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The Waves and Cycles of COVID-19 Pandemic: A Phase Synchronization Approach

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Abstract

This study pretends to contribute to a better understanding of the COVID-19 dynamics through the non-parametric technique of phase synchronization by comparing the fifteen most affected countries by the number of positive cases plus China, where the first outbreak took place in December 2019. It was possible to state the number of cycles and waves for each one of the studied countries and to determine periods of synchronization between them. The results also showed the average duration of the cycles and some coincidences regarding Nason (2020); Bontempi (2021); Coccia (2021); Rusiñol, Zammit, Itarte, Forés, Martínez-Puchol, Girones, Borrego, Corominas, and Bofill-Mas (2021). This study is limited by the reliability of the number of positive cases reported by national governments and health authorities because of an insufficient number of tests and a great number of asymptomatic persons but presents a legit alternative to predict the evolution of the pandemic in a country due to the forward looking behavior of another one, therefore studies like this could be useful to implement contention measures and to prepare the health systems in advance.

Keywords: phase synchronization, COVID-19, waves, cycles.

1. Introduction

The rapid transmission of the virus causing the ongoing COVID-19 pandemic has led researchers of all areas of knowledge to work in studies that could help to understand the dynamics of the virus and its spread around the world. Since the pandemic have been declared on 11th March 2020, it was clear for health professionals and researchers, that this infectious disease could prevail for a long time in the world population, and to present ups and downs in the number of contagions due to seasonal patterns, prevention measures, and quarantines. At this point, it is correct to ensure that COVID-19 will be endemically present and therefore, it is important to understand how it evolves and how its features could be tackled to maintain it under control while vaccination programs advance around the world. Previous pandemics, such as The Black Death (1347-1351) presented successive waves and outbreaks in Milan (1630), London (1665-1666), and Marseille (1720-1722), while the Spanish Flu (1918-1919) presented three waves within a 9 month interval Piret and Boivin (2020), therefore is pertinent to wait for successive waves until the pandemic can be fully controlled, and to anticipate resources and plans to contain its spread. The contention measures have included lockdowns, ban of international travels, restriction of movement between regions of the same country, school closures, ban of mass gatherings, mandatory use of facemasks, commerce operating at a minimum of their capacity, restricted sport and recreative activities, home office, etc. These measures have had a certain positive effect controlling the spread of the virus and, at some times have served as a measure to avoid the healthcare system collapse, but they have harmed some economic sectors such as tourism, airlines, restaurants, theatres, cinemas, amusement parks, and a lot of non-essential business. To avoid some of the negative effects it is important to understand the characteristics of the virus and to state the best contention measures due to it seems to be that the COVID-19 will be endemic, and therefore, a better understood of the contagion dynamics will be helpful until the herd immunity is reached boosted by the massive application of vaccines. To understand the COVID-19 spread and dynamics, it is important to model its transmission, predict diverse scenarios for new cases and deaths, find the best prevention measures, state seasonal and cyclical patterns and determine climate factors affecting the spread of the virus. This understanding will be useful for policymakers and authorities to elaborate plans for contention, prepare supplies for the healthcare systems, and detail budgets and social assistance aids to mitigate the negative effects of COVID-19 in the real economy. This research pretends to collaborate to the efforts to shed light beyond epidemiological questions or potential health implications in the long term and characterizes the cyclical component and synchronization between the number of new cases present in the most affected countries by March 18th, 2021, and China by applying a non-parametrical technique named phase synchronization. Phase synchronization allows to state one way causality from a system to another one and after a smoothing process also shows cycles and waves present in a time series. In the context of COVID-19 pandemics, it is important to establish similar dynamics, synchronization patterns, and forward looking behaviors between different countries, which is part of the importance and contribution of this study. This work is organized as follows: Section 2 presents a review of the literature about covid waves and cycles as well as studies about modeling; Section 3 details used data and country selection criteria; Section 4 describes the methodology used to perform the study; Section 5 presents the most remarkable results, and Section 6 concludes.

