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Fracture toughness of fibre-reinforced concrete determined by means of numerical analysis

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Abstract

As is well-known, the addition of fibres to concrete mix (Fibre Reinforced Concrete, FRC) produces a positive effect on cracking behaviour. In this work, the results of an experimental campaign on FRC specimens with randomly distributed micro-synthetic polypropylene fibrillated fibres are examined. The tests concern single-notched beams under three-point bending, where the fibre content varies. Such an experimental testing is numerically analysed through a non-linear finite element model, named 2D-PARC, where a proper constitutive law for fibre-reinforced concrete is implemented. The load-crack mouth opening displacement (CMOD) curves numerically obtained are employed to determine the critical stress-intensity factor (fracture toughness) for different values of fibre content, according to the two-parameter model. The comparison between such numerical results and those obtained by applying the two-parameter model to the experimental load-CMOD curves is performed.

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Keywords: critical stress-intensity factor; FRC specimens; 2D-PARC; two-parameter model

1. Introduction

Conventional or plain concrete is widely used in civil engineering practical applications due to its: (i) low production cost; (ii) excellent mechanical behaviour under compression; (iii) high form-fitting property to be casted

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in different shapes (Pantazopoulou et al., 2001). The main drawback of concrete is the weakness related to both tensile strength and toughness in presence of cracks.

In order to analyse the concrete fracture behaviour, a fracture mechanics approach different from that used for metals is needed. As a matter of fact, fracture mechanics is non-linear due to the presence of a zone ahead of stress-free crack tip (named fracture process zone, FPZ), where the material shows a non-linear behaviour. The fundamental fracture models available to study the fracture behaviour of plain concrete are: the cohesive crack model (CCM), the crack band model (CBM), the two-parameter model (TPM), the size effect model (SEM), the effective crack model (ECM), and the double-K fracture model (DKFM). More recent proposals are also available in the literature.

An effective way to improve concrete toughness is represented by the dispersion (during mixing) of discontinuous fibres into the concrete mix (Ivanova et al., 2016). As a matter of fact, fibres cross cracks and develop the so-called crack bridging effect on the crack surfaces (Rao et al., 2014).

The application of non-linear fracture models presented for plain concrete has also been extended to FRC. Within this research field, the results of an experimental campaign on FRC specimens with randomly distributed micro-synthetic polypropylene fibrillated fibres (Vantadori et al., 2016) are examined in the present work. The tests concern single-notched beams under three-point bending, where the fibre content varies.

This experimental testing is numerically modelled through non-linear FE analyses by adopting a proper constitutive model for FRC (Cerioni et al., 2008; Bernardi et al., 2013, 2016[a]). The model, proposed in terms of secant stiffness matrix, is formulated by imposing equilibrium and compatibility conditions both in uncracked and cracked stage. All the fundamental resistant contributions offered by both fibres and concrete are properly taken into account and, in this way, the structural material behaviour is simulated up to failure. Then, the load-crack mouth opening displacement (CMOD) curves numerically obtained are employed to determine fracture toughness for different values of fibre content, according to the two-parameter model (Jenq et al., 1985). Finally, the comparison between such numerical results and those obtained by applying the two-parameter model to the experimental load-CMOD curves is performed.

Nomenclature

$\underline{a}$	effective critical crack length
a_0	notch length
C_i	initial compliance
C_u	unloading compliance
$\mathbf{D}$	total stiffness matrix
$\mathbf{D_c}$	concrete stiffness matrix
$\mathbf{D_{c,cr1}}$	cracked stiffness matrix
E	elastic modulus
f	snubbing coefficient
K_{IC}^S	critical stress-intensity factor
P_{peak}	peak load
S	specimen loading span
W	specimen depth
$\alpha_0 = a_0 / W$	initial notch-depth ratio
$\beta = \underline{a} / W$	effective notch-depth ratio
$\boldsymbol{\varepsilon}$	total strain
$\boldsymbol{\varepsilon_c}$	strain related to fibre reinforced concrete (FRC) between cracks
$\boldsymbol{\varepsilon_{cr1}}$	strain related to resistant mechanisms developed in the fracture zone
σ_f	fibre action
τ_0	interfacial bond strength
η_0	orientation efficiency factor

