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# Effect of geometric irregularities on the dynamic response of masonry arches

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**Abstract**

In this paper, the effect of geometric irregularities on the dynamic response of circular masonry arches is considered. Irregular geometries are obtained through a random generation of the key geometric parameters, and the effect of these irregularities is shown by modelling the dynamic response to ground motion. The masonry arch is modelled as a four-link mechanism, i.e., a system made of three rigid blocks hinged at their ends, where the position of the hinges at the instant of activation of the motion is determined through limit analysis. Lagrange's non-linear equations of motion have been solved through numerical integration. The results show that geometrical uncertainties produce an alteration of the mechanical features of the rigid blocks which may reduce the seismic capacity.


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*Keywords:* masonry arch; geometric irregularities; non-linear dynamic analysis

## 1. Introduction

Masonry constructions represent a significant portion of the world's heritage building stock. During the XX century, seismic events produced severe damages, highlighting the necessity of a better understanding of the masonry structures behaviour in order to protect the human life and the cultural building heritage. Considering that arch-type elements can often be found in masonry constructions, the knowledge of their response in case of earthquake is a fundamental objective in order to understand how seismic loads can be transmitted though the masonry bearing elements that make up the complex structure. In this context, it should be observed that masonry constructions and arches often show the presence of defects of shape of the stones, associated with imprecisions of construction or deterioration due to environmental factors. Hence, the understanding of the influence of these geometrical irregularities on the collapse conditions is necessary for an adequate safety assessment, with special attention to the seismic action [1,2].

The single rocking block problem represents the basis of the dynamic analysis of the masonry arch. Housner [3] applied the dynamic laws in order to study the overturning of a rigid block subjected to horizontal forces. A dynamic analysis of the circular masonry arch was carried out by Oppenheim, considering an equivalent SDOF system made

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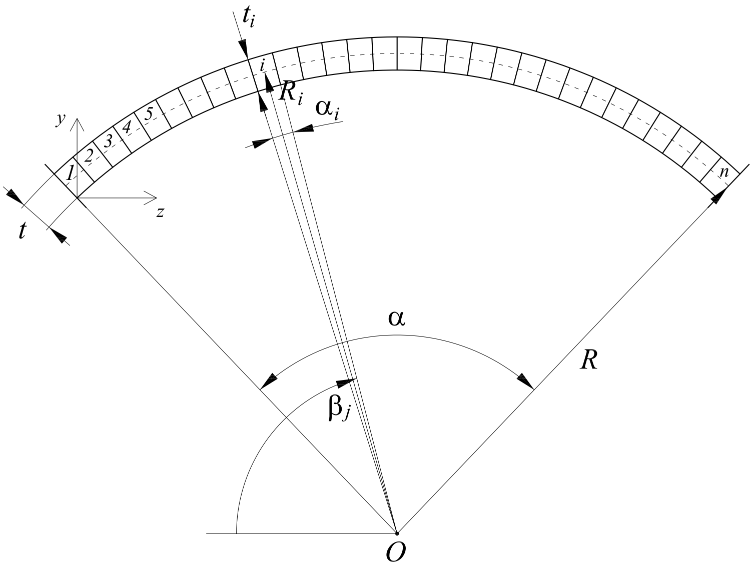


Fig. 1. Geometrical parameters for the definition of the arch shape.

of three rigid liks hinged at their ends [4]. The calculation, based on the application of Lagrange's Equations, led him to the definition of the dynamic equation of motion of the masonry arch under inertial loads. This approach was then followed by Clemente [5], who investigated the free vibrations and the response of the arch to harmonic base acceleration. Both authors considered only a single cycle of response and they neglected the loss energy due to the impact. Oppenheim's results have been compared by DeJong et al. [6] with discrete element modelling; the authors found that assuming a solution predicting failure due to direct overturning, without rocking, is unsafe. Oppenheim's analytical model was enriched by De Lorenzis et al. [7,8], taking into account the dissipation of energy caused by the impact. This improved modelling permits to study the motion of the arch through continued cycles of rocking. The analytical predictions have been subsequently compared to the results obtained by means of experimental tests on model arches [9]. An interesting application of Oppenheim's analytical model has been carried out to study the dynamic behaviour of the pointed arch [10]. More recently, a methodology to derive an equivalence between SDOF rocking structures (or mechanisms, e.g. the circular masonry arch) and the single rocking block has been developed by De Jong et al. [11].