2. Literature review

Research about COVID-19 waves, includes Rusiñol *et al.* (2021) with a study of wastewater treatment plants during first and second waves for the region of Catalonia, Spain, finding a strong correlation with the number of positive cases one week after the water sampling, and concluding that concentrations of SARS-CoV-2 RNA are a good predictor for future diagnosed cases. Coccia (2021) identified the two first waves in Italy, occurring the first from 24th February to July 31, 2020, while the second developing from August 1 to February 22nd, 2021. Bontempi (2021) studied the differences among waves present in a set of five European countries (France, United Kingdom, Italy, Spain, and Germany) focusing on the measures to contain the spread of the virus. He marked the second wave of contagion from July to September 2021. Piovani, Christodoulou, Hadjidemetriou, Pantavou, Zaza, Bagos, Bonovas, and Nikolopoulos (2021) studied social distancing measures and their effect on mortality during the first wave of the pandemic of the 37 members of the Organization for Economic Cooperation and Development (OECD) and concluding that reductions in the COVID-19 cumulative mortality are due to the early application of mass gathering bans and school closures. Pai, Bhaskar, and Rawoot (2020), investigated the dynamics of the pandemic for the case of India considering the lockdown measures and achieving a prediction model able to forecast peak values for decision-makers to help them to decide about control measures, to prepare health systems, and react to socioeconomic damage. Efforts to model and predict the COVID-19 dynamics sparse and the number of possible positive cases include Torrealba-Rodriguez, Conde-Gutiérrez, and Hernández-Javier (2020), who used a Gompertz and Logistic model as well as Artificial Neural Networks for Mexico, Muñoz-Fernández, Seoane, and Seoane-Sepúlveda (2021) with a Susceptible–Infected–Recovered (SIR) model for Italy, Spain, and the USA with fitted results for important epidemiological parameters such as active cases, daily new cases for the first and successive waves of the pandemic. Some other studies have had a focus on understanding the dynamics of the disease itself Engelbrecht and Scholes (2021) who tested for seasonality and climatic effects on the evolution of the pandemic and stated the possible existence of seasonality-induced waves due to socio-economic pressure and relaxation of lockdown measures. Anirudh (2020) used a variety of models to predict the spread, peaks, and reduction of COVID-19 number of cases but remarked the high uncertainty in the estimated models. Nason (2020) used a Bayesian spectral fusion method to accurately estimate short cycles by applying a signed log transform to daily cases of COVID-19 time series in first differences, and finding wavelengths of 2.7, 4.1, and 6.7 days, and suggesting lockdown and weekly effects. He studied 18 countries and clustered them based on their spectrum and finding some similarities in the peaking days. Ricon-Becker, Tarrasch, Blinder, and Ben-Eliyahu (2020), also found a weekly pattern for 7 of 12 developed Northamerican and European countries. Phase synchronization was used to evaluate the possible relationship between the evolution of the pandemic in two different territories. Phase synchronization is a non-parametrical technique that allows extracting the cyclical component of a time series and to evaluate two non-deterministic systems. Zhou (2013) defined phase synchronization as an oscillatory process for two or more cyclical signals with a repeating sequence of angles. These angles, named phase angles, are applied to two waveforms with the same frequency for each cycle. Perfect phase synchronization is present when the cycles of the two analyzed systems occur at the same time Borrego-Salcido, Juárez-Del-Toro, and Cruz-Aké (2020). Sometimes, analyzed systems are synchronized in phase but one of the two present a forward-looking behavior, which is called a controller or master system, while the other system tends to follow the master's movements but with a delay. The latter system is known as the slave and its behavior adjusts its movements to the controller one. Synchronization refers to two dynamic systems tending to adjust their movements and trajectories evolving around numeric values known as attractors. Synchronization can be defined as the adjustment of coupled systems and depends on the correlation among the analyzed systems Hung and Hsu (2016). For Zhou (2013) synchronization also implies coordination of events and defined phase synchronization as the process with two or more signals oscillating with a repeating sequence of angles, which is accurate to waveforms and cyclical signals where the coupled oscillators with similar waveforms present a phase shift between the synchronous movements. Zhou also identified that systems synchronized in phase present a change near to zero, while anti-phase and out of phase have a value of π and greater values, respectively. A cyclical behavior consists of the repetition of a pattern where the same events occur in the same order to complete a cycle, and then the events repeat once and again, therefore, a cycle is composed of the occurrence of a series of events. Although biological issues about the life cycle of the COVID-19 virus are out of the scope of this study, some of the dynamics of the virus and its life-cycle are represented through the data, such as a seven-day cycle with a peak in the number of new cases occurring on Thursday-Friday and a peak for deaths present on Wednesday-Thursday Ricon-Becker *et al.* (2020), which leads us to think that a visible cycle for statistical purposes follows the mentioned pattern with a similar cycle for all the studied countries.

3. Data description

To perform this study, selected countries are those fifteen countries with the most positive cases by March 18th, 2021, because of the belief of the fact that dynamics in countries with the greater number of positive cases, could condition the spread of the pandemics and its propagation to neighboring countries. Table 1 presents the countries and number of cases by this date. All data has been collected from Our World in Data . China is included as the number 16 country because it was the very first country to report a COVID-19 positive case, representing the origin of the ongoing pandemic, and provoking directly the first wave of the pandemic. The sample for each country is composed by 373 observations with daily frequency from March 11th, 2020 (when all the included countries reported at least a new case) to March 18th, 2021. It was used the number of new cases reported by each country, and time series presented has not been seasonally adjusted to represent accurately the dynamic of the pandemic.