2. Analysis of some experimental results

The experimental campaign (Vantadori et al., 2016) consists of three series of specimens: 4 plain concrete specimens, 4 concrete specimens reinforced by randomly-distributed fibres with a content equal to 0.5% by volume, and 4 specimens with 2.5% by volume. The micro-synthetic polypropylene fibrillated (that is, deformed and/or irregular in shape) fibres are made of 100% pure high-density polypropylene. They are generally used as secondary concrete reinforcement. The fibre aspect ratio is equal to 0.003 (length = 18mm), the tensile strength is equal to 450-600MPa, and the elastic modulus is equal to 3.5kN/mm^2

The specimen matrix is cementitious, characterised by the following proportions: cement: water: aggregates (by weight) = 1: 0.7 : 3.6. The cement is 42.5 CEM II/A-P, and the maximum aggregate size is 4mm. The experimental compressive strength of the mixture is 42MPa at 28 days after casting.

Three-point bend tests on single edge notched beams were performed at the 'Testing Laboratory of Material and Structures' of the University of Parma. The nominal sizes of each FRC specimen are shown in Figure 1.

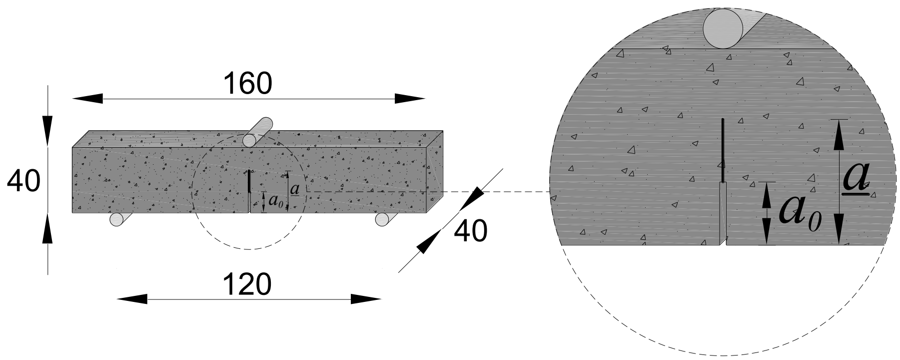

Figure 1. Three-point bend notched specimen (sizes in mm).

An Instron 8862 testing machine under crack mouth opening displacement (CMOD) control was used, employing a clip gauge (average speed = 0.1 mmh^{-1}) and measuring the load through a load cell.

Each specimen was monotonically loaded. After the peak load was achieved, the post-peak stage followed and, when the force was equal to about 95% of the peak load, the specimen was fully unloaded (up to a force value equal to about zero), by proceeding under load control. Then, the specimen was reloaded up to failure. All specimens exhibited a non-linear slow crack growth before the peak load was reached, as is shown in Figure 2.

3. The two-parameter model

The two-parameter model (Jenq et al.,1985) requires the registration of the applied load (P) against the crack mouth opening displacement. The initial compliance, C_i , is employed to determine the elastic modulus, E , whereas the unloading compliance, C_u , is used to estimate the effective crack length, $\underline{a}$ (Figure 1).

Firstly, the elastic modulus is computed through the following expression:

$$E = \frac{6 \cdot S \cdot a_0 \cdot V_1(\alpha)}{C_i \cdot W^2 \cdot B} \tag{1}$$

where S is the specimen loading span, a_0 is the initial notch length, C_i is the linear elastic compliance, W and B are the specimen depth and thickness, respectively, and $V_1(\alpha)$ is given by :

$$V_1(\alpha) = 0.76 - 2.28\alpha + 3.87\alpha^2 - 2.04\alpha^3 + \frac{0.66}{(1-\alpha)^2} \tag{2}$$

being $\alpha = a_0/W$.

