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A simple analytical model for the resonance frequency of perforated beams

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Abstract

In this work the bending properties of beams with periodic rectangular perforations are examined. Starting from the standard Euler-Bernoulli beam equation, compact analytical expressions for the equivalent bending stiffness in the filled and perforated sections are developed and used to compute the resonance frequencies of the perforated beam. The results are in good agreement with FEM simulations for most practical designs, as long as shear stress effects can be considered negligible.

Keywords: MEMS; resonant sensors; biosensors; MEMS modeling.

1. Introduction

The release of beams and plates of large area in MEMS surface technologies is limited by etch speed of the sacrificial layer under the beam or plate itself. Most MEMS approaches resolve this issue by creation of a regular pattern of holes in the structural layer. Despite being introduced for merely technological reasons, these perforations are expected to affect the mechanical behavior of MEMS structures in various ways. Among them, only the effect of holes on the air damping has been extensively investigated [1].

In this work, we examine the mechanical properties of beams with regular square perforations, and propose a simple analytical model to compute their in-plane stiffness behavior and resonance frequency. The model is aimed at the time-efficient design of beam-based MEMS resonant sensors [2]. Numerical methods can be and have been used to predict the behavior of such structures [3], but they tend to be computationally expensive, especially with the reduction of the perforation size. The mechanical

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properties of regularly structured materials can also be modeled by equivalent anisotropic materials [4, 5]. These approaches, however, are cumbersome and do not lead to simple closed expressions for the resonance frequency. Our approach is based on simple modifications of the standard Euler-Bernoulli (EB) beam equation, and retains its simplicity by substituting specialized expressions for the flexural stiffness of the different beam segments.

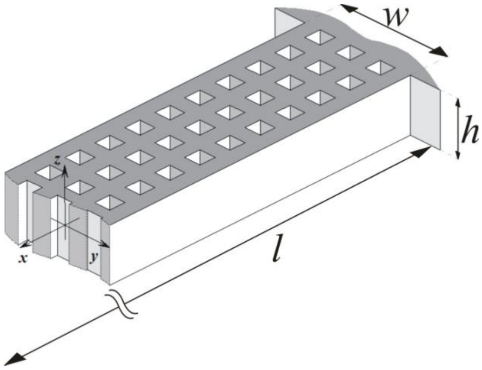

Fig. 1. View of a perforated beam with the coordinate system used in the text. Part of the beam is cut away for clarity.

2. Derivation of the model

The deflection *u(x)* of a beam with constant section under bending can be modeled by the EB beam equation:

$$EI\frac{\partial^2 u(x)}{\partial x^2} = M \qquad (1)$$

where E is the Young's modulus of the material, I the moment of inertia of the beam section, and M the external torque. Our model proposes the substitution of the moment of inertia I with an equivalent moment of inertia I_{EQ} for the perforated beam. Referring to the system of coordinates in Figs. 1, 2, EB theory assumes a linear distribution of the normal stress along the beam section of the form:

$$\sigma_{EB} = \frac{M}{I}y = K_{EB}y \qquad (2)$$

This distribution leads to the standard EB beam differential equation. We now consider the same beam with a pattern of square holes of period l_s and side $l_s - t_s$ (Fig. 2). We also define N as the number of holes along the section, and $\alpha = t_s/l_s$ as the filling ratio. We postulate a linear distribution of the stress in the (filled parts of the) perforated section, and a piecewise linear distribution (with two different slopes) in the filled section (Fig. 2). The stress distributions for the perforated section, the (under-stressed) parts of the filled section between holes, and the remainder of the filled section, are then:

$$\sigma_P = K_P y, \quad \sigma_{FP} = K_{FP} y, \quad \sigma_{FF} = K_{FF} y \qquad (3)$$

where the K's are constants to be determined. Both K_{FP} and K_{FF} have to converge to K_{EB} when $\alpha \rightarrow 1$. An expression for K_{FP} can be determined by solving a plane stress problem with appropriate boundary conditions in a single section of the filled part of the beam, following a general approach for plane stress problems [6].