Table 1: Countries with the most positive confirmed cases and China by March 18th, 2021. Source: own elaboration whit data from www.ourworldindata.org

Country	Numbers of confirmed cases (million)
US	29.67
Brazil	11.6
India	11.51
Russia	4.38
UK	4.29
France	4.24
Italy	3.31
Spain	3.21
Turkey	2.95
Germany	2.63
Colombia	2.32
Argentina	2.23
Mexico	2.18
Poland	1.98
Iran	1.76
China	0.101

4. Methodology

To perform this study, it was followed the phase synchronization methodology stated on Borrego-Salcido *et al.* (2020) and Aké (2017). In phase synchronization, the paired time series are not necessarily moving in the same direction, but it means that cycles have the same length Aké (2017). To begin with the analysis, each of the 16 studied COVID-19 time series are paired to compare their dynamic and trajectories. Comparing pairs of time series allows to state which system has a controller behavior, being the master system, and which one presents a slave behavior, being this the observer system, which reacts to the master system with a lag. To simplify the comparison between the magnitude of involved figures in the study, time series were normalized, which means that the values at time t are divided by the maximum value in the series. Then, it was chosen the smoothed parameter for all countries, which could take values between 0 and 1, with closer values to zero maintaining short-term trends, and values close to one keeping only the long-term trends Delgado López and Fonseca Zendejas (2021) It allows to extract the number of present cycles for each time series. The value of the smoothing parameter chosen for this study was 0.7 trying to maintain middle-term trends and oscillations coming from the daily frequency of the data. A complete cycle is identified when smoothed series present two sign changes, following a sine function, touching the zero twice “when it presents a fall and becomes negative, and when it rises getting a positive value again” (Borrego-Salcido *et al.* (2020), p. 42). Process for phase synchronization is presented in Borrego-Salcido *et al.* (2020) and Aké (2017)- Equation (1) presents the way to smooth a time series at time t where X_t represent the smoothed time series, ϵ is the smoothing level, X_{t-1} is the smoothed series lagged a period, and K_t represents the original time series. The equation in (2) allows to calculate the phase for each compared time series, with k representing the number of cycles for each series, and $t_{k+1} - t_k$ being the duration in days for each period, and Φ_X represents the phase differential at time t for compared countries. The process described in (2) is repeated throughout time series length to get the phase for each series. If the resulting phase differential for each t is the same, series s_1 and s_2, are synchronized in phase, and if this differential remains the same for a while (3), it represents strong synchronization, but if the differential $d\Phi_X$ in each time t, is close to zero, paired series present weak synchronization Borrego-Salcido *et al.* (2020). To standardize the determination of weak synchronization, Delgado López and Fonseca Zendejas (2021) calculated the mobile variance of phase differentials for each time t, and $t-1$ considering that if it takes a value up to 0.5, weak synchronization is present.

$$X_t = (\epsilon X_{t-1}) + (1-\epsilon)K_t \quad (1)$$

$$\Phi_X(t) = 2\pi(t-t_k)/(t_{k+1}-t_k) + 2\pi k \quad (2)$$

$$d\Phi_X(t) = \Phi_{s_1}(t) - \Phi_{s_2}(t) = \text{constant} \quad (3)$$

In phase synchronization, a signal is an independent variable that evolves through time and is characterized by $x(t) = \langle A\rangle \sin(\omega t)$ with an amplitude determined by $A = A_p/2$, where A_p is gotten by subtracting the minimum value from the maximum value of the signal. The angular frequency is denoted by $\omega = 1$, and the average number of periods is determined by $\langle T\rangle = t_N/N$, where t_N is the total number of observations while N represents the number of periods, and $\langle T\rangle$ refers to the average duration of the periods. The cyclical frequency is denoted by $f = 1/T$, and N is also seen as the number of cycles present in the signal, hence the average frequency is defined by $\langle f\rangle = N/t_N$. The angular frequency is given by $\langle f\rangle = N/t_N$. All these definitions and equations could be found in García Ruíz (2014).). This kind of study could be repeated with a larger sample to analyze the evolution of the pandemic and to state a complete pattern of behavior and establish if the master-slave relationships remain the same between the countries and by adding the effects of vaccination campaigns to differentiate subsequent waves and their effects.

5. Results

This section presents the most remarkable results and findings. Some other results with less significance, and the complete computations will be available upon request. The number of cycles, as well as signal properties, are presented in Table 2. The number of days a cycle lasts found by phase synchronization is similar to Nason (2020), with some coincidences in the duration of the cycles. The weekly effect found by Nason (2020) is present in Argentina, Brazil, Germany, Mexico, Poland, and the USA, with France showing a borderline similarity. Some other countries like China, Colombia, and Italy, present cycles with a duration of about 8 days. The rest of the analyzed countries present cycles with a longer duration, but Spain presents cycles of about 5 days, being this country the one which has the greater number of cycles with 71, and therefore the country presenting 0.19 cycles per day. Turkey is the country with the largest duration of its cycles, with a duration of about 12 days. Graphical analysis shows clearly all waves present for each country. Table 3 indicates how many waves were found for each studied country. These waves result from the smoothed time series (see Figure 1) and were determined when the number of cases reached a peak, and then fell. For the case of Germany and France, (see Figure 2) the onset of what could be the third wave is present at the end of the sample. India presents the beginning of its second wave by the end of the sample. Second waves have had a larger number of positive cases than the first ones in all studied countries but Argentina, and countries like Spain who have experienced the third wave, it is larger than the two priors. The UK and the USA (see Figure 2) are special cases of the third wave, with a second one not reaching a minimum number of cases when the third wave began, but the latter have been larger. Italy also shows a lack of control or minimum cases at the beginning of the second wave and then presenting its third one. These criteria could also be used to analyze the waves for China, (see Figure 3) with the third not reaching