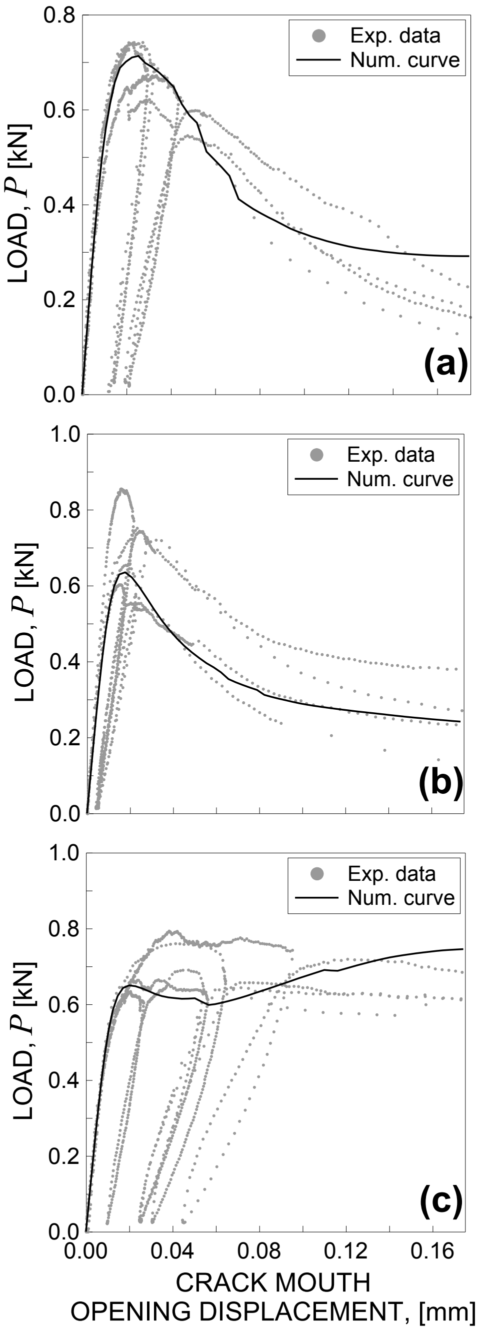

Figure 2. Experimental data and numerical curves in terms of load-CMOD for: (a) plain concrete specimens; (b) FRC specimens with a fibre content equal to 0.5% by volume; (c) FRC specimens with a fibre content equal to 2.5% by volume.

Then, the effective critical crack length, $\underline{a} = a_0 + \Delta a_c$ (being Δa_c the stable crack growth at peak load) can be determined from the following relationship:

$$E = \frac{6S \cdot \underline{a} \cdot V_1(\alpha)}{C_u \cdot W^2 B} \tag{3}$$

where $V_1(\alpha)$ is obtained from Eq. (2) by replacing a_0 with the critical quantity $\underline{a}$, and C_u is the unloading compliance at about 95% of the peak load.

Finally, the critical stress-intensity factor, K_{Ic}^S, is computed by employing the measured peak load, P_{max}, and the effective critical crack length, $\underline{a}$:

$$K_{Ic}^S = \frac{3P_{peak} \cdot S}{2 \cdot W^2 B} \sqrt{\pi \underline{a}} \cdot F(\beta) \tag{4}$$

with:

$$F(\beta) = \frac{1}{\sqrt{\pi}} \cdot \frac{1.99 - \beta(1-\beta)\left(2.15 - 3.93\beta + 2.7\beta^2\right)}{(1+2\beta)(1-\beta)^{3/2}} \tag{5}$$

being $\beta = \underline{a}/W$.

4. Fracture toughness by means of both the 2D-PARC model and the two-parameter model

4.1. A non-linear numerical model: 2D-PARC

The non-linear constitutive model 2D-PARC, able to represent the behaviour of FRC up to failure, is herein adopted. The model is expressed in the form of a secant stiffness matrix, which makes it suitable to be adopted in conjunction with Finite Element (FE) technique. This stiffness matrix, which is defined for each integration point, is able to consider material cracking by following a smeared cracking approach. Its theoretical formulation, deduced for an ordinary reinforced concrete (RC) membrane element subjected to general in-plane stresses, is explained in detail in Cerioni et al. (2008). Extensions of 2D-PARC model to Fibre Reinforced Concrete (FRC) have recently been proposed to include the resistant contribution offered by fibres dispersed in the concrete mix, both before (Bernardi et al. 2016[a]) and after cracking development (Bernardi et al. 2013, 2016[b]). The model has also been extended to RC structures externally retrofitted with fabric reinforced cementitious matrix composites (FRCM) in Bernardi et al. (2016[c]).

In the present work, the attention is focused on the case of fibre-reinforced concrete elements without ordinary steel reinforcement. In the uncracked stage, concrete is modelled following the non-linear elastic formulation originally proposed by Ottosen (1980). The non-linear stress-strain relationships for concrete under a general biaxial state of stress are determined by properly updating the secant values of the elastic modulus and Poisson ratio, inserted into the concrete stiffness matrix $\mathbf{D_c}$. The effect of fibres, which stiffen the material response in compression, is also included, as is described in Bernardi et al. (2016[a]).