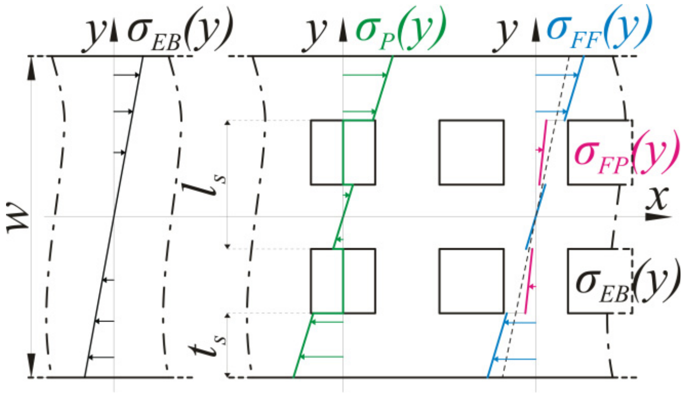

Fig. 2. Normal stress distribution in a full beam (left), and the proposed stress model in the perforated (center, green line) and filled (right, blue-pink line) sections of a perforated beam (right).

If we express K_{FP} (as it is convenient) as a fraction of K_{EB}, their ratio (which we will call the *filling function F*) can be shown to be:

$$\frac{K_{FP}}{K_{EB}} = F(N,\alpha) = \frac{I}{I_P}\left(\alpha - \frac{2}{\pi^3\alpha(1-\alpha)}\sum_{n=1}^{\infty}\frac{1}{n^3}\sin^2[n(1-\alpha)\pi]\tanh(n\alpha\pi)\right) \quad (4)$$

where I_P is the moment of inertia of the perforated section. Exploiting the conservation of the bending moment along the beam and (4), the constant K_{FF} can be computed, and in turn an equivalent moment of inertia (averaged between the filled and the perforated part) can be determined:

$$I_{EQ} = \frac{I \cdot I_P}{I + \alpha(I_P - I)F(N,\alpha)} \quad (5)$$

3. Results

The model was used to estimate the first in-plane (i.e. along y) resonant frequency of clamped-clamped beams using the standard formula:

$$f_1 \simeq \frac{1}{2\pi}\sqrt{\frac{4.73^4 EI_{EQ}}{\rho_l L^4}} \quad (6)$$

where ρ_l is the average mass per unit length of the beam. The infinite sum in (4) converges rapidly and was numerically arrested at $n = 10$. All the simulated beams were 300 μm in length and 10 μm in width. Different values of N (and, consequently, different hole sizes) were tested. The analytical results were compared with FEM simulations performed with ANSYS.

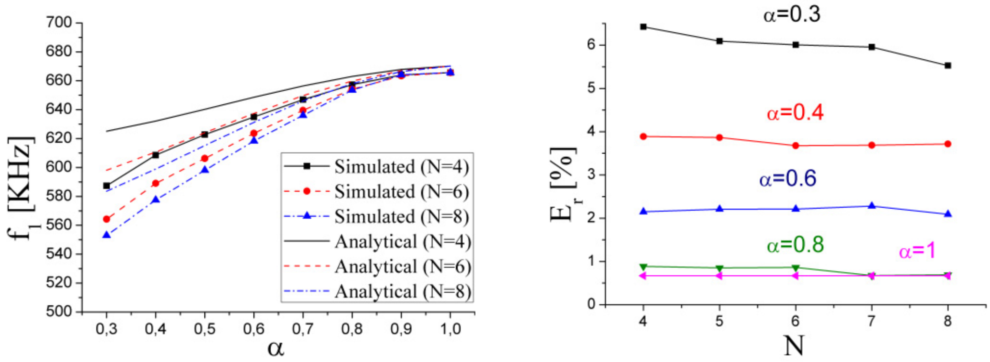

Fig. 3. Comparison between analytical and simulated resonance frequencies as a function of the filling ratio α for three different values of N (4, 6, 8) (left); relative error of the analytical frequency values with respect to the simulated ones as a function of N (right).

The results are summarized in Fig. 3. For $\alpha > 0.3$, which covers most practical MEMS designs, Eq. 6 gives approximations within 6% of the simulated values (Fig. 3, right). For lower values of α, i.e. for larger holes, the EB theory fails because of the importance of shear stresses in the beam. While the presented results involve only resonance frequencies, the model can also be used to evaluate static deflection and static elastic constants of perforated beams.

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