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# A MILP algorithm for the optimal sizing of an off-grid hybrid renewable energy system in South Tyrol

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## Abstract

The exploitation of renewable energy sources through sustainable energy technologies are taking the field to decrease the pollutions' emissions into the Earth's environment. To offset the limitations of such resources, hybrid energy systems are becoming fundamental in grid-connected applications as well as in off-grid ones. However, the unsteady behavior of renewable sources, such as Sun and Wind, complicates the prediction of the energy production's trend. The main factors and components involved in the design of hybrid energy systems are: (i) type of generators, (ii) their optimal number, (iii) storage systems and (iv) optimal management strategies. All of them have to be considered simultaneously to develop the optimal solution aimed at either reducing the dependence from fossil fuels or granting the supply of energy. In this paper, a methodology based on the Mixed Integer Linear Programming (MILP) is presented and adopted to meet the electric demand of a mountain lodge located in a remote area in South-Tyrol (Italy). The methodology has been developed implementing an algorithm through the Matlab ©software. The algorithm is capable of evaluating the optimal size of a hybrid off-grid Solar–Wind system with battery storage in order to replace an Internal Combustion Engine (ICE) fueled by diesel.

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*Keywords:* Hybrid off-grid energy system; Mixed integer linear programming; Matlab®; Optimization algorithm; Renewable energy

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## 1. Introduction

The continuous development of renewable energy technologies plays more and more a key role to avoid the consequences of the climate change and meet the goals that were set at the Paris agreement in 2015 [1]. Large renewable energy systems are generally connected to the power grid: however, there are some cases where the National power grid is not present. The best solution for the electrification of these rural areas regards the off-grid systems based on renewable energy sources [2]. During the design phase of these systems, the main challenges are

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related to the nature of renewable sources, which depends on both geographical and climatic parameters, thus affecting the sizing of hybrid systems' components. The nature of renewable sources pushes both researchers and engineers to consider various factors and elements involved in the design phase of a hybrid system. In such a context, the sizing of a hybrid system based on renewable technologies results to be an optimization problem in which an objective function must be optimized. Various optimization methods have been used so far, demonstrating the complexity of this investigation field. Conventional optimization techniques can be divided into mathematical optimization methods and metaheuristic methods [3]. Among the first category, Combinatorial Optimization (CO), Dynamic Optimization (DO), numerical techniques, Linear Programming (LP), Mixed Integer Linear Programming (MILP) and Dynamic Programming (DP) are employed. Among meta-heuristic methods, the most used are Particle Swarm Optimization (PSO) methods and Genetic Algorithms (GA) [4]. Some authors rely on commercial optimization software, while others implement their own methodologies through coding. One of the most used commercial simulation tool is HOMER<sup>®</sup> Energy, which allows to simulate and optimize different combinations of hybrid systems such as in Sen and Bhattacharyya [5]. Optimization methodologies that do not rely on commercial software can be based on different algorithms. The ones based on heuristic methods such as PSO and GA are used when nonlinear systems have to be considered, or when the problem deals with a multi objective optimization [6,7]. However, the solution provided by heuristic methods is not always the optimal one. When the models that have to be optimized comply with the class of linear systems, linear programming (LP) as in Kusakana et al. [8] or mixed integer linear programming algorithms (MILP) as in Lamedica et al. [9] are generally the best methods to find the best possible solution for an objective function. In this paper a model based on a MILP algorithm implemented by the authors of this paper using the commercial software Matlab<sup>®</sup> has been used to compute the optimal sizing of the components of a PV-Wind- Lead–Acid Battery system. To the authors' knowledge, works about this topic are generally focused on the optimization related to off-grid systems in developing countries, where load requirements are generally estimated. In this case, the novelty of the work is constituted by a measurement campaign that could provide information about the real energy consumption and lead to a more precise optimization of the system. Moreover, literature lacks of studies about off- grid solution applied to this type of loads in alpine regions, which is an area where off-grid loads are mostly fed by fossil fuels. The paper is organized in three main sections. In Section 2, the case study is presented along with a description of the components related to the model and the MILP problem. In Section 3, the results of the analysis are summarized and discussed, while Section 4 reports the conclusions of the work.

