Spring 5-5-2023

Areas of Same Cardinal Direction

Periyandy Thunendran
The University of Maine, thunendran.periyandy@maine.edu

AREAS OF SAME CARDINAL DIRECTION

By

Thunendran Periyandy

B. Sc. Sabaragamuwa University of Sri Lanka, 2018

A THESIS

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

(in Spatial Information Science and Engineering)

The Graduate School

The University of Maine

May 2023

Advisory Committee:

Max J. Egenhofer, Professor of Spatial Computing, Advisor

M. Kate Beard-Tisdale, Professor of Spatial Computing

Matthew P. Dube, Associate Professor of Data Science, Computer Information Systems, and Applied Mathematics

AREAS OF SAME CARDINAL DIRECTION

By Thunendran Periyandy
Thesis Advisor: Dr. Max J. Egenhofer

An Abstract of the Thesis Presented
in Partial Fulfillment of the Requirements for the
Degree of Master of Science
(in Spatial Information Science and Engineering)
May 2023

Cardinal directions, such as North, East, South, and West, are the foundation for qualitative spatial reasoning, a common field of GIS, Artificial Intelligence, and cognitive science. Such cardinal directions capture the relative spatial direction relation between a reference object and a target object, therefore, they are important search criteria in spatial databases. The projection-based model for such direction relations has been well investigated for point-like objects, yielding a relation algebra with strong inference power. The Direction Relation Matrix defines the simple region-to-region direction relations by approximating the reference object to a minimum bounding rectangle. Models that capture the direction between extended objects fall short when the two objects are close to each other. For instance, the forty-eight contiguous states of the US are colloquially considered to be South of Canada, yet they include regions that are to the North of some parts of Canada. This research considers the cardinal direction as a field that is distributed through space and may take on varying values depending on the location within a reference object. Therefore, the fundamental unit of space, the point, is used as a reference to form a point-based cardinal direction model. The model applies to capture the direction relation between point-to-region and region-to-region configurations. As such, the reference object is portioned into *areas of same cardinal direction* with respect

to the target object. This thesis demonstrates there is a set of 106 cardinal point-to-region relations, which can be normalized by considering mirroring and 90° rotations, to a subset of 22 relations. The differentiating factor of the model is that a set of base relations defines the direction relation anywhere in the field, and the conceptual neighborhood graph of the base relations offers the opportunity to exploit the strong inference of point-based direction reasoning for simple regions of arbitrary shape. Considers the tiles and pockets of same cardinal direction, while a coarse model provides a union of all possible qualitative direction values between a reference region and a target region.

DEDICATION

My thesis is dedicated to my parents,

MY LATE FATHER, MR PERIYANDY, M. AND MY MOTHER, MRS P, YOGESHWARY.

ACKNOWLEDGEMNTS

I would like to thank the following individuals and organizations for their guidance, help, and support through the pursuit of my graduate education:

- Advisor Dr. Max Egenhofer
- Thesis committee (Dr. Kate Beard and Dr Matthew P. Dube)
- The faculty of the Spatial Informatics Program (Dr. Max Egenhofer, Dr. Kate Beard, Dr. Silvia Nittel, Dr. Nimesha Ranasinghe, Dr. Torsten Hahmann, and Dr. Nicholas Giudice)
- United States-Sri Lanka Fulbright Commission (US-SLFC) for giving me the Fulbright Master's Grant for 2021-2023.
- Mentor Dr. DR Welikanna, for his significant life-changing support to my academic career
- The faculty who has served as academic references for my graduate applications (Dr. DR Welikanna, Dr. HMI Prasanna, Dr, R Bandara, and Dr. Raymond Hintz)

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Spatial relations play a fundamental role in individuals' daily lives, from navigating through geographic space to communicating with others and solving problems. The relative position of a scene of spatial objects is descried in up to three separated aspects: (1) distances, (2) topological relations, and (3) direction relations (Renz, 2002). For example, the weather forecast routinely will engage multiple of these aspects, "The eye of hurricane Ian is approximately located at 185 Kilometers due south of Tampa Bay, moving northeast at about eight mph, landfall occurs inside Florida by Wednesday," integrates distance (185 Kilometers), topology (inside), and direction (northeast) as an integral description of spatial communication.

Distances are captured quantitatively as metric forms between spatial objects and qualitatively as *far* and *near* (Hernández *et al.,* 1995; Zimmermann, 1995). Topological relations explore how two objects interact based on connectivity (Clarke, 1981; Randell *et al.,* 1992) and containment within the embedding space (Egenhofer and Herring, 1992). For example, the Fogler Library and Boardman Hall at the University of Maine have topological relation disjoint (Figure 1.1)

Directions provide a complementary view to topology (Sharma, 1996; Guo and Du, 2009; Dube, 2021). They define the relative orientation from a reference object to a target object following a reference direction. For instance, the magnetic compass defines the northeast direction from the Fogler library to Boardman Hall (Figure 1.1). Direction relations provide more detail than topology does (Dube, 2012). For example, Boardman Hall and Memorial Union are disjoint buildings from the Fogler Library, showing the same topological relation; however, they differ by direction relation (Figure 1.1).

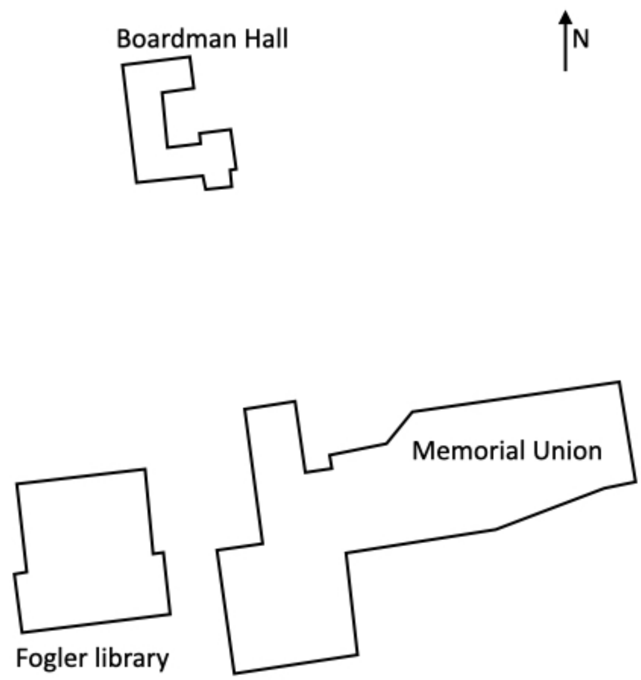


Figure 1.1 Three buildings at the University of Maine.

In geographic space, four cardinal and four intercardinal directions are standard in practice: North, East, South, West, Northwest, Southeast, and Northwest (Frank, 1996). In addition, the identity direction O refers to the reference and target object being at the same location. Cardinal directions are purely qualitative (Goyal, 2000), abstract, and generalized concepts among 127 global languages (Brown, 1983). These qualitative concepts, which construct spatial direction information from daily tasks and yield the human as an intelligent system in decision-making, explore the cardinal direction relations between spatial entities in the geographic space and provide the global framework.

Direction is a concept that applies throughout an entire spatial object, yielding potentially varying values. For instance, the cardinal direction from Orono, Maine to Canada is a concept that also applies to any other location in the State of Maine, although the actual cardinal direction values may be different for locations further north or further south in the state. Properties whose values may vary depending on the particular location at which they are taken are ubiquitous in geographic information science. The notion of a field is the typical term for this overarching concept (Goodchild, 1989; Couclelis, 1992). Prototypical spatial fields are temperature (measured in Celsius, Fahrenheit, or Kelvin) or elevation (typically measured in meters or feet). Values are typically sampled at selected locations, as a complete coverage with measurements is infeasible. Values at other locations that have no observations are commonly obtained through interpolation, yielding an approximate value. These properties of a field apply to cardinal directions. This thesis uses for this approach the term “direction as a field.”

Due to the essential construction that makes us consider spatial autocorrelation, sites that are close together typically have similar values. This correlation has been captured in Tobler’s First Law of Geography “Everything is related to everything else, but near things are more related than distant things” (Tobler, 1970). For the purpose of displaying an entire field, the values of a spatial field are typically aggregated, yielding areas[1] in which each location has the same value or a range of values. For example, a contour map, based on elevation samples, introduces isolines that delineate areas that are higher (on one side of the line) and lower (on the other side of the line) than a particular threshold. The cardinal direction from a reference region to a target region also exposes this behavior of a field. Cardinal directions, as a field, allow us to represent the overall arrangement of the geographic space.

The representation of overall arrangements of spatial entities under qualitative concepts leads to the long-term goal of implementing a human-like intelligence system for geographic information systems (GISs) (Freundschuh and Egenhofer, 1997). A GIS is a computer-based system for capturing, storing, manipulating, analyzing, and presenting spatial data (Maguire, 1991). It allows one to view, understand, and query the relations between spatial entities, spatial data, mirroring principles in geographic information science. The overall success of a GIS from the user’s perspective depends on its ability to answer people’s questions without directly having to comprehend the mechanics and structure of the GIS as both its constituent algorithms and implicit data structure. Naive Geography (Egenhofer and Mark, 1995) attempts to create human-like GIS, (Frank, 1993; Mark, 1993), which is challenging for GIS designers. The task

[1] The term area has two different meanings: (1) a region or part of a town, a country, or the world, as in, “the rural areas of Maine” and (2) the extent or measurement of a surface or piece of land, as in, “the area of a triangle.” In this thesis, the term area has only the first meaning.

requires models of spatial concepts that follow the human conception of space. The formalization of spatial concepts has been an active area of interest in fields like GIS, geography, computer vision, cognitive science, psychology, data science, and artificial intelligence (Peuquet, 1994). Human-like intelligent GISs need methods to formulate and process qualitative spatial relations, because so much of human behavior as mirrored by language (Dube and Egenhofer, 2012; Egenhofer and Mark, 1994; Shariff et al., 1998; Egenhofer and Mark, 1995) is predicated in the qualitative domain.

Typically, GISs use quantitative methods to capture spatial relations, modeling geographic space as Euclidean space (Couclelis, 1992; Nunes, 1991). The formalization of spatial relations in Euclidean space leads to computational complexity. It requires sophisticated algorithms to handle geometric computations and tabulate relations between objects. Spatial relations in Euclidean space consider the specific application domain and the intended user's needs (Andrienko *et al.,* 2008); however, an intelligent human system captures geographic space in the qualitative domain (Egenhofer and Mark, 1995). Qualitative methods apply binary base relations, jointly exclusive and pairwise disjoint. For example, the direction relation between spatial entities exactly holds one of the base relations (Renz, 2002). This thesis pursues an approach to capturing the field of a region's cardinal directions from a reference object to a target region. It aims at identifying the locations at which sample values will differ, thereby determining the boundaries between adjacent subregions of the reference object. These subregions are such that each will carry homogeneously the same cardinal direction throughout a subregion's interior.

1.1 Motivation for Formalizing Direction Relations

Formalizing direction relations refers to how different objects are spatially related to each other regarding their direction relations. Broad applications require formalizing direction relations, including robotics (Cohn and Renz, 2008), natural-language processing (Egenhofer and Shariff, 1998), computer vision (Chen *et al.,* 2015), and GIS (Frank, 1996). Formalizations can enhance human knowledge of the physical world by providing a standardized framework for expressing and reasoning about spatial relations (Goyal and Egenhofer, 2001). The need for formalization of cardinal directions remains for broader applications in GIS.

The cardinal direction between two regions builds on point-to-region relations. Direction is a concept that applies throughout an entire spatial object, yielding potentially varying values. For example, Canada being north of the USA is a common interpretation of the direction relation in Naive geography (Egenhofer and Mark, 1995). However, some states of the USA represent single or multiple direction relations. For example, Colorado and South Dakota represent two different single cardinal direction relation throughout their state to Canada, such neighbors of point location on those regions show similar values. Likewise, Maine and Michigan occupy several tuple cardinal direction relations with respect to Canada, for instance, Maine has two regions of same cardinal direction, still, neighboring points occupy the similar cardinal direction values (Figure 1.2). The boundary points between Canada and the USA occupy different cardinal direction relations, preserving the objects' geometric shapes. The set of points in the reference region shows similar cardinal direction values to the target. These are motivated to construct a model to capture the region-to-region direction relations, considering the fundamental properties of space while preserving the geometric shape of the reference and target objects.

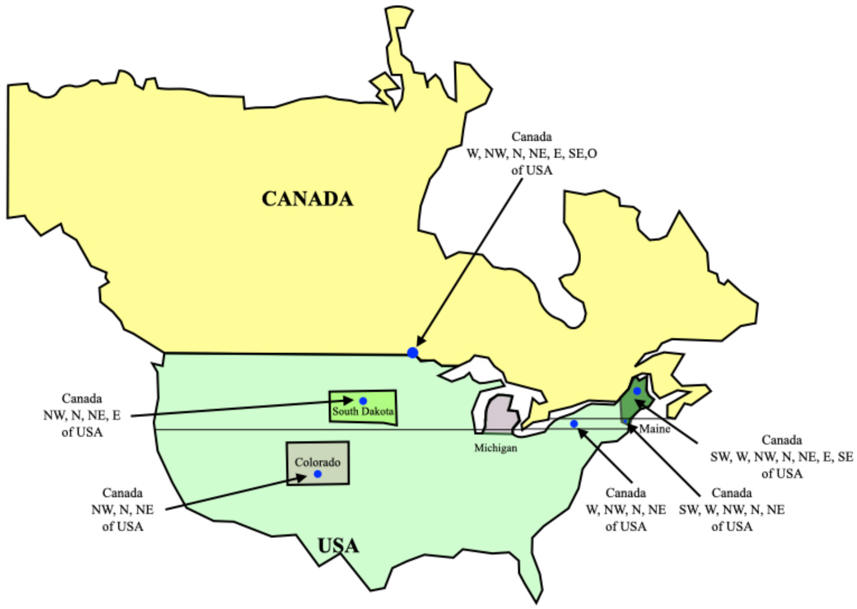


Figure 1.2 Mercator Projection of the US and Canada to represent the cardinal direction relations.

1.2 Cardinal Direction Relations between Simple Regions

The direction relations between point-like objects are well investigated (Peuquest and Zhan 1987; Frank 1996), but people define the direction relations for extended objects without approximations over them (Kritzman and Hahmann, 2018). Two decades of research have determined the direction between extended objects, introducing computational models of cardinal directions. The researchers' crude approximations of the object geometries, such as the point-like approximation (Haar, 1976; Peuquet and Zhang, 1987; Frank, 1991, 1992, 1996) or their minimum bounding rectangles (Abdelmoty and Williams, 1994; Papadias *et al*., 1995; Goyal and Egenhofer, 2001; Skiadopoulos and Koubarakis, 2004; Schneider *et al.,* 2012), fall short for preserveing geometric shapes. For example, the MBR-based model uses minimum bounding rectangles for reference and target objects (Papadias *et al.,* 1995). The direction relation matrix (DRM) considers approximations for only the reference object while preserving the geometry of the target object (Goyal and Egenhofer, 2001). These models consist of approximations since the geometry of reference and target objects influence direction relations at a coarser range.

1.2.1 Problem Statement

To minimize approximations of the current computational models of cardinal directions, a model for cardinal directions must have the following properties:

- The model must provide the qualitative direction from each point in a reference region to the entire target region.
- The model must be free from approximations and needs to preserve the shapes of the reference and target object.
- The model must yield, for reference region, areas of same cardinal direction to a target region.
- The model must be standard among multi-dimensional spatial entities. For example, a single model must be capable of referring to a spatial entity as a set of points.
- The model must refer to the region-to-region direction relation as the union of all of its points' elements.

These properties are the foundation for the present investigation in modeling the computational model for cardinal directions.

1.2.2 Goal and Hypothesis

The primary goals of the thesis are (1) to define a model for capturing the cardinal direction between extended objects, considering the outlined problem statement (Section 1.2.1), and (2) to introduce algorithms to determine for two arbitrarily shaped simple regions just the areas of same cardinal direction, which allow reference objects to have subcomponents with the same cardinal directions. A simple region is any simply connected region bounded by a single Jordan Curve (Egenhofer and Franzosa, 1991). Hence, the hypothesis of this thesis is as follows:

Considering the point-reference direction of all points in the region object overcomes the ambiguities that DRM has with target objects that fall into the reference object's MBR.

The point-reference model creates the continuous direction relations in the embedded space by a finite set of atomic relations (Section 3.3) and their conceptual neighborhood graph (Section 3.5). Chapter 4 defines the region-to-region relations while considering them as point-sets. Chapter 5 validates the algorithms of the region-to-region direction relations. These results lead to testing the hypothesis in section 6.1 by comparing the point-reference and existing MBR-based models of cardinal directions.

1.2.3 Scope of the Thesis

This thesis uses a projection-based model to capture point-reference direction relations between two simple regions. The symbolic representation of the projection-based model records the cardinal direction between a point and a simple region. The point reference model does not apply approximation over the reference and target objects. This research builds the foundation for point-set cardinal direction relations. Such a reference object is considered as a point-set, with each point on the reference object representing the direction relation to the target.

1.2.4 Topics Excluded from the Present Investigation

The thesis mainly focuses on cardinal direction relations, not on direction reasoning (Frank, 1992). The following aspects of direction relations are excluded from this investigation:

- This thesis excludes quantitative, a combination of qualitative and quantitative approaches to define the direction relations are excluded from the investigation.
- This thesis excludes cardinal direction relations for the surface of the sphere or a 3-dimensional space.
- This thesis excludes point-to-line, line-to-line, line-to-region direction relations.
- This thesis excludes considerations about the converseness of the direction relations and composition.
- This thesis excludes the cardinal direction relations between non-simple regions.

- This thesis excludes the Python program to define direction relations from the boundary of the reference region.
- This thesis excludes the direction relations based on the cone-shaped cardinal direction model (Haar, 1976).

1.3 Intended Audience

This research is potentially attractive to many individuals, including researchers in geographic information science, artificial intelligence, sensor design, spatial databases, spatial informatics, computational geometry, computer science, data science, and other fields.

1.4 Major Results

The primary outcome of this thesis is a point-reference cardinal direction model in line with the projection-based model that records the direction between a point-reference and an irregularly shaped simple object without approximating its geometry.

Significant contributions of this thesis are:

- The point-reference cardinal direction model provides a structure to record multiple directions.
- 106 point-to-region direction relations introduce 22 normalized atomic relations.
- The 22 atomic relations are closed under union and intersection, introducing the conceptual neighborhood graph for atomic relations.
- The normalization concepts of 90° rotation and horizontal and vertical mirroring and their combinations introduce the eight sets of 22 atomic relations that lead to the conceptual neighborhood graph for all 106 elements.
- All possible relations of the 22 atomic relations are a power-set of the base relations of point-to-region direction relations.

- The point-reference model and its atomic point-to-region relations define the region-to-region direction relations that allow reference regions to have regions with the same point-to-region relations.
- Point-to-region direction relations and their neighborhood connections confirm cardinal direction as a field.
- Point-set consideration of the reference region conform to the first law of geography. Near points occupy the same direction relations, and distant points vary by the direction relations.

A significant contribution of this thesis is the confirm of the hypothesis. The point-reference cardinal direction model confirms the cardinal direction as a field and overcomes the ambiguities of the DRM.

1.5 Thesis Organization

The structure of the remaining thesis is as follows:

Chapter 2 reviews the literature on direction relations, direction relations in geographic space, qualitative direction relations, and existing projection-based computational models of cardinal directions. This analysis explores the necessity of a point-reference model for directional relations.

Chapter 3 explains the projection-based model for point-to-region direction relations, the symbolic representation of the projection-based model that records point-to-region direction relations and introduces realizable point-to-region relations. This chapter develops 22 normalized atomic relations for point-to-region relations and constructs its conceptual neighborhood graph.

Chapter 4 uses the results of Chapter 3 to define region-to-region relations and introduces cardinal directions as a field. The chapter also introduces an algorithm to define the areas of same cardinal direction and the interpolation between cardinal direction relations.

Chapter 5 uses the algorithms from Chapter 4 to construct a Python program to create a spatial database for recording the areas of same cardinal direction around a target object. The program defines the areas of same cardinal direction at the exterior of the target object.

Chapter 6 compares the point-reference approach with the existing cardinal direction models, such as the direction relation matrix and the object interaction matrix.

Chapter 7 provides the conclusions and proposes future work in direction relations motivated by this thesis.

CHAPTER 2

COMPUTATIONAL MODELS OF CARDINAL DIRECTION RELATIONS

"The whole practice and philosophy of geography depends upon the development of a conceptual framework for handling the distribution of the objects and events in space" (Harvey, 1969).

This chapter provides the foundation for the rest of this thesis by introducing the key spatial principles that will be referenced throughout the thesis.

The human perception of space is qualitative (Egenhofer and Mark, 1995), and it is an intelligent system with great computational capability that senses spatial knowledge in daily life. A long-term objective is to develop a human-like intelligent system for geographic information systems (Robinson *et al.*, 1986). Such systems would be capable of handling any sophisticated queries about spatial features on the surface of the Earth and spatial data management in human-centric terms. This thesis develops a model for qualitative direction relations between spatial entities in this context.

This chapter reviews direction relations, geographic space, quantitative and qualitative approaches to modelling direction relations in geographic space, and cardinal directions to describe direction relations for point-to-point and region-to-region objects, each foundational to the adequate grounding of the thesis.

2.1 Direction Relations

Relations exhibit the connections or associations between things or people. In mathematics, relations are classified into two types: (1) internal relations and (2) external relations. *Internal relation* has some relation about a set to its elements. *External relation*s exist between the sets, such that they have ordered pairs as members with the specific relations (Pincock, 2004).

Sets denote the ordered elements, whereas relations associate elements of sets. If the order of the elements changes, the relation would change, except for the equal relation. In detail, R and T are two sets of ordered pairs. If R has relations with T, then the values of R are referred to as the *domain*, and the values of T as the *range* (Cetin and Dubinsky, 2017). The relations that exist between a domain and range are a subset of the Cartesian product of the sets R and T (RxT). They are captured as a tuple (domain, relation, range).

Ordered pairs in a set with relations define the relative location of the pairs (Lu and Milios, 1997). For example, a set of real numbers A has an internal relation R. If R exists in a set A, then R is a subset of the Cartesian product of A with itself (AxA). R defines the sequence of numbers that specify locations in an n-dimensional space. When n is equal to 1, the set of locations represents the 1-dimensional space. The relations between the elements in ordered pairs determine the relative position of the domain and range and how the elements in the set are related to each other.

The *base relation* is common to define any ordered pairs on or between sets. For example, in a set of real numbers, the relation *greater than* applies between any non-equal pairs. The base relations for a set could be a single base relation or a set of base relations. The minimum number of base relations describes the set more concisely, making it easier to prove theorems and perform operations on the set. For example, a set may be defined as a set of real numbers in which all elements are *greater than* a certain value (Bekessy and Demetrovics, 1979).

The mathematical relation-based set concepts are applicable to convey relations between physical objects (Pincock, 2004). In the physical world, internal relations are often based on the spatial distribution of objects or entities in space. The relative position of the objects is explained by their spatial relations. They are captured as a spatial relation tuple (reference object, relation, target object).

Spatial relations capture three aspects: (1) topological relations, (2) direction relations, and (3) distance relations (Renz, 2002). Topological relations explore how two objects interact on the basis of connectivity and containment within the embedding space. The base topological relations are defined by the 9-intersection between simple regions (Egenhofer and Herring, 1992). They are used to explain any topological configurations between two simple regions in R^2. Distances are captured quantitatively as metric forms between the objects and qualitatively as *far* and *near* (Hernandez *et al.*, 1995). Distances may be used as a substitute to refine topological and direction relations (Egenhofer and Dube, 2009; Dube *et al.,* 2015).

Directions are an important component of understanding space in people's daily routines. Directions provide a complimentary view to topology (Sharma, 1996). Direction relations define the relative orientation of objects with respect to a reference direction. Typical direction relations are quantitative point-to-point relations in contrast to vectors (Moratz, 2006) or a qualitative approach to capture direction relations (Peuquet and Ci-Xiang, 1987; Frank, 1991; Goyal and Egenhofer, 2001). This thesis focuses on qualitative direction relations in terms of cardinal directions. Cardinal direction relations refer from a reference object to a target object in a fixed reference frame.

2.2 Geographic Space

Geographic space is the combined extent of all the spaces that people interact with in daily wayfinding and navigation activities (Mark and Frank, 1996). Since the arrangement of the spatial entities in geographic space cannot be observed from a single viewpoint, it is referred to a large-scale space (Kuipers, 1978). Geographic space can be represented through two conceptual models: (1) object-based representations and (2) location-based representations (Peuquet, 1988). An object-based conceptual model groups the objects based on the relations between the entities and properties. For example, a building object, would be of different types such as residential home, factory, or school. The location-based conceptual representation emphasizes the way people conceptualize the space in daily routines. This method allows for continuous space, multiple scales and context of detail, and spatial relations (Peuquet, 1988). The location-based view spontaneously conceptualizes the geographic space, but individuals apply the location-based spatial relations within their visual range.

Naive geography introduces a set of theories about how people conceptualize geographic space. (Egenhofer and Mark, 1995b). The naive view of representing geographic space is a key to implementing intelligent systems that can handle geographic information consistent with human reasoning (Frank, 1993; Mark, 1993). Representation of geographic space is essential in geographic information science (Yuan, 2001), earth sciences (Walsh, 2007), artificial intelligence (Smith, 1984), cognitive science (Mark *et al.*, 1999), robotics (Cohn and Renz, 2008), spatial analysis (Miller, 2000), and spatial database systems (Rigaux *et al.,* 2002). This thesis handles geographic space in line with Naive Geography, focusing on direction relations.

2.3 Direction Relations in Geographic Space

An intelligent system for geographic information is essential to having a representation of geographic space (Peuquet, 1999). Direction relations between spatial entities enable the configuration of geographic space (Frank, 1992). The fixed reference is important to define the allocentric directions, which must be common within the geographic space of interest, such as the direction of the sunrise or the Earth's rotational axis (Clementini *et al.*, 1997). A fixed reference aligns direction relations (Shekhar *et al.*, 1999). Direction relations specify the corresponding location of the target objects with respect to the reference object.

Spatial entities often contain two components: (1) the spatial component, referred to as the shape and location and (2) properties, such as the name or a unique id (Devogele *et al.*, 1998). The spatial component often refers to the given fixed direction of the corresponding space. Spatial details define direction relations between the objects.

Two methods capture the direction relations between the spatial entities: (1) quantitative methods, which treat geographic space as a continuous Euclidean space in the 2-dimentional plane, and (2) qualitative methods, which are not numerical and align with human conceptions of geographic space. For example, object A is left of object B (Renz and Mitra, 2004). In the following sections, direction relations between reference and target objects are described quantitatively (section 2.3.1) and qualitatively (section 2.3.2).

2.3.1 Quantitative Methods of Direction Relations

In current quantitative approaches, geographic space is treated as Euclidean space (Couclelis, 1992; Nunes, 1991). Its fundamental unit is a point, which has a unique numerical location based on a local or global reference system (Whitney, 1955). The cross product of horizontal and vertical one-dimensional Euclidean space yields a two-dimensional Euclidean space (Figure 2.1). It partitions the geographic space into four quadrants, and the intersection point is often referred to as the reference point (Belkin and Niyogi, 2003).

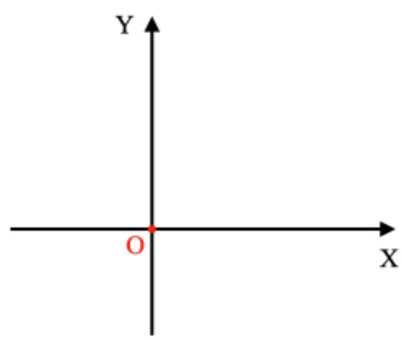


Figure 2.1 Two-dimensional space.

Euclidean space uses points to represent the spatial entities whose shape is negligible. If the geometry is necessary for the spatial information, then a series of points represents the geometry of the objects (Bertolotto, 1998). For example, on a cartographic generalization large-scale map, one typically represents cities as point-like objects, while on small-scale map, a city would be a polygon.

In Euclidean space, the point approximation works well for capturing direction relations. The direction of a target point is expressed in a plane angle α in 2-dimensional Euclidean space. It is an angle between the vertical fixed reference and a line connecting the reference point O and a target point T (Figure 2.2). The plane angle defines the direction relation between points. This approach treats the Earth's surface as flat space. The large-scale representation of geographic space suffers from flat surface approximation, necessitating the use of a global coordinate system (Burrough *et al.*, 2015).

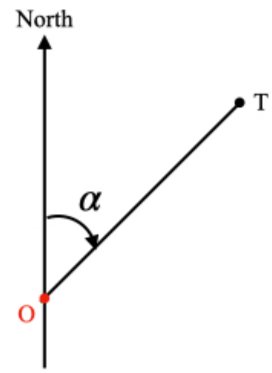


Figure 2.2 Direction relation between reference object and a target object T by a plane angle representation.

The global coordinate system is important for figuring out direction relations on the Earth, which is roughly shaped like a sphere. The global system deals with geographic space as an embedding space on the spherical surface of the Earth (Dutton, 2006). The direction relations to a point-like target object on the Earth with respect to a reference point is defined by azimuth, or a pair of angles (latitude, longitude) (Laurini and Thompson, 1992).

The azimuth is an angle measurement typically ranging between 0º and 360º. The value zero indicates true North, which is defined by the Earth's rotation and passes through the North and South poles of the Earth. Azimuth is a clockwise measurement from true North to a great circle, called a *meridian*, which passes through the reference point and a point-like target object (Figure 2.3a). The pair of angles to define the direction relation follows two fix references, such as Earth's Greenwich meridian and the Equator (Figure 2.3b).

