,rad/s$. To show the effect of the elastic bond on the beam response, alternatively the response of the beam bridge with rigid interlayer bond is also derived. In this case, the natural frequencies become larger (i.e. $\omega_1 = 31.83\,rad/s$, $\omega_2 = 76.36\,rad/s$, $\omega_3 = 173.77\,rad/s$, $\omega_4 = 305.44\,rad/s$) because of increased global lateral beam stiffness. Fig. 3 shows the first four displacement eigenfunctions of the beam with elastic interlayer slip (Fig. 3a) and alternatively of the beam with rigid bond (Fig. 3b), derived by the proposed method and the classical method. In the latter method, the free-vibration response is represented by twelve integration constants, six for each segment, respectively left and right of the translational support. Instead, in the proposed more efficient method only six integration constants appear. The fundamental mode of the actual beam with interlayer slip exhibits a slope (see Fig. 3a), indicating the impact of the flexible support. In contrast, the corresponding mode of the rigidly bonded beam looks like more the mode of the simply supported beam without elastic spring at mid-span (see Fig. 3b). The higher modes are less influenced by the elastic support, compared to the ones of simply supported beam. The dynamic lateral beam deflection induced by the series of repetitive loads is shown in Fig. 4. In the left subplot the time history of the deflection at $x = L/2$ is depicted. As observed, the beam with interlayer slip (red solid line) is excited to resonance because the loads move with a critical speed related to the first mode of this structure. In contrast, the beam model with rigid interlayer bond underpredicts significantly the actual response, emphasizing the importance of correct consideration of the interlayer slip stiffness. This response behavior can be led back to detuning of the critical speed from the actual fundamental frequency, as it has been shown above. The right subplot shows the beam deflection over span at six time instants, specified in Fig. 4a by numbers 1 to 6. Additionally, also the displacement considering only the first eigenfunction for time instant 3 is shown (red dashed line). No significant changes are found when considering more than four modes in the proposed solution, thus, the results shown are based on a five mode series approximation.

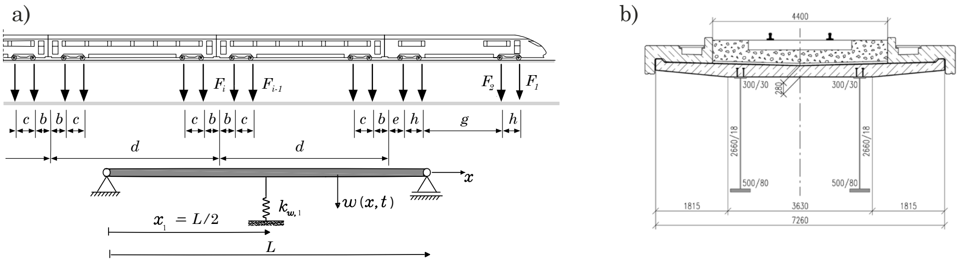

Fig. 2. (a) Compound beam bridge subjected to a series of concentrated forces (modified from [20]). (b) Cross-section ([19])

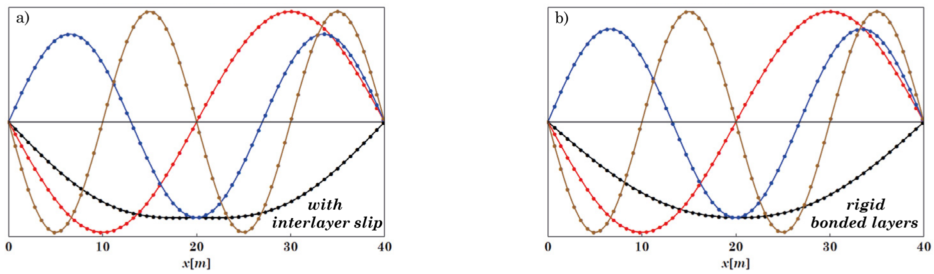

Fig. 3. First four displacement eigenfunctions of the beam shown in Fig. 2 in two different configurations: (a) composite beam with elastic bond, (b) composite beam with rigid bond. Solution based on the proposed method (solid line) and on the classical procedure (black markers)

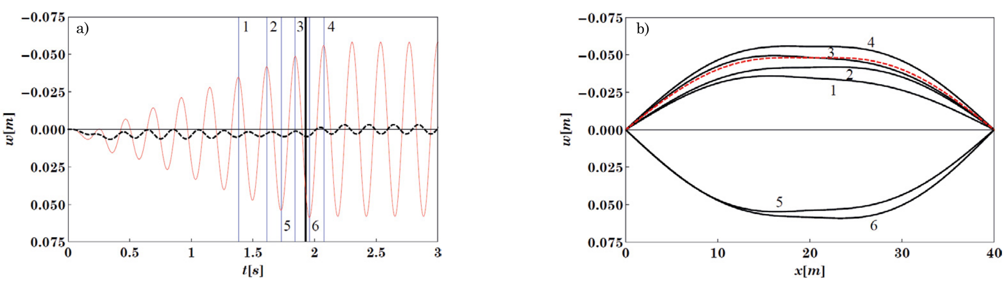

Fig. 4. Response to moving multi-loads at critical speed $V_0 = 104.91 m/s$. (a) Deflection time history at $x = L/2$. Red solid line: beam with elastic bond. Black dashed line: beam with rigid bond. (b) Deflection over span of the beam with elastic bond. Black solid lines: multi-mode response at six time instants specified in (a). Red dashed line: first mode response at time instant 3

5. Summary and conclusion

A method has been presented to analyze the moving multi-loads problem on layered beam structures with interlayer slip resting on elastic translational supports. A generalized function approach has been proposed to derive the exact eigenfunctions of all response variables, independently of the number of discontinuities associated with the elastic supports. Based on pertinent orthogonality conditions for deflection modes, the response under moving multi-loads has been built in the time domain by modal superposition. This allows considerable computational advantages over standard numerical solutions built by the finite element method. The efficiency and accuracy has been proven considering a discontinuous compound beam bridge with flexibly interconnected layers, subjected to a high-speed train traveling with a critical speed.

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