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Fracture Assessment of Blunt V-Notches under Prevalent Mode II Loading by Means of Local Energy

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Abstract

The main purpose of this research is to re-analyse experimental results of fracture loads for blunt V-notched samples under mixed mode (I+II) loading considering different combinations of mode mixity ranging from pure mode I to pure mode II. The specimens are made of polymethyl-metacrylate (PMMA) and tested at room temperature. The suitability of fracture criterion based on the Strain Energy Density (SED) when applied to these data is checked in the paper. Dealing with notched samples, characterized by different notch angles and notch root radii, the SED criterion used in combination with the concept of local mode I, valid in the proximity of the zone of crack nucleation, permits to provide a simple approximate but accurate equation for the SED evaluation in the control volume.

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Keywords: Brittle Fracture; Mixe mode; Strain Energy Density; Control Volume; Equivalent Mode I

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1. Introduction

For many years the Strain Energy Density (SED) has been used to formulate failure criteria for materials exhibiting both ductile and brittle behavior. The point-wise criterion formalised by Sih gave a sound theoretical basis to Gillemot's experimental findings [1]. Sih proposed the SED parameter S, which is the product of the strain energy density and a small distance from the point of singularity [2]. The concept of elementary structural volume was used by Lazzarin [3, 4] and collaborators to formalize a SED approach applied to finite size volumes. The approach was successfully used under static loading conditions to

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Nomenclature

H	Function used in the evaluation of averaged SED for blunt notches under mode I loading
H*	Function used in the evaluation of averaged SED for blunt notches under mixed mode loading
R_c	SED critical radius
$\overline{W}$	Averaged strain energy density
Symbols	
2α	Opening angle of V-notch
β	Loading angle of Brazilian disk specimens
ρ	Notch root radius
σ_{tip}	Maximum principal stress at the notch tip
σ_{max}	Maximum principal stress along the notch edge

assess the strength of notches subjected to predominant mode I. Under mixed mode loading, particularly for notches with a non-negligible radius, to provide a suitable fracture criterion is more complex than under mode I loading because the maximum elastic stress is out of the notch bisector line and its position varies as a function of mode I to mode II stress distributions, along the notch edge. For this reason, the problem of brittle or quasi-brittle fracture of blunt V-notched components loaded under mixed mode (I+II) requires further investigations. Recently the use of SED criterion has been extended to failure from U-notched components and sharp V-notches (characterized by small notch radii and considered as sharp notches) loaded in mixed mode (I+II) for brittle or quasi-brittle materials [5, 6]. Dealing with U-notches the equivalent local mode I concept has been applied by moving the control volume along the notch edge in such a way that it is centred in relation to the maximum elastic stress and a simply but accurate expression has been found to evaluate the SED once the maximum value of the principal stress along the notch edge is known. The proposal of mode I dominance for crack plates was suggested first by Erdogan and Sih (1963) when dealing with cracked plates under plane loading and transverse shear, where the crack grows in the direction almost perpendicular to the maximum tangential stress in radial direction from its tip [7].

By taking advantage of the complete database of experimental data from specimens made of PMMA (polymethyl-metacrylate) and tested at room temperature reported in Ref.[8, 9], the SED criterion is applied here to blunt V-notches under mixed mode loading and prevalent mode II loading. The data present different values of load mixity and V-notch angles. The original work summarizes more than 160 fracture tests from Brazilian disk specimens weakened by blunt V-notches. To show the applicability of the SED criterion for mixed mode fracture, previously developed and applied mainly to U-notched and pointed V-notched samples, the assessment of the fracture loads taken from Ref. [8] has been carried out by means of local SED approach. As previously made for U-notches, the same finite size volume already defined for Mode I loading has been used here by moving it along the notch edge in such a way that it is centred in relation to the maximum elastic stress. With the aim to show that the equivalent local mode I concept can be applied also to blunt V-notches and not only to U-notches, a simple equation for the SED has been provided as a function of the maximum stress along the notch edge directly evaluated from a free mesh.