In this paper, the dynamic of rigid body of the masonry arch has been analysed, by means of the Oppenheim's four-link mechanism, enriched with the De Lorenzis' impact model, taking into account the geometrical irregularities. In the first part, the uncertain arch geometry has been defined and the analytical model adopted for the dynamic analysis has been described. In the second part, the dynamic analysis has been performed on the arch geometry considered by Oppenheim [4] taking also into account the geometrical uncertainties related to the imprecisions of construction, the defects of shape of the voussoirs or the deterioration level. The response of the Oppenheim's arch to step impulse base acceleration has been compared to those of two uncertain geometries, generated from the nominal one.

## 2. Geometrical model

The circular masonry arch in its plane has been considered. The nominal (or deterministic) geometry has been defined by assigning the radius $R$, the angle of embrace $\alpha$ and the thickness $t$. The arch has been discretized into $n$ voussoirs by radial lines passing though the centre $O$ (Fig. 1) [12].

The uncertain geometry of the masonry arch has been generated by assuming the following hypotheses: *i*) radial joints, *ii*) deterministic value of the angle of embrace $\alpha$ of the whole arch and *iii*) uniform probability density functions for the random geometrical parameters (independent functions). The parameters that define the nominal geometry of the arch have been related to each voussoir: $\alpha_i$, $t_i$ and $R_i$ denote respectively the angle of embrace of the generic $i$th

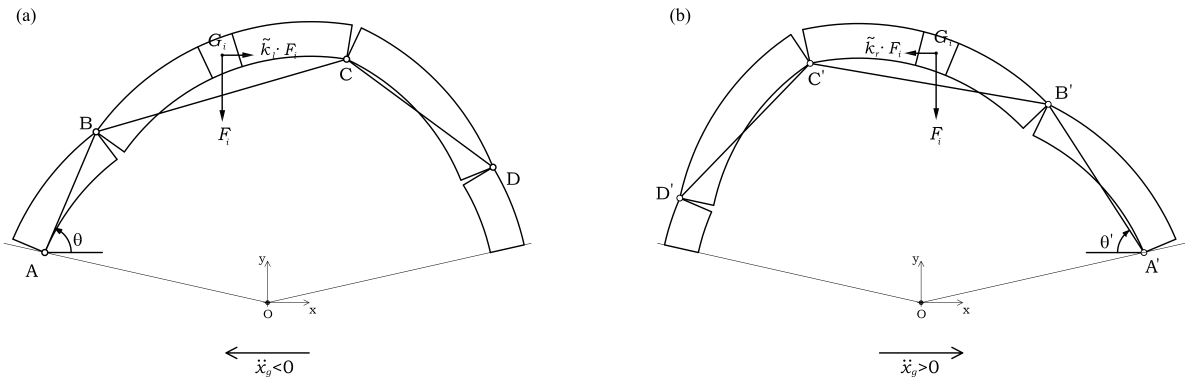


Fig. 2. Mechanism $M_l$ (a) and $M_r$ (b) with the corresponding forces acting on the $i$th voussoir.

voussoir, the thickness and the radius of the mean circular construction line of the same voussoir. The uncertainties related to the shape of the voussoirs have been modelled by considering these parameters as random variables with uniform probability density functions. Further details on the definition of the uncertain geometry can be found in [12].

In this paper, the arch geometry analysed by Oppenheim [4] has been considered, with a radius $R = 10$ m, an angle of embrace $\alpha = 157.5°$, a thickness $t = 1.5$ m and a number of voussoirs $n = 7$. This deterministic geometry will be hereinafter denoted as the "Oppenheim's arch", while in presence of geometrical uncertainties it will be called "random Oppenheim's arch".

## 3. Loading system and activation conditions of the mechanism

The arch has been subjected to a horizontal base acceleration whose time history has been defined by assigning the magnitude $\ddot{x}_g$ depending on time. The evaluation of the mechanism activation conditions under horizontal loads has been carried out through the limit analysis, adopting the well known Heyman's hypotheses [13,14]. In order to evaluate the horizontal loads multiplier $k$, the satisfaction of the equilibrium condition, the yield criterion and the mechanism condition has been imposed at the collapse. A thrust line in equilibrium with the acting loads, lying inside the boundaries of the arch and corresponding to a four-hinge mechanism has been found [15].