## 2. Materials and methods

### 2.1. Case study and user load

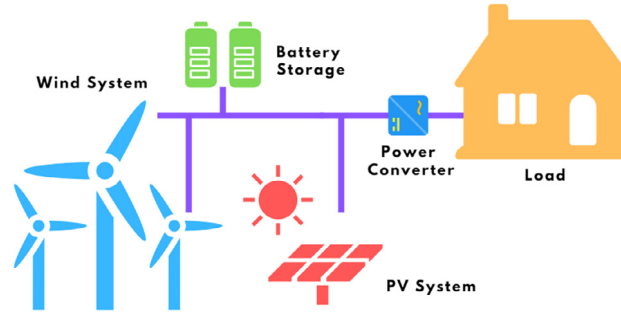
South-Tyrol is a mountain region located in the North of Italy. Due to the morphological conformation of this region, it is very common to find places at high altitudes that are not reached by the National power grid. Typical loads that can be found in these areas are mountain huts or ski resorts, which are generally electrified through diesel generators. The substitution of this type of power system with new technologies that exploit renewable sources would constitute an important improvement in terms of CO<sub>2</sub> and pollutants' emissions in places that are often located in protected environments. The aim of this case study is to provide an optimized solution for a hybrid system constituted by a PV-Wind generator with battery storage for a mountain hut located at 2200 m a.s.l. The data, used to run the simulation, were taken from a measurement campaign related to the average hourly power absorbed by the load during the month of June 2018 and the wind speed, which was measured each minute of the same month. Table 1 shows the average daily load requirement of the hut. The data are focused in this period of the year as the hut is typically open only during the summer period due to climatic reasons. Data of the Sun radiation in June 2018 were downloaded from the database of the Photovoltaic Geographical Information System [10]. It is worth to notice that the values of the power load absorption, the wind speed and the Sun radiation are hourly averaged; thus, the discretization of the considered time period is composed by 24 values of one hour each, such as in Malheiro et al. [11].

### 2.2. Scheme of the HRES

Photovoltaic generators, wind turbines and lead–acid battery storage will constitute the renewable energy system. The scheme of the Hybrid Renewable Energy System (HRES) is shown in Fig. 1. The models of the PV, wind and battery systems are described in Sections 2.3, 2.4 and 2.5.

**Table 1.** Average daily load requirement.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load [kW]	2.3	2.3	2.2	2.3	2.7	5.9	4.9	5.5	7.6	7.5	7.5	7.5
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Load [kW]	7	6.3	6.4	6.1	6	5.5	4.1	3.8	5	2.8	2.5	2.4



**Fig. 1.** HRES scheme.

### 2.3. Photovoltaic (PV) system model

The photovoltaic (PV) system was model according with [12] and considering a Sharp poly-crystalline module characterized by a net cell opening area ( $A_{eff}$ ) of  $1.47 \text{ m}^2$ , an efficiency in Standard Test Conditions (STC) equal to 14.6%, a peak power of 240 W and an efficiency temperature coefficient of  $0.44\%/^{\circ}\text{C}$ . The Direct Current (DC) power, which is delivered by the PV system, was computed through Eq. (1) where  $G$  is the solar radiation and  $\eta_c$  is the cell efficiency that, for a simplified model, is assumed equal to the one in STC. In order to calculate the effective AC power produced by a PV system, the so-called Balance of System (BOS) losses have to be considered since they usually reduce the DC power by 15% [12]. These BOS losses can be taken into account in the computation of the AC power delivered by the PV panels, as described by Eq. (2), through the BOS efficiency  $\eta_{BOS}$  that was assumed equal to 85%.

$$P_{PV}(DC) = \eta_c A_{eff} G \tag{1}$$

$$P_{PV}(AC) = P_{PV}(DC) \eta_{BOS} \tag{2}$$

### 2.4. Wind system model

The wind generator was modeled considering the power curve of a typical turbine having 10 kW as shown in Table 2 and as reported in Maleki and Askarzadeh [13]. A wind turbine of 10 kW was chosen in order to satisfy the power's peak of the load and to respect environmental constraints that fix a limit to the hub's height. Eq. (3) shows the operating principle of the wind generator, where  $w_t$  is the wind speed at the hub of the turbine. The wind turbine does not generate power if either  $w_t$  does not reach a minimum value, i.e.  $w_{cut-in}$ , or if it is higher than the maximum allowed value  $w_{cut-out}$ . The wind turbine generates an output power  $P_t$  proportional to  $w_t$  if  $w_t$  varies from  $w_{cut-in}$  to a rated value  $w_R$ . If  $w_t$  varies from  $w_R$  to  $w_{cut-out}$ , the wind turbine generates its rated power  $P_R$ .

$$P_{WT} = \begin{cases} 0, & \text{if } w_t < w_{cut-in} \text{ or } w_t > w_{cut-out} \\ P_i, & \text{if } w_{cut-in} \leq w_t < w_R \\ P_R, & \text{if } w_R \leq w_t \leq w_{cut-out} \end{cases} \tag{3}$$