2. The SED approach applied to static loadings

The SED approach is based both on a precise definition of the control volume and the fact that the critical energy does not depend on the notch sharpness. To apply the SED fracture criterion, two independent parameters are needed: the critical value of the strain energy density, W_c, and the critical length, R_c. For an ideally linear elastic material

$$W_c = \sigma_t^2 / 2E \tag{1}$$

being σ_t the ultimate tensile stress and E the elastic modulus. The critical length, R_c, can be evaluated according to the following expressions:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 \tag{2}$$

under plane strain conditions [3, 4], K_{Ic} being the fracture toughness and ν the Poisson's ratio.

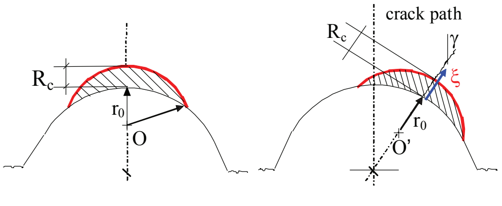
The critical volume in U-notched and blunt V-notched specimens under mode I loading conditions is centred in relation to the notch bisector line (Figure 1a). Under mixed mode loading the critical volume is no longer centred on the notch tip, but rather on the point where the principal stress reaches its maximum value along the edge of the notch (Figure 1b). It is assumed that the crescent shape volume rotates rigidly under mixed mode, with no change in shape and size [5]. This is the governing idea of the '*equivalent local mode I*' approach, as proposed in this research for blunt V-notches and as previously applied to U-notches by Berto *et al.* [5]. As made for U-notches for mixed mode loading an equivalent expression for the averaged strain energy density is proposed here for blunt V-notches:

$$\overline{W} = F(2\alpha)\times H^*(2\alpha,\frac{R_c}{\rho})\times\frac{\sigma_{max}^2}{E} \tag{3}$$

where σ_{max} is the maximum value of the principal stress along the notch edge and H^* depends again on the normalised radius R/R_c, the Poisson's ratio ν and notch opening angle.

Dealing with U-notches and different configurations of mode mixity, the function H, analytically obtained under mode I loading, was shown to be very close to H^* confirming the idea of equivalent local mode I as discussed in a previous work [5].

In the present investigation the maximum stress (σ_{max}) occurring along the edges of V-notches has been calculated numerically by using the FE code ANSYS 11.0. Two different procedures have been used to evaluate the strain energy density averaged over the control volume. The SED has been directly evaluated by means of finite element analyses by creating in the numerical models the control volume of radius R_c. This procedure requires a first model to identify the angle where the maximum principal stress occurs along the notch edge and a second model with the accurate definition of the control volume. The second approximate procedure based on local mode I has been applied here to blunt V-notches. Eq. (3) has been used by considering the maximum value of the principal stress along the notch edge, σ_{max} and by imposing $H^*=H$ as obtained from mode I.



(a) (b)

Figure 1. Control volume (area) for sharp V-notch (a), crack (b) and blunt V-notch (c) under mode I loading. Distance $r_0 = \rho\times(\pi-2\alpha)/(2\pi-2\alpha)$. For a U-notch $r_0=\rho/2$.

3. Experiments

The present paper considers a rounded-tip V-notched Brazilian disc specimen, called RV-BD (Figure 2). The material used was the glassy polymer PMMA which is relatively homogenous and isotropic material. This set of data has recently been provided by Ayatollahi and Torabi [8]. A high precision 2-D CNC

water jet cutting machine was utilized to fabricate the specimens from a PMMA sheet of 10-millimeter thick. The disc diameter (*D*), the notch length (*d/2*) and the thickness were 80 mm, 20 mm and 10 mm, respectively. To study the effects of the notch opening angle and the notch tip radius on the fracture behavior of the RV-BD specimens, three values of notch opening angle 2α = 30°, 60°, 90° and three values of notch radius ρ = 1, 2, 4 mm were considered for preparing the specimens. A total number of 162 mixed mode fracture tests were performed for various notch geometry parameters and different loading angles β from 0 (pure mode I) to β_{II} (pure mode II). For $2\alpha = 30$, experiments were performed according to the loading angles (β) equal to 0, 5, 10, 15, 20, 25. Similarly, for 2α = 60 and 90, fracture tests were conducted for various angles β of 0, 5, 10, 15, 25, 30 and 0, 5, 10, 15, 25, 35, respectively. For each geometry shape and loading angle at least two fracture tests were performed. The Young's modulus E (2960 MPa) and tensile strength σ_t (70.5 MPa) for PMMA were determined using a standard tensile test according to code ASTM D638-99. The parameters ν (0.38) and K_{Ic} (1.6 MPam$^{0.5}$) for PMMA were also obtained using the codes ASTM E132-04 and ASTM D5045-99, respectively.
The critical energy, evaluated considering the ultimate strength of the plain material, is equal to W_c=0.84 MJ/m^3. In parallel, Eq. (3) gives a critical radius for the material equal to 0.1 mm.