The activation of the kinematic chain corresponds to a value $\ddot{x}_g/g$ greater than or equal to the horizontal loads multiplier $k$, being $g$ the gravity acceleration. For a nominal geometry, a unique load multiplier $k$ can be calculated, corresponding to a certain collapse mechanism $M$. When an uncertain geometry is considered, two values of the horizontal loads multiplier can be determined, denoted by $\tilde{k}_l$ and $\tilde{k}_r$, because of the non-symmetry of the structure respect to the vertical axis passing through the crown. These two values of the horizontal loads multiplier correspond to different mechanisms, which can be denoted by $M_l$ (Fig. 2(a)) and $M_r$ (Fig. 2(b)) respectively. In this case, the activation of the motion could appear in correspondence of $\tilde{k}_l$ or $\tilde{k}_r$, depending on the relative direction of the ground motion [16]:

$$\begin{cases} \ddot{x}_g(t_0)/g <= -|\tilde{k}_l| & \text{motion starts with mechanism } M_l \\ \ddot{x}_g(t_0)/g >= |\tilde{k}_r| & \text{motion starts with mechanism } M_r \end{cases} \tag{1}$$

where the acceleration $\ddot{x}_g$ has a positive sign if directed from left to right and $t_0$ the instant of the beginning of motion.

In this paper, the results of the dynamic analysis related to the Oppenheim's arch, whose hinges and thrust line at collapse have been represented in Fig. 3, have been compared to those obtained by considering two random Oppenheim's arches, denoted as $RA - 1$ and $RA - 2$. The horizontal loads multiplier for the Oppenheim's arch was found to be equal to $k = 0.3700$. For the random arch $RA - 1$ the load multipliers are equal to $\tilde{k}_l = 0.4111$ and $\tilde{k}_r = 0.3104$, while for the random arch $RA - 2$ it results $\tilde{k}_l = 0.3946$ and $\tilde{k}_r = 0.3501$.

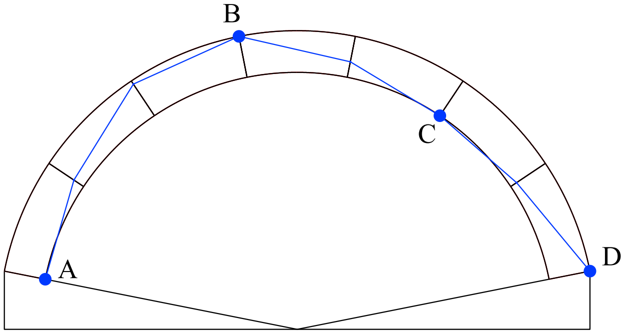


Fig. 3. Collapse hinges and thrust line of the Oppenheim's arch.

## 4. Analytical model for the dynamic analysis

The behaviour of the arch in the dynamic field has been studied by modelling the structure as a four-link mechanism, following the approach proposed by Oppenheim [4] and with the impact modelling by De Lorenzis et al. [7]. An equivalent SDOF system made of three rigid blocks hinged at their ends has been considered. Let us assume that mechanism $M_l$ has been activated first, with hinges in position $A$, $B$, $C$, $D$ and with the generic displaced configuration defined by the Lagrangian coordinate $\theta$ (Fig. 2(a)). The equation of motion can be determined starting from Lagrange's Equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q \tag{2}$$

where $T$ is the kinetic energy of the system, $V$ is the potential energy and $Q$ the generalized forcing function related to the non-conservative forces. The equation of motion corresponding to a kinematics governed by mechanism $M_l$ can be written in the following form

$$M_l(\theta)\,\ddot{\theta} \;+\; L_l(\theta)\,\dot{\theta}^2 \;+ F_l(\theta)\,g \;=\; P_l(\theta)\,\ddot{x}_g \tag{3}$$

The arch will move following this mechanism $M_l$ until an impact occur. At the instant of the impact, mechanism $M_r$ will be activated, with hinges in position $A'$, $B'$, $C'$, $D'$ and a Lagrangian coordinate $\theta'$ (Fig. 2(b)). Because of the geometrical uncertainties, after the impact the hinges will not form in a symmetric configuration. Hence, when the kinematics is determined by mechanism $M_r$ the equation of motion becomes:

$$M_r(\theta')\,\ddot{\theta}' \;+\; L_r(\theta')\,\dot{\theta}'^2 \;+ F_r(\theta')\,g \;=\; -P_r(\theta')\,\ddot{x}_g \tag{4}$$