When the maximum principal stress exceeds the adopted failure envelope in the tension region, for the considered integration point a stiffness matrix accounting for cracking contributions is computed. Cracking is assumed to develop at right angle with respect to the principal tensile stress direction, and a strain decomposition procedure is adopted. The total strain $\boldsymbol{\varepsilon}$ is subdivided into two components $\boldsymbol{\varepsilon_c}$ and $\boldsymbol{\varepsilon_{cr1}}$, related to fibre reinforced concrete (FRC) between cracks and to all the resistant mechanisms that develop in the fracture zone, respectively. The behaviour of FRC between cracks is described through the matrix $\mathbf{D_c}$, derived for the uncracked stage, with slight modifications (i.e. by applying a proper reduction to the concrete strength envelope) to account for the degradation induced by cracking.

All the resistant mechanisms related to the kinematics developed at crack location (i.e. aggregate bridging and interlock, fibre effects) are included in matrix $\mathbf{D_{c,cr1}}$, as a function of two main local variables, namely crack opening w_1 and sliding v_1, by using effective laws available in the technical literature to individually represent each physical phenomenon.

The procedure adopted for modelling of aggregate bridging and interlock is described in Cerioni et al. (2008). For such a reason, the attention is herein focused on the improvement provided by fibres to the post-cracking behaviour of concrete structures. This improvement is mainly related to the so-called fibre-bridging, which consists in the transmission of additional tensile stresses across crack surfaces. This mechanism is taken into account by modifying the cracked stiffness matrix $\mathbf{D_{c,cr1}}$ through the introduction of an additional term (due to fibre effects across the crack), which is summed up to aggregate bridging contribution.

Fibre action σ_f, which depends on fibre geometric and mechanical characteristics, as well as on their content in the admixture, is evaluated by adopting the micro-mechanical formulation proposed by Li et al. (1993). It is worth noticing that the evaluation of fibre action σ_f requires the knowledge of three main parameters: the snubbing coefficient f, the interfacial bond strength τ_0, and the orientation efficiency factor η_0 (Li et al. 1993; Bernardi et al. 2013). Their values are here selected within the range of variability given in the literature for polypropylene fibres.

Considering both the equilibrium and compatibility conditions for the cracked material, the stress field in the cracked stage can be written as follows by developing some mathematical steps:

$$\boldsymbol{\sigma} = \left(\mathbf{D_c}^{-1} + \mathbf{D_{c,cr1}}^{-1}\right)^{-1} \boldsymbol{\varepsilon} = \mathbf{D}\boldsymbol{\varepsilon} \tag{6}$$

D being the total stiffness matrix.

4.2. Numerical simulation of experimental tests by means of the 2D-PARC model

The 2D-PARC model is applied in order to numerically simulate the fracture behaviour of FRC specimens described in Section 2. To this aim, a FE mesh constituted by triangular 6-node membrane finite elements is adopted. Figure 3 shows the adopted discretization, which refers only to one half of the notched specimen, taking advantage of the symmetry of the problem. As can be seen, the mesh is properly refined around the notch, where a stress concentration is expected. Numerical analyses are performed under displacement control, by imposing an increasing vertical displacement to the central loaded point, in order to achieve a better numerical convergence.

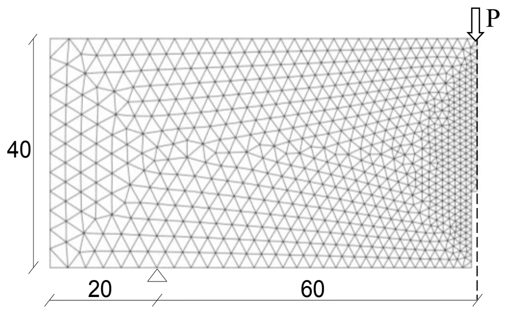

Figure 3. FE mesh of FRC specimens (sizes in mm).

The load-crack mouth opening displacement (CMOD) curves, numerically obtained and shown in Figure 2, highlight that the numerical model employed is able to correctly describe the fracture behaviour of all the analysed specimens, characterised by different values of fibre content.

By employing such numerical curves and applying the two-parameter model (see Section 3), the elastic modulus

(E_{num}), the peak load ($P_{peak,num}$), the effective critical crack length ($\underline{a}_{num}$) and the critical stress-intensity factor, $K^S_{Ic,num}$, for the above three specimen series are computed and listed in Table 1. Note that the numerical unloading compliance C_u is approximately computed as the secant compliance at peak load, that is, according to the alternative procedure of the TPM to be applied when tests are performed employing an actuator which cannot control unloading.