Figure 2: Geometry of the Brazilian disk weakened by blunt V-notches

4. Results

To give a theoretical support to the concept of equivalent local mode I applied to blunt V-notches, the mode I theoretical stress component, $\sigma_{\theta\theta}$, as obtained from Ref. [10] (considering θ=0) has been compared with the numerical values. The stress component $\sigma_{\theta\theta}$ normalised to its maximum value occurring along the notch edge is plotted in Fig. 3a as a function of the normalised distance ξ/ρ. The inclined path is perpendicular to the notch edge and starts from the point of the maximum of $\sigma_{\theta\theta}$ stress component along the notch profile. The finite element results are compared with the mode I theoretical solution as reported in (Filippi et al., 2002) [10]. The agreement is satisfactory under prevalent mode II and independent of the notch radius. In parallel, the shear stress component has been verified to be close to zero, as it happens along the notch bisector under Mode I loading. This observation leads to the conclusion that under mixed mode loading the line normal to the notch edge and starting from the point of maximum principal stress behaves as a virtual bisector line under pure mode I, confirming the applicability of the equivalent local mode I concept. Table 1 summarizes the outlines of the experimental, numerical and theoretical findings (only the case 2α=30°, ρ=1 mm has been reported for sake of brevity). In particular, the table summarizes the average experimental critical load (P_{EXP}) for every loading angle β and the maximum value of the principal stress (σ_{max}) as derived from FE models. The maximum stress has been obtained by applying to the numerical models the average value of the critical loads summarised in the table. The average experimental crack initiation angles, measured from the bisector line ($<\varphi_{EXP}>$),

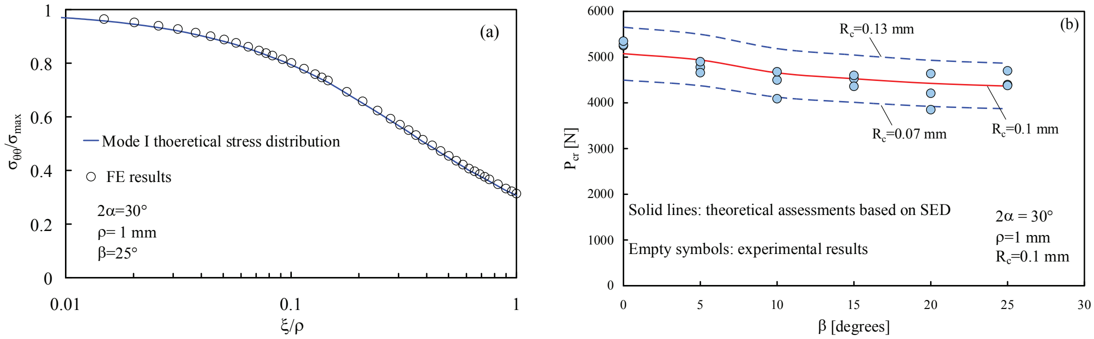


Figure 3: Comparison between theoretical mode I stress distribution and FE results along the normal from the point of maximum principal stress (a); Comparison between experimental data and theoretical assessment (b).

are also reported. The calculated values of SED as derived from the two procedures have been compared in Table 1, where also the mixed-mode exact function H* has been evaluated by inverting Eq. (3) and considering the local energy $\overline{W}_{FE}$ obtained from numerical models. The percentage deviation Δ between the function H (mode I) and the function H* (mixed-mode), defined as $\Delta = (H^* - H)/H^* \mathrm{x}100$ is listed in the table. The comparison clearly shows that the approximate procedure is accurate being the maximum deviation less than 7%. Figure 3b compares, the *experimental* values (open dots) of the critical loads as a function of the loading angle β for a constant value of the notch radius and of the notch opening angle (ρ=1 mm 2α=30°), with the *theoretical* predictions based on the SED model (solid line). As can be noted, the agreement between the experimental results and the theoretical predictions based on a constant value of the local strain energy is generally very satisfactory being the maximum deviation less than 10%. The dashed lines represent the assessments based on R_c=0.07 and 0.13 mm. The two curves are plotted to show the influence of the critical radius on the final fracture assessment.