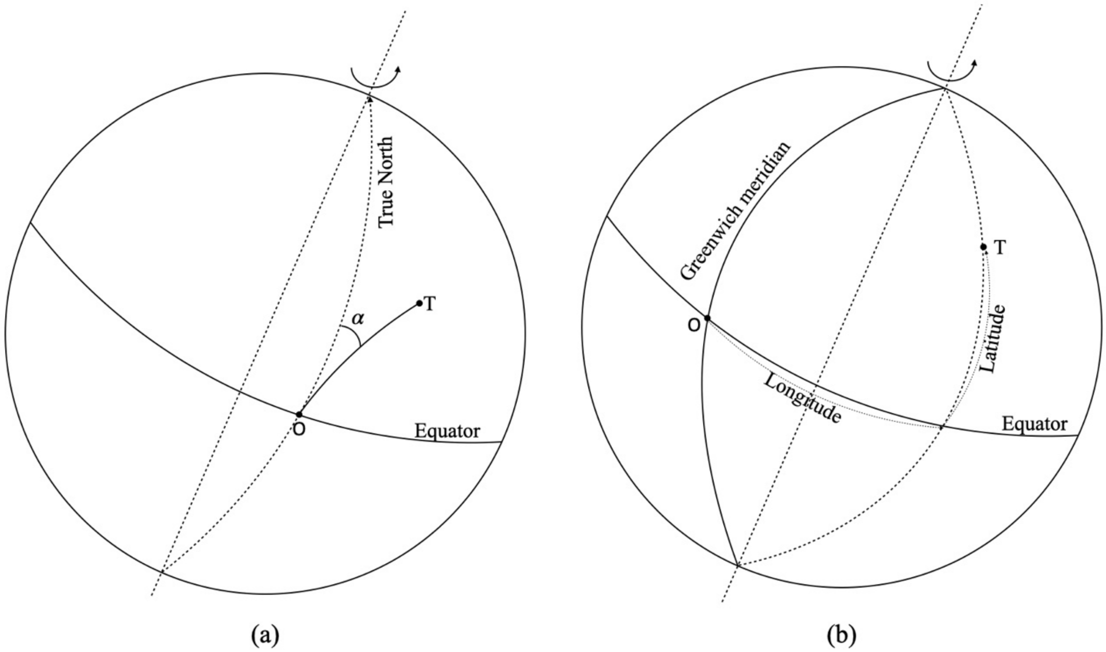


Figure 2.3 The global reference for direction relation of a point-like target object T: (a) by azimuth; (b) express by longitude and latitude.

Quantitative methods of representation are particularly incapable of conveying a sense of geographic space by utilizing the spatial objects' natural geometry (Nunes, 1991). The method is unlikely to be related to human spatial perception (Freksa, 1991; Freundschuh and Egenhofer, 1997). The representation of geographic space needs to focus on the human conceptual frame over spatial entities and their relations (Harvey, 1969).

2.3.2 Qualitative Methods of Direction Relations

The way people perceive geographic space is qualitative, largely but not exclusively (Egenhofer and Mark, 1995). It represents the spatial relations for everyday activities (Harvey, 1990). For example, a daily routine task like spatial navigation in geographic space relies on an individual's cognitive maps (Tolman, 1948), which are mental models of spatial relations. Cognitive maps store the fundamental and important spatial relations that are relevant to the specific task. This process improves inference power and does not depend on numerical calculations. The process of human conception used to generate a cognitive map is the key information for geographic information systems (Freundschuh and Egenhofer, 1997). A cognitive map is frequently considered to contain three levels of spatial knowledge: (1) declarative, (2) procedural, and (3) configurational knowledge (Kitchin and Blades, 2002). *Declarative knowledge* is based on the collection of spatial features like points, landmarks, lines, routes, and areas (Liben, 1981).

Procedural knowledge is wayfinding information, that acquires order, distance, and direction between spatial entities. The detailed spatial information is captured in a cognitive map using configurational knowledge. *Configurational knowledge* is a combination of information, such as angles, orientation, direction, and distance that is based on the spatial relations and relative positions of the spatial entities (Golledge *et al.,* 1987). Individuals' learning environments are configured with experience and configurational knowledge (Freundschuh, 1991).

Human cognitive maps preserve spatial knowledge based on reference frames, a component of descriptive knowledge (Tolman, 1948; McNamara *et al.,* 2003; Epstein and Ward, 2010). Frames are used to represent the relative positions of the spatial entities (Avraamides and Kelly, 2008), such as the *egocentric frame* (in front of, in back of, to the left of, and to the right of), which captures the body-centered relations and subject-to-object relations (Elkind, 1967). The *allocentric frame*, which determines the relations between the objects, enables navigation in different environments (Burns, 1999; Colombo *et al.,* 2017). Those frames are subjective and not fixed between the individuals (Foreman and Gillett, 1997).

The fixed frames of reference are defined by landscape axes: (1) axis of symmetry (referring to natural features); (2) main line axis (line features); and (3) landmark axis (salient features) (Tversky, 1981). The fixed reference frame introduces distortions of alignment and rotation in the fixed frames, and leads to spatial communication biases (Lloyd, 1989). Spatial prepositions, such as above, in, and behind (Retz-Schmidt, 1988), and contextual information introduce the standard conceptual frames.

Three types of frames deal with spatial prepositions and contextual information of reference frames: (1) intrinsic, (2) deictic, and (3) extrinsic (Table 2.1). These frames correspond to the reference and target objects. The extrinsic global frame is a survey description that resembles the Earth's poles and leads to a system of four cardinal directions (North, East, South, and West) that aid people in navigating in an unfamiliar environment (Mark *et al.,* 1999; Goyal and Egenhofer, 2001; Kitchin and Blades, 2002).

In geographic space, human conception is subjective and relies on the egocentric and allocentric frames. Since the Stone-Age, the subjective representation of geographic space has been combined with the extrinsic frame, such as cardinal directions (Schmidt and Schlosser, 1984). The Stone-Age cardinal direction-based representation of geographic space provides evidence for cognitively equal direction captures and point-reference concepts.

Table 2.1 Reference frames and arguments on the corresponding space (Retz-Schmidt, 1988).

Reference Frame	Space	Arguments
intrinsic	The embedded space of the reference objects is divided into regions.	Reference Object Target object
extrinsic	Space is defined by the accessibility of the reference object, its motion, or gravitation of the Earth (vertical axes determination).	Reference Object Target Object Contextual factor (Point of view)
deictic	Space contains primary and reference objects and the point of view of the reference frame.	Reference object Target object Point of view (speaker or hearer can serve as point of view)

This thesis focuses on developing the global frame of cardinal directions to capture direction relations in geographic space.

2.4 Cardinal Directions

Cardinal directions are qualitative perceptions that capture direction relations. These are generalized spatial concepts (Goyal, 2000), applicable among 127 global languages to communicate about spatial relations (Brown, 1983). The frame of the cardinal directions often has a fixed reference object, such as the sun, moon, or stars.

The concept of a sun-oriented representation of spatial knowledge has long been important in geographic space and is used as a standard reference point. Initially, East and West cardinal directions referenced the direction of sunrise and sunset respectively. With the observer's egocentric frame, these two cardinal direction symbols yield East as right and West as left. This configuration allows us to define the front and back of the observer to the North and South (Frank, 1991). The observer's entire space is covered by these four cardinal directions.

The compass is frequently used to represent relative direction relations on maps. A compass uses magnetic north, which is determined by the Earth's magnetic field. The compass yields more subdivisions based on the angular directions between the primary cardinal directions, such as North, East, South, and West (Frank, 1992).

These fundamentals of cardinal directions introduce different qualitative frames of reference models, such as human conception of geographic space (Frank, 1996). The representation of geographic space continues to adhere to the fundamentals of cardinal directions in order to improve the inference power of direction-based computational models.

2.5 Point-Based Objects

The cardinal direction relations between point-like objects are captured by two fundamental models: (1) cone-shaped models and (2) projection-based models. The cone-shaped cardinal direction model concerns the reference object as a point. The model assigns the relative direction of the target object by subdividing the embeddeing space of the reference object into four nonoverlapping equal cones, where represent the four primary cardinal directions: North (N), East (E), South (S), and West (W) (Figure 2.4a) (Haar, 1976).

Then, the corresponding direction zone of the target objects, define the direction relation from the reference to the target objects. This model suffers from the distance between reference and target objects.

The cones are considered an extension of the triangle model in 2D plane space (Peuquet and Ci-Xiang, 1987). Depending on the distance between the reference and the target object, the model defines the target object being in a specific direction (Frank, 1992). The model further considers eight directions. Intermediate secondary directions are Northeast (NE), Southeast (SE), Southwest (SW), and Northwest (NW) between primary directions (Figure 2.4b).

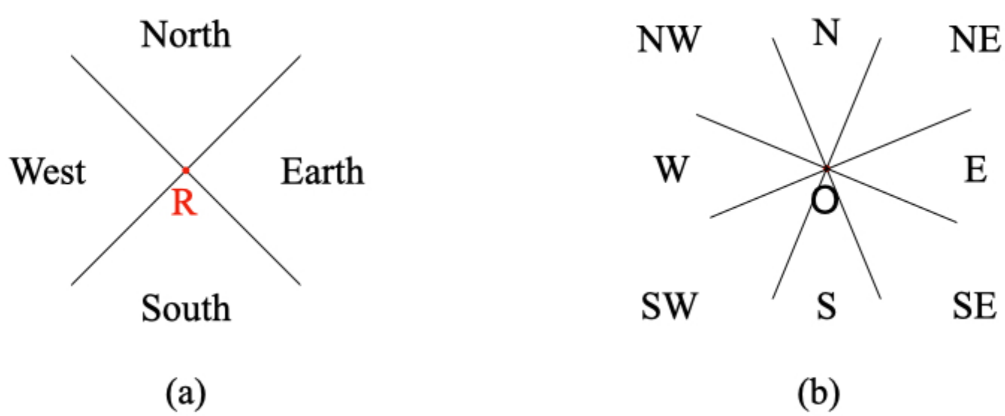


Figure 2.4 The cone-shaped models: (a) four directions model and (b) eight-directions model

If the point-like reference and target objects coincide, the direction between them is neutralized and introduced as the *same* direction, an identity symbol. The primary direction symbols become five, including an identity element. The identity element removes the constraints on inferring the direction relations between reference R and target T. For example, a direction from R to T (dir(R, T)) and its converse, do not have to be the same to infer its composition direction. Most of the compositions from the eight directions are approximate (40 out of 64) (Frank, 1992).

The projection-based model (Frank, 1991), is based on two half-planes. A horizontal subdivision of space yields North and South (Figure 2.5a), while a vertical partition yields East and West (Figure 2.5b). Those four half-planes together form a single projection-based system for cardinal directions, tiling the space into equal quadrants and introducing four secondary cardinal directions: Northeast (NE), Southeast (SE), Southwest (SW), and Northwest (NW) (Figure 2.5c).

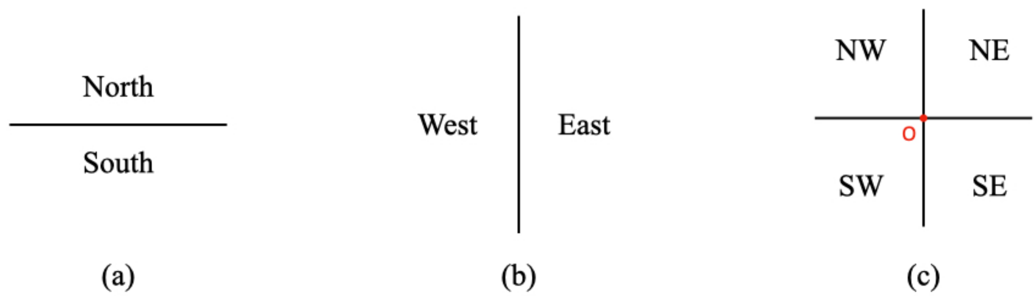


Figure 2.5 Construction of projection-based model. (a) North-South half-planes, (b) East-West half-planes, and (c) the projection-based cardinal direction model (Frank, 1991).

The model has an intersection point O, called a *neutral zone*, which represents the direction from the reference point to itself. Two vertical and two horizontal direction lines begin from the intersection point, representing the primary cardinal directions (N, E, S, and W), and the adjacent directional boundaries for four regions that represent the secondary directions.

The projection-based model provides crisper inference then the cone-shape model. For example, the composition of all nine directions shows 56 exact cases out of 81. Therefore, this thesis follows the projection-based cardinal direction model as its foundation for cardinal directions between non-point objects.

2.6 Extended Objects

Extended objects include linear and areal shapes in the plane. Extended objects may have variety of shapes; the rectangles whose edges are horizontal and vertical. They align well with the projection-based model. For other shapes, especially concave or convex, that is not the case. Therefore, one often introduces the minimum bounding rectangles (MBRs) to define direction relations between extended objects.

Direction relations between extended objects are typically considered from convex or concave shapes to MBRs as approximations. The MBR is a simple form of a complex geometric shape. The boundaries of the MBR are horizontal and vertical, which are parallel and pass through the extrema (North most, East most, South most, and West most) of the approximated object.

The MBR-based direction model considers reference and target objects' MBRs. Spatial relations between reference and target MBRs define the direction relation between the corresponding extended objects. The MBR approximations are projected along each axis, which is a 1-dimensional ordering that is exploited to define MBR relations by a pair of 1-dimensional relations (Figure 2.6). A set of thirteen 1- dimensional interval relations (Allen, 1983) is applied between MBR projections. The MBR model distinguishes 13 * 13 = 169 direction relations (Figure 2.7) (Papadias *et al.,* 1995).

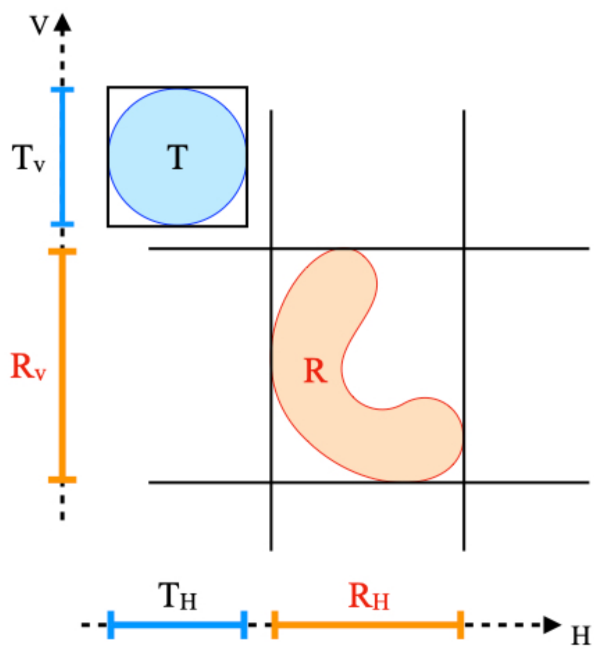


Figure 2.6 Projection of two minimum bounding rectangles on the horizontal and vertical axes (Papadias *et al.,* 1995).

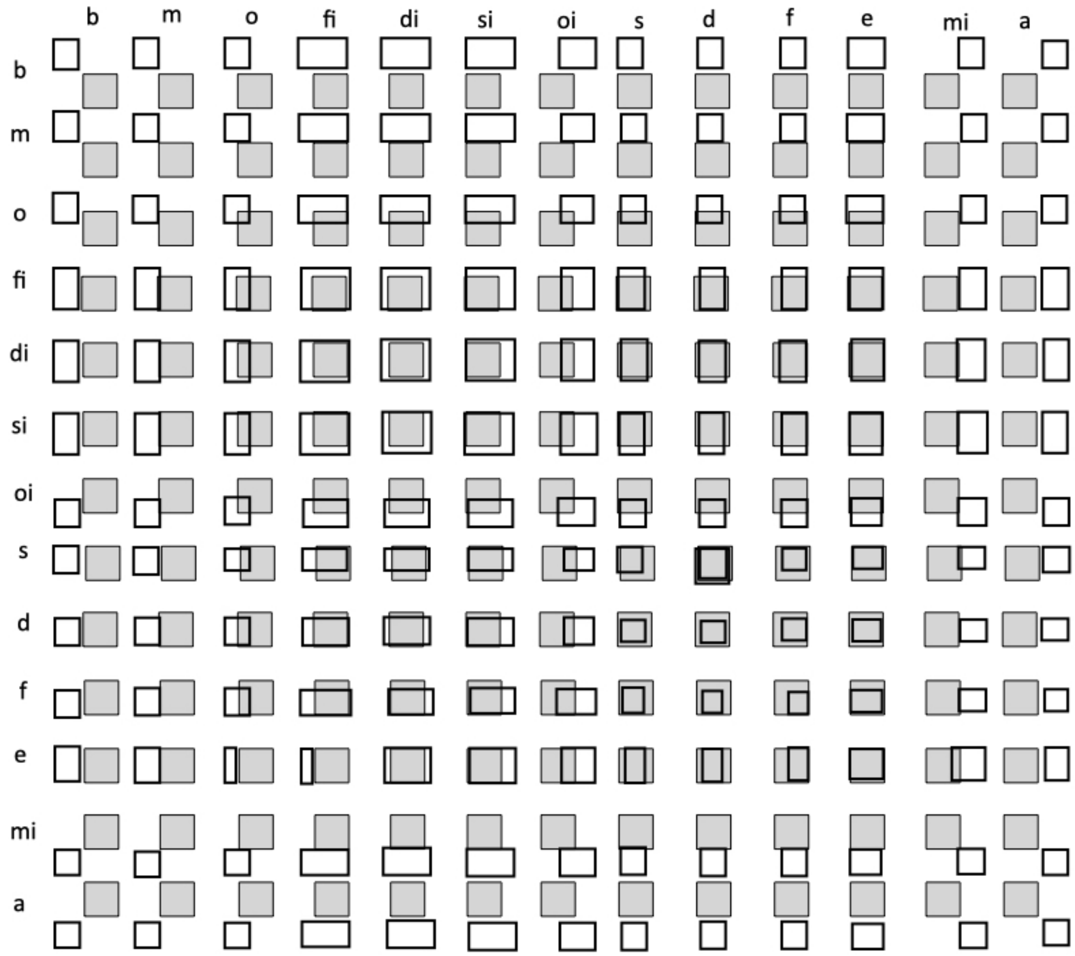


Figure 2.7 Allen interval relations: b = before, m = meet, o = overlap, s = starts, d = during, f = finished, fi = finished by, di = during inverse, si = started by, oi = overlapped by, mi = met by, and a = after, are applied to determine 169 distinct direction relations between reference and target MBRs (Papadias *et al.,* 1995).

The MBR-model produces fewer distinct direction relations between the reference and target objects. For instance, if the reference MBR contains the target MBR, then the MBR-model defines one of the 169 relation pairs as (during, during) (Figure 2.8a); however, the same relation applies between other direction configurations with respect to the reference object R (Figure 2.8b).

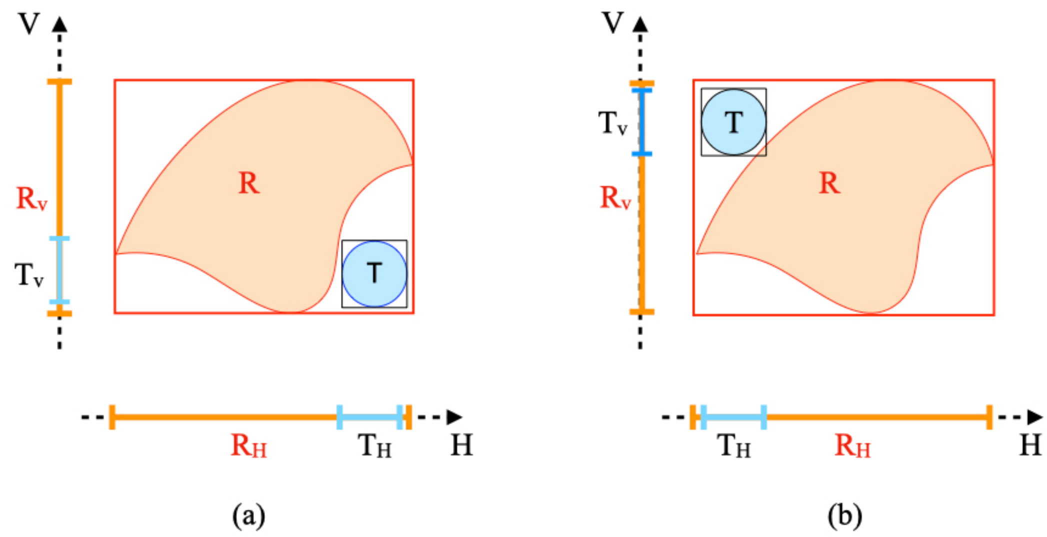


Figure 2.8 The MBR-based model defines the same pair of relations (during, during) between R and T for different direction configurations, (a) and (b).

The Direction Relation Matrix (DRM) addresses the inconsistency of the MBR-model in defining the direction relations between two simple objects. Rather than using MBRs to approximate both the reference and target objects, the DRM uses MBR only to approximate the reference object, while preserving the geometric shape of the target object (Goyal, 2000). DRM uses both the reference MBR and the projection-based method to determine the cardinal direction between two simple regions.

The MBR of the reference object forms the neutral zone of DRM. The extended edges of the MBR subdivide the exterior of the reference MBR into eight tiles. Then the projection-based model assigns the cardinal direction of each tile as N, NE, E, SE, S, SW, W, and NW. The direction of the reference MBR is O (the neutral zone, which equals the MBR) (Figure 2.9).

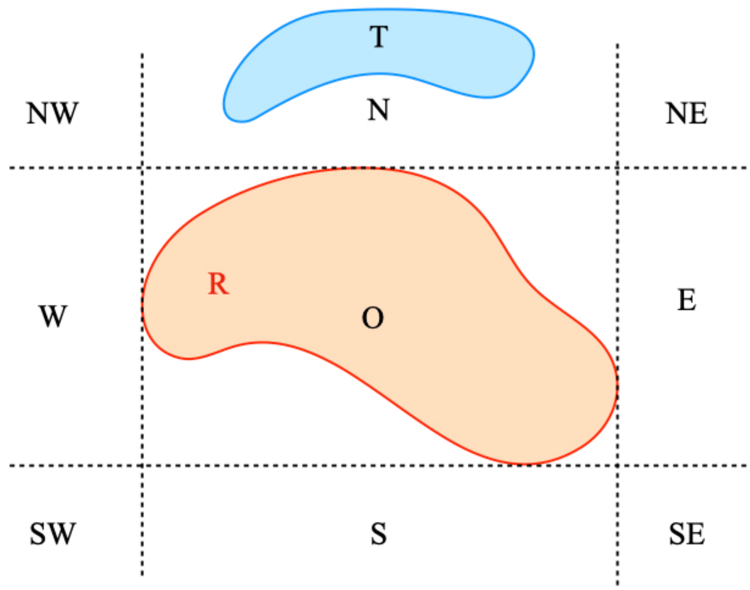


Figure 2.9 Direction Relation Matrix to define the cardinal direction relation from reference object R to target object T (Goyal and Egenhofer, 2001).

The DRM captures the direction relation of the target object in a 3x3 binary matrix. From 2^9=512 possible binary configurations, the DRM identifies 218 possible symbolic representations between two simple regions (Goyal and Egenhofer, 2001) (Figure 2.10).

The DRM is superior at preserving the geometric shape of the target object since it eliminates the MBR approximation over the target object. The shape of the reference object, however, causes an inconsistency in defining the direction relation between two simple regions. For example, if the target object falls into the reference MBR without intersecting the reference object, DRM specifies the direction as the *same*, but since direction relations exist between the reference and target objects (Figure 2.11a), in addition, the *same* direction relation exists when target and reference objects intersect within the reference MBR (Figure 2.11b).

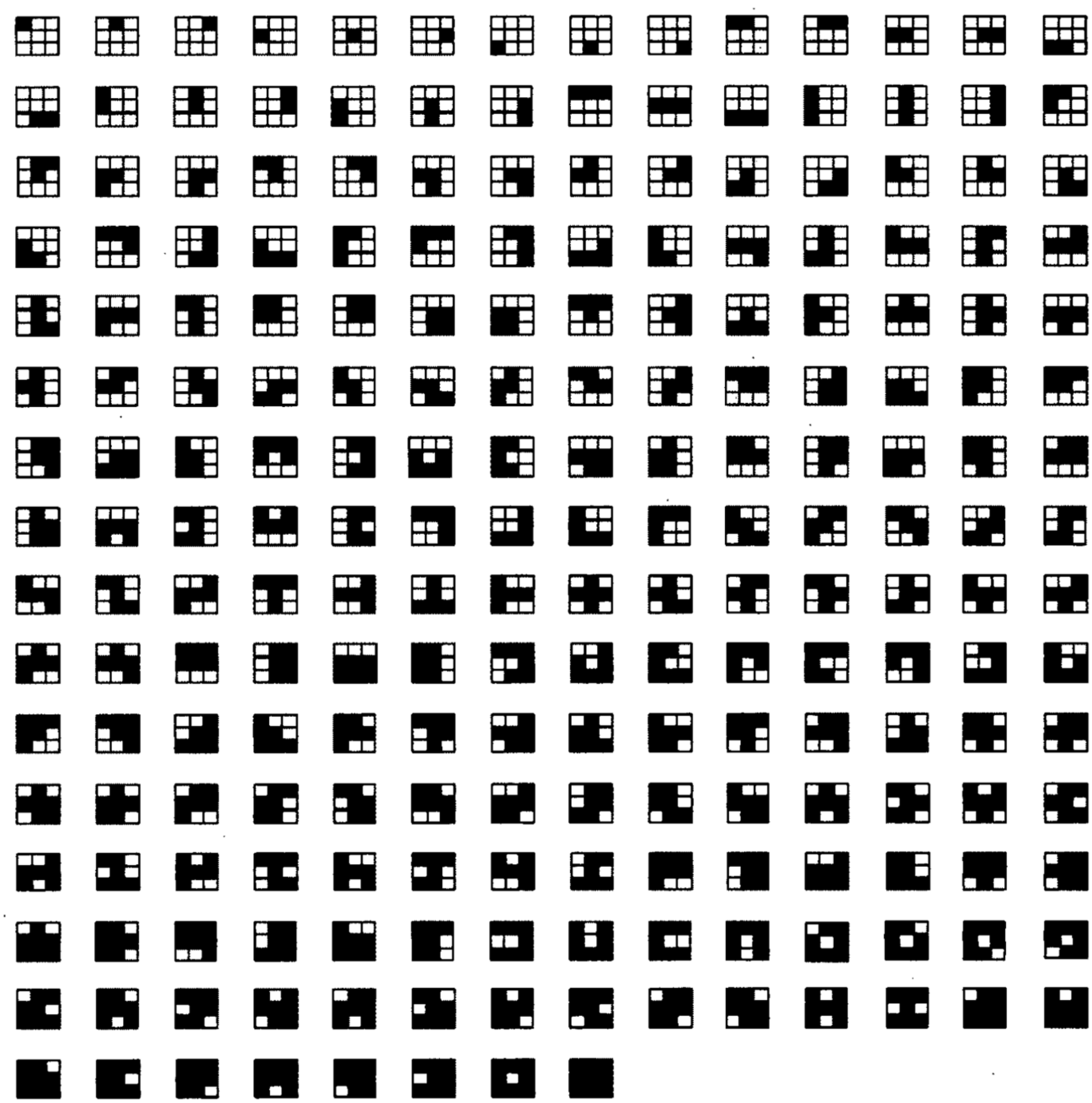


Figure 2.10 Direction-relation matrix symbolic representation between simply connected objects (Goyal and Egenhofer, 2001).

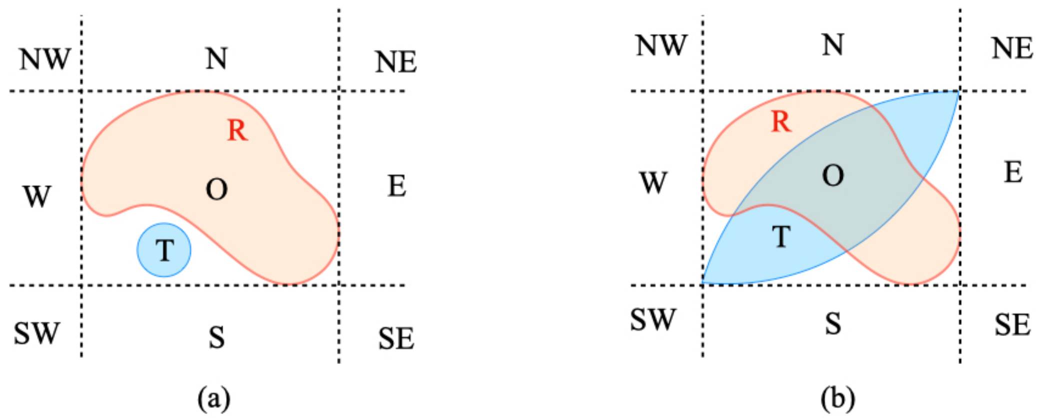


Figure 2.11 DRM-based direction relations for different configurations are (a) dir (R,T) = (O), and (b) dir (R,T) = (O).

The heterogeneous cardinal direction relation model (Kurata and Shi, 2009) attempts to overcome DRM shortcomings. It uses DRM and the half-plane model, subdivides the reference object's DRM, based on the intersection between the target region and DRM's direction tiles. This method, however, projects the spatial object onto the horizontal and vertical axes to assign cardinal directions, converseness, and compositions. Projections of spatial objects approximate the geometry of reference and target objects.

The object interaction matrix (OIM) (Schneider *et al.,* 2012) is another approach to overcome the coarse approximation problem, to define direction relations between complex regions. The model generates a grid based on MBRs of the reference and the target regions (Figure 2.12a). It is a two-phase model: (1) The tiling phase determines the tiles of nine cardinal directions of reference and target objects and intersections between them. This model records the existence of operant objects in the grid cells as 0 for an empty cell, 1 for a reference, 2 for a target object, and 3 for both a reference and a target object. (2) The interpretation phase, which determines the cardinal directions for the results obtained from the first phase (Figure 2.12b).

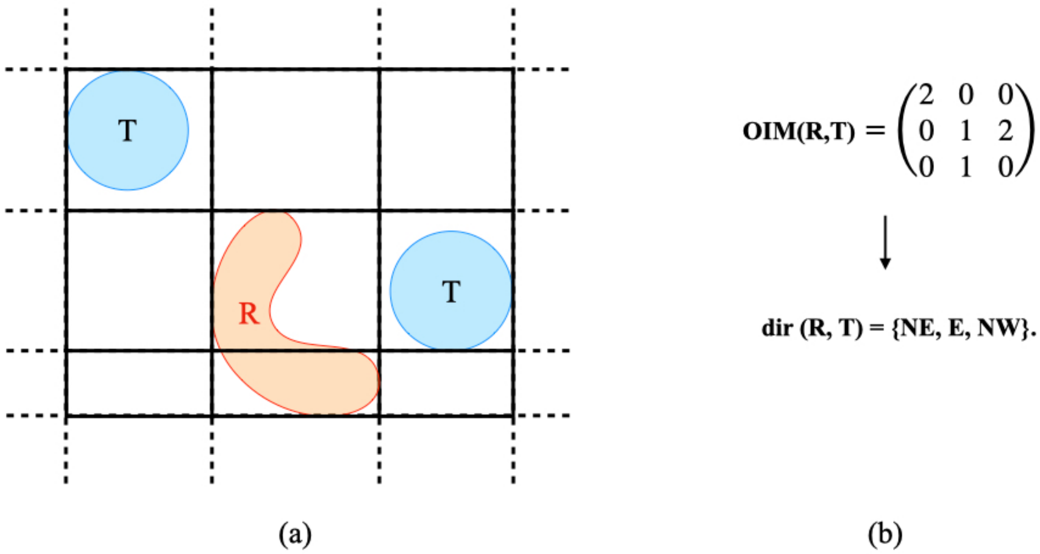


Figure 2.12 OIM (a) defines the object interaction grid for reference region R and complex region T and (b) derived object interaction matrix and corresponding direction (Schneider *et al.*, 2012).