In this paper, the effect of the energy dissipation associated with the impact has been considered, following the procedure proposed by De Lorenzis et al. [7]. The motion before impact, described by the function $\theta(t)$, can be determined by solving equation (3) with the corresponding initial conditions in terms of $\theta_0$ and $\dot{\theta}_0$, while the motion after impact is known once $\theta'(t)$ has been determined by solving equation (4) with the initial conditions after impact (expressed by $\theta'_0$ and $\dot{\theta}'_0$). Hence, in order to solve the impact problem, the rotational velocity after the impact $\dot{\theta}'_f$ must be determined. The problem can be solved by imposing the equilibrium of linear and angular momentum. A system of five equations of equilibrium results, in which the presence of the impulsive forces, applied at the opposite edge of each hinge, is considered. The evaluation of $\dot{\theta}'_f$ permits to determine the value of the coefficient of restitution $c_v = \dot{\theta}'_f/\dot{\theta}_i$, which depends only on the geometry of the arch, being $\dot{\theta}_i$ the rotational velocity immediately before impact [7].

Once the geometrical parameters that define the shape of a nominal arch have been fixed (the angle of embrace $\alpha$, the radius $R$ of the mean circular construction line and the thickness $t$), the coefficient of restitution $c_v$ can be evaluated. Then, as a first approximation, this value related to a deterministic condition has been adopted also for the uncertain geometries obtained starting from that nominal arch.