Table 1: Outline of experimental and numerical results for 2α=30°

ρ [mm]	β [°]	$<P_{EXP}>$ [N]	σ_{max} [MPa]	$<\varphi_{EXP}>$ [°]	H	$\overline{W}_{th}$ [MJ/m^3]	$\overline{W}_{FE}$ [MJ/m^3]	H*	Δ %
1	0	5300	90.8	0.00	0.475	0.91	0.88	0.460	-3.4
1	5	4780	84.2	12.47	0.475	0.79	0.77	0.465	-2.1
1	10	4300	80.3	24.96	0.475	0.72	0.71	0.472	-0.7
1	15	4500	86.4	37.48	0.475	0.83	0.8	0.459	-3.5
1	20	4200	82.5	46.87	0.475	0.76	0.72	0.453	-4.9
1	25	4500	89.6	52.92	0.475	0.89	0.83	0.444	-6.9

A synthesis in terms of the square root value of the local energy averaged over the control volume (of radius R_c), normalised with respect to the critical energy of the material as a function of the loading angle β is shown in Figure 4. The plotted parameter is proportional to the fracture loads. The aim is to investigate the influence of the mode mixity on the fracture assessment based on SED. From the figure it is clear that the scatter of the data is very limited and almost independent of the loading angle. All the values fall inside a scatter ranging from 0.95 to 1.10. In the same figure also the data from cracked plates have been reported for comparison [9]. They have been plotted for the sake of visibility at β=2.5°. The synthesis confirms also the choice of the control volume which seems to be suitable to characterize the material behaviour under mixed mode loading. Future work should be focused to show that the assumption of the constancy of the control radius under mode I and mode II loading is verified also for other materials and with an acceptable accuracy. The cases of pure compression or combined compression and shear, for example, would require a reformulation for R_c and should also take into account the variability of the critical strain energy density W_c with respect to the case of uniaxial tension loads. This is also a very intriguing and challenging topic.

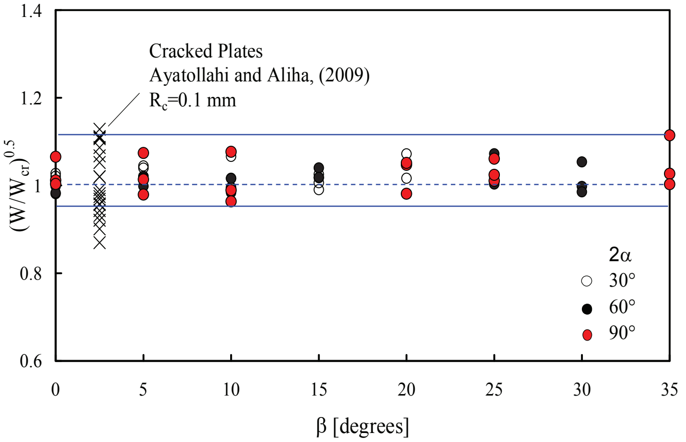


Figure 4: Synthesis of the data in terms of SED as a function of the loading angles.

5. Conclusions

This research re-analysed in terms of local SED criterion some recent experimental fracture results obtained from the Brazilian disk specimens weakened by blunt V-notches and characterized by different degrees of load mixity. The equivalent local mode I concept used in parallel with the SED approach was found to be suitable for the fracture assessment independent of the loading angle ranging from pure mode I to pure mode II. The approximate procedure to evaluate SED based on local mode I was justified by the analysis of the stress field along the inclined path perpendicular to the notch edge and starting from the point of the maximum elastic stress.

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