**Table 2.** Power curve of the wind turbine.

Wind [m/s]	1	2	3	4	5	6	7	8	9	10	11	12
Power [kW]	0	0	0.15	0.37	0.78	1.38	2.25	3.48	5.12	7.2	9.25	0

### 2.5. Battery storage model

The battery storage was modeled according to Eq. (4), which shows the energy that would be delivered by the battery to the load and vice-versa ( $E_{batt}$ ). During each discretized value of the time interval  $i$ , the battery bank feeds the load if the sum of the energy produced by the photovoltaic generator ( $E_{PV} \cdot N_{PV}$ ) and the wind turbines ( $E_{WT} \cdot N_{WT}$ ) is lower than the energy required by the load ( $E_{Load}$ ). On the other hand, if this overall energy is higher than the one required by the load, the excess of the produced energy charges the batteries. Another parameter, which has to be considered when modeling the energy that is delivered or absorbed by the batteries, is the State of Charge of the battery bank ( $SOC_{Batt}$ ). For each time interval  $i$ , the SOC indicates the amount of energy stored in the batteries, thus the amount of energy that can be either delivered or absorbed. Eq. (5) describes the operating principle of the charging and discharging phases of the batteries.

$$E_{batt}(i) = E_{Load}(i) - [E_{PV}(i) \cdot N_{PV} + E_{WT}(i) \cdot N_{WT}] \quad (4)$$

$$SOC_{Batt}(i) = SOC_{Batt}(i-1) + [E_{PV}(i) \cdot N_{PV} + E_{WT}(i) \cdot N_{WT} - E_{Load}(i)] \quad (5)$$

### 2.6. MILP model

The developed algorithm based on a MILP model receives as inputs the load requirements, the power delivered by the generators and the power delivered or absorbed by the batteries, providing as output the optimal value of the objective function, which is the total cost of the hybrid system. Generally, a MILP model consists in the definition of an objective function, a set of constraints that have to be expressed as linear equalities or inequalities and a set of decision variable. In the case presented in this work, it is possible to list: (i) the integer number of PV panels to be installed ( $N_{PV}$ ), (ii) the integer number of wind turbines to be installed ( $N_{WT}$ ), (iii) the integer number of the batteries to be installed ( $N_{Batt}$ ), (iv) the amount of energy delivered or absorbed by the batteries at each time interval  $i$  as decision variables. Both objective function and constraints relate the input data of the problem with the set of decision variables. The objective function, expressed by Eq. (6), was chosen considering the overall costs ( $C_{TOT}$ ) of the hybrid system's components, which are the sum of the capital ( $C_{cap}$ ) and Operation and Maintenance (O&M) costs ( $C_{O\&M}$ ) as reported in Table 3.

$$\min [C_{TOT}(PV) \cdot N_{PV} + C_{TOT}(WT) \cdot N_{WT} + C_{TOT}(Batt) \cdot N_{Batt}] \quad (6)$$

**Table 3.** Capital and O&M costs of the components of the system.

	PV	Wind	Batteries
$C_{cap}$	1400 [€/kWp]	2000 [€/kW]	1223 [€/kWh]
$C_{O\&M}$	29.6 [€/kWp/year]	34.7 [€/kW/year]	308 [€/kWh/year]

The algorithm computes the values of the decision variables that minimize the objective function. However, the decision variables are subjected to technical constraints, which resemble the limitations on designing real HRES. In this case, the constraints are related to the energy that, for each discretized time interval  $i$ , has to be delivered to the load, expressed by Eqs. (7) and (8), and the energy that can be absorbed or delivered by the battery, expressed by Eqs. (9) and (10). Eq. (7) establishes that the system always produces at least the amount of energy that satisfies the half of the load requirements, letting the batteries to behave as generators when the amount of energy produced by PV panels and wind turbines is not enough to satisfy them. On the other hand, batteries behave as an additional load when there is an excess of PV and wind energy production. Eq. (8), establishes that the total energy produced by PV panels and wind turbines has to be equal to the total energy required by the load in the considered time period. The aim of these constraints is to design a system considering the fluctuating nature of renewable sources without oversizing the battery bank. Eq. (9) limits the amount of energy that, for each discretized time interval  $i$ , can be delivered by the batteries ( $E_{Batt-out}$ ), keeping in mind that it cannot decrease under the 20% of the battery's capacity ( $C_{Batt}$ ) and exceed the amount of energy that is present in the battery bank at the previous time interval  $i-1$ , i.e. the SOC of the battery. Eq. (10) limits the energy quantity that can be absorbed at each discretized time interval by the batteries: in particular, it establishes that the energy quantity cannot exceed the total capacity of the battery bank.