Table 2: Signal properties. Source: own elaboration

Country	$\langle A\rangle$	N	$\langle T\rangle$	$\langle f\rangle$	$\langle\omega\rangle$
ARG	0.4281849	54	6.9074074	0.1447721	0.90963
BRA	0.4259445	53	7.0377358	0.1420912	0.892785
CHN	0.6090426	45	8.2888889	0.1206434	0.7580027
COL	0.4382315	51	8.6744186	0.1152815	0.7243137
FRA	0.3157035	58	6.4310345	0.155496	0.9769812
GER	0.2894764	54	6.9074074	0.1447721	0.90963
IND	0.4783968	40	9.325	0.1072386	0.6738
IRA	0.4763748	33	11.30303	0.0884718	0.555885
ITA	0.4445823	46	8.1086957	0.1233244	0.77487
MEX	0.6912915	48	7.7708333	0.1286863	0.8085362
POL	0.380906	50	7.46	0.1340483	0.84225
RUS	0.4833623	20	18.65	0.0536193	0.3369
SPN	0.3136299	71	5.2535211	0.1903485	1.1959951
TUR	0.3588796	30	12.433333	0.080429	0.50535
UK	0.4402051	41	9.097561	0.1099196	0.690645
USA	0.4320264	50	7.46	0.1340483	0.8422252

a peak when the fourth began. the first wave in Poland (see Figure 3) is the last to appear in the sampled countries.

Table 3: Number of waves for each country. Source: own elaboration

ARG	BRA	CHN	COL	FRA	GER	IND	IRA
2	2	4	2	3	3	2	2
ITA	MEX	POL	RUS	SPN	TUR	UK	USA
3	2	2	2	3	3	3	3

For the case of Italy, there is a coincidence between the duration of the waves in Coccia (2021) and the findings in this study, with the first wave already happening at the beginning of the sample, and the second appearing just about 10 days after the initial date in Coccia. After the end of the second wave, phase synchronization allowed to capture the onset of the third wave beginning by February 15th, 2021 and is still present at the end of the sample used in this research. For the case of Spain, (see Figure 3) this study found the peak of the first wave on March, 29th, 2020 almost coinciding with the peak in Rusiñol *et al.* (2021) who marked it just one day before, while the beginning of the second wave through phase synchronization was found in July 16th, 2020 and coinciding with the date found by Rusiñol et al. Results in this research match those in Bontempi (2021), with all countries included in both studies (Spain, Italy, UK, France, and Germany) experiencing a minimum number of contagions during June and July, but showing an increase from mid-July, with Spain being the very first to see a rise in the number of positive cases and indicating the beginning of the second wave. In the case of India, (see Figure 3) results in this research show a peak in the number of contagions by September 17th, differing by about a month that results in Pai *et al.* (2020), where the peak is predicted by the mid of August 2020. The phase synchronization revealed the onset of the second wave in India, beginning by February 16th, 2021, when the number of positive cases began to increase after being at minimum levels after the first wave.
There is evidence of synchronization of the smoothed series at the beginning of the sample for some cases, but the COVID-19 dynamics in some countries have not a synchronized behavior with others, particularly China, which does not present synchronization with any country in the sample. It is important to highlight the fact that geographic aspects were present at the onset of the pandemic, due to travels and commerce activities and other interconnections between border countries. This is the case for countries from the same region, specifically