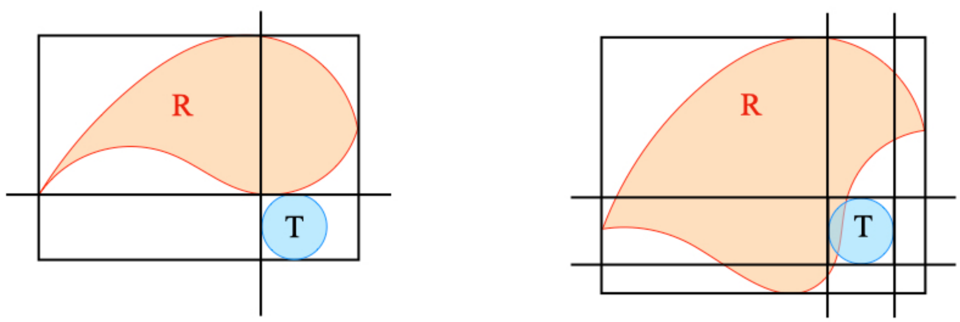
The OIM's tiling phase is similar to the MBR approximations of the DRM. The common grid cell of both reference and target objects does not show the direction relations. For example, the cardinal direction relations between Brazil and Argentina are defined by the OIM grid cells, but the common grid cell, which captures the parts common to both countries, does not assign the direction relations to some parts of the two countries (Figure 2.13).

The OIM is not homogeneous based on the number of the grid cells and changes away with different configurations. For example, a common extremum such as an equal minimum and maximum of reference and target MBRs generates a 2x2 grid cells (Figure 2.14a), while overlapping MBRs introduce a 3x3 grid (Figure 2.14b). This method addresses the converse problem; however, it is similar to the DRM in the tiling phase, and it shows less accuracy in the interpretation phase (Li and Liu, 2015).

$$\mathrm{OIM}(A,B) = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 3 & 2 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathrm{dir}(B,A) = \{S, W, SW, SE, O\}$$

Figure 2.13 OIM model based cardinal direction relation between Argentina (A) and Brazil (B).



$$\mathrm{OIM}\ (R,T) = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

(a)

$$\mathrm{OIM}\ (R,T) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(b)

Figure 2.14 OIM grid size variation for different configurations of reference (R) and target objects (T).

2.7 Summary

Qualitative direction relations are an important spatial concept to represent geographic space. This spatial concept coincides with the human conception of space, which may lead to a human-like intelligent system for geographic information systems. Cardinal directions are frequently used to define direction relations in geographic space. The generalized concepts of cardinal directions generate computational models to capture qualitative direction relations.

The projection-based cardinal direction model captures the direction relations between point-like objects. Direction relations between extended objects initiated with crude approximation such as minimum bounding rectangle (MBR). The MBR-based model treats both reference and target objects as respective MBRs. Shortcomings of the MBR-based model are overcome by the direction relation matrix, which uses a projection-based model and an MBR approximation over the reference object, while preserving the geometric shape of the target object.

This thesis focuses on using the projection-based model as a foundation to define cardinal direction relations between the extended objects without any approximation over the reference and target objects.

CHAPTER 3

CARDINAL DIRECTIONS FROM POINT TO SIMPLE REGION

"The point-set approach is the most general model for representation of spatial relations."

(Egenhofer and Franzosa, 1991)

The representation of the direction relations follows the point-reference concept, because they may vary between locations; therefore, the fundamental unit of the space, the point, is used to capture the direction relations.

The projection-based model defines point-to-point direction relations and introduces nine atomic relations (Frank, 1991). In accordance with the projection-based model, this chapter details the point reference model to define point-to-region atomic direction relations.

Section 3.1 details the projection-based model for point-to-region direction relations. Section 3.2 introduces the fixed set of point-to-region base direction relations. In Section 3.3, direction relations are normalized for atomic point-to-region direction relations. Following, Section 3.4 applies set operators, such as union and intersection over the atomic relations. Subsequently, Section 3.5 introduces the conceptual neighborhood graph for the atomic relations and for a set of base relations.

3.1 Projection-Based Cardinal Directions Model

The projection-based model is a point-to-point model (Figure 3.1a), yielding nine atomic cardinal directions from a reference point to a target point. If the target object replaces a simple region, the direction values may expand to more than one atomic direction relations (Figure 3.1b). In this thesis, simple regions are considered as point set.

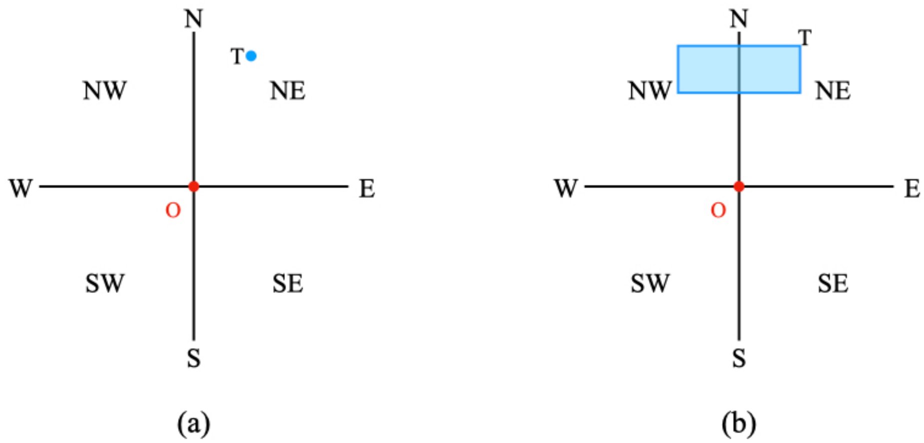


Figure 3.1 Projection-based cardinal directions model to define (a) point-to-point and (b) point-to-region.

The direction relations from a point to a region, captured by a projection-based model, are represented in a 3x3 iconic model that includes circles and squares. A circle for a point or horizontal or vertical line that extends from center, denotes primary cardinal direction symbol (Figure 3.2a). The central circle represents only point-reference. The square (Figure 3.2b) is the secondary cardinal direction symbol for a region or line that does not consider the point-like objects. The highlighted circles and squares represent the existence of the target. The complete model captures the direction relation from a point reference to a target object (Figure 3.2c).

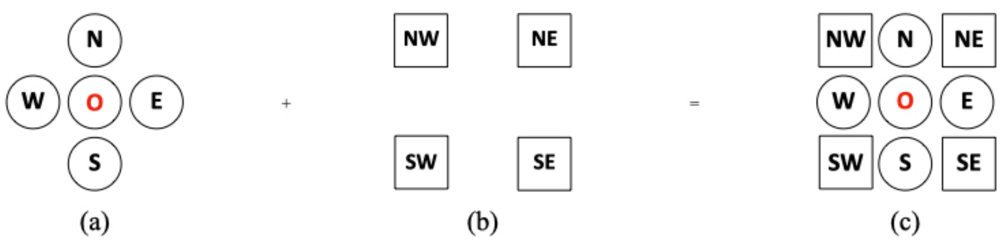


Figure 3.2 The iconic representation of cardinal directions: (a) circle show primary cardinal directions and a neutral point (O), (b) squares show secondary cardinal directions, and (c) integrated point-based cardinal direction model.

A single target point can only lie in exactly one of the nine cardinal directions (Figure 3.3a). If a target point coincides with the reference R, it occupies the central element of the model. As such, the nine cardinal directions are mutually exclusive for point targets. An extended target object (i.e., a region or a line) can extend through more than one of the nine cardinal directions (Figure 3.3b). At some instances, the target region may extend through all nine directions at the same time (Figure 3.3c).

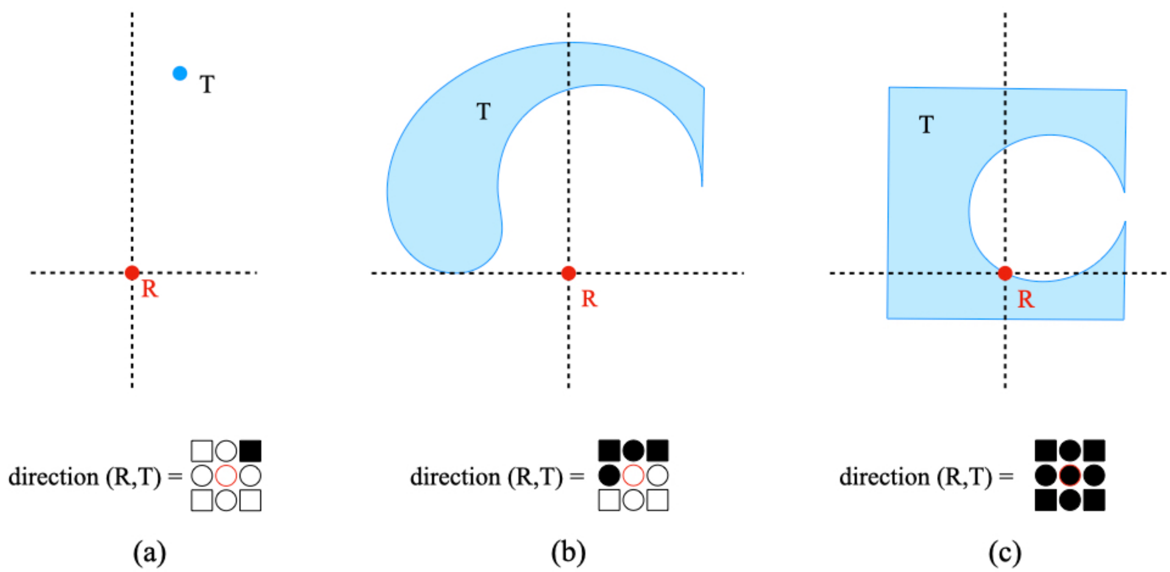


Figure 3.3 A point-based cardinal direction model captures (a) point-to-point direction relation, (b) point-to-region direction relation, and (c) if target object extends all nine directions.

3.2 Realizable Point-to-Region Cardinal Direction Relations

The nine elements of the point-based cardinal direction model could be represented by a 3x3 binary matrix. If the target object exists, highlighted symbols denote by one, otherwise, they are zero. These binary relations yield 2^9 = 512 possible different configurations. However, not all the configurations represent simple regions. For example, if a configuration shows a target object only in the NW and NE cells, it would be a separated region, because, without a connection between region tiles, a simple region could not continue from NW to NE, therefore, NW and NE alone are one of the non-realizable configurations.

This section determines the consistency constraints to define a set of base direction relations, which are only realizable between a point reference and simple regions.

3.2.1 Consistency Constraints

The 3x3 matrix represents the set of elements T(i,j), where i =row, j = column and (i,j) ∈ [1,3]. Each element corresponds to one of the cardinal directions, such as,

$T\ (i,j)\ |_{i=1}^{3}|_{j=1}^{3} = \{NW, N, NE, W, O, E, SW, S, SE\}$ and $T(2,2) = reference\ point\ (O)$ (Figure 3.4). The value of each element in the matrix is either one or zero.

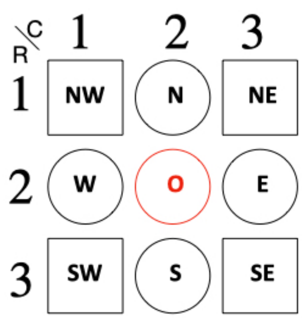


Figure 3.4 The matrix refinement of projection-based model.

Postulates 3.1 (Region elements)

Let S be a symbol in the point-reference model for region R with respect to point O. If the cardinality of S is 1, then R is confined to NW, NE, SW, or SE.

$n(\{T\ (i,j)\ |_{i=1}^{3}|_{j=1}^{3} = 1\}) = 1$, if only one of $\{T(1,1), T(1,3), T(3,1), T(3,3)\}$ exists, those configurations represent the simple region without any neighborhood connections (Figure 3.5).

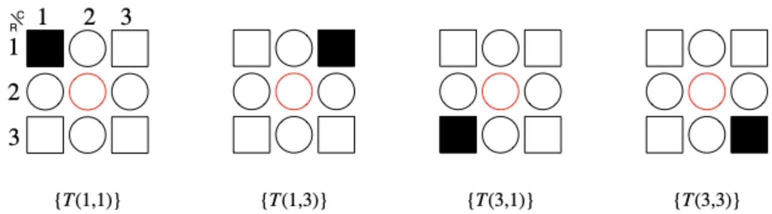


Figure 3.5 Region elements, where the simple target region falls into a direction tile.

There is no way a region can fit in a line or a point.

Postulates 3.2 (Connectivity Constraints)

If the target object extends over more than one tile, it must be connected to its neighbors in one of the following four way for it to be a simple region:

1. $T(1,1)$: $\{T(1,2), T(2,2), T(2,1)\}$, such an $T(1,1)$ region element allows neighboring connections with elements one of $\{T(1,2), T(2,2), T(2,1)\}$ to represent a simple region as:

   $\{T(1,1), T(1,2)\}, \{T(1,1), T(2,2)\}, and\ \{T(1,1), T(2,1)\}$.

   The neighborhood connections of region element express 2-tuple (Figure 3.6b), 3-tuple (Figure 3.6c), and 4-tuple (Figure 3.6d) to represent a simple region.

Similarly,

2. $T(1,3)$: $\{T(1,2), T(2,2), T(2,3)\}$
3. $T(3,1)$: $\{T(2,1), T(2,2), T(3,2)\}$
4. $T(3,3)$: $\{T(3,2), T(2,2), T(2,3)\}$

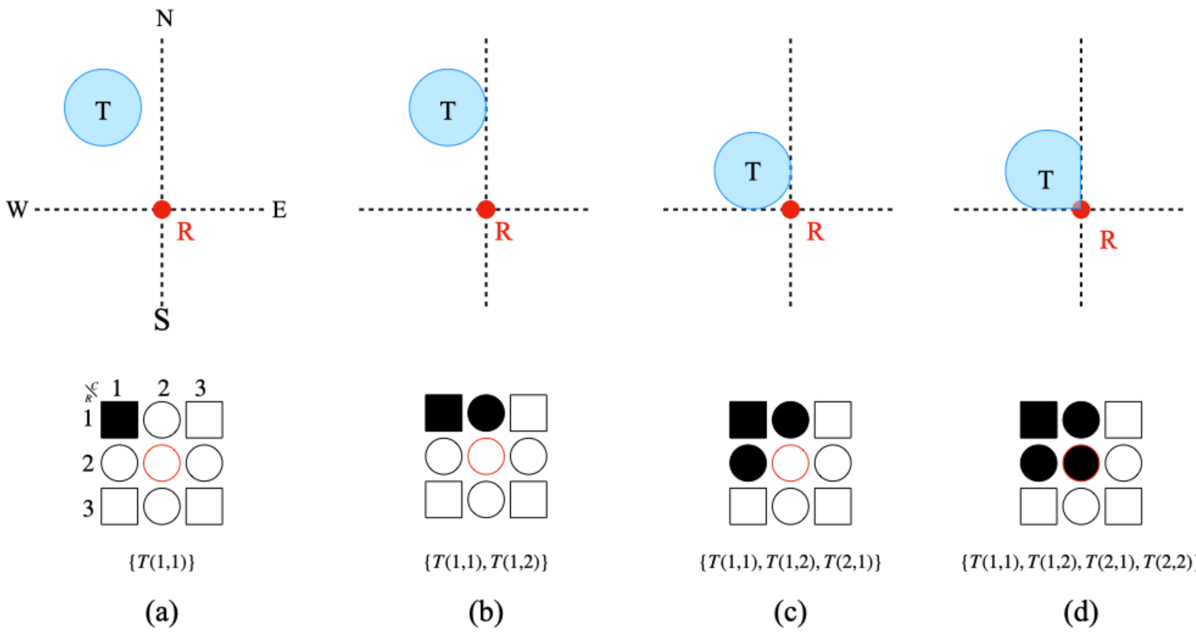


Figure 3.6 Point-to-region direction relations express by (a) a single element, (b) 2-tuple, (c) 3-tuple, and (d) 4-tuple elements; highlighted symbols describe the existence of the target.

Postulates 3.3 (Valid Configurations)

Region elements = $\{T(1,1), T(1,3), T(3,1), T(3,3)\}$

Line elements = $\{T(1,2), T(2,1), T(2,3), T(3,2)\}$

Point element = $\{T(3,3)\}$

Region, line, and point elements construct the entire point-based direction model. If $n\left(\{T\ (i,j)\ |_{i=1}^{3}|_{j=1}^{3} = 1\}\right) \geq\ 2$, the elements must contain a region element and an element from either in line elements or a point element. Each element of the configurations must satisfy the *definition 3.2* to be a valid configuration.

Example 1:

One of the configurations (Figure 3.7a), $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ shows the $n\left(\left\{T\ (i,j)\ |_{i=1}^{3}|_{j=1}^{3} = 1\right\}\right) = 2$, it has a region element T (1,1), and the second element T (2,3) is one of the line elements. In *Postulates 3.2*, T (1,1) could possibly have only one of the elements $\{T(1,2), T(2,2), T(2,1)\}$ as a neighbor to define the simple region; T (2,3) is not a neighboring element of T (1,1). This configuration does not represent a simple region.

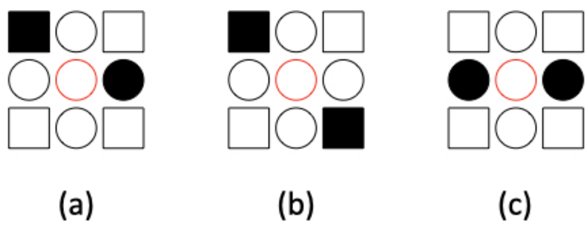


Figure 3.7 Invalid 2-tuple configurations to represent simple region.

Example 2:

The 3-tuple configuration (Figure 3.8a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, shows the region element T (1,1) and line elements T (1,2), and T (2,3). The region element has a valid neighboring connection (Postulates 3.2). Element T (2,3), however, does not have a valid connection. This is not a valid configuration to represent a simple region.

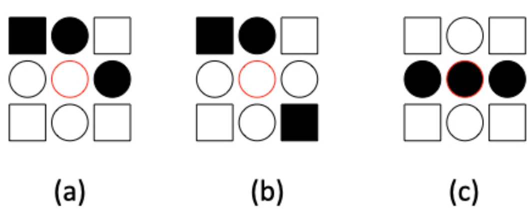


Figure 3.8 Invalid 3-tuple configurations to represent the simple region.

3.2.2 Iconic Representation of Point-to-Region Direction Relations

Consistency constraints yield, out of the $2^9 = 512$ combinations, a total of 106 feasible point-to-region relations (Figure 3.9). The valid binary 3x3 matrix respectively transforms into symbolic direction representation. The highlighted circles and squares indicate the simple target region from a reference point. The color of symbol such as black and white circles or squares, represent the non-empty and empty states of target, respectively. If the target object exists in a specific direction, non-empty symbols are shown in black.

The 106 point-to-region direction relations are divided into two classes: (1) if the reference point is not part of the target region (empty self), 53 atomic relations are such that eight subsets: as four singletons, eight 2-tuples, eight 3-tuples, eight 4-tuples, eight 5-tuples, eight 6-tuples, eight 7-tuples, and one 8-tuple (Figure 3.9a). (2) if the reference point is part of the target region, 53 atomic relations with non-empty self. It has eight subsets: four 2-tuples, eight 3-tuples, eight 4-tuples, eight 5-tuples, eight 6-tuples, eight 7-tuples, eight 8-tuples, and one 9-tuple (Figure 3.9b). These 106 realizations have been confirmed computationally, with a Python program which confirms the valid configurations.

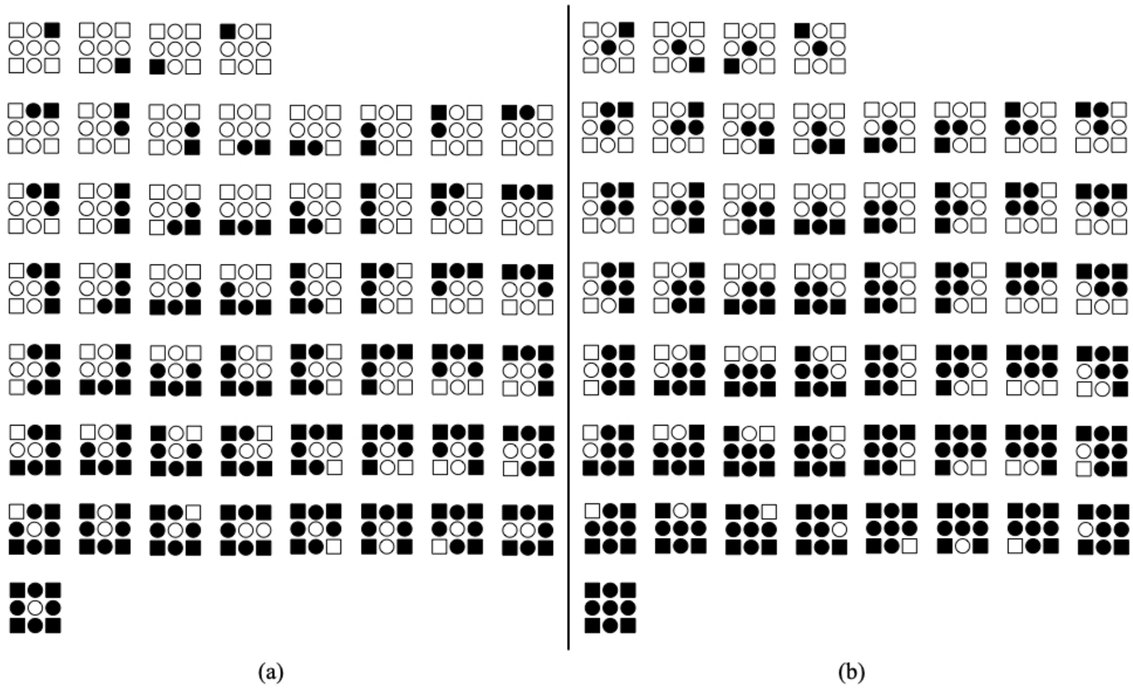


Figure 3.9 An iconic representation of the 106 point-to-region direction relations: (a) 53 feasible atomic relations from empty self (b) 53 realizations from non-empty self.

3.3 Atomic Point-to-Region Direction Relations

The 3x3 matrix of a point-reference cardinal direction relation has several symmetric properties, such as horizontal mirroring, vertical mirroring, and 90^{o} rotation. The symmetric properties of the 106 point-to-region direction relations allow for normalization. The mirroring and 90° rotation apply to normalize the base relations, introduce 22 atomic directions. A set of empty-self normalized relations has eleven atomic relations (Figure 3.10a), similarly, a non-empty set has eleven atomic relations (Figure 3.10b).

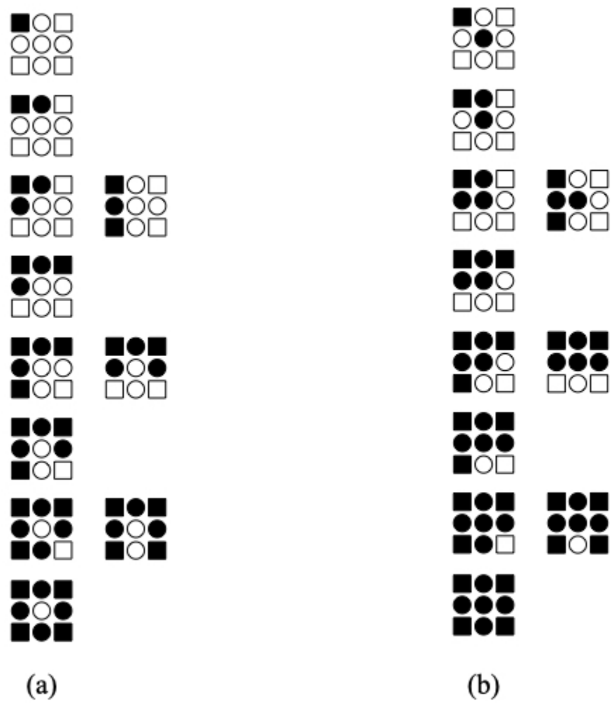


Figure 3.10 Normalized point-to-region base relations: (a) eleven empty-self atomic relations and (b) eleven non-empty-self atomic relations.

3.4 Union, Intersection, and Complementation of Atomic Direction Relations

For sets of qualitative relations, it is interest how the individual relations behave certain logical operations, has they may become critical ingredients for later investigation of spatial reasoning.

The typical basic operations are (1) unions, (2) intersections, (3) complementation, and (4) set differences. Most advanced is the behavior of pairs of relations under composition. The more regularly these operations behave, the stronger the relations' inferences. Here, the behavior under the first three operations is investigated for the complete set of 106, as well as for the atomic point-to-region relations. The goal is to find whether the sets of relations are closed under the operations, that is, whether the result obtained is always an element in the set itself.

For the union of the 106 relations, this can be quickly dismissed with a single example. Using the relations NE and SE, then union would yield NE or SE, which is not possible as a cardinal direction for a point-to-simple region relation. Likewise, for the 106 relations, the intersect can be shown to be not closed. The intersection of the two cardinal directions E or SE and E or NE would yield E, a relation that is not feasible for a point-to-region relation. Therefore, the set of 106 relations always lacks closure for two basic operations. However, the 22 atomic relations (Section 3.3) are closed under the union (Figure 3.11) and intersection (Figure 3.12).

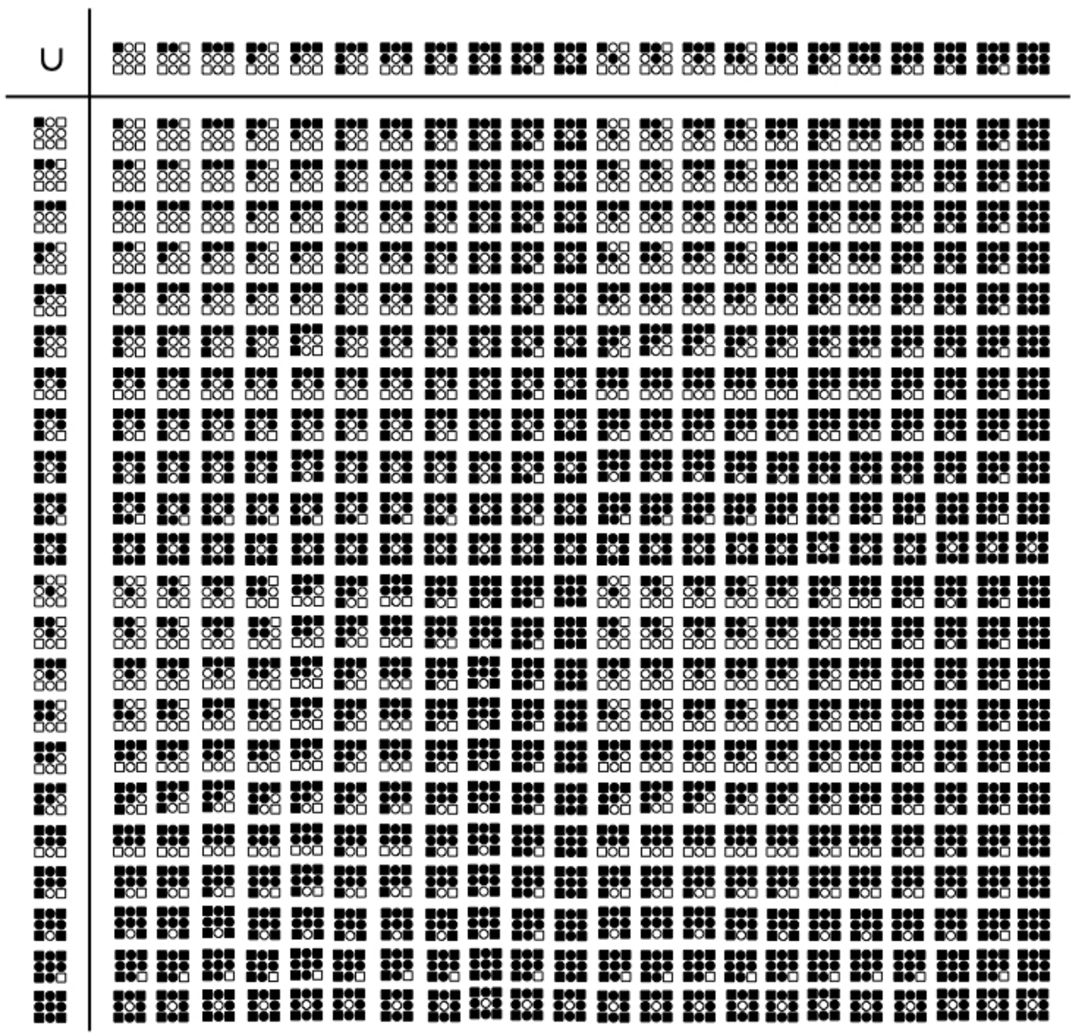


Figure 3.11 Closer of the union of 22 atomic relations.