$$E_{Batt}(i) \cdot N_{Batt} + E_{PV}(i) \cdot N_{PV} + E_{WT}(i) \cdot N_{WT} \geq 0.5 \cdot E_{Load}(i) \quad (7)$$

$$\sum E_{PV}(i) \cdot N_{PV} + \sum E_{WT}(i) \cdot N_{WT} = \sum E_{Load}(i) \quad (8)$$

$$0.2 \cdot C_{Batt} \cdot N_{Batt} \leq E_{Batt-out}(i) \leq SOC(i-1) \quad (9)$$

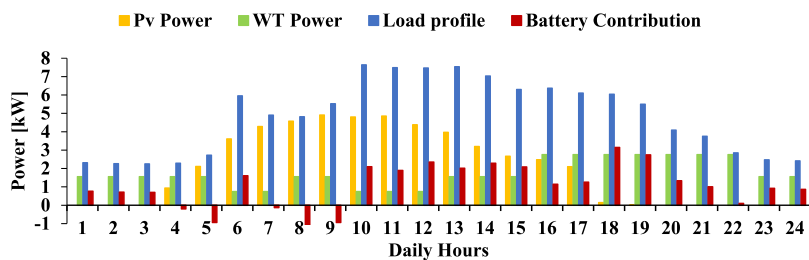
$$E_{Batt-in}(i) \leq C_{Batt} \cdot N_{Batt} - SOC(i) \quad (10)$$

### 3. Results and discussion

The simulation was run considering the data provided in the month of June 2018. However, the code could be also applied when dealing with other months or other dataset related to different extension. In this paper, a simulation on the average daily load demand was chosen to better depict the obtained results, such as the behavior of the generators and the charging and discharging phases of the battery, as shown in Fig. 2. Hourly average values of load, sun radiation and wind speed have been computed and used in the simulation. Table 4 lists the results of the simulation applied for the entire month of June. In particular, it depicts the total number of PV panels, wind turbines and the respective total power that minimizes the objective function. Moreover, it shows the total energy of battery, expressed in kWh, that have to be installed and the total cost of the components over the entire lifecycle of the installation. The high cost of the system can be justified by two reasons: firstly, the cost of batteries that are characterized by high investment and O&M costs if compared with costs of the other devices. The electric supply of these users is met using diesel generators, which are programmable power sources and allow to reduce the size of the battery bank and, consequently, the total cost. However, this solution was not considered because the aim of the work is to investigate the feasibility of a fully renewable system for the electrification of remote alpine areas. Moreover, due to the remoteness of the area, O&M costs of all the components are considered to be higher than usual values. The high final cost is justified by the high environmental benefit and the total independence from fossil fuels that comes at the expenses of high initial investments. This also means that the battery technology is still far from being mature and available at a reasonable price. In order to reduce the total cost of the system, further developments of this investigation can be a configuration that includes a backup diesel generator to reduce the size of the battery storage or a pump-hydro storage. The algorithm could be adapted with a multi-objective optimization approach to increase the price competitiveness and to maintain the environmental sustainability of this solution by optimizing both the total cost and CO<sub>2</sub> emissions.

**Table 4.** Simulation's output for designing the hybrid system.

	PV	Wind	Batteries
Units	86	1	
Installed capacity	20.64 kWp	10 kW	30 kWh
Cost [€]	44,170	28,669	267,690



**Fig. 2.** Trend of the electric load, power produced by PV panels, wind turbines and power produced or absorbed by the batteries.

### 4. Conclusions

The paper analyzes the optimal design of a Hybrid Renewable Energy System (HRES) composed by photovoltaic panels, wind turbines and batteries that store the excess of the energy production. The system was studied to electrify a mountain lodge placed at an altitude of 2200 m in South-Tyrol (Italy), to avoid the use of traditional power generators. A measurement campaign was realized to determine the average hourly load absorption during the month of June 2018 and the average hourly wind speed. Data regarding the PV system were taken from the online database PVGIS. An objective function based on the minimization of the total costs of the system along with the

decision variables and constraints was defined. Finally, a MILP optimization algorithm was developed using the Matlab<sup>®</sup> software to compute the optimal solution of the problem that consists of finding the correct number of PV panels, wind turbines and battery units that minimizes the objective function.

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