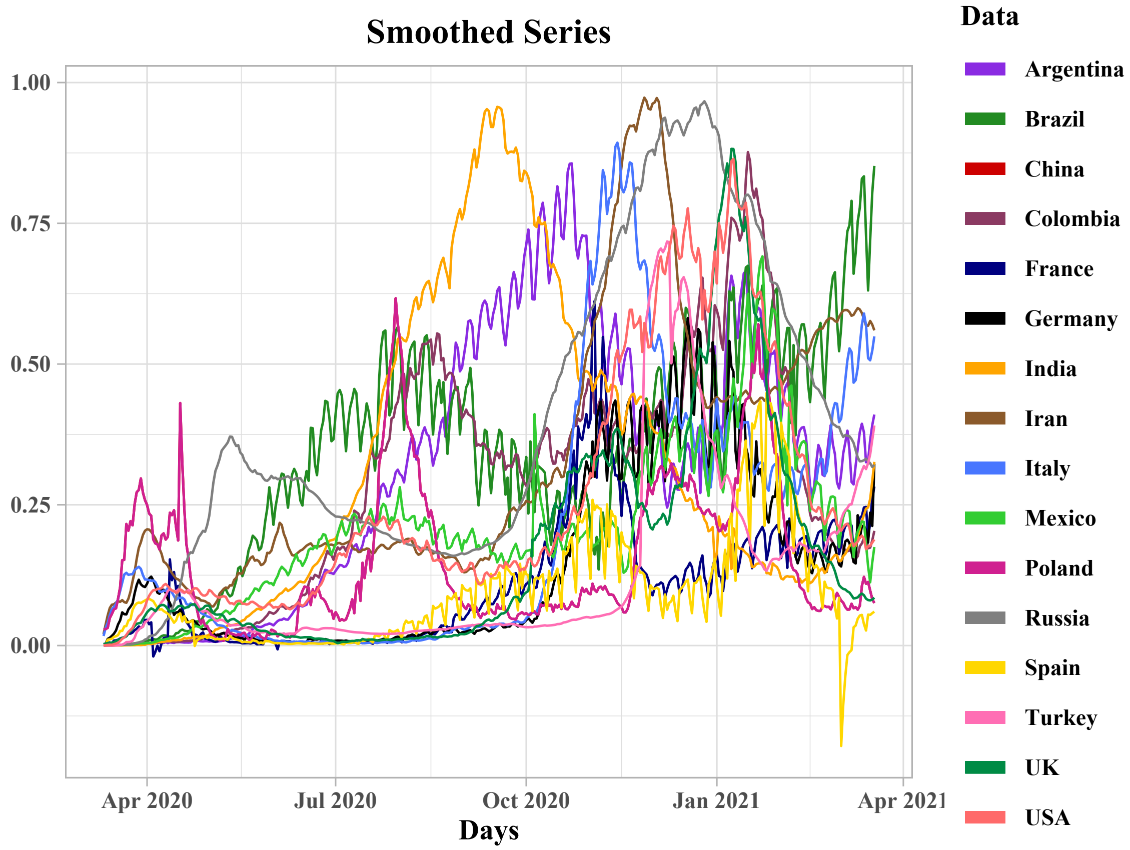

Figure 1: Smoothed series and waves for studied countries. Source: own elaboration with R-Software

Latinamerican countries like Argentina, Brazil, and Colombia, and for the case of Mexico and the USA, which share the most active border in the world. For the case of European countries, France and Spain present a synchronized behavior, and it is assumed as a border effect and the free transit of persons, with Spain also presenting a synchronized behavior with the UK and Germany which could be seen as an effect of the interconnectedness of the Euro Zone . Some other synchronized behaviors are present for some countries, but it is not necessarily due to geographic aspects. The master-slave relationships are explained using the example of Brazil. This country leads the behavior for the case of Mexico (see Figure 4), which presents a delay regarding the master system during most of the time of the sample. Comparing Brazil and Colombia (see Figure 4) led to state that these two countries have synchronized behaviors during almost all the sample, but for a short period, Brazil was the master system. Continuing with the example of Brazil, it also leads the USA (see Figure 4) excepting at the end of the sampling which presented synchronized behavior. To illustrate a mixed behavior of a time series regarding another one, Figure 4 presents the comparison between Brazil and Poland, with the first being the master system at the beginning of the pandemic and switching to a slave behavior at the middle of the sample, to become the master system again, and to finally synchronize their cycles at the end of the sample. Some other findings of other pairs of countries at a regional level are France as master of UK, Italy, and Poland, while the UK is synchronized with Poland and with Italy, for most of the part of the sample. Turkey and Iran are master systems to Russia, but Iran is a slave system to Turkey for most of the sample but changing its behavior in the middle to synchronize it but become the master system by the end. Another case of changing behavior is that present in India as a master system at the onset of the pandemic but becoming a slave to China.