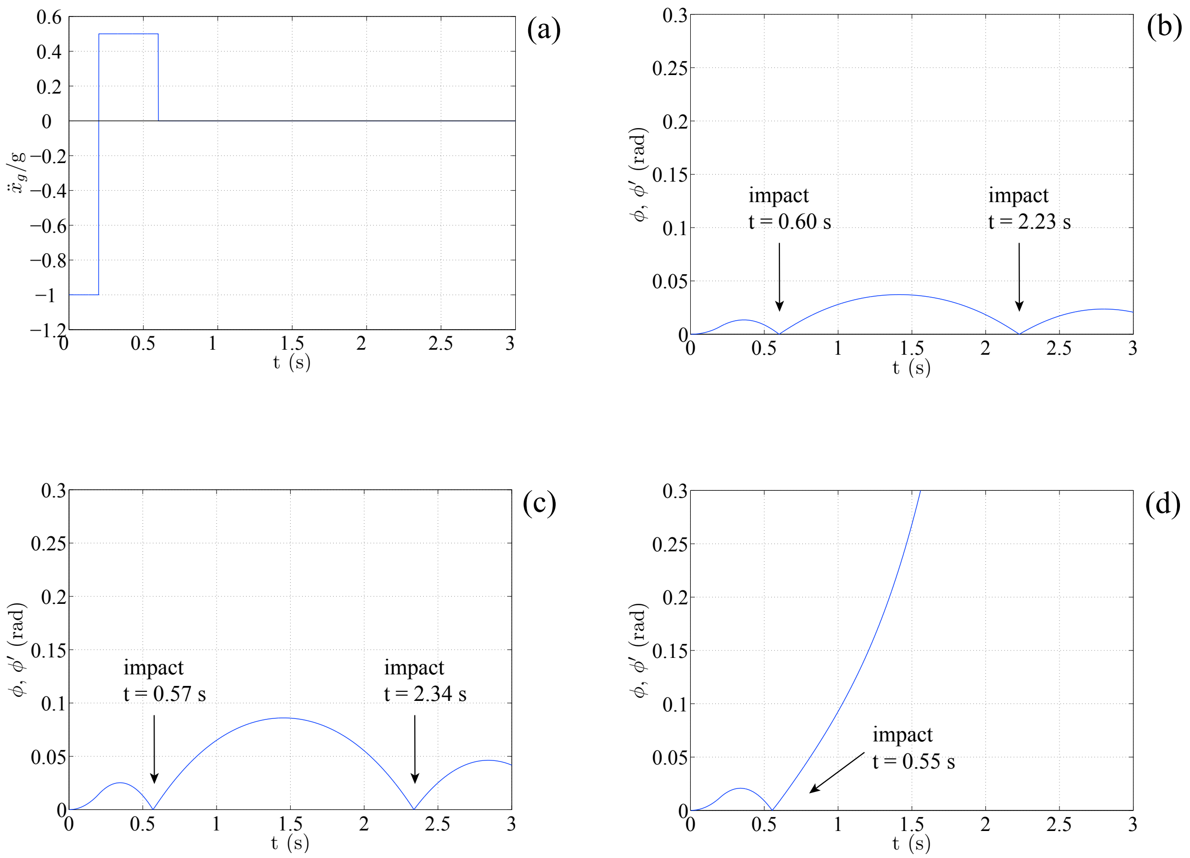


Fig. 4. Time history of the ground acceleration with $t_p = 0.20$ seconds and $\ddot{x}_g = -1\,g$ (a) and dynamic response in terms of $\phi$ ($\phi'$) for the Oppenheim's arch (b), the random arch $RA-1$ (c) and the random arch $RA-2$ (d).

## 5. Results

### 5.1. Time history and initial conditions

The idealized forcing function adopted by Oppenheim [4] has been considered. The arches have been subjected to a pulse with constant ground acceleration $\ddot{x}_g$ and duration $t_p$, followed by a pulse in the opposite direction having the half magnitude and twice the duration. The dissipation of energy due to the impact has been considered and the value of the coefficient of restitution $c_v$ has been calculated by referring to the Oppenheim's arch and it was found to be equal to 0.8754. The arch motion has been determined depending on time, by solving alternatively equations (3) and (4), with the corresponding initial conditions related to mechanism $M_l$ ($\theta_0 = \theta_u$ and $\dot{\theta}_0 = 0$ at time $t_0$, being $\theta_u$ the Lagrangian coordinate of mechanism $M_l$ in the undisplaced condition) or $M_r$ ($\theta'_0 = \theta'_u$ and $\dot{\theta}'_0 = 0$ at time $t_0$, being $\theta'_u$ the Lagrangian coordinate of mechanism $M_r$ in the undisplaced condition).

### 5.2. Dynamic response

In order to highlight the effects of the geometrical uncertainties on the dynamic response, the results related to the Oppenheim's arch have been compared to those of the random arches, when subjected to the impulse base motions adopted by De Lorenzis et al. [7]. A value of the ground acceleration $\ddot{x}_g$ equal to $-1\,g$ has been considered with a duration $t_p$ equal to 0.20 seconds. The results are shown in Fig. 4. The acceleration time history has been represented in Fig. 4(a); literature results by De Lorenzis et al. [7] regarding Oppenheim's arch have been found and they have been shown in Fig. 4(b). In Fig. 4(c)-(d) the dynamic response in terms of $\phi = \theta - \theta_u$ ($\phi' = \theta' - \theta'_u$) has been shown for the random geometries, being $\theta_u$ ($\theta'_u$) the Lagrangian coordinate in the undisplaced configuration. It should be noted that while Oppenheim's arch and the random arch $RA - 1$ recover after subsequent impacts (although it has not been shown in Fig. 4, since the response time history has been represented only until $t = 3$ seconds), the random arch $RA - 2$ fails during the second half cycle of motion after an impact [16]. The uncertainties modified significantly the dynamic response of the structure.

## 6. Conclusions

In this paper, the dynamic behaviour of the masonry arch has been investigated taking into account the geometrical uncertainties related to defects of shape of the voussoirs, imprecisions of construction or deterioration due to environmental factors. The Oppenheim's analytical model has been adopted, based on an equivalent SDOF system made of three rigid blocks, whose equation of motion has been obtained from the Lagrange's Equations. The dissipation of energy due to the impact between the rigid blocks has been modelled following the approach proposed by De Lorenzis et al., estimating the resulting reduction of the rotational velocity. The innovative contribution compared to literature results is represented by the modelling of the geometrical uncertainties in the dynamic analysis procedure. The voussoir random geometry has been generated by considering the thickness, the radius of the mean circular construction line of the arch and the angle of embrace of each voussoir as random variables with independent uniform probability density functions. The dynamic response of the arch to an idealized forcing function, which reproduces in a simplified way the ground acceleration that may occur during an earthquake, has been analysed. The dynamic response of a nominal arch geometry has been compared to those of two random arches. The results showed that geometrical uncertainties may modify the dynamic behaviour of the arch. If the deterministic calculation forecasts the recovery after subsequent impacts, failure may occur, however, if the geometrical uncertainties are considered. Hence, in the context of an adequate seismic assessment of masonry arches, the results highlighted the necessity to model the irregularities of the geometry.

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