A visual sense of number

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Evidence exists for a non-verbal capacity to apprehend number, in humans[1] (including infants[2, 3]) and in other primates[4-6]. Here we show that perceived numerosity is susceptible to adaptation, along with primary visual properties of a scene like colour, contrast, size and speed. Apparent numerosity was decreased by adapting to large numbers of dots and increased by adapting to small numbers, the effect depended entirely on the numerosity of the adapter, not on contrast, size, orientation or pixel density, and occurred with very low adapter contrasts. We suggest that numerosity is also an independent primary visual property, not reducible to others like spatial frequency or density of texture[7].

Jevons[8], a 19th Century economist, rather than counting beans, assessed his accuracy in estimating the number of beans in a box at a single glance. He made no errors at four or below but became increasingly inaccurate as the number of beans increased beyond four. Subsequent studies have confirmed his findings and the lack of errors below five has led to the concept of *subitizing*[9-12], usually presumed to be a separate process allowing immediate apprehension of the numerosity of collections containing fewer than five objects. The perception of larger numbers is usually assumed to involve other more cognitive processes, like counting.

All primary visual properties are susceptible to *adaptation*, sometimes giving rise to dramatic aftereffects, like the waterfall illusion[13], and changes in colour, size, distance, spatial frequency and orientation. If numerosity were a primary property, like colour or motion, it too should be prone to adaptation. The on-line demonstration shows that it is. After 30 seconds adaptation to the two adapter patches, the two subsequent patches appear to differ considerably in numerosity (while inspection or counting after adaptation wears off shows that they both comprise 30 dots). We quantified adaptation effects by asking subjects whether a *test* stimulus (of variable numerosity), presented to the region that had been adapted, appeared more or less numerous than a *probe* stimulus (of fixed numerosity), presented to a different position a little later. The proportion of trials where the test appeared more numerous than the probe was plotted against test numerosity, and fitted with Gaussian functions whose mean estimates the point of subjective equality (PSE) between test and probe, and standard deviation the threshold for discriminating between the two (the just-noticeable difference: jnd). Fig. 1B shows sample psychometric functions for a 30-element probe, with and without adaptation to a 400-element stimulus. The PSE of the test increases from a veridical 30 with no adaptation to more than 100 after adaptation (the test number increased to compensate for the reduction in apparent numerosity). Note also that that after adaptation the psychometric function is steeper (on logarithmic coordinates), implying a smaller jnd.

We first measured the effect of adapting to a large number (400) of dots as a function of number of dots in the probe (Fig. 1B). The amount of adaptation was fairly constant with probe numerosity down to about 12 dots, then decreased as the probe approached the subitizing range. The precision of the match, given by the jnd or Weber

fraction (jnd) expressed as a fraction of dot number), did not deteriorate during adaptation, average percentage Weber fractions for unadapted and adapted conditions, being on average 28% for unadapted and 26% for the adapted conditions, similar to published Weber fractions for numerosity[14, 15].

We next investigated whether adapting to small numbers can cause an increase in apparent numerosity. The red circles of Fig. 2 show that adaptation occurred in both directions: adapting to small numbers increased apparent numerosity (so the matched number decreased), and adapting to large numbers decreased apparent numerosity. Adapting to 50 dots (the number of the probe) had no effect, with the amount of adaptation increasing with the difference between adapt and probe number. The curves of both subjects were well fit by linear regression on log coordinates, with a slope around 0.25.

In order to test whether adaptation depends on numerosity *per se*, or is derived from other factors, like texture density[7] we performed a number of controls. We firstly varied the size of the adapter and test dots, in order to vary pixel density. In the above-described study (red circles of Fig. 2), both adapter and test dots were circles of 6 pixel (20 arcmin) diameter (28 pixels area). We repeated the experiment with square adapter stimuli of 8 X 8 pixels (64 pixels) and test stimuli of 3X3 pixels (9 pixels, 1/7 as many as the adapter). If pixel density were the relevant attribute, the curves of Fig. 2 should shift leftwards by a factor of 7, so the null point occurs when adapter and test pixel density are matched (for adaptation dot number of 7). This clearly does not occur. For naïve observer PB the curves remain superimposed, for DB there is a slight shift in the opposite direction.