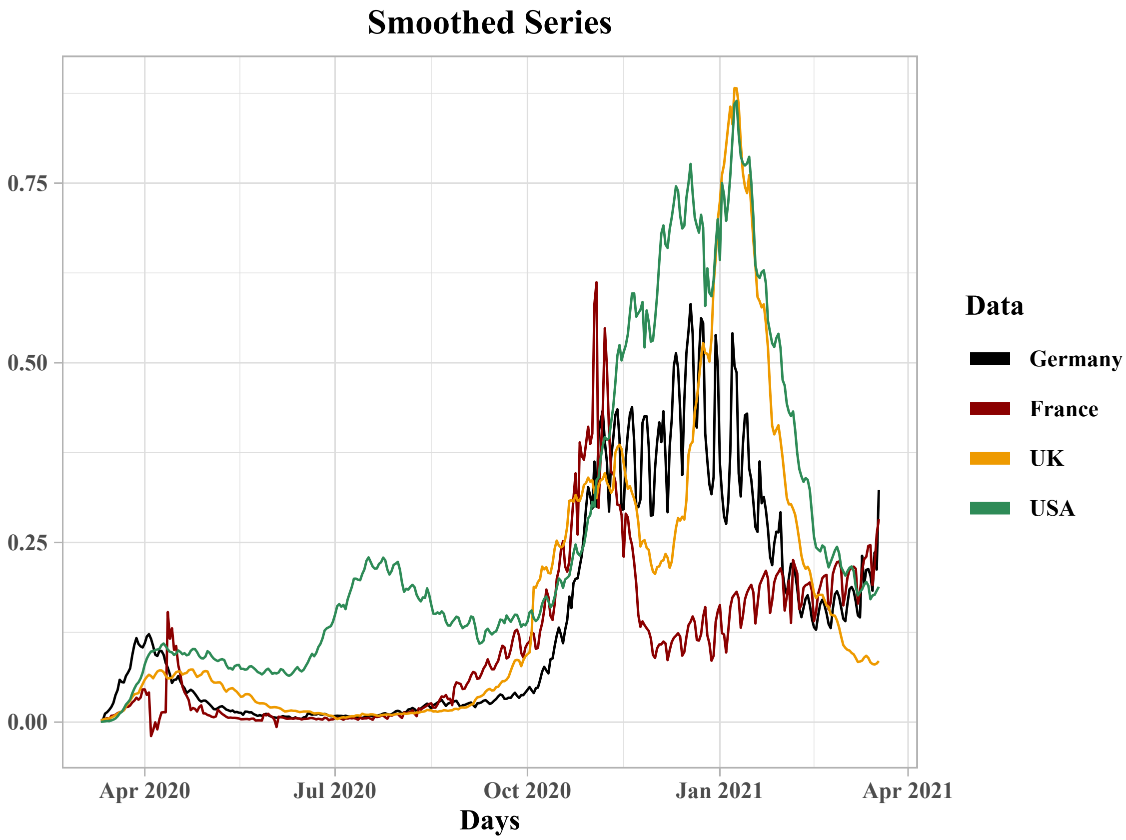

Figure 2: Smoothed series for Germany, France, UK, and USA. Source: own elaboration with R-Software

Results regarding the days of synchronization are presented in Tables 4-7. The reader can see that countries with the mayor number of synchronized days are Poland and Italy with 124 days of strong synchronization, followed by 118 days from Argentina and USA and Russia and Turkey with 111. The less strong synchronized pairs of countries are USA and Turkey without a single period of strong synchronization followed by Turkey and France with just one day and Turkey and Argentina and Turkey and UK with two days. These results could lead to think that Turkey is the less synchronized country, but reality is that this country presents weak synchronization with Iran and Italy for more than 300 days and with the USA for 367 days, being this the highest figure of weak synchronization and followed by similar numbers of days from the pairs formed by Turkey with Argentina, Mexico, Germany, Brazil, France. Other pairs of countries with high figures in weak synchronization are Iran and Mexico with 305 days, Iran, and Russia with 335, Mexico and Russia with 310, Russia and UK with 306 and UK and USA with 308. The less synchronized countries come from the no days of synchronization, being these pairs China and Iran, China and Poland, Argentina, and China with 226, 199 and 181 days respectively, followed by Spain and Poland and Spain and India with 177 and 169 days. Number of days of synchronization confirms the regional pattern presented by smoothed series.

Regarding the effects of vaccination, it is not possible to appreciate it in most countries because the first register of the number of vaccinated persons is December 27th , 2020 and by the end of the studied sample, only the US presented a share of people fully vaccinated of 12%. All other countries presented figures not higher than 5.2% of fully vaccinated people which corresponds to Turkey. Although the share of people partially vaccinated in the UK is the higher one, 2.8% by January 10th, 2021, it is consistent with the highest point of its third