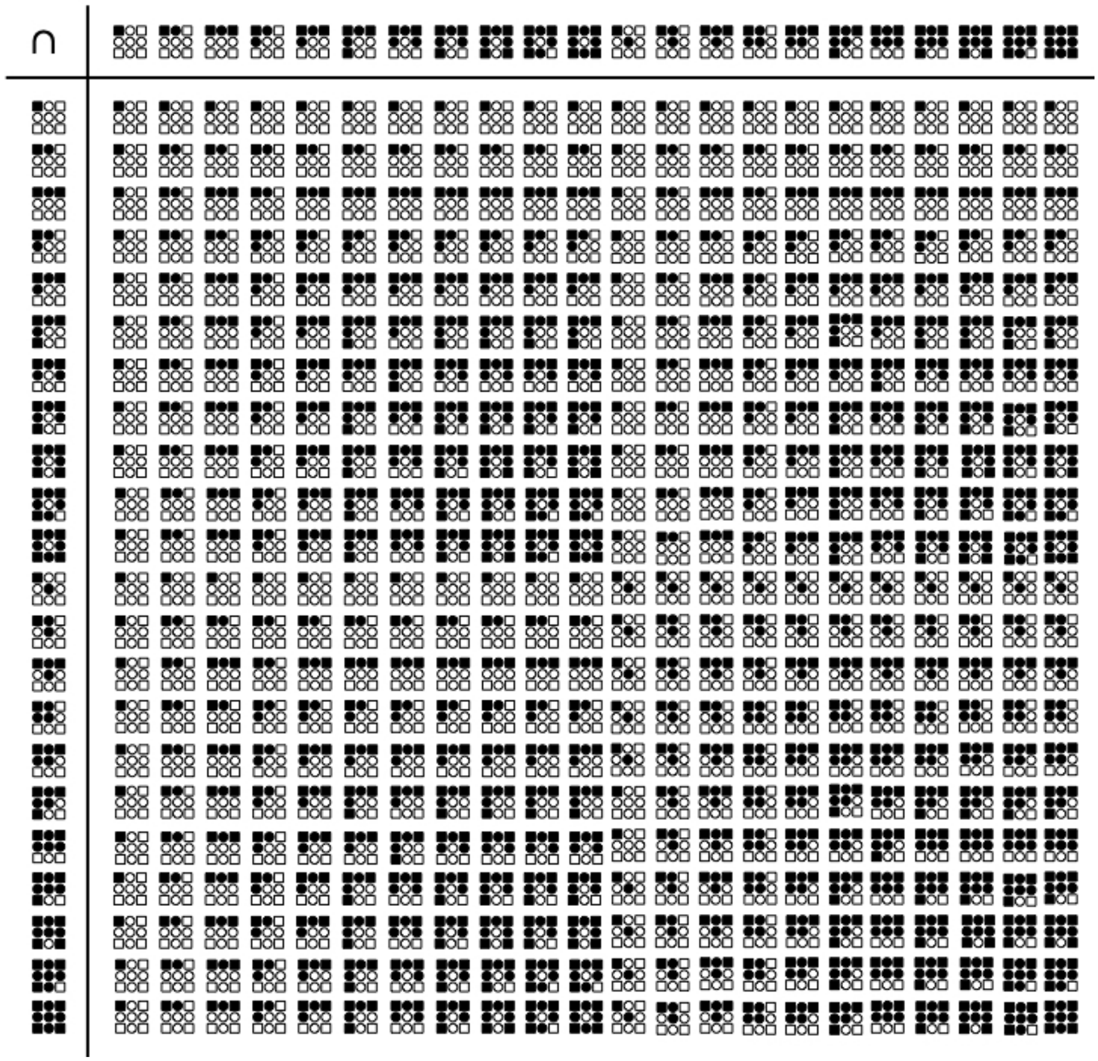


Figure 3.12 Closure of the intersection of 22 atomic relations.

The normalization concepts such as horizontal mirroring, vertical mirroring, 90º rotation, and their combinations introduce eight different configurations (Figure 3.13) and explore eight sets of 22 atomic relations (Figure 3.14). These eight sets are closed under union and intersection.

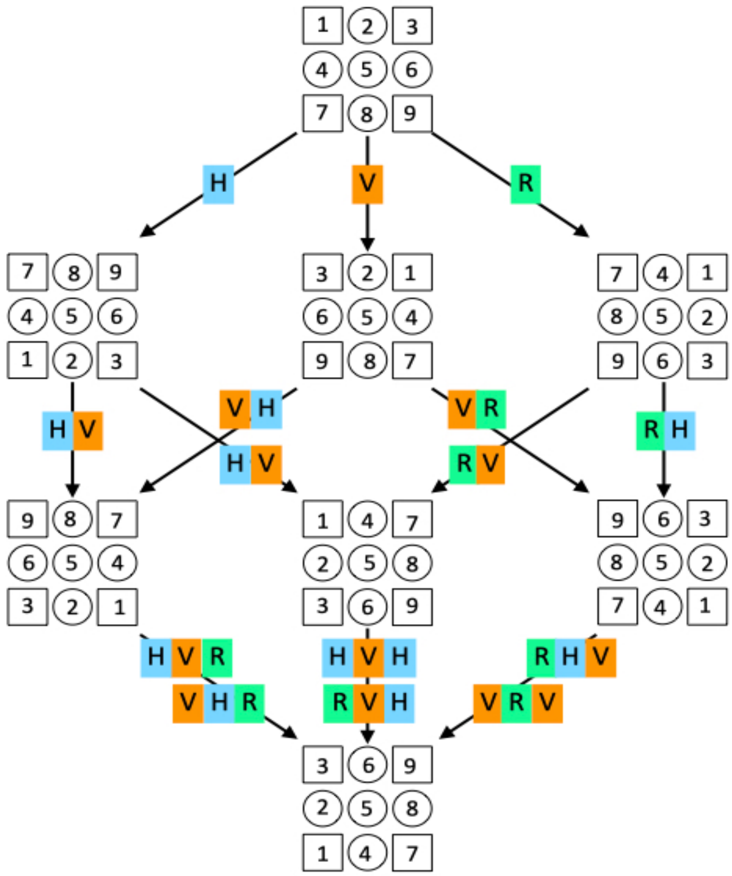


Figure 3.13 Closure of eight 3x3 matrices under horizontal (H), vertical (V) mirroring, and 90º rotation (R).

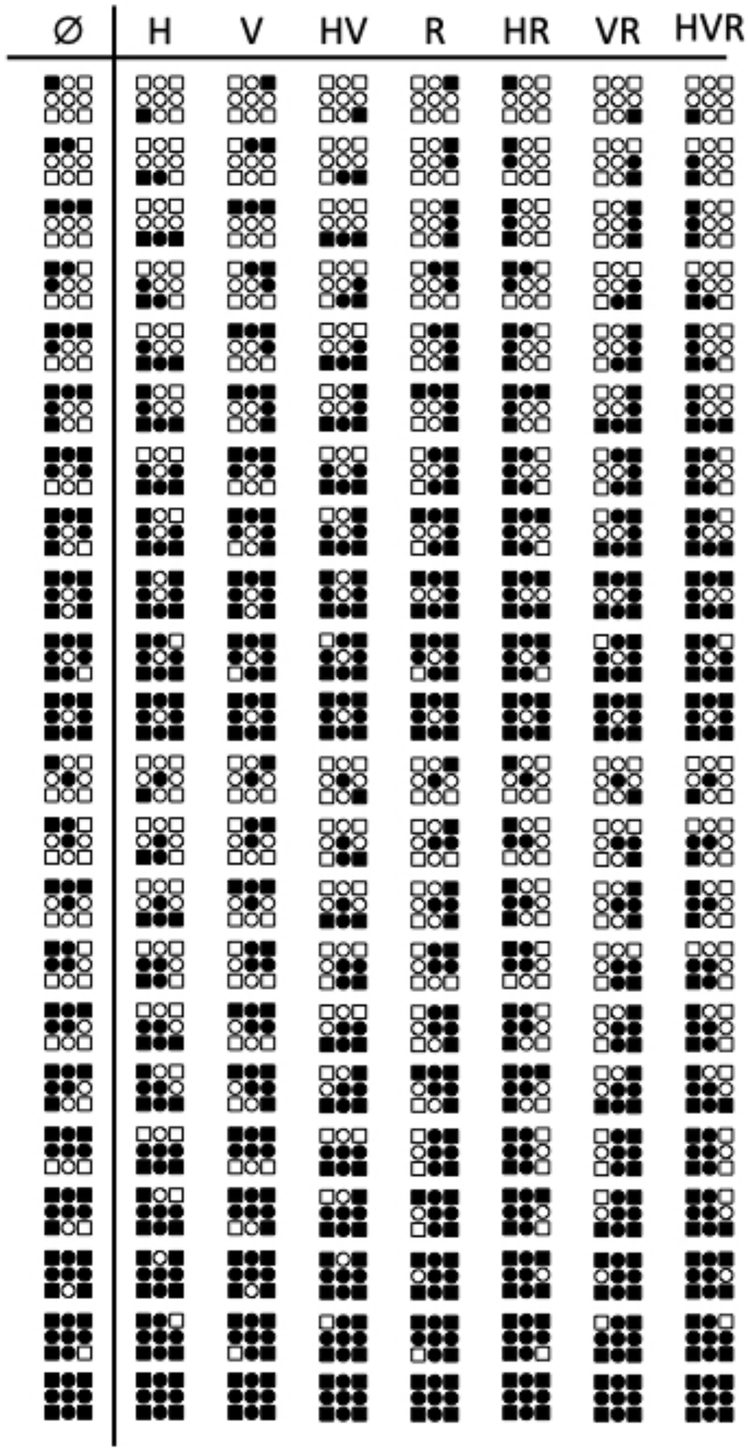
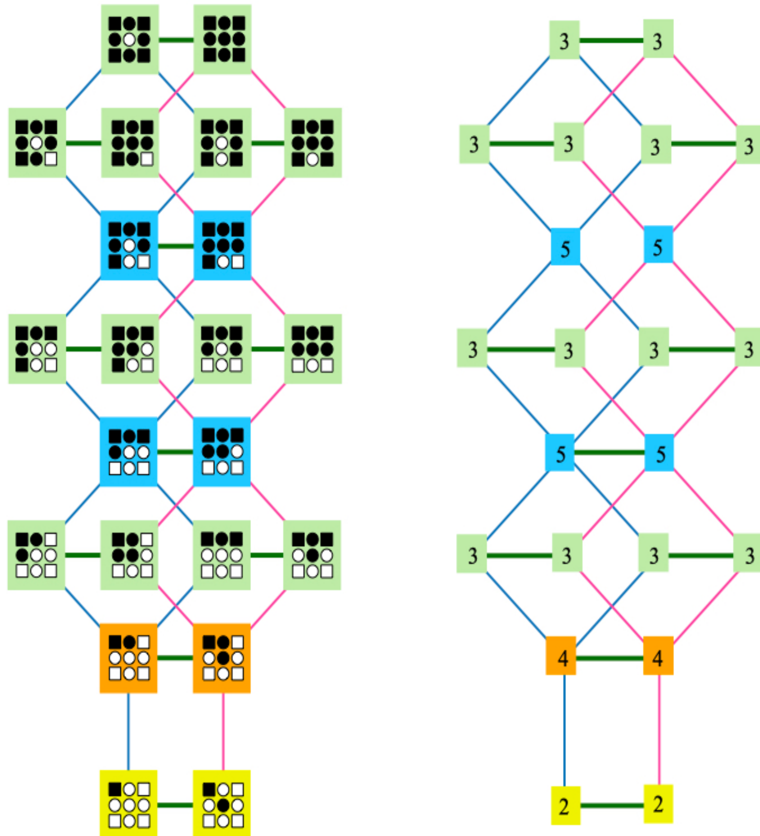


Figure 3.14 Eight sets of 22 normalized atomic relations.

For the complementation of the atomic relations, this can be quickly dismissed with a single example. Using the one of the base relations, (NW, W, SW, S, SE, E, NE), the complement of the relation would not be a simple region (O, N), complement is a line representation, therefore the complementation of atomic relations is not closed.

3.5 Conceptual Neighborhood Graph

The idea of transfer one relation to other and providing the hypothetical construction, such which relation come next handle qualitatively by the *conceptual neighborhood graph* (Freksa, 1992). The normalized atomic relations and their closure for union and intersection lead to the conceptual neighborhood graph (Figure 3.15a). For a set of 22 atomic relations, a singleton and a non-empty-self 2- tuple have a minimum of two neighboring connections. The empty self of 4-tuple and 6-tuple relations shows the maximum of five neighboring connections similarly, the non-empty self of the 5-tuple and 7- tuple have five neighboring connections (Figure 3.15b). The union between relations is followed in the direction from singleton to 8-tuple, intersection directs opposite to the union.



(a) (b)

Figure 3.15 Conceptual neighborhood graph of normalized atomic relations: (a) union and the intersection of each element and (b) number of neighbors of each atomic relation.

The eight sets of normalized 22 atomic relations construct a conceptual neighborhood graph for all 106 base relations, organized into two layers. The singletons of *layer 1* construct the 2-tuples with the primary direction elements to show a valid simple region, similarly, the 2-tuples of *layer 2* construct the 3- tuples. The remaining relations follow the neighborhood graph of 22 atomic relations. Each relation in *layer 1* has a direct connection with a corresponding element in *layer 2*, such the intersection between *layer1* and *layer 2* relations is a central element. For example, the intersection between an empty-self 3- triple and its corresponding non-empty-self 4-tuple is a self-direction (Figure 3.16).

A subset of the 106 point-to-region relations is not closed under union and intersection, explore the invalid neighborhood connections. For example, in a point-to-region relation, (SE) does not show the closed in union with the (SW), (NW), (NE), (W, SW), and (NW, N, W, O, SW). The composition table of base relations (106x106) introduces 688 configurations out of 11236 (106x106), those are not a power set of realizable 106 point-to-region relations. The normalization applies to the composition table of base relations, considering a mirroring and 90^{o} rotation over the relations in the columns, while keeping the row index relations. The normalized six base relations and 13 relations in rows, introduce 33 invalid configurations for the empty-self point-to region relations (53). The mirroring and 90^{o} rotation over the atomic relations generate either a set of four or eight relations (Figure 3.17). Considering the number of corresponding possibilities of each normalized atomic relations, such as 4 for singletons, 8 for 2-tuples, 4 for 3-tuples, 8 for 4-tuples, and 4 for 5-tuples, one could construct the 344 invalid configurations for 53 empty-self relations (Figure 3.18a), similarly, non-empty-self relations exhibit the same number 344 invalid configurations. The invalid combinations of pairs of relations are not direct neighbors in the conceptual neighborhood graph (Figure 3.18b).

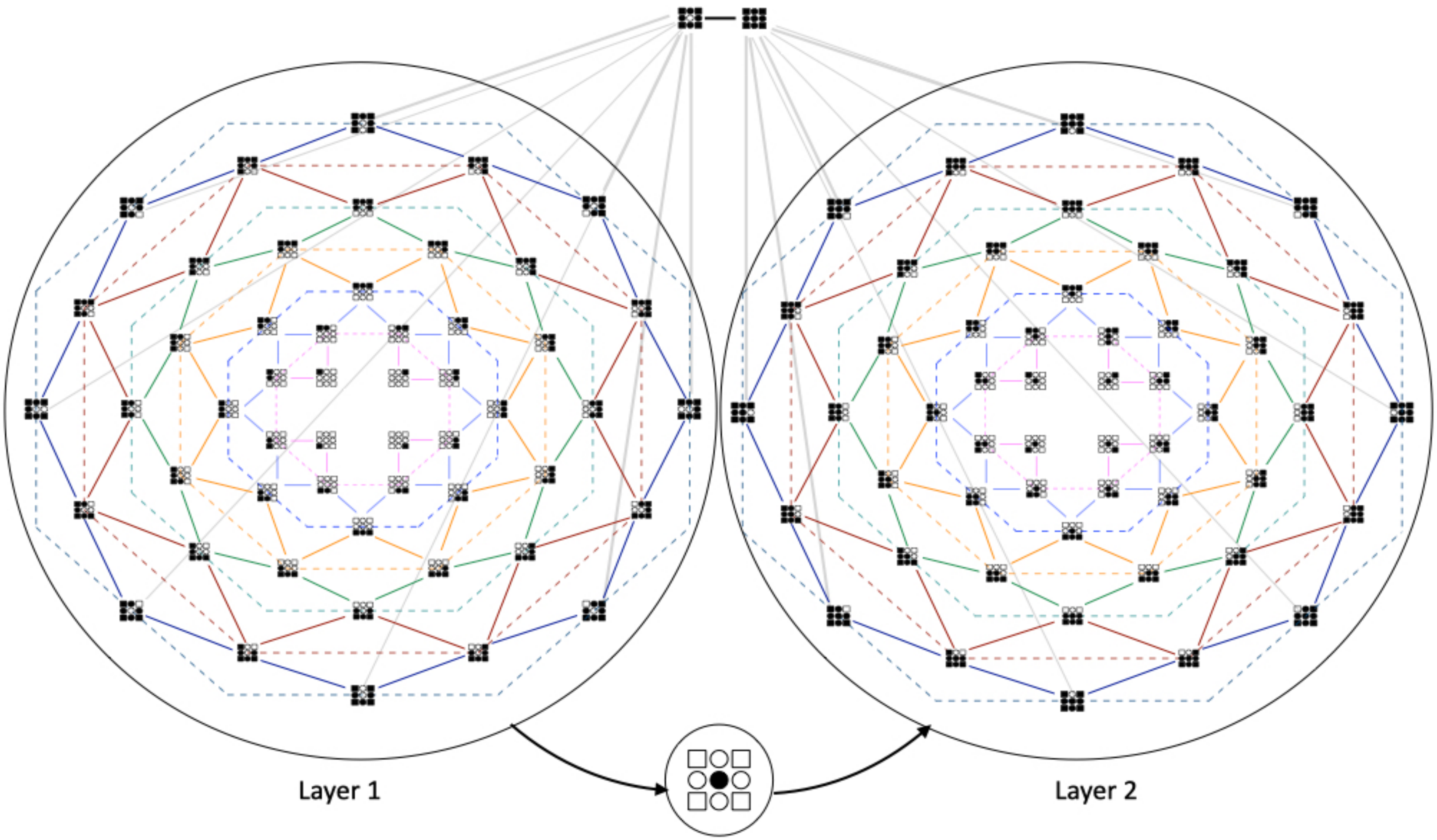


Figure 3.16 Conceptual neighborhood graph for all 106 point-to-region relations.

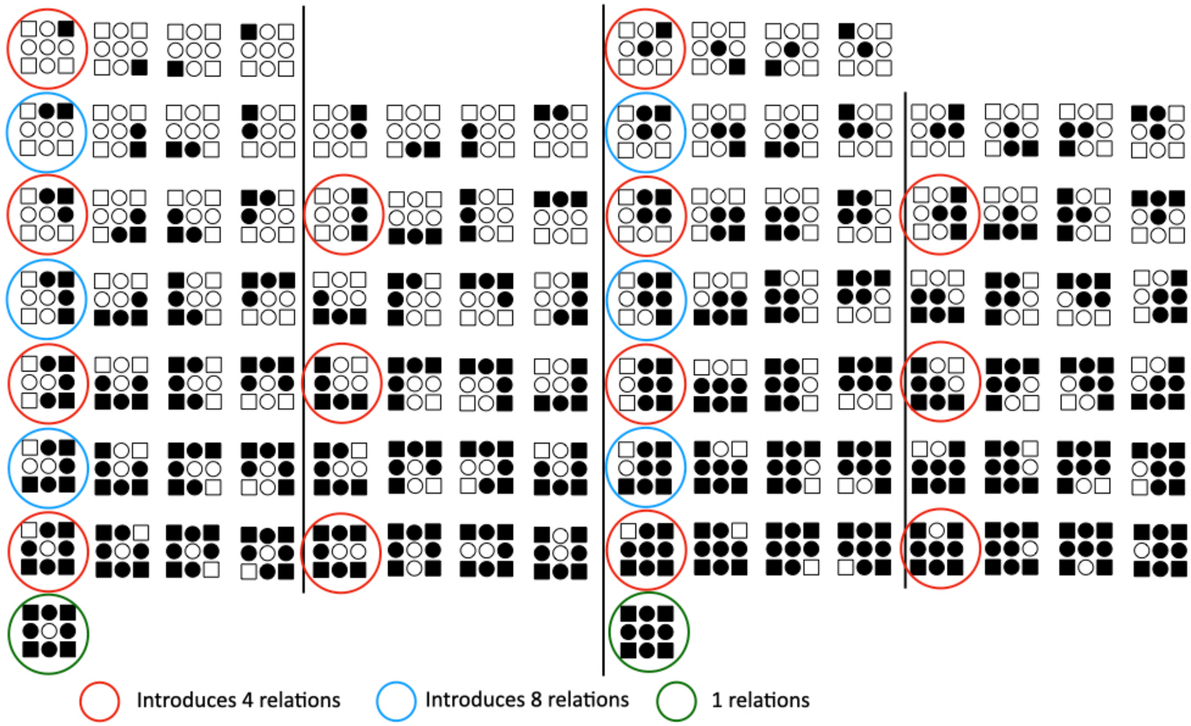


Figure 3.17 Atomic relations and their impact, considering mirroring and 90º rotation to construct the remaining 84 point-to-region relations.

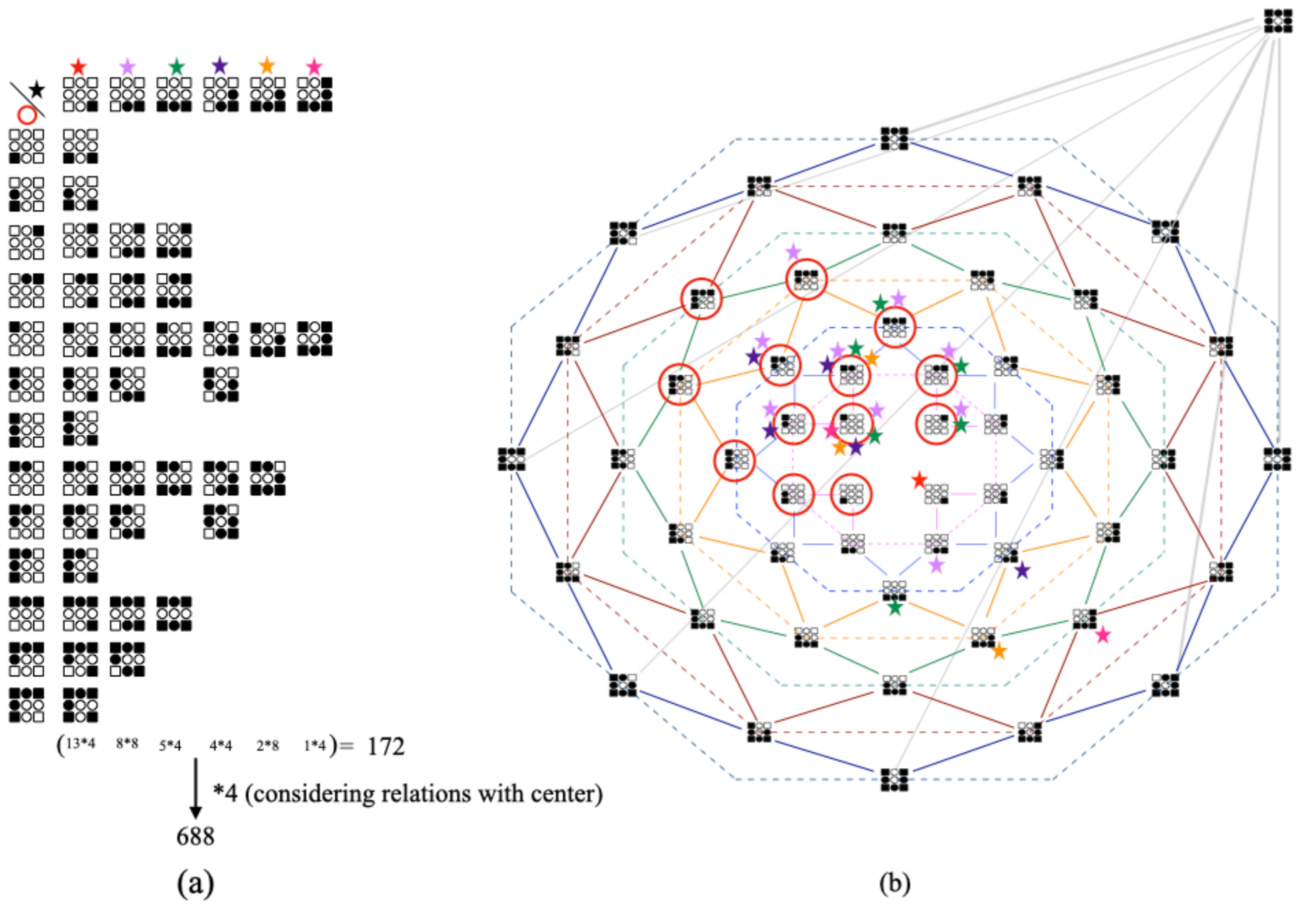


Figure 3.18 A set of base relations that exhibits 106 base relations is not closed under union and intersection, resulting in (a) a normalized set of relations and (b) invalid neighboring connections between relations denoted by the stars in a conceptual neighborhood graph.

3.6 Point-to-Region Relations at PLE intersections

In a 2-dimensional plane, each simple region has Northmost, Eastmost, Southmost, and Westmost extrema with respect to its centroid. The projection-based model at extrema introduces two sets of horizontal and vertical partitioning lines, called as partitioning lines through extrema (PLEs) (Figure 3.19a). The intersections between horizontal and vertical PLEs introduce four intersection points such as (1) Northmost and Eastmost PLEs (Figure 3.19b), (2) Northmost and Westmost PLEs, (3) Southmost and Eastmost PLEs, and (4) Southmost and Westmost PLEs. A simple region could have only four PLE intersections (Figure 3.19c).

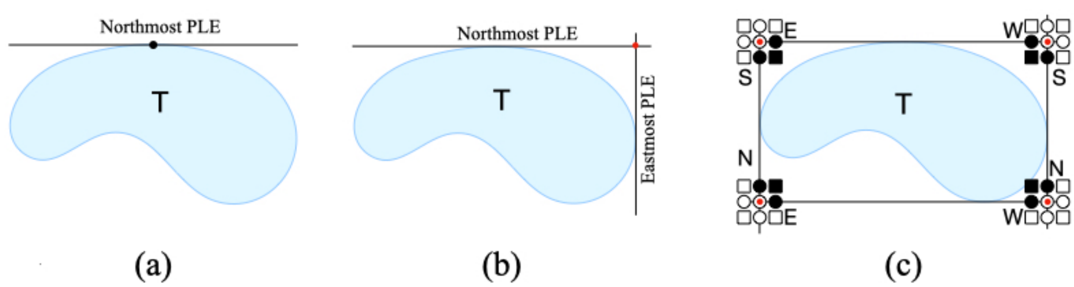


Figure 3.19 Simple target region's (a) horizontal PLE, (b) the intersection of horizontal and vertical PLEs, introduces a PLE intersection point, and (c) four PLE intersections.

Point-to-region direction relations from PLE intersections to a target object use a set 20 base relations out of 106 realizations (Figure 3.20), normalized to four atomic relations. The 20 base relations contain minimum of two elements in a 3x3 configuration, which includes a region element and a center element to construct the simple region, and a maximum of four elements, 3-tuple with center element (Figure 3.20b).

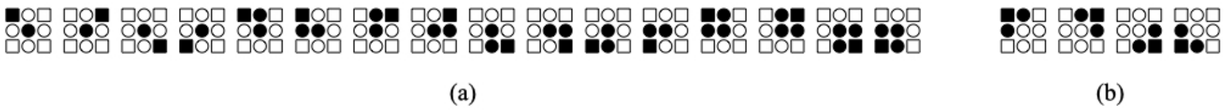


Figure 3.20 Point-to-region direction relations from viewpoints uses 20 base relations out of 106 realizations: (a) 16 non-empty-self relations; (b) 4 elements with empty self.

The sets of four point-to-region direction relations at the PLE intersections, define the direction relations for a target object, follow a set of consistency constraints. These constraints extract valid four 3x3 symbolic configurations for point-to-region relations from the PLE intersections. The PLE intersections, are labeled as upper left (U_L), right (U_R), and lower left (L_L), right (L_R), such they form from

$U_L = R(most\ west, most\ north)$, $U_R = R(most\ east, most\ north)$, $L_R = R(most\ east, most\ south)$, and $R(most\ west, most\ south)$.

Postulates 3.4. (Neighbors of direction relations at viewpoints)

$P_{PLE} \in \{U_L, U_R, L_L, L_R\}$ has horizontal and vertical neighboring such as,

$U_L = [U_R, L_L], U_R = [U_L, L_R], L_R = [L_L, U_R], L_L = [L_R, U_L]$. If $(P_{PLE} \cap R)\ exits, P_{PLE} \in \{U_L, U_R, L_L, L_R\}$, neighboring elements of PLE intersection must contain either the horizontal or vertical primary cardinal direction relations.

For example, if the point-to-region at U_L has a target region at the center and extends in the southeast (SE) direction, then the horizontal neighboring element U_R must have a horizontal primary cardinal direction that is direct towards U_L, west. The vertical neighbor L_L must have a vertical primary cardinal direction, such as north (Figure 3.21a).

Invalid configuration, a set of relations such as one 2-tuple element, two 3-tuples with a center, and one without a center 3-tuple (Figure 3.21b). A relation that occupies the center, must have neighboring relations with horizontal or vertical primary cardinal directions (Definition 3.4).

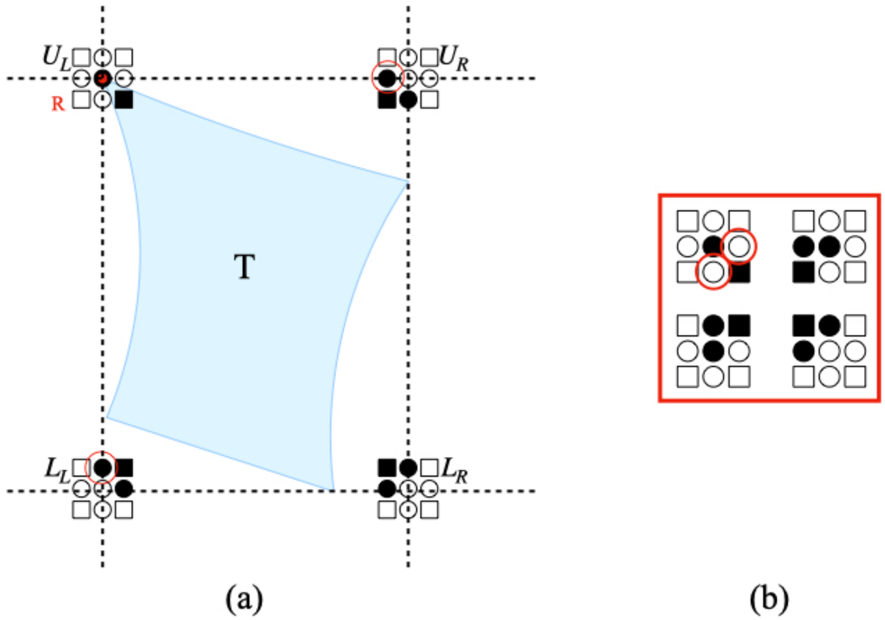


Figure 3.21 A set of point-to-region base relations at PLE intersections: (a) valid set (definition 3.4) and (b) invalid set of relations.

Postulates 3.5. (Union of direction relations at viewpoints)

The union of four point-to-region direction relations at PLE intersections of a region must be equal to either the set of all nine direction elements or eight directions without the center element.

For example, $U_L = (E, SE, S), U_R = (W, O, SW), L_R = (N, NW, W), L_L = (O, NE)$ denotes the simple region from the PLE intersections. The union of U_L, U_R, L_R, and L_L is universal, nine elements of point-based model occupy the directions. $U_L \cup U_R \cup L_R \cup L_L = (NW, N, NE, E, SE, S, SW, W, O)$ (Figure 3.22).