We also examined the effect of adapter contrast. As Fig. 2C shows, contrast of adapter dots had little effect on the magnitude of adaptation. At contrasts as low as 12%, the adaptation effect is still nearly two-fold, dropping only near detection threshold. It appears that the only factor that affects adaptation is numerosity, not density, orientation, or contrast.

As a direct control for the effects of texture we next adapted to vertical elements and tested either vertical or horizontal elements. As the bar graphs of Fig. 3A show, there

was little difference in the magnitude of the effects. If texture or spatial frequency were being adapted, one would expect some specificity for orientation. We also performed discriminations (without adaptation) for patterns of completely different pixel density, orientation, Fourier transform etc. An example is shown in the psychometric functions of Fig 3B, where the test or probe could be either small 5 X 5 pixel (16.5 X 16.5 arcmin) squares or 20 X 5 pixel rectangles, randomly vertical or horizontal. Neither the PSE nor the width of the curves depended on the type of stimuli being compared, even though the stimuli were visually completely different, varied by a factor of 4 in pixel density and Michelson contrast, and had completely different Fourier power spectra. All these results agree with a recent study[16] showing that apparent numerosity of a field of dots can be reduced by adding links between some dots: the linked pairs contribute to the numerosity as single entities, rather as two separate dots.

We propose that just as we have a direct visual sense of the *reddishness* of half a dozen ripe cherries so we do of their *sixishness*. In other words there are distinct qualia[17] for numerosity, as there are for colour, brightness and contrast. One distinctive feature of the numerosity sense is that the Weber fraction (jnd expressed as a fraction of dot number) is considerably higher for numerosity (around 25%) than, for example, luminance (near 0.2%), possibly because of high prior uncertainty about the stimulus and the informational limitations of the visual system[18]. The high Weber value accounts for subitizing, without having to postulate a separate mechanism, as for numbers below 4 the quantal leap to the next number is at least 25%, more than the Weber fraction (supporting several recent studies that fail to find evidence for separate mechanisms for the subitizing and counting range of numbers[12]).

One of the more fascinating aspects of this study – readers can verify it for themselves with the on-line demonstration – is that although the total apparent number of dots is greatly reduced after adaptation, one would be hard pressed to know which dots disappear. This reinforces much recent evidence[18-20] in suggesting that the perceived richness of our perceptual world is very much an illusion. Although we seem to perceive 30 or 50 or 100 individual dots occupying very specific positions, this cannot be the case, as adaptation could not reduce or increase the total number of dots without annihilating or creating them. Rather, it would seem that what is encoded is a statistical description of

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the scene, where some aspects of the elements (colour, shape, contrast etc), together with a rough (±30%) estimate of their numerosity.

Recent studies have demonstrated the existence of neurons broadly tuned for number in the parietal cortex of macaque monkeys[4, 21, 22]. fMRI studies also point to their existence in a the intraparietal sulcus in humans, both for symbolic[12, 23, 24] and non-symbolic[25] representation of numbers. These neurones are likely candidates for the physiological substrate of the visual sense of number and, like most neurones, they are probably adaptable. Vision has formidable in-built computational powers, correcting for variation in image size with distance, in image shape with tilt and in image spectral composition with changes in illuminant to allow approximately constant perception of object size, shape and colour; it can also segment images, a difficult computational task[26]. It should occasion little surprise that it can provide approximate estimates of number.

Methods

Stimuli Stimuli were generated by a framestore (Cambridge Research Systems VSG Visage) and displayed on the face of a Hitachi Accuvue monitor at 170 Hz framerate, with a resolution of 640 X 480 pixels and luminance of 18 cd.m^{-2}. The 37 X 28 cm screen subtended 35 X 26.5 deg at the viewing distance of 60 cm (each pixel 3.3 arcmin wide). The stimuli were fields of small disks (of 6˚ diameter, unless otherwise stated), randomly positioned within a circle of 10˚ diameter (similar in appearance to those of Fig. 1A). The disks were half bright half dark, of 100% contrast (unless otherwise stated).