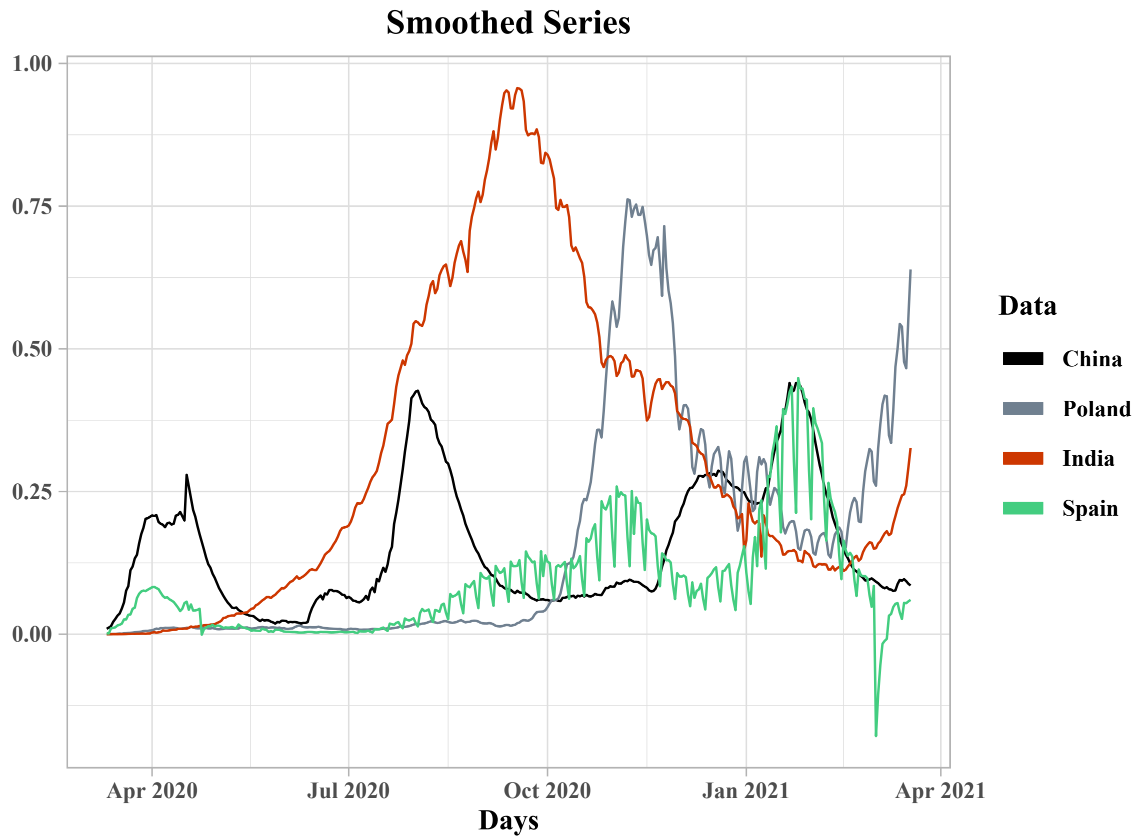

Figure 3: Smoothed series for China, Poland, India, and Spain. Source: own elaboration with R-Software

wave and it is not possible to state its effect, because by the date the only 0.57% of people had a complete scheme of vaccination. China has no information about vaccination by the end of the sample, while Colombia and Iran had the smallest figure of fully or partially vaccinated people. The prior does not provide enough data to state a visible relationship between the vaccination programs and the decline in the number of cases for the studied period. Progress in vaccination programs could mean a reduction in the number of cases and would allow an effective comparison between the number of cases present in the previous year and the second half of 2021 when some countries present vaccination rates close to what could be considered necessary to obtain herd immunity.

6. Conclusions

This study pretends to contribute to the great number of research regarding the COVID-19 pandemic by applying a technique that as far as we know, has not been used before. This kind of study is important because a better understanding of the COVID-19 dynamics could contribute to control the number of positive cases, identifying the best contention measures, and understanding how the virus is evolving beyond its biological characteristics. Mathematical models and statistical techniques are very useful to do so. Phase synchronization has been used to understand the dynamics between two non-deterministic systems or signals. Synchronization between geographically close was an expected result, but some other similarities like the number of average days in a cycle or some synchronization patterns between farther countries are also present and are like those in Bontempi (2021); Coccia (2021); Rusiñol *et al.* (2021); Nason (2020). These results support the utility of phase synchronization as a useful

Table 4: Number of days of synchronization by compared pairs of countries. Source: own elaboration

	ARG			BRA			CHN			COL		
	N	W	S	N	W	S	N	W	S	N	W	S
ARG	-	-	-	61	208	99	181	164	24	62	276	36
BRA	-	-	-	-	-	-	85	243	39	54	291	29
CHN	-	-	-	-	-	-	-	-	-	153	206	15
COL	-	-	-	-	-	-	-	-	-	-	-	-
⋮						⋮						⋮
USA	-	-	-	-	-	-	-	-	-	-	-	-

Table 5: Number of days of synchronization by compared pairs of countries. Source: own elaboration.

	FRA			GER			IND			IRA		
	N	W	S	N	W	S	N	W	S	N	W	S
ARG	106	201	61	57	225	87	92	268	7	72	260	36
BRA	66	220	81	49	218	100	74	271	23	43	278	46
CHN	205	238	25	88	259	22	111	239	19	226	133	9
COL	77	243	48	73	263	33	94	268	7	55	307	7
FRA	-	-	-	103	197	68	101	235	32	93	246	29
GER	-	-	-	-	-	-	152	174	43	55	261	52
IND	-	-	-	-	-	-	-	-	-	79	274	16
IRA	-	-	-	-	-	-	-	-	-	-	-	-
⋮						⋮						⋮
USA	-	-	-	-	-	-	-	-	-	-	-	-

tool to lead to a better understanding of patterns, dynamics, and behavior of the COVID-19 pandemic and to identify countries which could condition the evolution of the disease in some others. The study could be replicated or repeated using seasonally adjusted series as seasonal behavior of the pandemic is understood. Future research could enlarge the sample, analyze different pairs of countries, or compare complete sets of neighbor countries to find new relationships. The prior is important due to the fast evolution of the pandemic and the appearance of new and more dangerous variants. It will be important to compare if the same behaviors are still present as time goes and pandemic evolves and how the later responds to vaccination. This study could be repeated to compare the same periods for different years considering the impact of high vaccination figures for most of the countries studied in the sample.