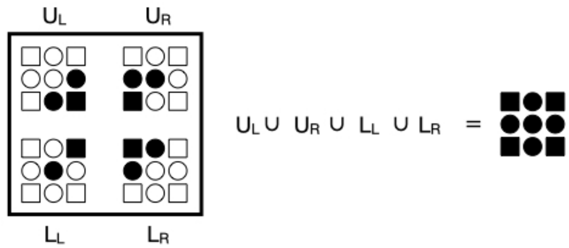


Figure 3.22 Union of four point-to-region relations from a target regions PLE intersections.

Postulates 3.6. (Point-to-region direction relations without center occupancy)

A set of four 3x3 configurations that represents point-to-region relations from target region's PLE intersections, do not have central element that must contain the horizontal and vertical primary cardinal directions. For example, $U_L = \{E, SE, S\}, U_R = \{W, SW, S\}, L_R = \{N, NW, W\}, L_L = \{N, NE, E\}$ (Figure 3.23).

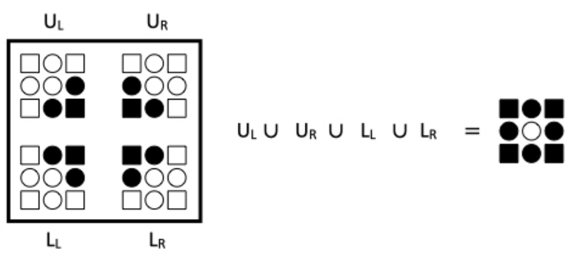


Figure 3.23: Point-to-region direction relations from PLE intersection of the target, without center element occupancy.

The four point-to-region direction relations at PLE intersections for a target region are a subset of 20 base relations, which makes 116,280 $\left(\frac{20!}{(20-4)!}\right)$ possible combinations. However, three constraints exhibit 80 unique valid 3x3 4-tuple configurations to represent the simple region from the PLE intersections (Figure 3.24).

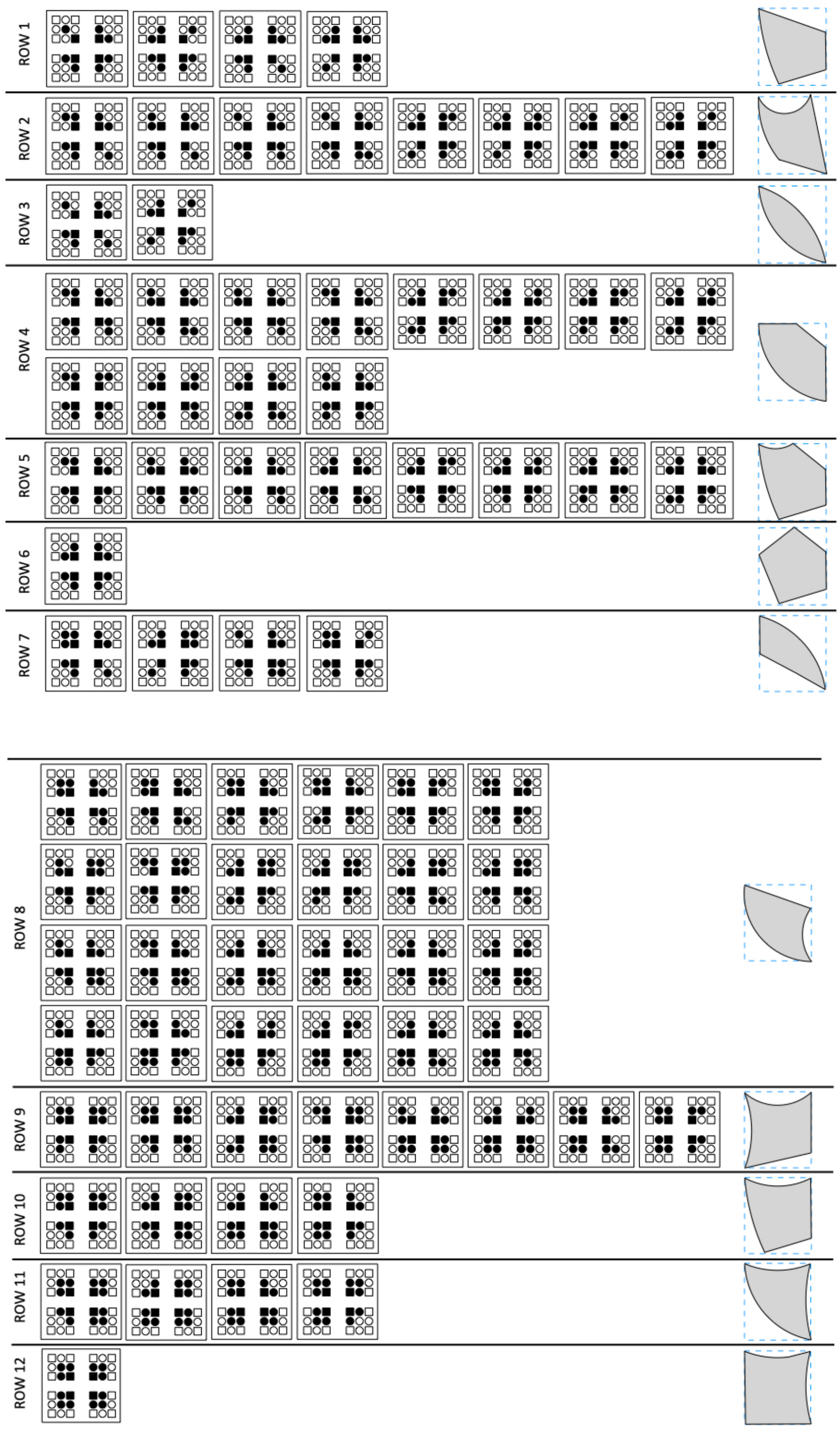


Figure 3.24 Valid set of point-to-region direction relations from PLE intersections to the target object, 80 configurations represented by 12 unique rows.

Valid configurations that extracted, considering three classes of 20 base relations: (1) 2-tuple (2) 3-tuple (3) 4-tuple. 3-tuple are subclassified based on the existence of a central element. The group of elements that belong to each class constructs a valid set of four 4-tuple configurations (Table 3.1).

Table 3.1 Combinations construct a set of four point-to-region relations at PLE intersections from the different classes.

Row	2-tuple (4)	3-tuple		4-tuple (4)	Configurations
		With center (8)	Without center (4)		
1	1		3		4
2	1	1	2		8
3	2		2		2
4		2	2		12
5		1	3		8
6			4		1
7	1		2	1	4
8		1	2	1	24
9		1	1	2	8
10			2	2	4
11			1	3	4
12				4	1
Total					80

The 80 configurations define the point-to-region direction relations from the PLE intersections to a simple target region in sense to the geometric shape of the objects (Figure 3.24). For example, the unique set represents the participation of the target boundary to define point-to-region direction relations.

3.7 Summary

This chapter focused the projection-based cardinal direction model for point-to-region direction relations. The direction relation between a point and a target is recorded by the 3x3 symbolic matrix. It has nine elements, yielding 2^9 =512 combinations. Three consistency constraints filter a set of 106 valid relations out of 512 configurations to represent the point-to-simple region direction relations. Those 106 direction relations are normalized to 22 atomic relations, considering mirroring and 90° rotation.

The 106 base relations are not closed under the union and intersection, but a set of 22 atomic relations is closed. The horizontal and vertical mirroring, 90° rotation, and their combinations introduce eight sets of 22 atomic relations, each closed under union and intersection. The atomic and 106 base relations are not closed under complementation.

Set of 22 atomic relations organized into a conceptual neighborhood graph, the union and intersection between neighboring relations introduce the valid neighborhood connections. Atomic relations are ordered from singleton to 8-tuple. The intersection follows the opposite direction to the union. The eight sets of atomic relations and their neighboring connections yield for a conceptual neighborhood graph for all 106 elements.

The conceptual neighborhood graph explains how different locations of reference points are differentiated to define the direction relation to a target object. The point-to-region direction relations and its neighborhood connections are the foundation for the definition of direction relations between two simple regions (Chapter 4).

CHAPTER 4

CARDINAL DIRECTIONS BETWEEN TWO SIMPLE REGIONS

"Everything is related to everything else, but near things are more related than distant things."

(Tobler,1970)

The point-to-region direction relations, representing a field (Chapter 1, page 1). At each point in a reference region, one could obtain a cardinal direction value to a target region, that is, one of the 106 point-region relations. In theory, one could take sufficiently detailed samples throughout the entire reference region to obtain an accurate picture of which direction values apply. While such an approach would lead to a model for the cardinal direction from a region to another region, it is tedious, and its accuracy highly depends on the samples' densities. In some scenarios, the same direction value may apply throughout the entire reference region (Figure 4.1a), while in other cases, two or more direction values may apply (Figure 4.1b).

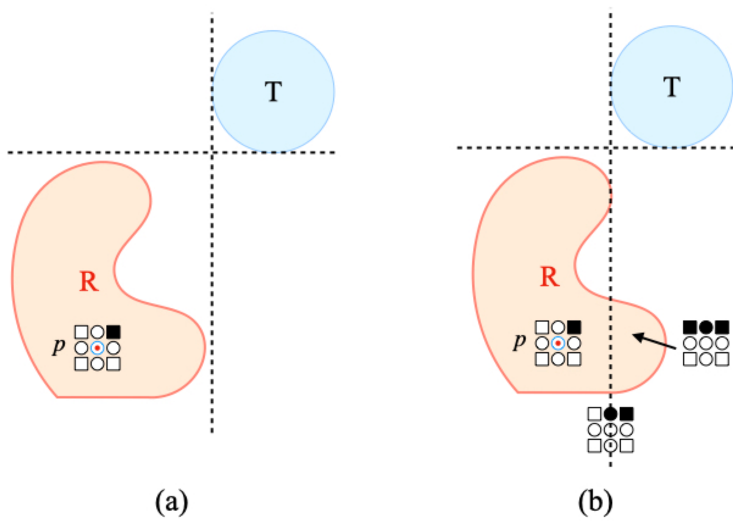


Figure 4.1 Direction relations between two simple regions: (a) the same direction value applies over the reference region, and (b) the reference region has three direction values.

This thesis pursues an approach to capturing the field of a region's cardinal directions for a target region. It aims at identifying the locations at which sample values will differ, thereby determining the boundaries between adjacent subregions of the reference object. These subregions are such that each will carry homogeneously the same cardinal direction throughout a subregion's interior. This spatial range of the same qualitative direction value will be called *areas of same cardinal direction*. It is expected that a reference object will have a limited number of such subregions of same cardinal directions.

4.1 Extrema of a Simple Region

The set of points that construct a simple region introduces two subsets: (1) interior points, where every point at the interior has eight directions of neighbors, such direction relations being common for all the points, and (2) boundary points, which separate the exterior from the interior of a region. In the context of cardinal direction relations, boundary points of a target allow one to define the direction relation to its interior. Direction relations varies with the location of the point, such boundary points act as reference points. The boundary points are classified based on their location-based impact to having different point-to-region base relations: (1) *absolute extrema* those are in boundary of the region, such fall into the northmost, eastmost, southmost, and westmost (Figure 4.2a), and (2) *local extrema*, are in boundary that make a *disjoint* neighboring relation with its interior along the PLEs, which pass through the corresponding vertex of the region (Figure 4.2b).

Absolute extrema exist for all simple regions, while local extrema appear only for convex regions.

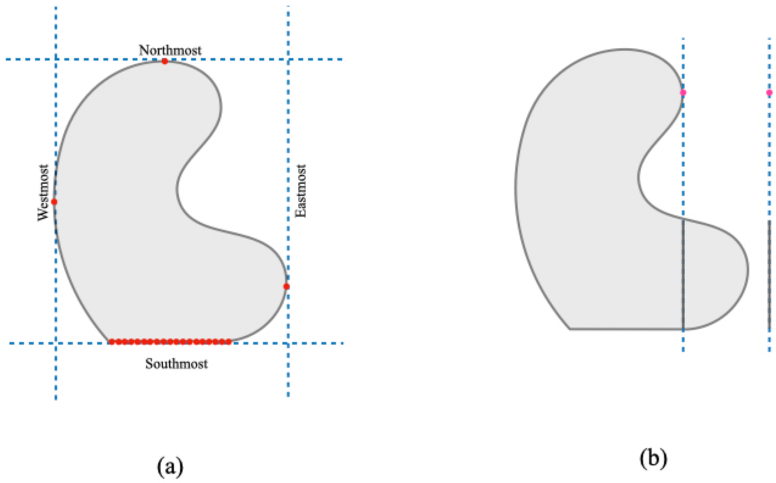


Figure 4.2 Classification of boundary points: (a) absolute extrema and (b) local extrema.

4.2 Projection-Based Cardinal Direction Model for Region-to-Region Relations

The projection-based model at absolute extrema introduces partitioning line through extrema (PLE). The PLEs tiles the exterior of a region into rectangle-shaped partitions. The central element of the point-reference model is the absolute extremum, while the primary cardinal directions of the model align to the horizontal and vertical PLEs that partition the exterior of the region into nine tiles. The adjacent primary cardinal directions of the model initiate boundaries of rectangle tiles. For example, the Northeast tile has the boundaries such as adjacent and orthogonal primary cardinal directions North and East (Figure 4.3a).

The point-based model at a PLE intersection of the target object, tiles the exterior of the target into rectangle tiles, which define the direction relation between the extended reference object and the target. For instance, the direction relation between reference R and target T generated by the point p, is one of the PLE intersections. Projection-based model at point p tiles the exterior of target into four quadrants, and one of them occupies the target object T. The remaining tiles partition the reference region R into two regions (Figure 4.3). Those regions and common boundary between them use three point-to-region base cardinal directions out of 106 to define the direction relations from the reference to target. The interior of two regions of directions is called as areas of same cardinal direction. The union of areas of same cardinal direction and boundaries preserves the geometric shape of the reference.

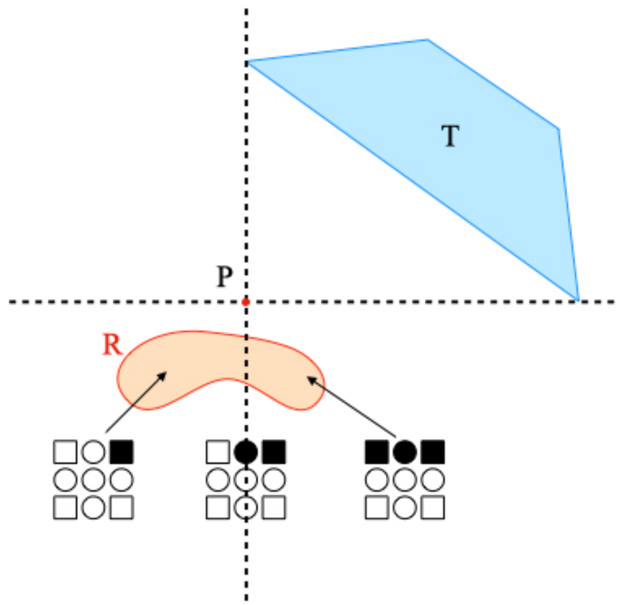


Figure 4.3 The projection-based model for partitioning the extended reference object R, and the exterior of target region T.

Spatial regions often have irregular shapes, such vertices that form convexities (at V_{11}) and concavities (at V_8) with respect to neighboring vertices, which impact the direction relations (Figure 4.4). Therefore, a detailed partitioning algorithm is required to define the areas of same cardinal direction at a coarse range in the direction field.

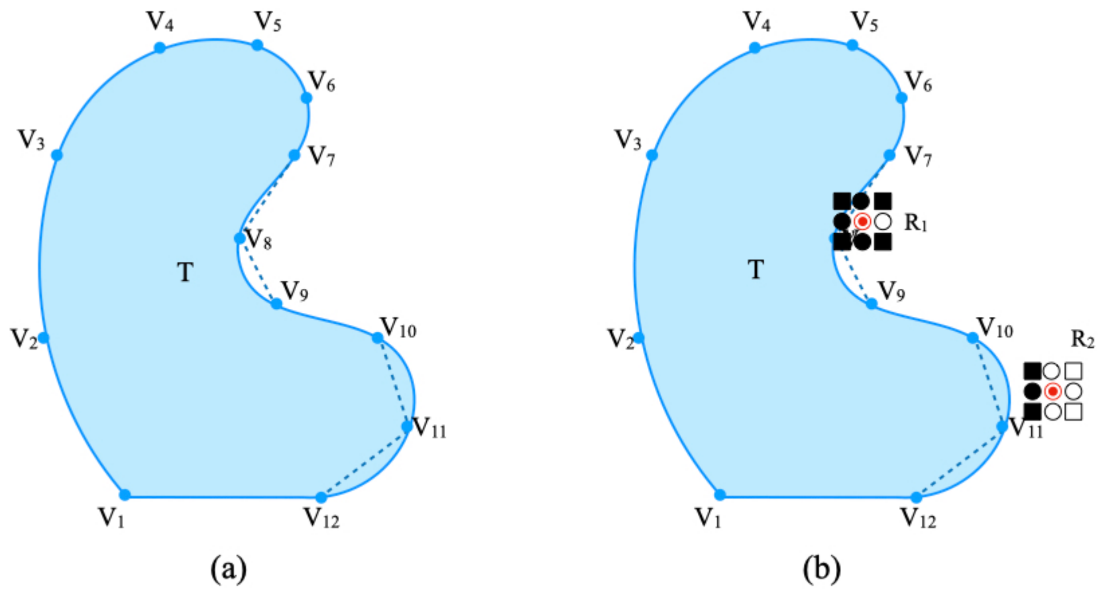


Figure 4.4 Irregular shapes and the influence on point-to-region direction relation: (a) concavity at vertex V_8 and convexity at vertex V_{11}, and (b) impact of concavity and convexity on direction relations at reference point R_1 and R_2.

4.3 Extrema and Partitioning

The projection-base model at the extrema splits the exterior of the target region into two: (1) the exterior of the rectangular region of the target object, which is constructed by PLEs such as northmost, eastmost, southmost, and westmost of the target object, considered as a union of rectangle tiles (Figure 4.5a). (2) The pockets of the exterior exist for the concave (Figure 4.5b) and convex target objects at the interior of the rectangular region of the target. The following sections introduce the algorithms to define the areas of same cardinal direction for the reference region.

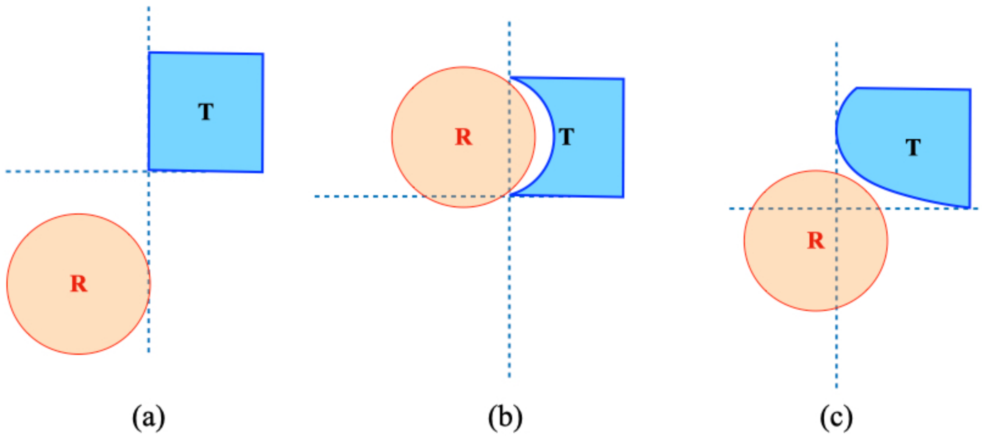


Figure 4.5: Areas of same cardinal direction (a) tiles, (b) pockets of same cardinal direction for concave reference, and (c) pockets of same cardinal direction for convex reference.

4.3.1 Tiles: Areas of Same Cardinal Direction

The PLEs tiles the exterior of the target into areas of same cardinal direction. For example, the model at the upper-left (U_L) of PLEs intersection of the target, tiles the exterior at U_L into four (NW, NE, SE, and SW). Three (NW, NE, and SW) of the four represent the exterior from the rectangle region, and the SE tile represents the target object. The projection-based model at four PLEs intersections, tiles the exterior of the target into eight directional tiles (Figure 4.6a). The tiles are labeled based on the region element's presence from the projection-based model's symbolic representation. For example, the NW tile refers to the existence of the NW region element at U_L, while the NE-NW tile includes region elements from U_L and U_R.

The eight embedded tiles preserve the point-to-region relations from 106 base relations. Each adjacent tile has a common cardinal direction boundary that is a subset of 106 base relations. For example, the cardinal direction at common boundary between NW and NE-NW tiles is (S, SE) (Figure 4.6b).

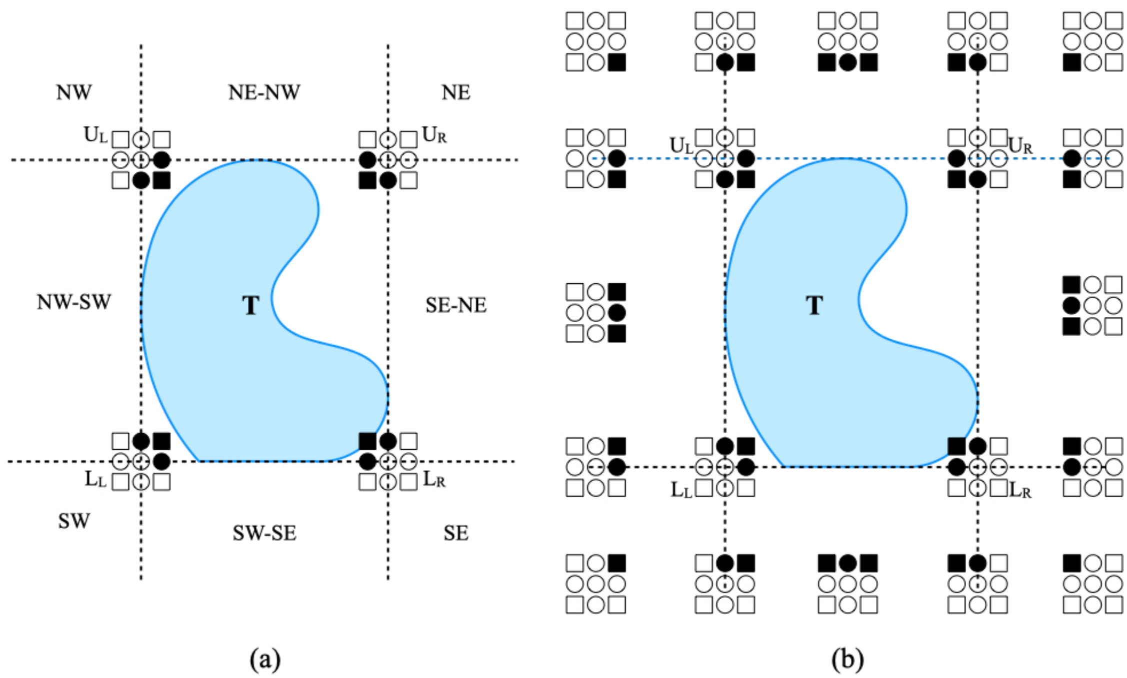


Figure 4.6 PLEs tile the exterior of the target region into (a) eight direction tiles and (b) assign respective point-to-region direction relations.

Sixteen base relations out of 106 manage the entire exterior of the rectangular target region by PLEs. If a reference point R falls in one of the direction tiles of the target T, the direction relation between R and T refers to the base relation that corresponds to the tile (Figure 4.7a).

The extended reference region falls into a specific direction tile without intersecting the boundaries of the tile, reference region refers to the cardinal direction of the tile. For instance, when a region falls into the NE-NW tile, every point reference within the reference region uses the cardinal direction from the corresponding tile (SW, S, SE) (Figure 4.7b).

If a region extends to more than one tile, the axis between the tiles partitions the reference region interior into areas of same cardinal direction. For example, region R extends from NE-NW to NE direction tiles of target T. The axis between the tiles splits the reference region into two area of same cardinal direction, such as (SW, S, SE) and (SW) (Figure 4.7c).

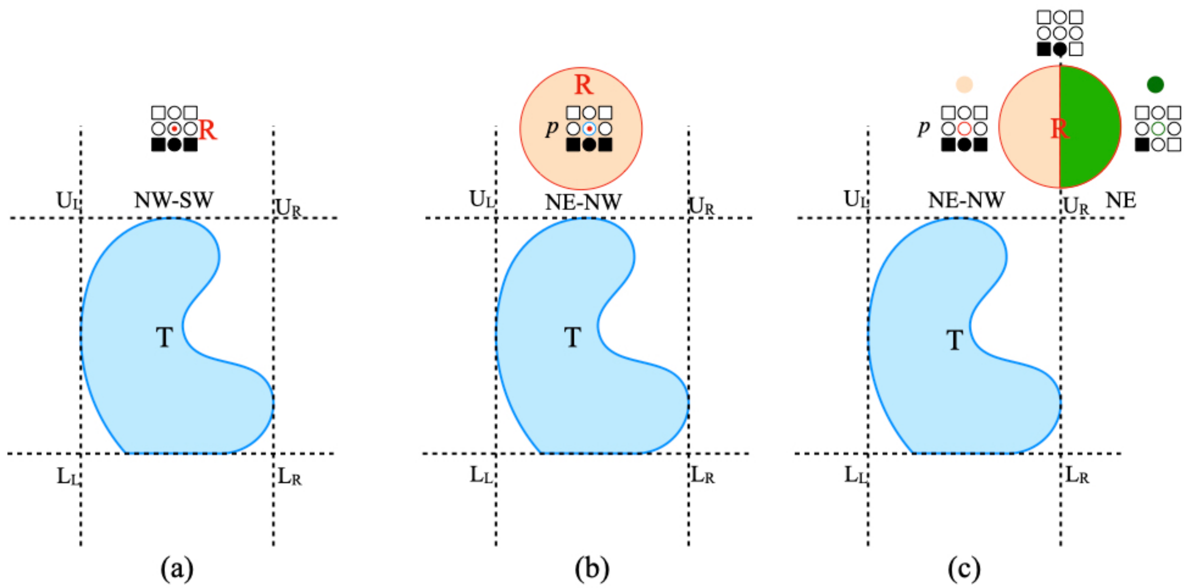


Figure 4.7 The point-to-region relations apply for region-to-region relations: (a) point-reference at NE tile, (b) extended object that occupies only one tile (NE-NW), and (c) instant that reference object falls into multiple tiles.

The tiling algorithm initially identifies common extrema, considering reference and target objects to construct a rectangular region, which contains both reference and target objects, called PLE_{TR}. The PLE_{TR} limits the exterior of the target object and narrow the computational area to define the areas of same cardinal directions (ASCD) (Figure 4.8a).

The horizontal and vertical PLEs of the target, parallel and equal to the PLE_{TR} boundaries, are the reference axes for ASCD and tile the exterior of the target region into direction tiles. The PLEs named horizontal top (H_T) and bottom (H_B) and vertical left (V_L) and right (V_R) form the rectangle direction tiles based on the half-plane model while passing through the extrema of the target region. For example, the NW tile is bounded by the V_L and H_T PLEs (Figure 4.8b). Similarly, the NE-NW tile is bounded by the V_L, V_R, and H_T PLEs. The different configuration of PLEs occupies the corresponding point-to-region cardinal direction base relations (Figure 4.8c).

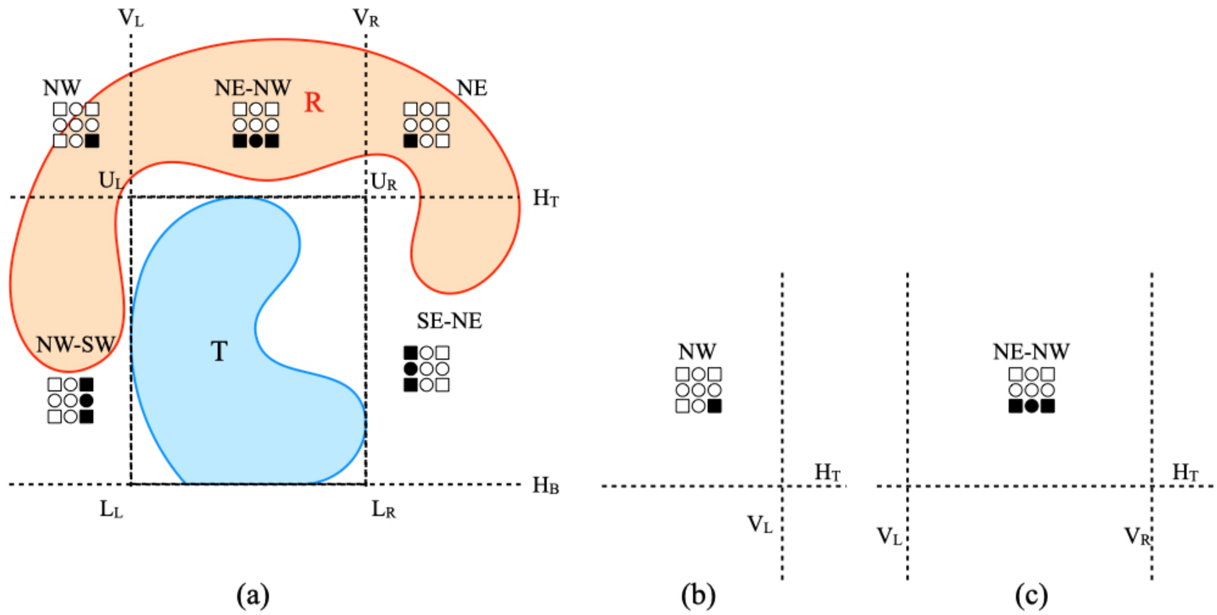


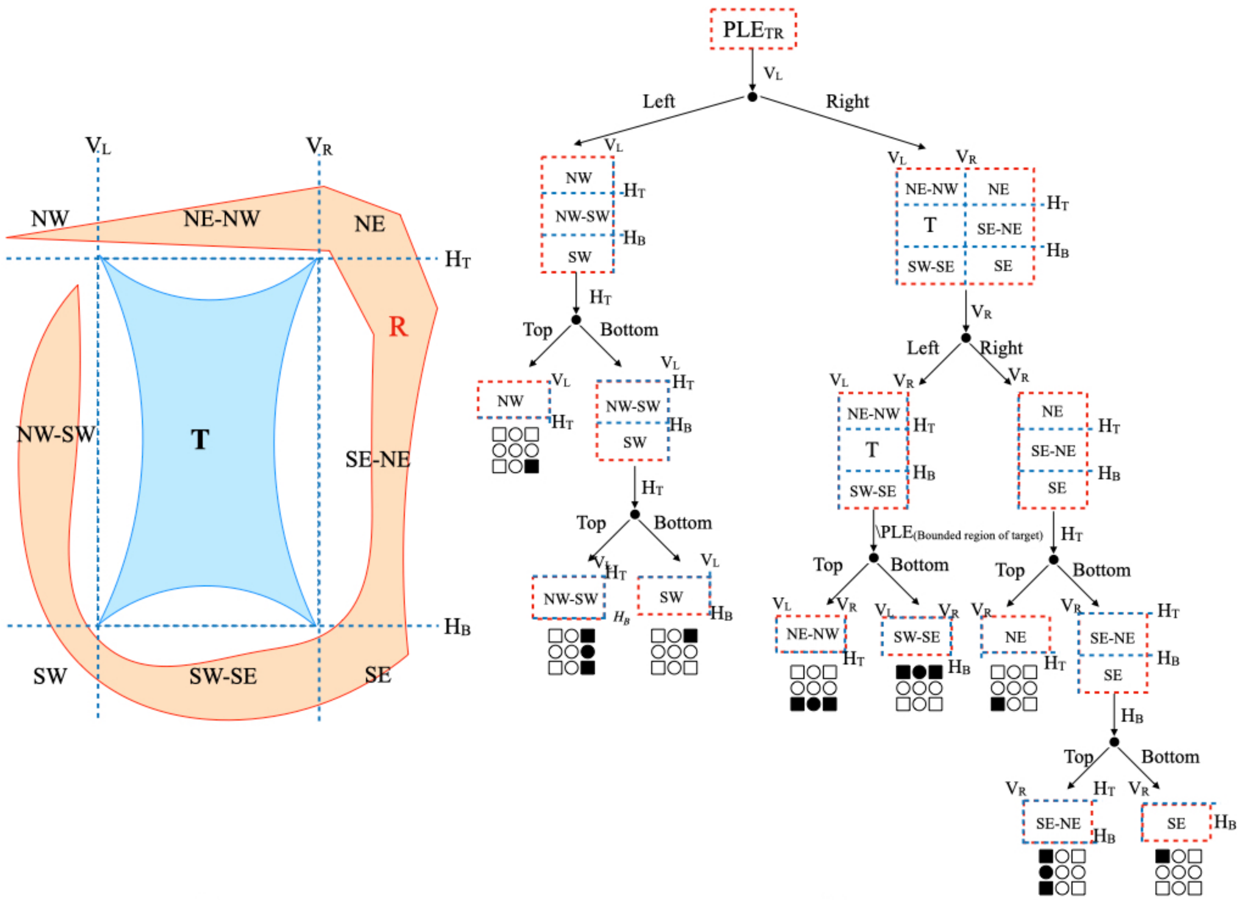
Figure 4.8 The horizontal and vertical PLEs tile the exterior of the target region. The different combinations of the PLEs define the unique direction tiles. (b) The NW tile bounds are defined by the vertical left and horizontal top PLEs. (c) NE-NW tile bounds by the vertical left and right and horizontal top PLEs.