Experimental Procedure Subjects fixated a fixation spot at the centre of the screen. The adaptation stimuli were centred 7˚ away from fixation, above left for half the sessions, below right for the others. The *test* stimulus was displayed in the same position as the adapter for 600 ms, then the *probe* for 600 ms, directly below or below the test (all stimuli separated by a pause of 400 ms). Subjects adapted for 30 sec at the beginning of each session, with 7 sec top-up adaptation between trials. On each trial subjects were required to report whether the probe appeared more or less numerous than the test,

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guessing if unsure. After each trial, an adaptive algorithm (QUEST[27]) estimated the PSE which, after addition of a random quantity (drawn from a log Gaussian distribution of standard deviation 0.15 log-units) determined the probe number for the following trial. The technique ensured an approximately equal number of right and left button presses, as well as placing most trials at a numerosity to estimate best PSE and curve slope. The proportion of "greater" trials was plotted against the logarithm of probe numerosity, and fit with a cumulative Gaussian function (see Fig. 1B), whose mean yielded an estimate of PSE and standard deviation an estimate of jnd.

Contrast thresholds (reported in Fig. 2C) were measured by a two-alterative forced choice procedure. Half the dots (above or below a diagonal line radiating from fixation) were removed, and subjects were required to identify in which half the dots were confined. Again the QUEST[27] algorithm homed in near threshold, and threshold was calculated by Gaussian fit (allowing for guessing).

Subjects Four subjects were measured systematically for most conditions, the two authors and two others naïve to the goals of the study (PB and ED). Sample results are shown in the figures.
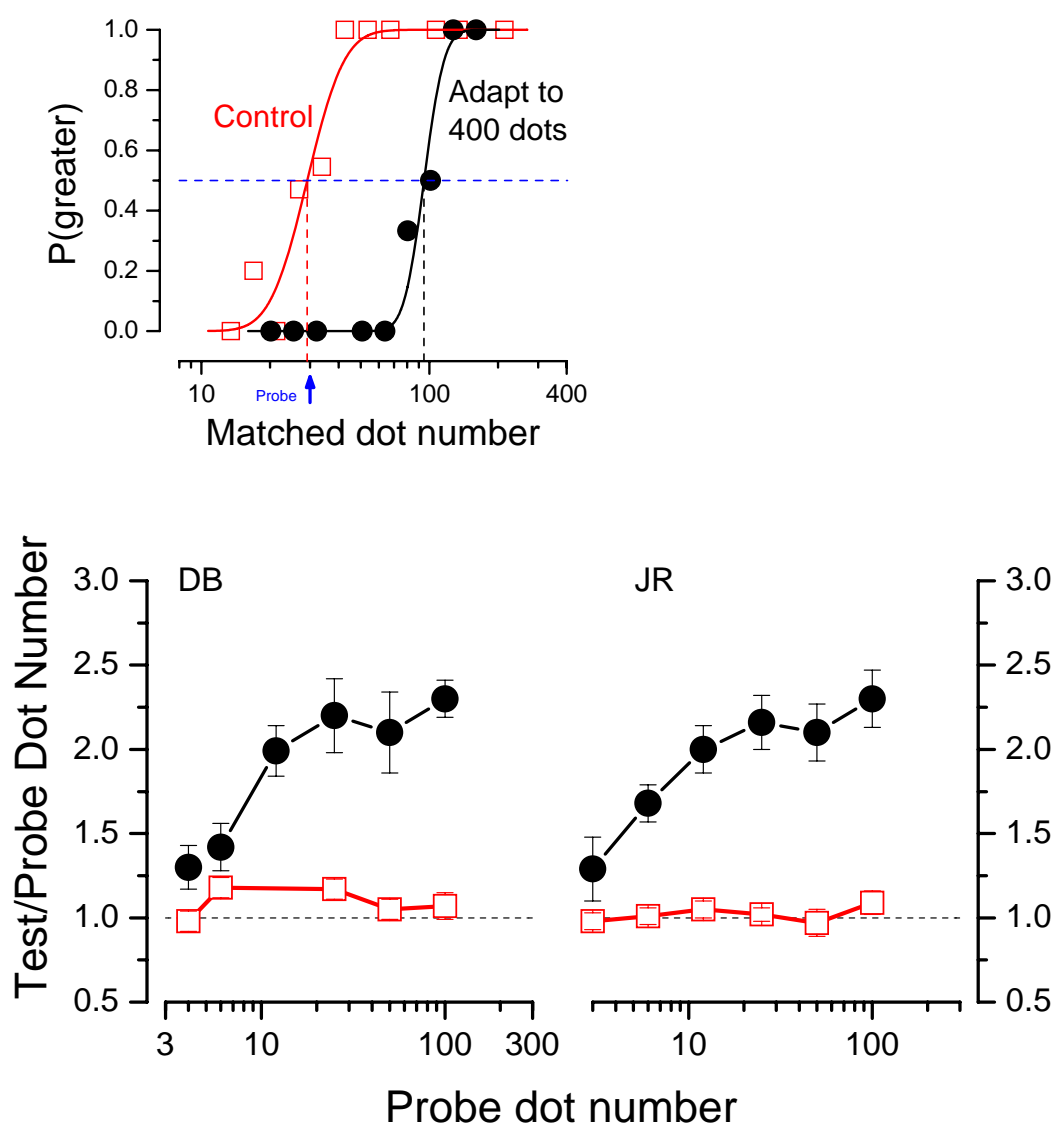
Fig. 1