The corresponding averaged values, computed from the experimental load-crack mouth opening displacement (CMOD) curves (Figure 2), are also reported in Table 1.

Table 1. Numerical and experimental results for the three specimen series in terms of: elastic modulus, peak load, effective critical crack length and critical stress-intensity factor.

Series	E_{num} [MPa]	E_{exp} [MPa]	$P_{peak,num}$ [N]	$P_{peak,exp}$ [N]	$\underline{a}_{num}$ [mm]	$\underline{a}_{exp}$ [mm]	$K^S_{IC,num}$ [MPa m$^{1/2}$]	$K^S_{IC,exp}$ [MPa m$^{1/2}$]
P	17073.06	17952.35	713.01	658.00	19.29	18.82	0.673	0.553
R05	16783.24	17647.56	634.83	734.75	17.39	17.82	0.520	0.569
R25	16115.84	16945.80	649.85	755.43	17.76	19.02	0.547	0.720

From Table 1, it can be remarked that the $K^S_{Ic,num}$ values are in a satisfactory agreement with the averaged experimental ones, and the scatter between them may be attributable to the different procedure employed to compute the parameter C_u.

5. Conclusions

In the present work, an experimental campaign performed on concrete specimens reinforced with polypropylene fibres has been examined. The tests concern single-notched beams under three-point bending. Such an experimental testing has been numerically modelled through non-linear finite element analyses, where a proper constitutive law for fibre-reinforced concrete is implemented. The numerical load-crack mouth opening displacement (CMOD) curves have been employed to determine the critical stress-intensity factor, according to the two-parameter model. The comparison between such numerical results and those obtained by employing both the experimental load-CMOD curves and the two-parameter model has shown a quite satisfactory agreement.

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References

Bernardi, P., Cerioni, R., Michelini, E., 2013. Analysis of post-cracking stage in SFRC elements through a non-linear numerical approach. Engineering Fracture Mechanics 108,238–250.

Bernardi, P., Cerioni, R., Michelini, E., Sirico, A., 2016 [a]. Numerical modeling of the cracking behavior of RC and SFRC shear-critical beams. Engineering Fracture Mechanics 167, 151-166.

Bernardi, P., Michelini, E., Minelli, F., Tiberti G., 2016 [b]. Experimental and numerical study on cracking process in RC and R/FRC ties. Material and structures 49, 261-277.

Bernardi, P., Ferretti, D., Leurini, F., Michelini, E., 2016 [c]. A non-linear constitutive relation for the analysis of FRCM elements. Procedia Structural Integrity, 2:2674-2681.

Carpinteri, A., Fortese, G., Ronchei, C., Scorza, D., Vantadori, S., 2017. Mode I fracture toughness of fibre reinforced concrete. Theoretical and Applied Fracture Mechanics, doi: 10.1016/j.tafmec.2017.03.015.

Cerioni, R., Iori, I., Michelini, E., Bernardi, P., 2008. Multi-directional modeling of crack pattern in 2D R/C members. Engineering Fracture Mechanics 75, 615–628.

Ivanova, I., Assih, J., 2016. The effect of fatigue test on short reinforced-concrete corbel strengthened by externally bonded composite fibre fabrics. Engineering Fracture Mechanics 167, 167–175.

Li, V.C., Stang, H., Krenchel, H., 1993. Micromechanics of crack bridging in fibre-reinforced concrete. Materials and Structures 26, 486–494.

Ottosen, N.S., 1980. Nonlinear Finite Element Analysis of Concrete Structures. PhD Thesis, Risø National Laboratory, Denmark.

Pantazopoulou, S. J., Zanganeh, M., 2001. Triaxial tests of fiber-reinforced concrete. Journal of Materials in Civil Engineering 13, 340–348.

Rao, S., Appa Rao, 2014. A Fracture Mechanics of Fiber Reinforced Concrete: An Overview. International Journal of Engineering Innovation & Research 3, 2277-5668A.

Jenq, Y.S., Shah, S.P., 1985. Two parameter fracture model for concrete. J. Engng Mech., ASCE 111, 1227–1241.

Vantadori, S., Carpinteri, A., Fortese, G., Ronchei, C., Scorza, D., 2016. Mode I fracture toughness of fibre-reinforced concrete by means of a modified version of the two-parameter model. Procedia Structural Integrity 2, 2889-2895.