The vertical and horizontal PLEs have the property of partitioning the region into two regions; (1) the vertical PLE partitions the region at interior as right and left regions, and (2) the horizontal PLEs partitions it as top and bottom regions. For example, the V_L, V_R, H_T, and H_B axes, while passing through the target absolute extrema, can partition the PLE_{TR} into two partitions (Figure 4.9a). V_L and V_R partition the left and right tiles, respectively. H_T and H_B define top and bottom tiles.

Each tile represents the unique cardinal direction base relation relative to the target object. Those base relations are assigned to the tiles, considering the combinations of the reference PLEs. For example, the vertical left PLE, V_L, tiles the PLE_{TR} into two tiles: (1) the left set of tiles and (2) the right set of tiles. The left set of tiles contains V_L, H_T and H_B PLEs and a right set of tiles contains the V_R, H_T, and H_B PLEs. The left set of tiles have three cardinal direction tiles. The top and bottom tiles of left set of tiles define by the horizontal PLEs (H_T, H_B). For instance, if a tile that is divided by the H_T does not have an intersection with the H_B, it is a top tile. If a tile has only V_L and H_T bounds, it is labeled as a NW tile. Its cardinal direction base relation is SE, so any point reference from the NW tile represents the direction relation as SE (Figure 4.9a).

The right tile of PLE_{TR}, which is generated by the V_L, is partitioned by the V_R. If a partitioned tile does not have an intersection with the V_L, then that is the right tile. The right tile could be tiled by the H_T, resulting in the top and bottom tiles. A tile with V_R and H_T bounds that represents the SW direction tile.
The left tile, which is formed by the right vertical line, could have three tiles, including the target PLEs bounded region. The geometry difference between the left set of tiles and the target PLE bounded region introduces two tiles. One of them intersects with bottom horizontal line, then the tile is SE-SW, and its direction is NW, N, NE. The remaining tile is a NE-NW tile, and its cardinal direction relation is SW, S, SE (Figure 4.9a).

Each rectangle tile of the exterior of the target region has unique horizontal and vertical PLEs configurations (Figure 4.9b). These unique representations occupy the specific point-to-region base relations that are independent from the geometric shape of target and reference objects.



(a) (b)

Figure 4.9 PLEs defines the (a) exterior of the target region with the direction tiles and (b) the flow of the algorithm to assign the unique base relations for respective tiles.

The algorithm for partitioning the exterior of the target region works well, when the reference region does not pass target PLEs. If the reference region falls into the rectangular target region, formed by the extrema, the next algorithm will define areas of same cardinal direction as pockets.

4.3.2 Pockets: Areas of Same Cardinal Direction

The pocket-based algorithm, to define the areas of same cardinal direction at the interior of rectangle region of target, formed by its extrema, is the solution for coarse range cardinal direction relation determinations. If a reference region falls into a rectangle region of target, existing models of cardinal directions suffer from capturing the direction relations (Chapter 2). The following algorithm determines the unique set of direction relations between the reference and target regions, where the reference object falls into the rectangular target region.

Pockets, which are sensitive to the geometric shape of the target object, constructed by the orthogonal horizontal and vertical PLEs, which pass through the vertices of the target, define areas of same cardinal direction.

The algorithm initially defines the vertical and horizontal PLEs (Figure 4.10a). The PLEs pass through each vertex of the target region, they record the point vertex and the target region intersection. If the record of PLE indicates the disjoint relation between the vertex and target region along the PLEs, introduce the *local extreme* point (Figure 4.10b).

The *local extreme* and the closest boundary of the region along the PLEs define the vertical and horizontal split lines for segmenting the exterior at the interior of the rectangular region of target (Figure 4.10c). The exterior, which is the geometric difference between the rectangular region and target, considered as *region of exterior*.

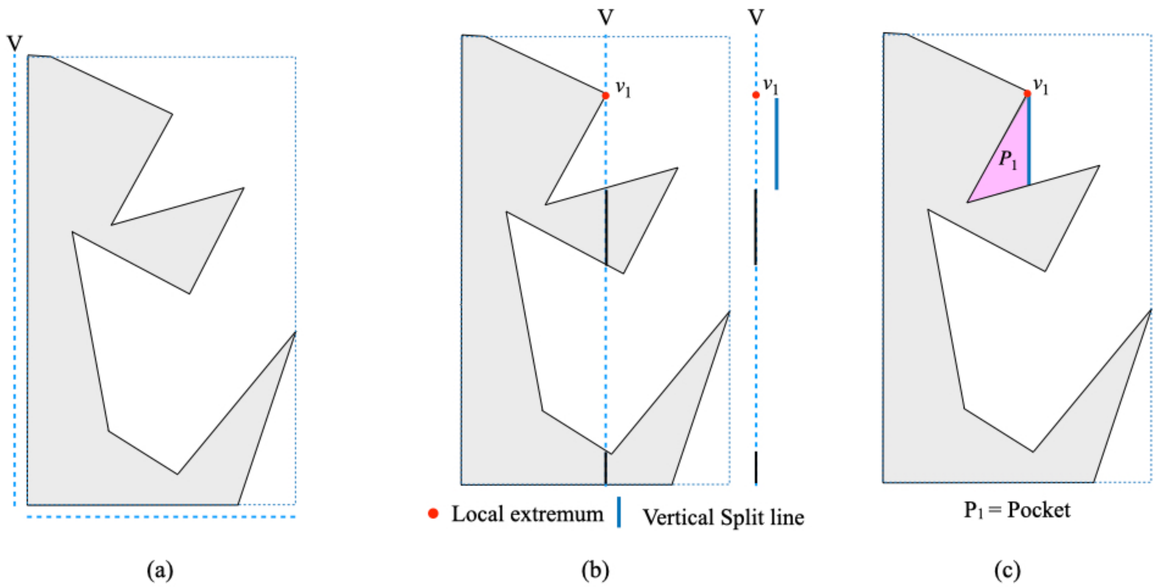


Figure 4.10 The local extrema points and the split lines are defined by following steps: (a) horizontal and vertical PLEs formation, (b) records of the local extremum and the region intersection, and (c) split lines to define areas of same cardinal direction.

Vertical and horizontal split lines segment the *region of exterior* into left and right, top and bottom partitions, respectively, that allow to define the areas of same cardinal direction (Figure 4.11a). The algorithm traces the intersection between the vertical and horizontal segments. If the intersection exists, the common region between them occupies the base relation (NW, N, NE, E, SE, S, SW, W) (Figure 4.11b), then the common region subtracts from the corresponding vertical and horizontal pockets. The results of geomatic subtraction introduce the pockets for of same cardinal direction (Figure 4.11c and Figure 4.11d).

If a resultant partition is bound by the pair of orthogonal PLEs of the target region, which preserves the unique base relation, for example, if a partition, which is the result of a vertical split line, is bound by the Northmost and Eastmost PLEs the region represents a unique base relation (Figure 4.11e).

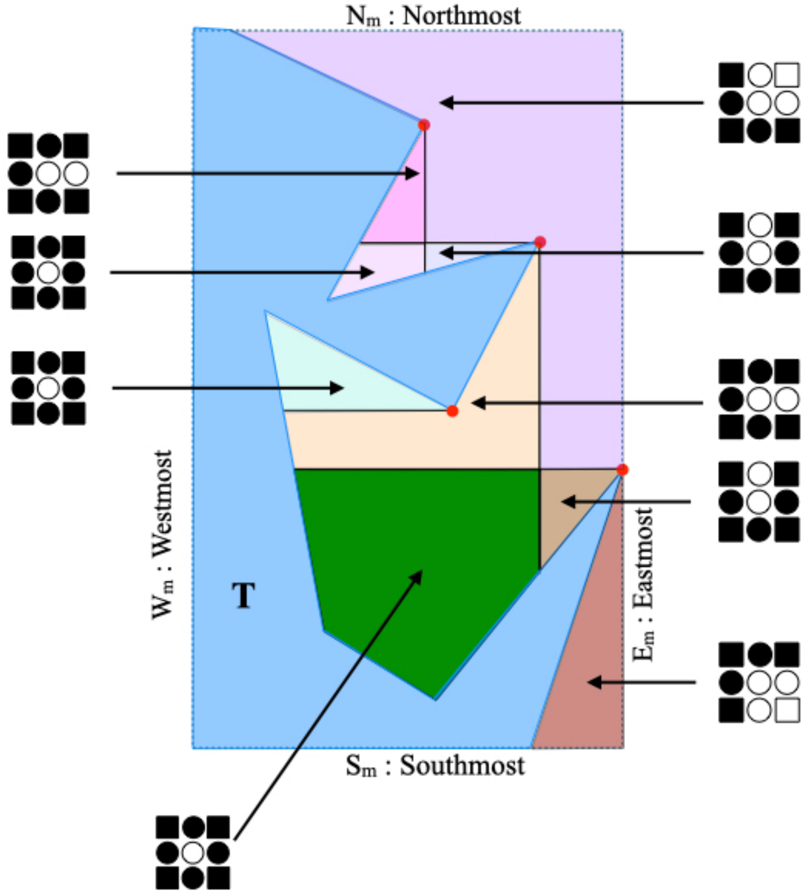


Figure 4.11 Pockets of same cardinal directions.

Vertical and horizontal split lines segment the *region of exterior*. Each segment preserves the unique point-based cardinal direction base relations (Figure 4.12). In line, common boundaries between the neighboring pockets of cardinal direction represent unique set of base relations following point-to-region conceptual neighborhood graph. Those sets of unique base relations at *region of exterior* allow to interpolation.

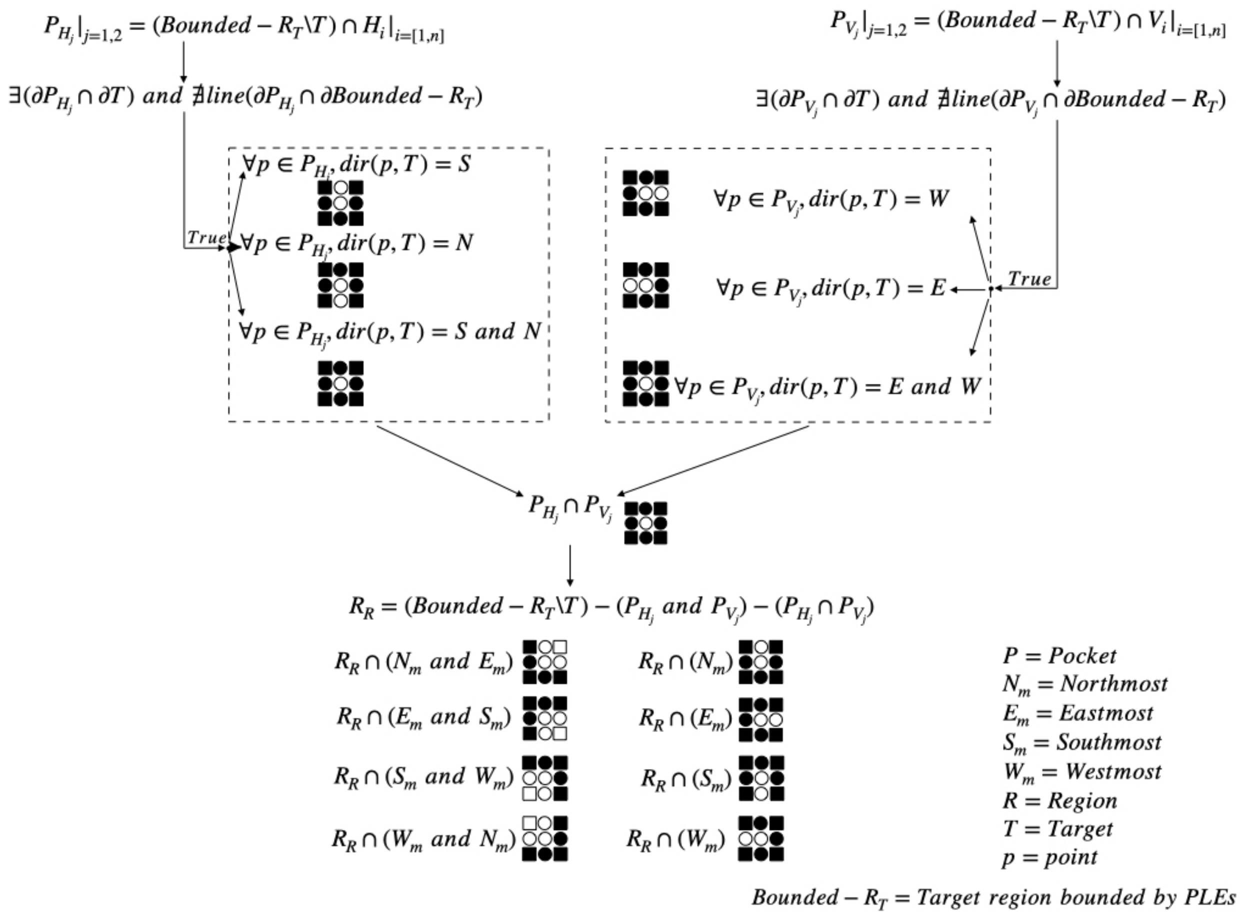


Figure 4.12 The algorithm to define the pockets of same cardinal direction.

4.4 Interpolation of Cardinal Directions

The cardinal direction as a field is continuous and exists everywhere in the operant regions embedded 2-dimensional plane space. The projection-based model partitions the cardinal direction field into areas of same cardinal direction. The relations between the neighboring areas of same cardinal direction are important in future spatial reasoning because they enable the interpolation of a direction relation for an unknown location in the direction field. This section ensures the set operators, such as union and intersection, between the areas of the same cardinal directions for the interpolation.

The PLEs intersections are the primary reference points for determining the direction of the target from its reference direction field. The participation of the PLEs intersections in tiling the exterior of the target region into areas of same cardinal direction classifies the tiles into two categories: (1) tiles with a single PLE intersection point, and (2) tiles with two PLEs intersections.

Tiles of same cardinal direction with a PLEs intersection are closed by the intersection of base relations at its bounding PLEs. For example, if the NE-tile of the target object, the direction relation from the NE-tile to the target object is SW. The NE-tile has a single intersection point and two adjacent primary cardinal direction PLEs as boundaries. Along the PLEs at the exterior of the target, two unique point-to-region base relations are preserved: (1) the vertical PLE (S, SW) and (2) the horizontal PLE (W, SW). The intersection between those base relations is equal to the direction of the tile (SW) (Figure 4.13a). Similar property follows for all other tiles who have single intersection points by its PLEs.

.

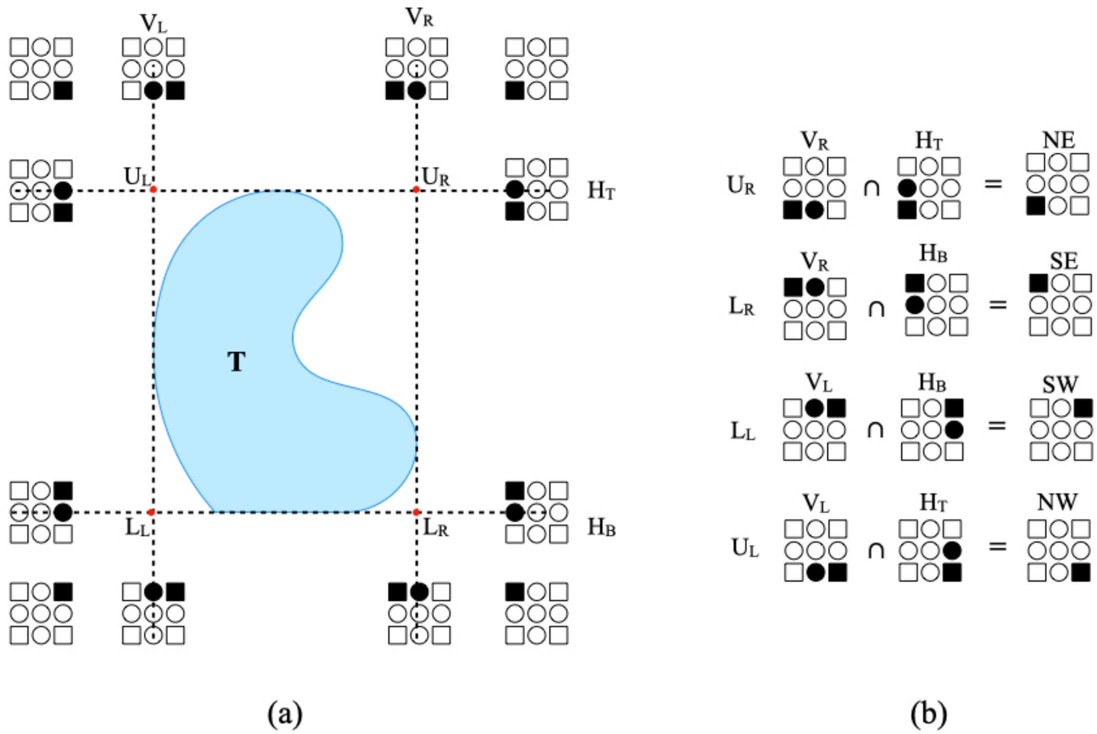


Figure 4.13 Tiles of same cardinal direction explore (a) tiles formed by the single intersection point of PLEs and (b) the intersection of their boundary's direction relations.

The direction tiles with two PLEs intersection are closed under the union of pairs of their horizontal or vertical PLEs directions. For example, the NE-NW tile has two intersection points, such as the upper left and upper right of the bounded target region (Figure 4.14a). The combination of two base relations from two distinguishable vertical PLEs results in the direction of the tile, such as NE-NW as (SW, S, SE) (Figure 4.14b).

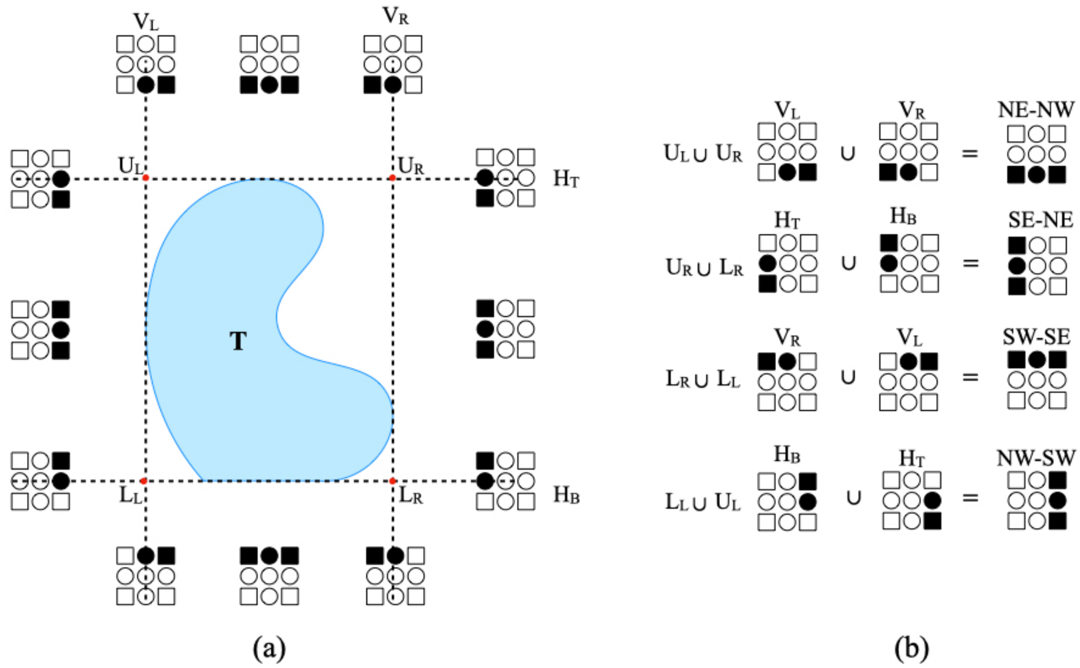


Figure 4.14 Tiles of same cardinal direction explore (a) tiles that are formed by the two intersection points of PLEs and (b) the union of their boundary's direction relations.

The union and intersection of the base relations in the cardinal direction field lead to the interpolation. The conceptual neighborhood graph of the point-to-region relations exhibits the valid relations between the areas of same cardinal direction and their boundaries in a direction field. It allows interpolation of neighboring direction relations. For example, the tiles of areas of same cardinal direction, and their boundaries preserve 16 point-to-region base relations (Figure 4.15). Those base relations show valid neighborhood connections in the conceptual neighborhood graph (Figure 4.16).

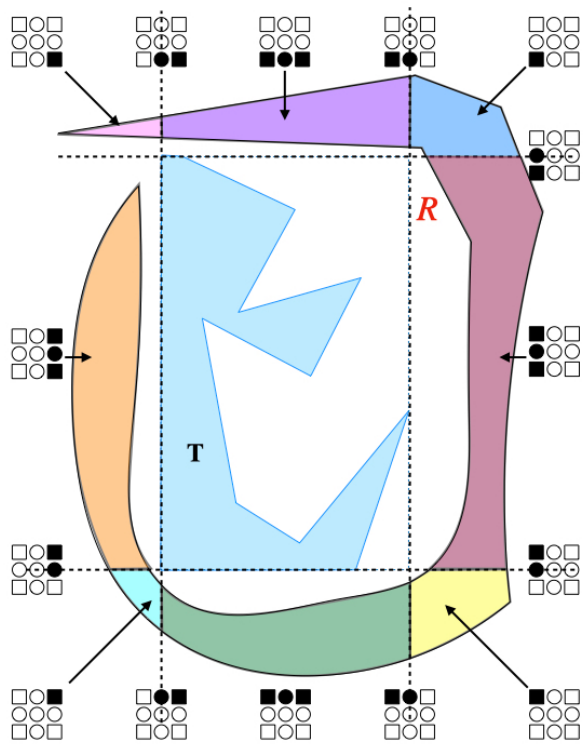


Figure 4.15 Areas of same cardinal direction on the reference object R.

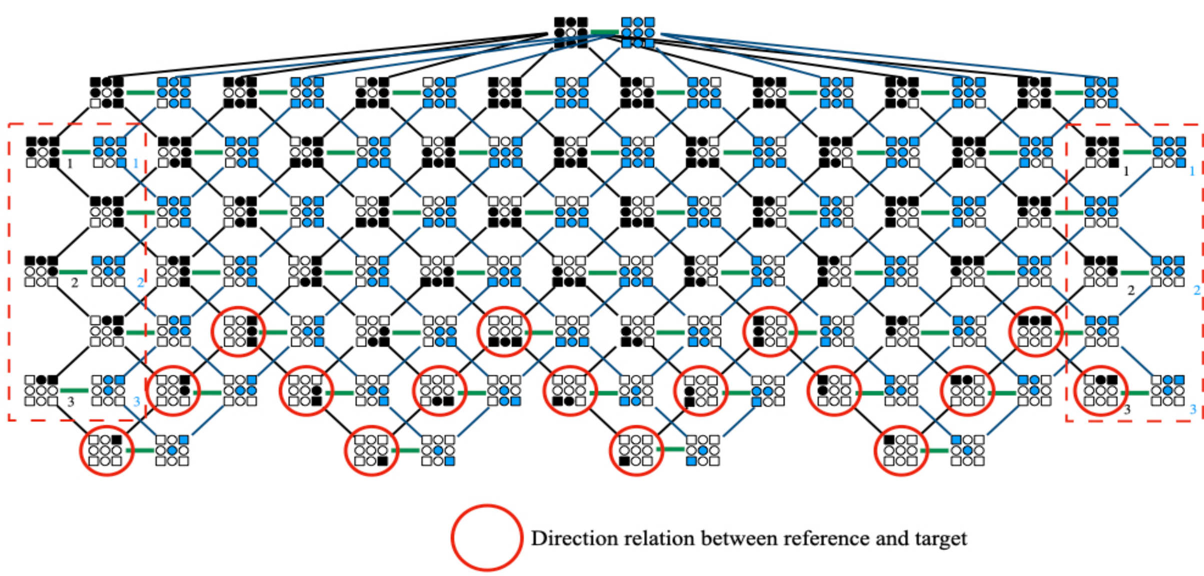


Figure 4.16 Sixteen point-to-region base relation's valid neighboring connections at the exterior of the target object.

The conceptual neighborhood graph explores the valid neighborhood connections between the pockets of same cardinal direction. The point-to-region neighborhood graph encourages interpolation of direction relations with few known relations among the pockets. In addition, the pocket with neighborhood connections explains that its exterior is continuous to define the direction relations, which conform to the cardinal direction as a field (Figure 4.17).

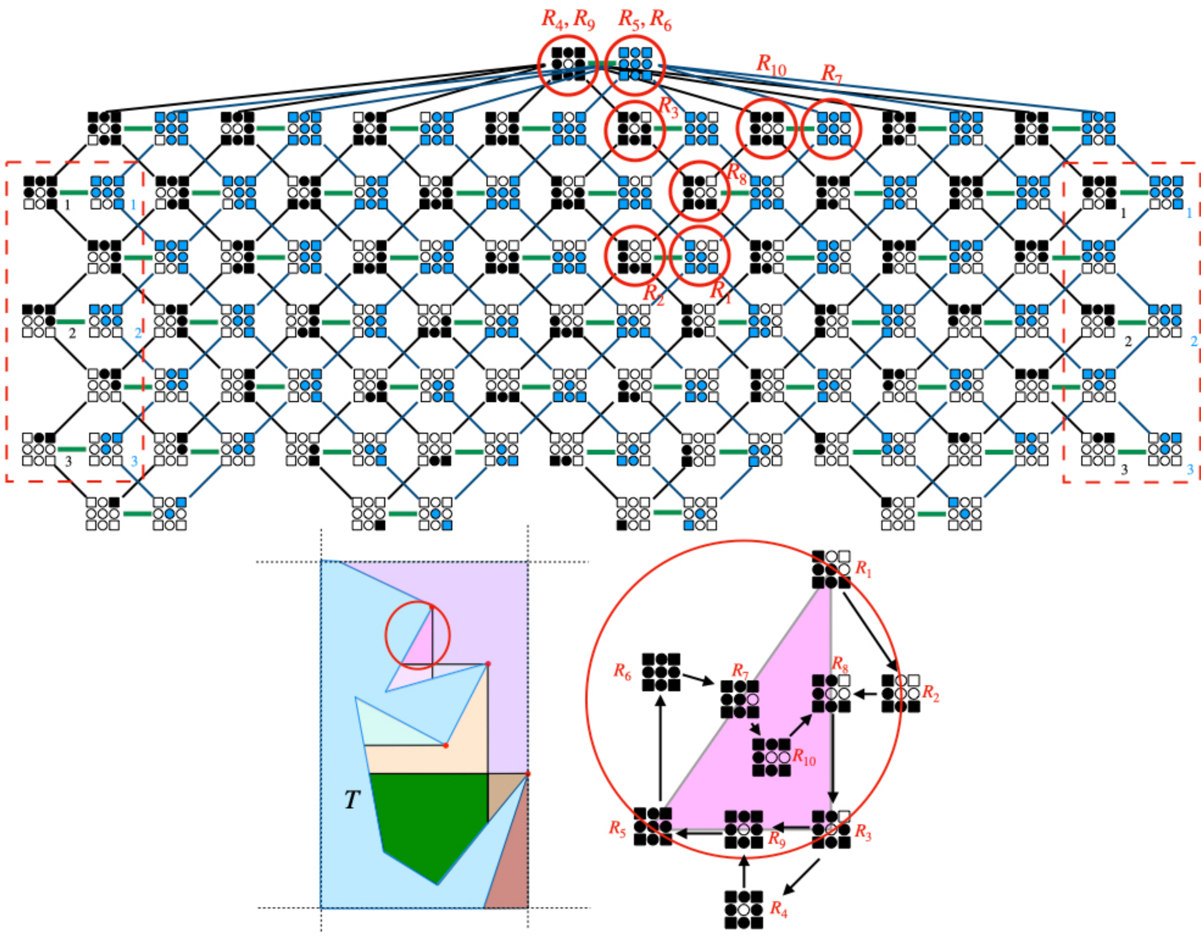


Figure 4.17 Areas of same cardinal direction, pockets that exhibit the neighborhood connections through the conceptual neighborhood graph with their boundaries and neighboring pockets.