The effect of adaptation on numerosity. **A** Sample psychometric functions plotting the proportion of trials where the *probe* seemed more numerous, as a function of number of *test* dots. The vertical dashed lines indicate the PSE of the match, about three times higher than the probe number (indicated by the arrow) after adaptation. **B** Magnitude of adaptation (test/probe dot number at PSE) as a function of the number of dots in the probe. For a wide range of numerosities, adaptation caused a doubling of the matched number.
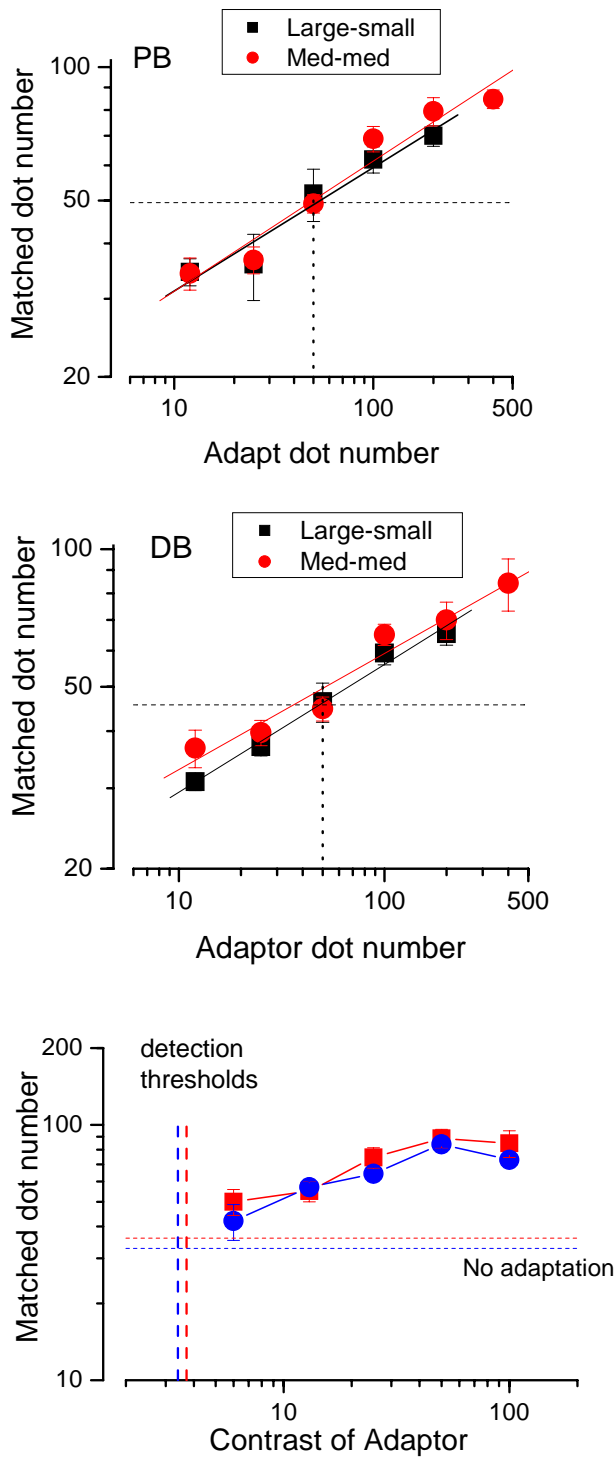
Fig. 2