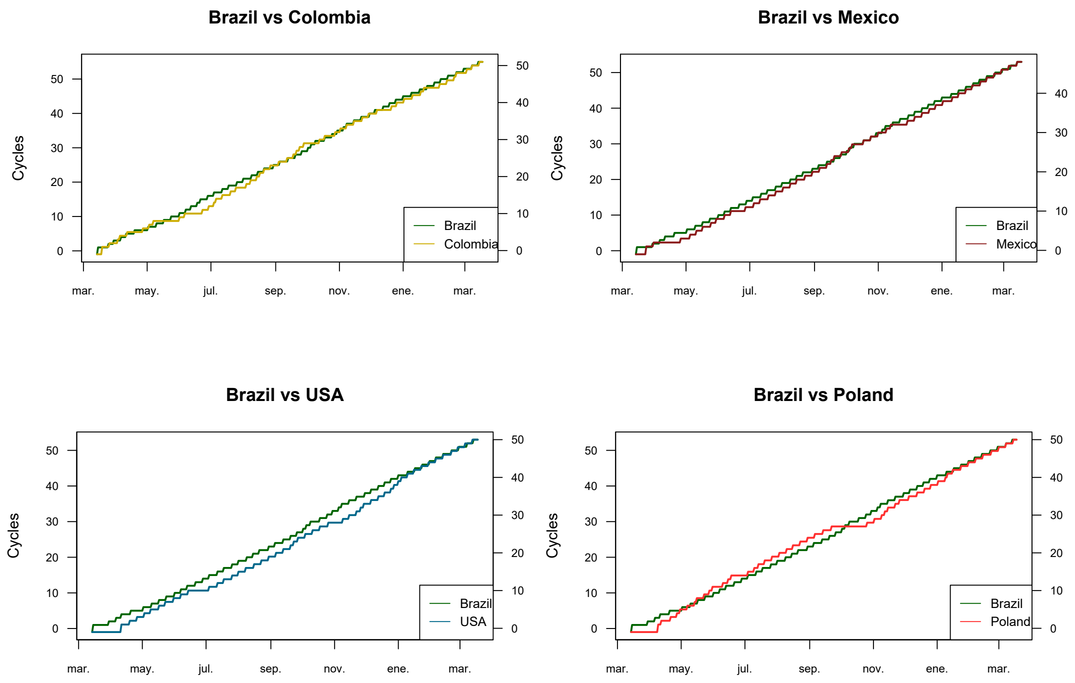

Figure 4: Comparison between Brazil-Colombia, Brazil-Mexico, Brazil-USA and Brazil-Poland. Source: own elaboration with R-Software

Table 6: Number of days of synchronization by compared pairs of countries. Source: own elaboration.

		ITA			MEX			POL			RUS	
	N	W	S	N	W	S	N	W	S	N	W	S
ARG	86	228	55	66	249	54	101	210	58	59	297	14
BRA	61	240	68	42	248	78	81	213	74	77	270	21
CHN	87	261	21	87	265	17	199	147	23	115	229	25
COL	66	274	29	63	286	25	70	273	26	63	292	14
FRA	79	233	56	76	231	61	103	195	70	83	272	13
GER	73	217	79	58	225	86	107	189	73	87	257	25
IND	79	257	33	73	241	55	85	233	51	70	225	74
IRA	54	276	39	42	305	22	57	276	36	31	335	3
ITA	-	-	-	56	240	73	53	192	124	55	303	11
MEX	-	-	-	-	-	-	69	213	87	54	310	5
POL	-	-	-	-	-	-	-	-	-	62	294	13
RUS	-	-	-	-	-	-	-	-	-	-	-	-
⋮						⋮						⋮
USA	-	-	-	-	-	-	-	-	-	-	-	-

Table 7: Number of days of synchronization by compared pairs of countries. Source: own elaboration.

	SPN			TUR			UK			USA		
	N	W	S	N	W	S	N	W	S	N	W	S
ARG	150	165	53	67	299	2	75	263	31	63	188	118
BRA	140	175	54	62	294	12	52	285	31	58	206	104
CHN	152	191	26	127	234	13	77	280	12	96	257	21
COL	133	204	32	75	285	14	62	293	14	67	287	15
FRA	149	148	71	76	291	1	71	272	25	111	188	69
GER	138	147	84	55	295	18	71	271	27	79	190	100
IND	169	179	18	57	290	21	85	269	15	78	243	48
IRA	137	200	32	37	301	25	60	278	31	59	262	48
ITA	153	174	42	53	303	12	57	287	25	66	237	66
MEX	158	171	40	45	297	32	62	285	22	40	239	90
POL	177	145	47	71	273	24	80	263	26	58	228	83
RUS	145	202	22	41	216	111	57	306	6	56	287	26
SPN	-	-	-	135	220	13	126	221	22	160	157	52
TUR	-	-	-	-	-	-	47	319	2	1	367	0
UK	-	-	-	-	-	-	-	-	-	46	308	15
USA	-	-	-	-	-	-	-	-	-	-	-	-

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