4.5 Detailed and Coarse Direction Relations Between Two Simple Regions

The point-based direction model for a specific relation between a simple reference region and a simple target region yields a non-empty set of point-to-region relations, which wholistically cover the directions from all possible locations in the reference region. This set is called the set of applicable cardinal directions (SACD).

In a unique scenario, SACD will consist of exactly one of the 106 possible point-to-region relations, a scenario that implies that from each point in the reference region the same cardinal direction holds to the target region. In an extreme scenario, SACD would comprise all 106 relations. Whether this is possible for two simple regions is not part of the current investigations.

4.5.1 Detailed Cardinal Direction Between Two Regions

Usually, a particular point-to-region relation will apply to several areas with the same cardinal direction (Figure 4.18a); therefore, the SACD alone will not necessarily define the cardinal direction between two regions completely at its finest granularity. Such a detailed model would require not only the point-to-region relations that apply to each region of the same cardinal direction, but it also needs to capture how these partitions, each associated with an element of SACD, are topologically related.

For this purpose, the neighborhood of the areas of same cardinal direction are captured as a graph in which an edge in the graph (Figure 4.18b), while disjoint areas of the same cardinal direction are not directly connected in that graph. The graph's nodes are the applicable direction relation icons.

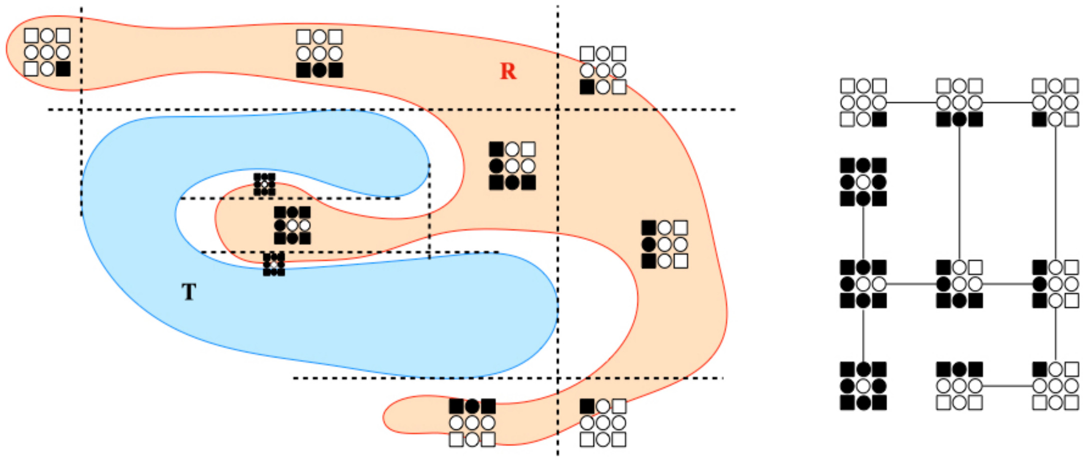


Figure 4.18 Detailed cardinal direction relation between a reference R and the target T: (a) intersect object R with the tiling field and (b) that's the direction relation.

4.5.2 Coarse Cardinal Direction Relations

Tiles and pockets define multiple areas of same cardinal direction in the reference region. The areas of same cardinal direction and their connected graph explain the deep direction relations between a reference and the target objects. The opposite of obtaining the most detailed cardinal direction is the coarsening to obtain a single representative cardinal direction. There are multiple semantics possible for obtaining such a representative cardinal direction that would yield a single relation (possibly empty). From any point in the reference region, the point-to-region relation to the target region is included among the possible relations, but further relations, may apply.

The attempt to obtain a complete coverage of all direction tiles that apply the union of all elements of SACD (Figure 4.19). The semantics of this union is that any point-to-region direction relation that is not covered by the union cannot be obtained from any possible location in the reference region to the target region. Such a union may quickly reach the universal cardinal direction relation, that is, all nine point-to-region relations are possible. The universal relation would require that the reference and the target regions share at least one common point so that the centerpiece "same direction" is included.

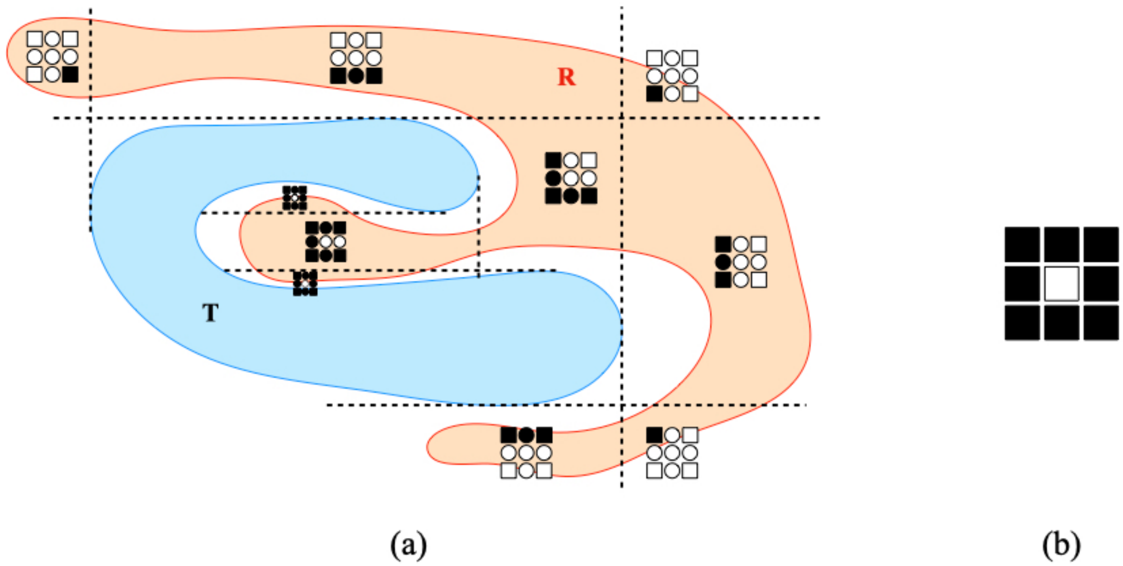


Figure 4.19 The point-reference cardinal direction model defines the direction relation between two simple regions by (a) areas of same cardinal direction, and (b) their coarse direction relation is the union of all applicable point-to-region relations.

This thesis uses the union of all elements of SACD as a process to obtain a single representative cardinal direction relation. When this coarse relation is insufficient, as a first step, the set of all applicable cardinal directions can be used. Finally, if SACD is still insufficient for a particular task, the graph linking the neighboring areas of same cardinal directions, each with its direction symbol, may be used.

4.6 Summary

This chapter details the region-to-region cardinal direction relations. The properties of cardinal directions in the 2-dimensitional plane space, such as continuity and the fact that they exist everywhere, led to the introduction of cardinal direction as a field. The fundamental unit of space, the point, is considered to extract the direction relations from the field. The direction relations between two simple regions are defined by considering the region as a point set. At each point in the interior of a reference region, one could obtain a cardinal direction value to a target region, that is, one of the 106 point-region relations.

The projection-based model allows for partitioning the exterior of the target into areas of same cardinal direction. Two algorithms are introduced to define the areas of same cardinal direction: (1) tiling the exterior of the rectangular target region, formed by the extrema, and (2) the pockets of same cardinal direction, where the exterior of target is at the interior of the rectangular target region. Tiles and pockets prove the continuity of the direction relations in a cardinal direction field. The neighborhood areas of same cardinal direction follow the point-to-region conceptual neighborhood graph. The conceptual neighborhood graph of point-to-region direction relations enable the interpolation of the direction relations in the field.

CHAPTER 5

IMPLEMENTATION OF AREAS OF SAME CARDINAL DIRECTION

Cardinal directions are a generalized concept to define the direction relations between spatial entities. In spatial database applications, cardinal directions are abstract search criteria to filter for relevant data (Gunther and Buchmann, 1990). For example, climate scientists working on the sea level rise and coastal production in Maine may require information, such as which spatial direction of coastal residents of Maine would be impacted by sea level rise. A landscape map immediately indicates which direction of the land area is impacted first, such that low elevation impacts first, then higher elevation. Researchers may find the direction as the east and west coasts of the Cranberry Isles, Maine. Those cardinal directions may apply to filtering the residents on the east and west coasts of Cranberry Isles rather than searching through the surrounding coastal boundary. Cardinal directions are an effective tool for spatial communications, for instance, on weather reports.

This chapter discusses the implementation of a projection-based model for point-to-region and region-to-region direction relations. A Python program was developed based on the algorithms that define tiles in Section 5.1 and pockets of same cardinal direction in Section 5.2. The program generates a new attribute for the spatial entity and stores it in the spatial database. In Section 5.3, the spatial database linked to QGIS, the application of areas of same cardinal direction in GIS software.

5.1 Implementation of Tiling-Based Algorithm

Python program of areas of same cardinal direction contains two sets of programs: (1) It contains exterior partitioning codes (Figure 5.1) and (2) interior partitioning codes. Initially, the program finds the individual extrema sets for reference and target regions. The two sets of extrema combine into a set to define the combined bounded region of reference and target. In exterior partitioning, perform through the vertical and horizontal sweep lines, which pass through the target region's extrema, generating the tiles of same cardinal directions. The program uses the intersection, touches, split, and clip functions from the pygeos, to define the sweep line algorithm.

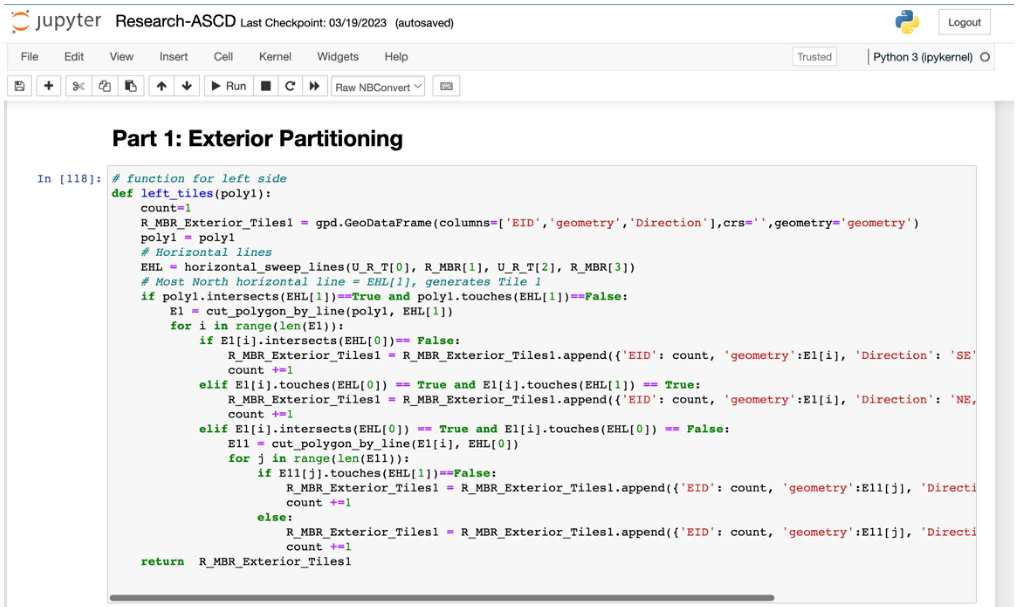


Figure 5.1 Program to define tiles of same cardinal directions.

The arbitrary two simple regions are generated as spatial entities. Those regions are stored in the spatial database (Figure 5.2), and pgAdmin is used for database management. The tiling-based Python program is adaptive to generate the cardinal direction tiles. For example, if the reference object only falls into the Northeast direction tile, the program would generate only the Northeast tile to define the direction relations between the reference and target. The defined tiles are clipped with the reference region to define the areas of same cardinal direction (Figure 5.3).

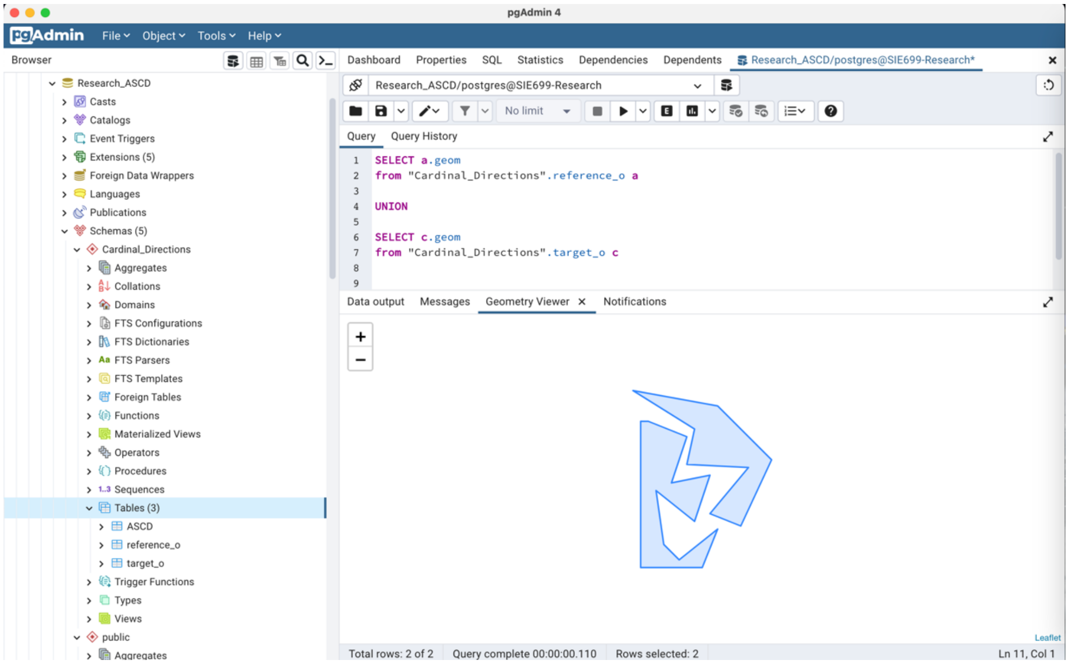


Figure 5.2 Reference and target objects are stored in a spatial database.

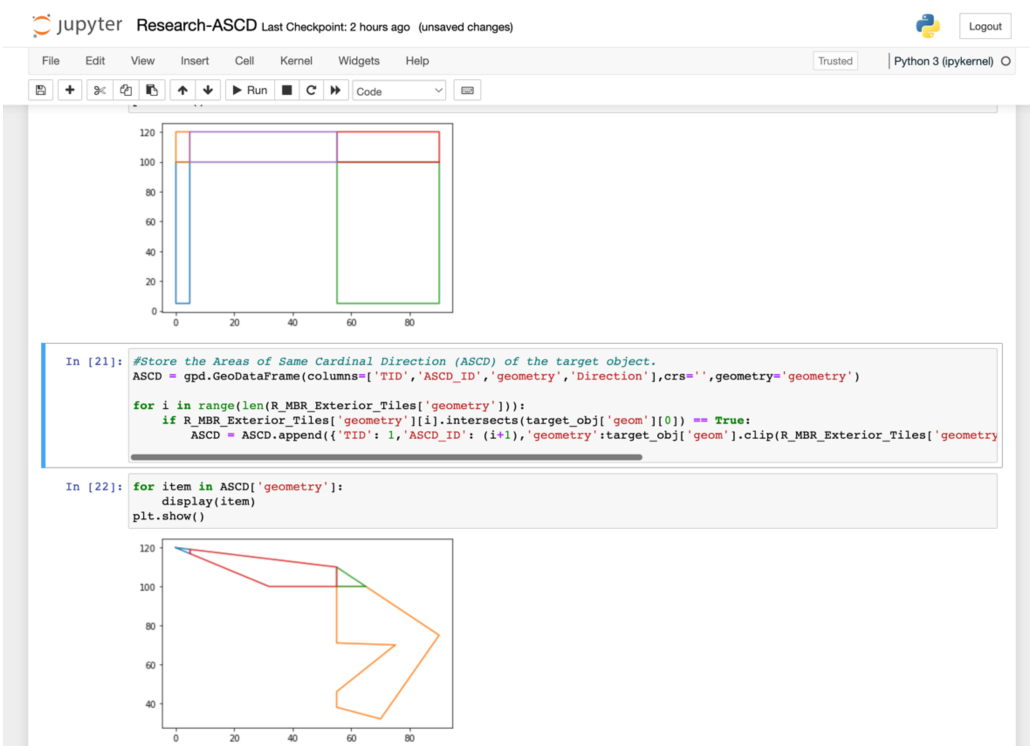


Figure 5.3 The algorithm defines tiles of same cardinal direction.

5.2 Implementation of Pockets of Same Cardinal Direction

The program for pockets of same cardinal direction generates sweep lines, based on the extrema of the target region, then the vertical and horizontal sweep lines pass through each vertex to define the local extrema and split lines to generate pockets of same cardinal directions (Figure 5.4).

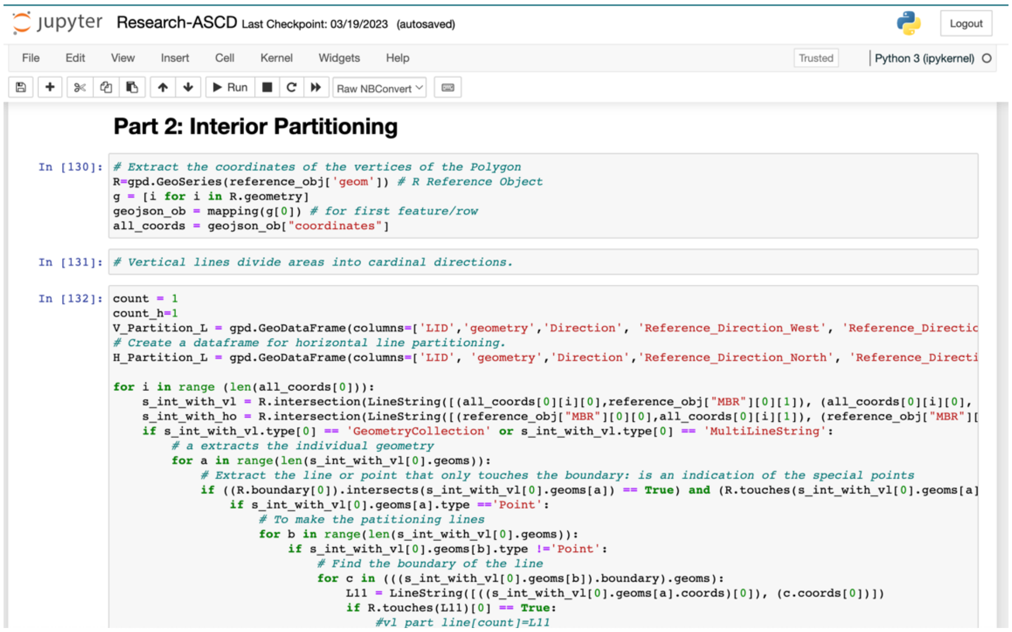
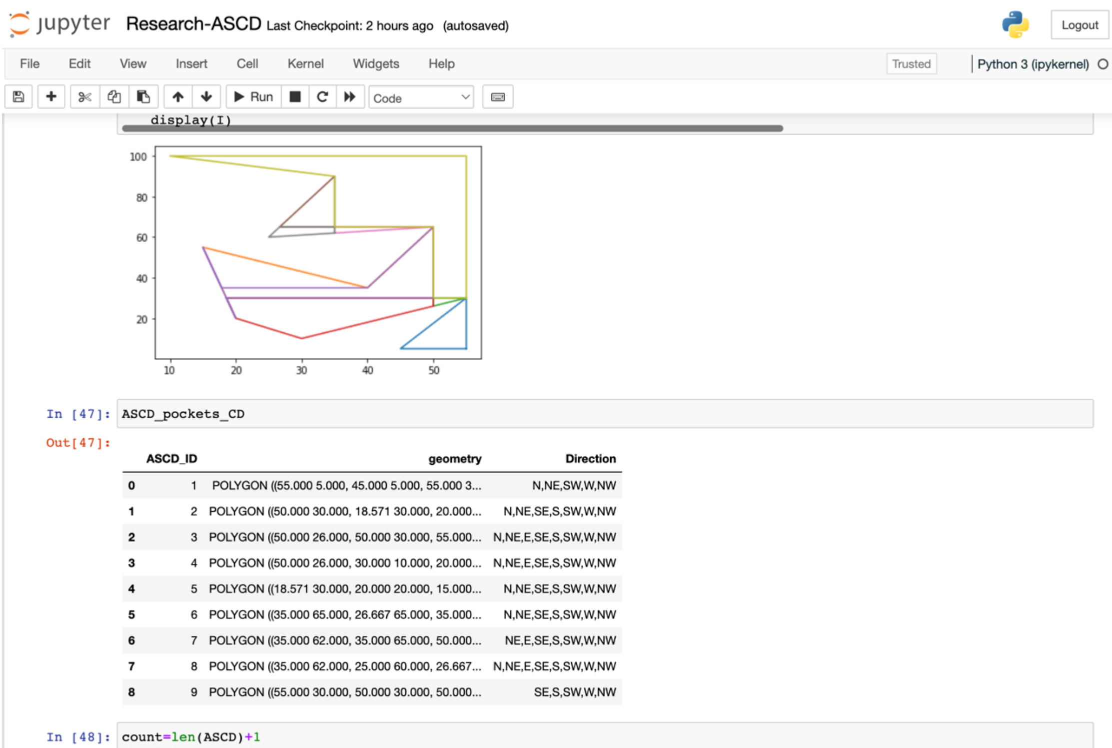


Figure 5.4 Program of pockets of same cardinal directions.

The algorithm to define the pockets of same cardinal direction applies to constructing the pockets of the target region's exterior. The number of pockets depends on the shape of the spatial object, but those numbers are fixed for a target object. Any simple region could have its number of pockets, which allows one-time computation for the specific spatial database. The program updates the pockets and their direction as attributes of a target object in the database (Figure 5.5).



	ASCD_ID	geometry	Direction
0	1	POLYGON ((55.000 5.000, 45.000 5.000, 55.000 3...	N,NE,SW,W,NW
1	2	POLYGON ((50.000 30.000, 18.571 30.000, 20.000...	N,NE,SE,S,SW,W,NW
2	3	POLYGON ((50.000 26.000, 50.000 30.000, 55.000...	N,NE,E,SE,S,SW,W,NW
3	4	POLYGON ((50.000 26.000, 30.000 10.000, 20.000...	N,NE,E,SE,S,SW,W,NW
4	5	POLYGON ((18.571 30.000, 20.000 20.000, 15.000...	N,NE,SE,S,SW,W,NW
5	6	POLYGON ((35.000 65.000, 26.667 65.000, 35.000...	N,NE,SE,S,SW,W,NW
6	7	POLYGON ((35.000 62.000, 35.000 65.000, 50.000...	NE,E,SE,S,SW,W,NW
7	8	POLYGON ((35.000 62.000, 25.000 60.000, 26.667...	N,NE,E,SE,S,SW,W,NW
8	9	POLYGON ((55.000 30.000, 50.000 30.000, 50.000...	SE,S,SW,W,NW

Figure 5.5 Pockets of same cardinal directions.

If the reference object passes the northmost, eastmost, southmost, and westmost PLEs of the target object, then the pockets clip the reference object and introduce pockets of same cardinal directions for the reference object (Figure 5.6).

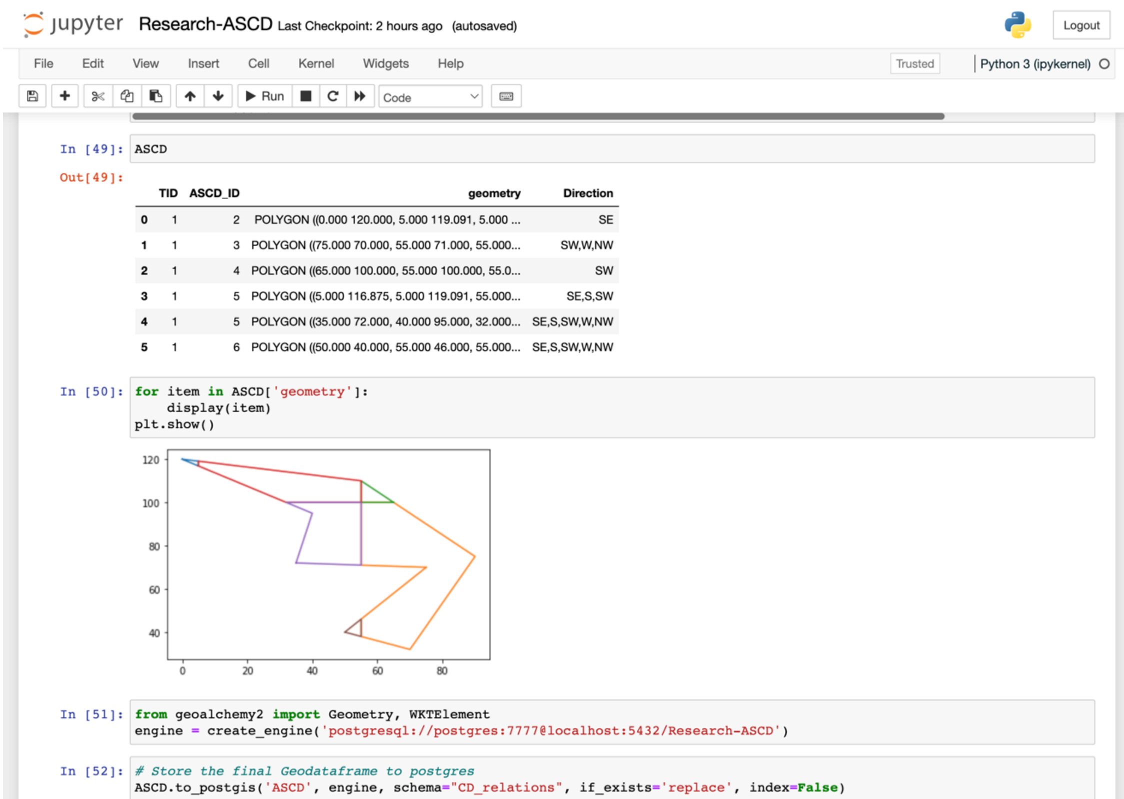


Figure 5.6 Areas of same cardinal direction of reference object.

5.3 Spatial Database and Implementation in QGIS

The areas of same cardinal direction are recorded in the spatial database in a different table. The table contains target object ids, ids of areas of same cardinal direction (ASCD), and respective directions (Figure 5.7). A query, such as “select all areas of same cardinal direction of the reference object,” is evaluated on the pgAdmin (Figure 5.8).

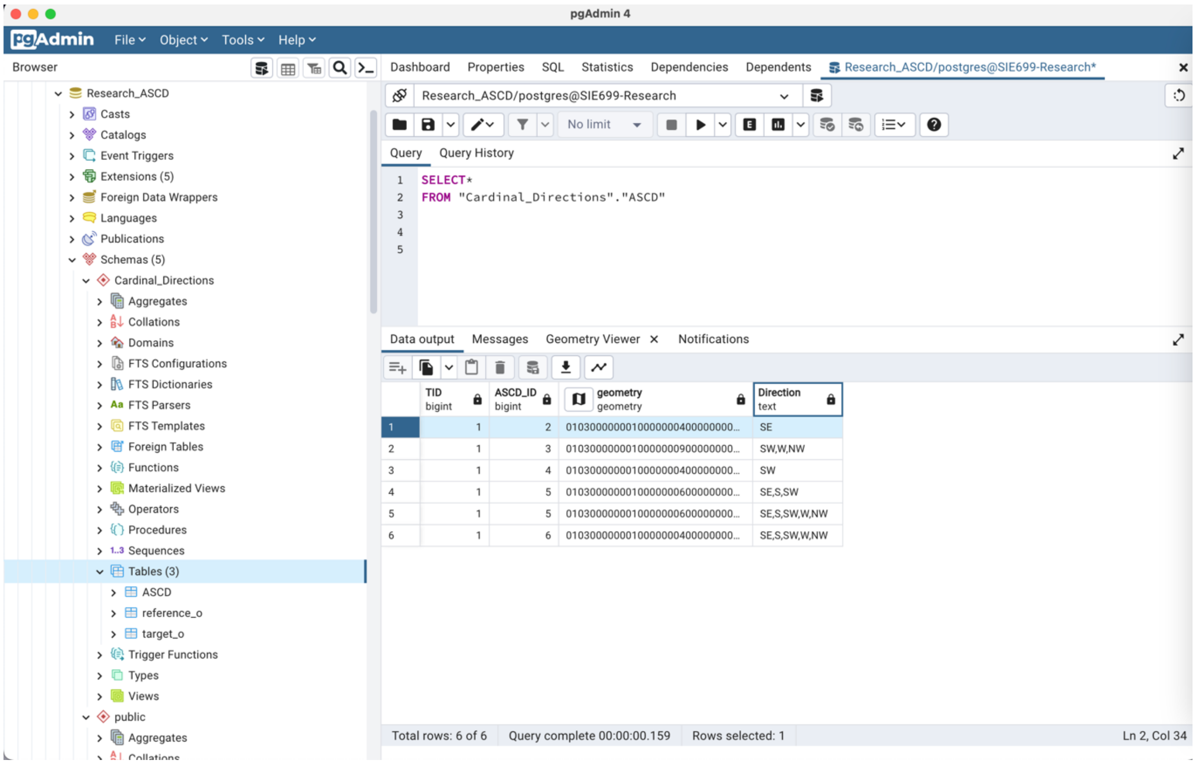


Figure 5.7 Spatial database of areas of same cardinal direction.