Effect of numerosity and contrast of the adapter. **A & B** Effect of adapter numerosity and density on apparent numerosity of a 50-dot probe. The red circles refer to adapter and test dots of 6 pixel (20 arcmin) diameter, the squares to adapters of 8 X 8 and tests of 3 X 3 pixels (7 times more adapt than test pixels for matched numerosity). In all cases the adapters were of 50% Michelson contrast, the tests 100%. Adaptation occurs for both high and low adaptation numbers, and is independent of pixel density. **C** Effect of adapter contrast on apparent numerosity of a 30-dot probe (red symbols DB, blue PB). The vertical dashed lines indicate the contrast threshold for detecting the patterns (see methods), the horizontal lines the matches with no adaptation. Adaptation effects were pronounced down to near-threshold contrasts.
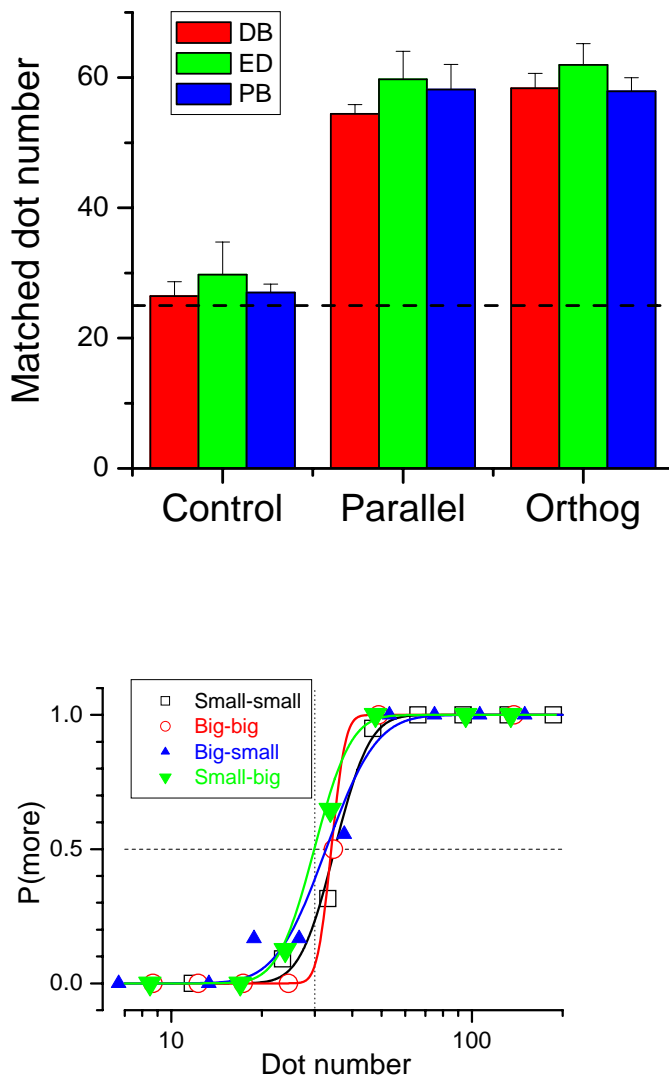
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Fig. 3

Effect of element size and shape. **A** Effect of adapter orientation. Subjects adapted to a field of 200 vertical elements (3 X 10 pixels), and matched a field of either vertical or horizontal same-sized elements to a probe (same orientation as test). The effects of orthogonal and parallel adapters were little different. **B** psychometric curves for matching numerosity of element arrays that were either the same (5 X 5, or 5 X 20 pixels), or small with large, or large with small. Element size and shape has very little effect of either PSE or Weber fraction (given by the function width), suggesting that the matches were based solely on number of elements.

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