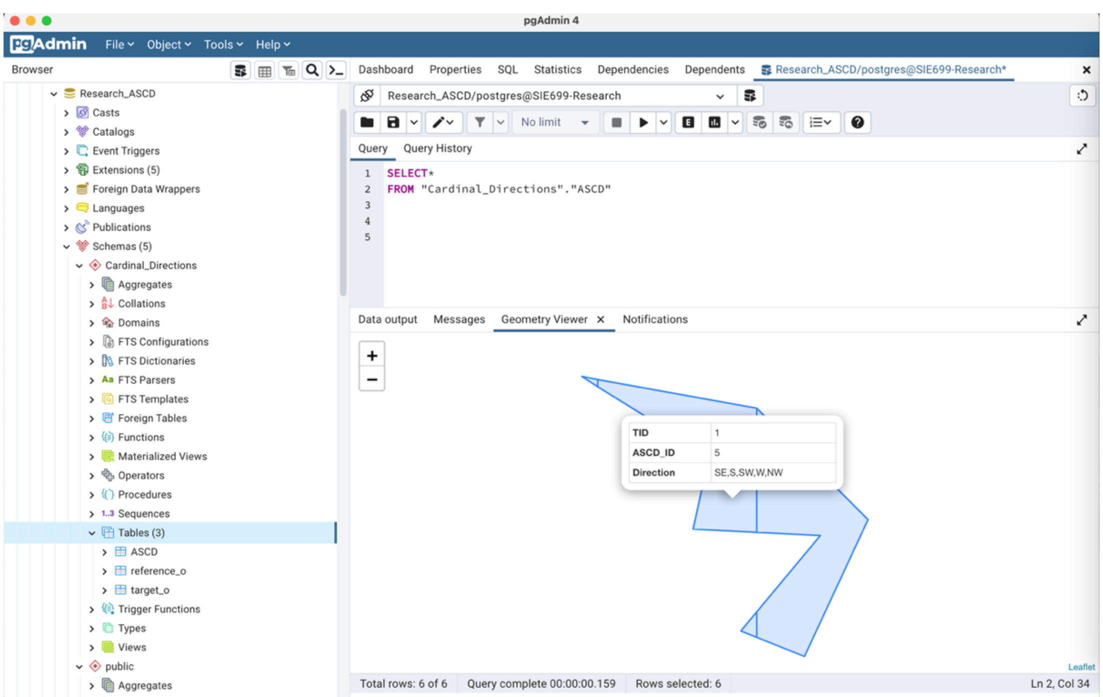


Figure 5.8 Spatial database implementation for areas of same cardinal direction.

The spatial database is capable of sharing the data with multiple applications (Figure 5.9).

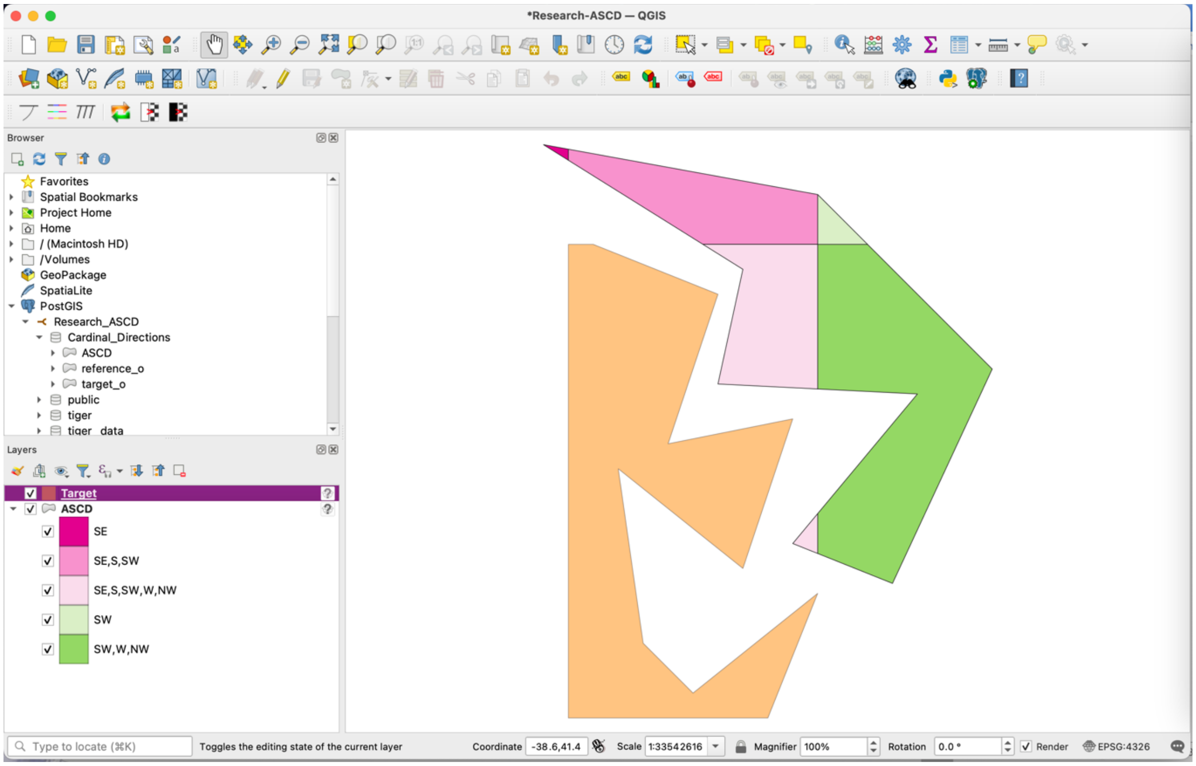


Figure 5.9 Areas of same cardinal direction as a layer in QGIS.

QGIS allow for the SQL queries, if one interest to select all southeast direction regions, direction(reference, target) =SE, then the SQL query would be:

SELECT*

FROM “Cardinal_Directions”. “ASCD”

WHERE “ASCD”. “Direction” = ‘SE’

The from defines the cardinal directions database and ASCD, areas of same cardinal direction table. Where specifies the condition to select all southeast regions (Figure 5.10).

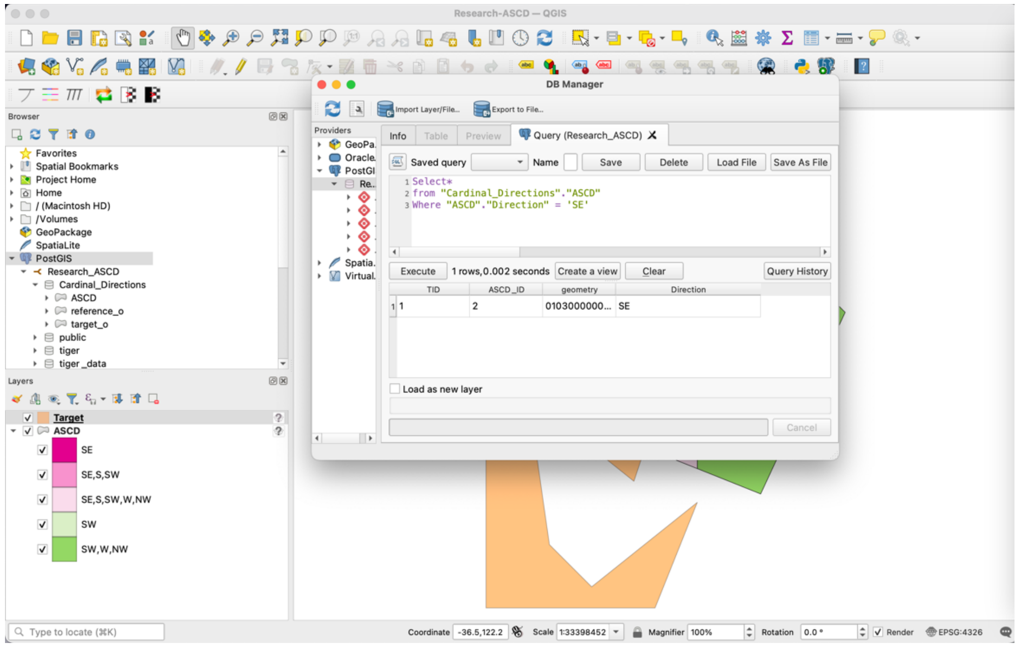


Figure 5.10 Spatial query in QGIS.

QGIS allows the query output into a separated layer and allows for spatial analysis (Figure 5.11).

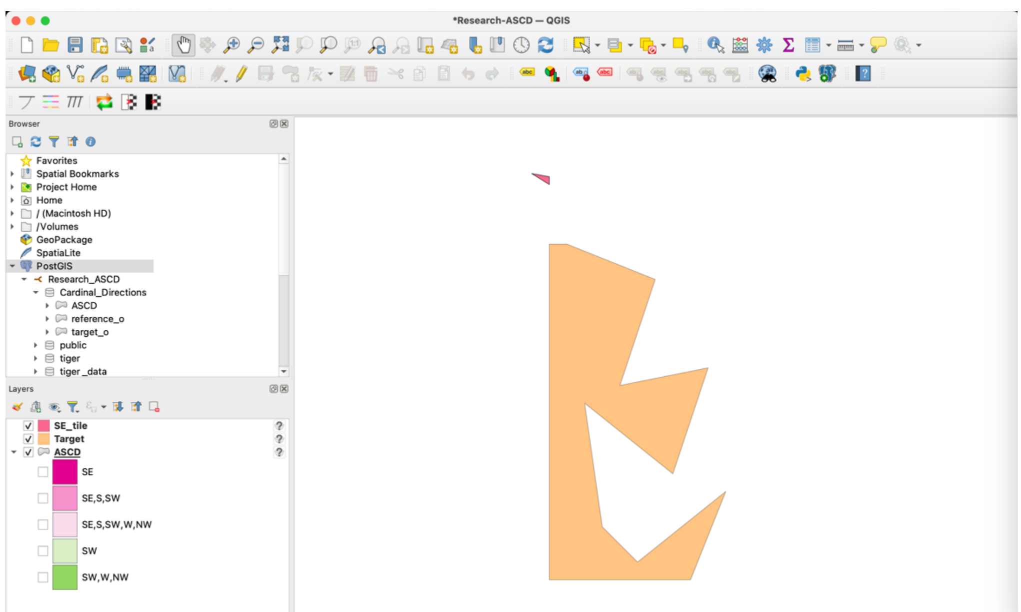


Figure 5.11 Query results as separated layer for spatial analysis.

5.4 Summary

This chapter detailed the implementation of areas of same cardinal direction. The algorithms for tiles and to define the pockets of same cardinal direction are implemented and tested through the Python program. Two simple regions, the target and reference regions, are stored as spatial entities. The program creates a spatial database to store the spatial entities with areas of same cardinal direction. The spatial database allows for data sharing between the different GIS applications. The generated areas of same cardinal direction are converted as a layer in QGIS. The spatial data layer of areas of same cardinal direction allows for spatial analysis.

CHAPTER 6

COMPARISON WITH EXISTING MODELS

This chapter tests the hypothesis of the thesis by comparing the point-reference cardinal direction model, which defines simple region-to-region direction relations, called areas of same cardinal direction, with the major existing computational models of cardinal directions: (1) DRM and (2) OIM. The DRM approximates the reference object into its minimum bounding rectangle and falls short of assigning direction relations at a coarse range. OIM approximates the reference and target objects into object interaction grids, which are same as their minimum bounding rectangle approximation.

The point-reference model defines the point-to-region and region-to-region direction relations and introduces a set of 106 direction relations, including 22 normalized atomic relations. The eight topological relations between simple reference and simple target objects are used as context to compare the point-reference model with DRM and OIM models.

6.1 Direction Relation Matrix

The DRM defines the direction relation between two extended objects by approximating the reference object through its minimum bounding rectangle (MBR) while preserving the target object's shape (Goyal and Egenhofer, 2001). This approximation introduces inconsistencies when defining direction relations. The model assigns the same relation whenever the target object falls into the reference MBR.

The DRM approximation falls short between an information system and human conceptualization, because the model shows inconsistency in coarse range direction relations. For example, topological relations such as covers (Figure 6.1a), contains (Figure 6.1b), and equal (Figure 6.1c) between the reference MBR and target region show the similar cardinal direction relation.

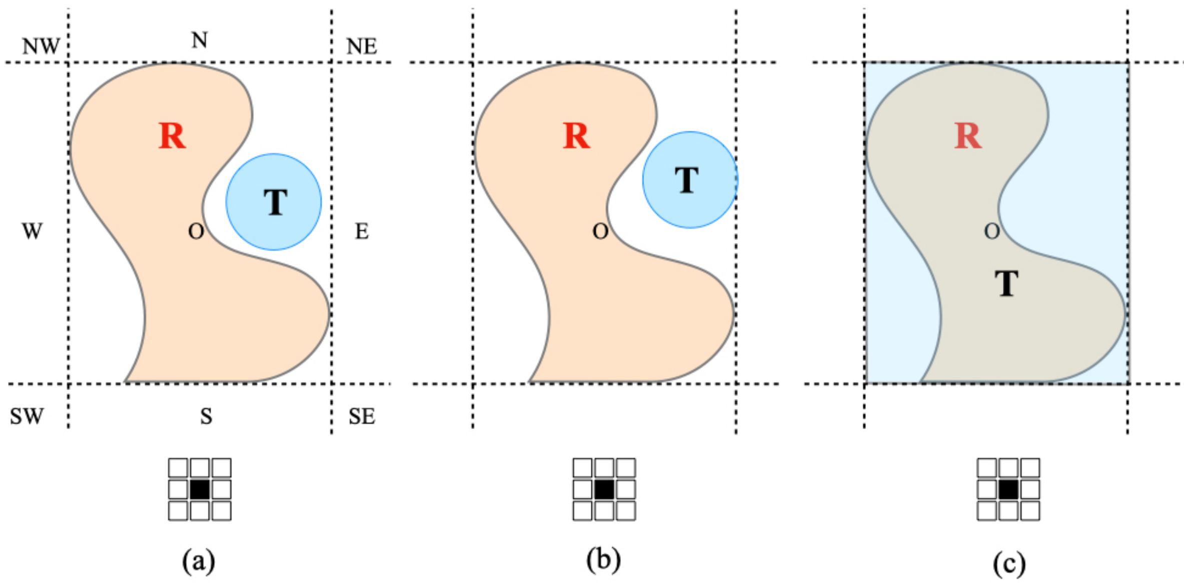


Figure 6.1 Direction relation matrix for topological relations between reference MBR and target region: (a) contains, (b) covers, and (c) equal.

The base topological relations are defined by the 9-intersection between simple regions (Egenhofer and Herring, 1992). They are used to explain any topological configurations between two simple regions in R^2. The eight simple topological relations between reference and target region apply to confirm the DRM that inconsistency to defines the direction relations between simple region at coarse range. DRM provides similar direction captures for eight topological relations except inside (Figure 6.2h), assigns same direction for disjoint (Figure 6.2a), meet (Figure 6.2b), overlap (Figure 6.2c), equal (Figure 6.2d), contains (Figure 6.2e), covers (Figure 6.2f), and coveredBy (Figure 6.2g). These examples indicate that DRM has ambiguities with the target objects whenever the target region falls into the reference MBR.

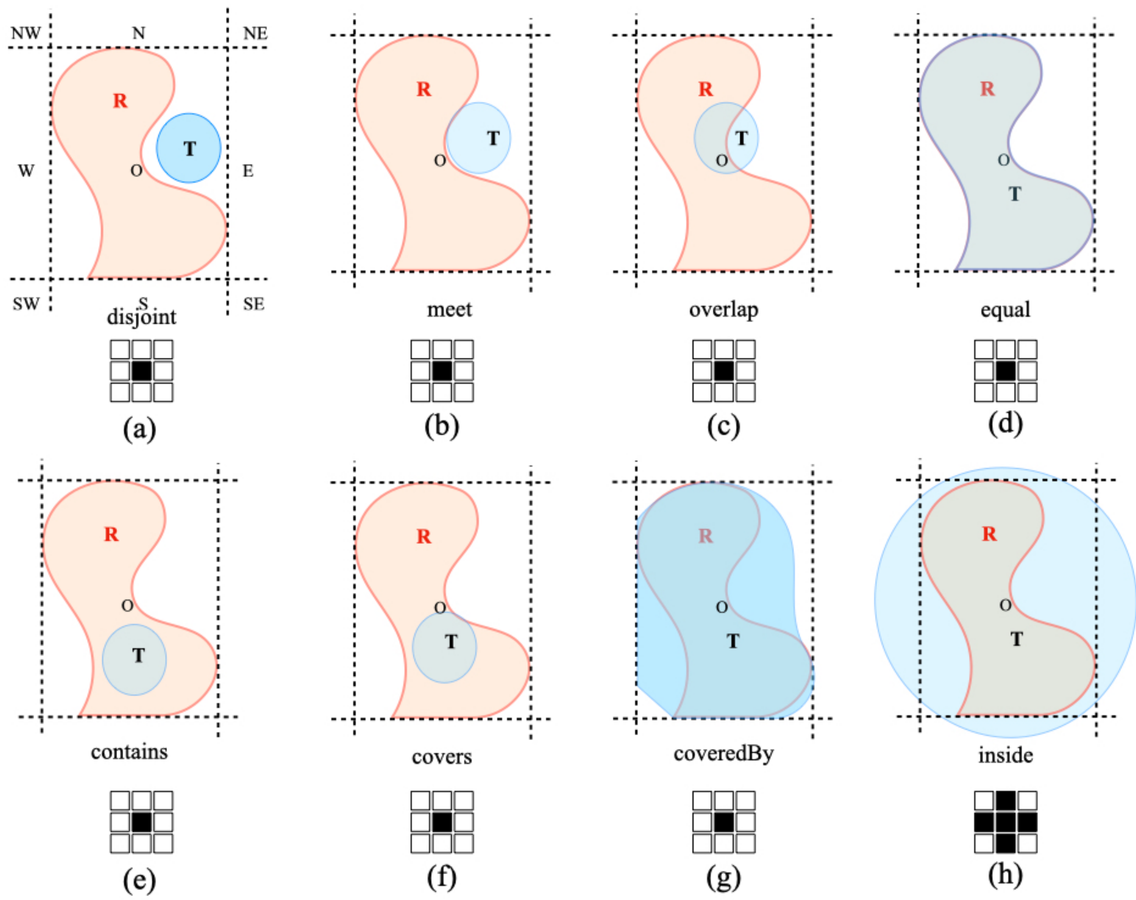


Figure 6.2 DRM for eight simple topological relations between reference and target object: (a) disjoint, (b) meet, (c) overlap, (d) equal, (e) contains, (f) covers, (g) coveredBy, and (h) inside.

6.2 Object Interaction Matrix

The object interaction matrix (OIM) considered extrema of both reference and target objects to construct the object interaction grid. OIM records the existence of the operant objects by numbers. If the grid cell contains a reference object, it is 1, 2 for the target object, and 3 for the reference and target object. The eight simple topological relations between reference and target region apply to confirm the OIM that works less to prove cardinal direction as a field except inside (Figure 6.3h). The OIM suffer to distinguish the eight topological relations; disjoint (Figure 6.3a), meet (Figure 6.3b), overlap (Figure 6.3c), contains (Figure 6.3e), and covers (Figure 6.3f) show similar direction relations. The OIM for equal (Figure 6.3d) and coveredBy (Figure 6.3g) relations is similar to the DRM, assigns as it the same direction.

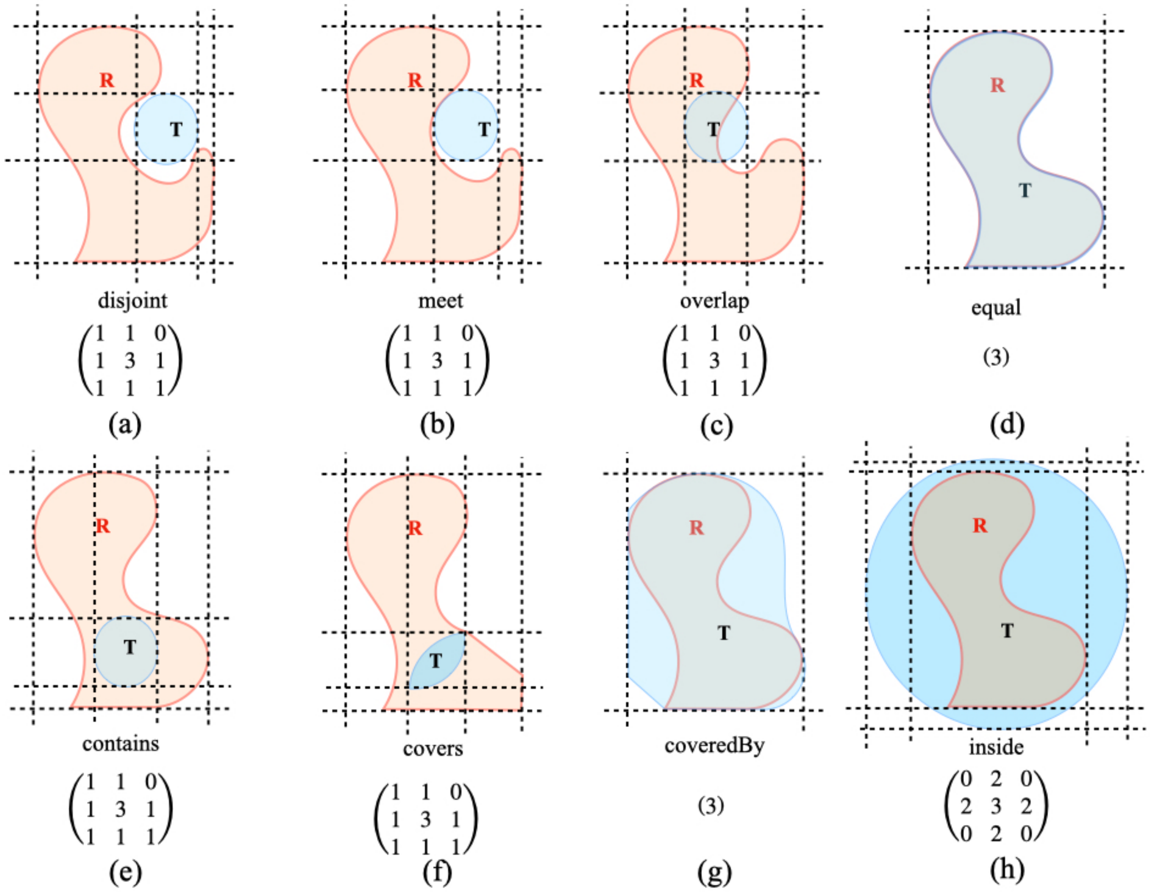


Figure 6.3 OIM for eight simple topological relations between reference and target object: (a) disjoint, (b) meet, (c) overlap, (d) equal, (e) contains, (f) covers, (g) coveredBy, and (h) inside.

6.3 Point-Based Cardinal Direction Model

The point-reference cardinal direction model confirms the cardinal directions as a field by capturing continuing direction changes over the reference object. Point-reference cardinal direction model distinguish the eight topological relations between a simple reference and the target objects (Figure 6.4 and Figure 6.5).

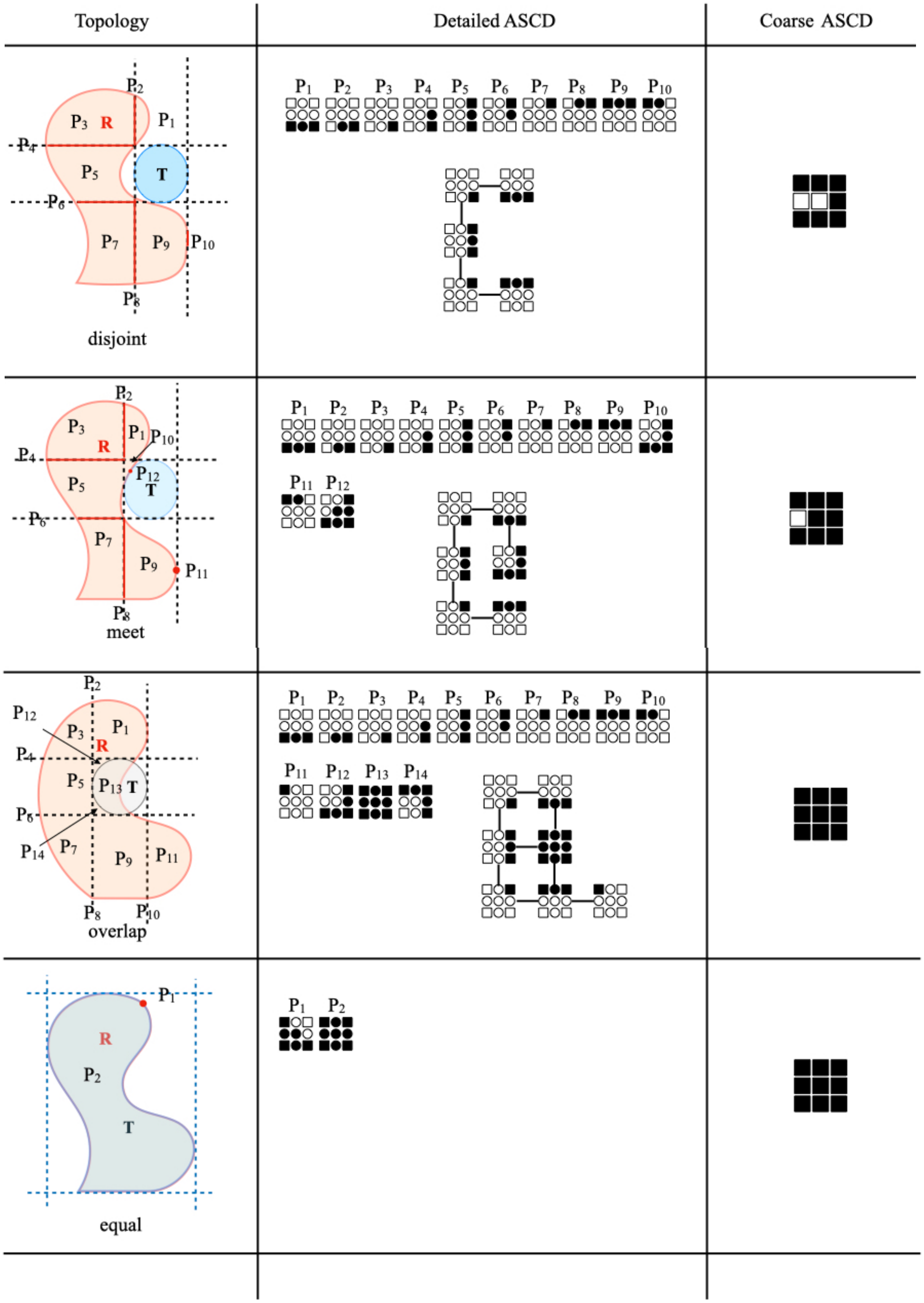


Figure 6.4 Areas of same cardinal direction distinguish the topological relations; disjoint, meet, overlap, and equal.

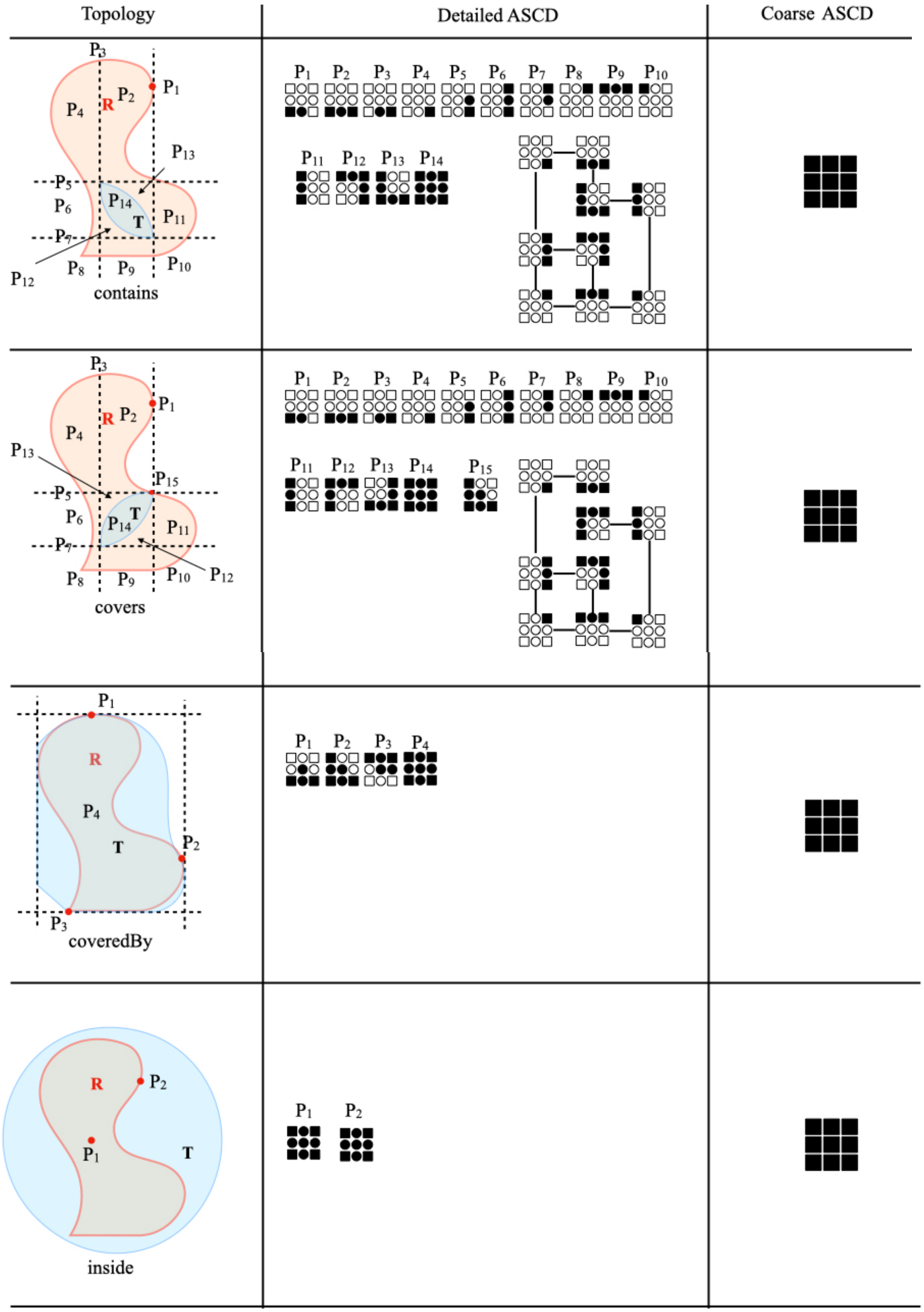


Figure 6.5 Areas of same cardinal direction distinguish the topological relations; contains, covers, coveredBy, and inside.

The point-reference cardinal direction model removes the ambiguities of the DRM by defining the direction relations between two simple regions at a coarse range. For example, if the target object falls into reference MBR since topological relations exist, DRM and OIM create inconsistency in direction relations for eight topological relations; however, the point-reference model overcomes the ambiguines of DRM and OIM and support the hypothesis of this thesis by distinguishing the eight topological relations.

6.4 DRM, OIM vs. Coarse of Direction Relations

The DRM uses the MBR approximation over the reference object to define the direction relation between the reference MBR and the target object. The central element of the iconic representation of the DRM indicates both the reference and target objects (Figure 6.6a). The coarse direction relation represents the union of all areas of same cardinal direction (Figure 6.6b). Those two iconic representations show the different contexts for representing the direction relations.

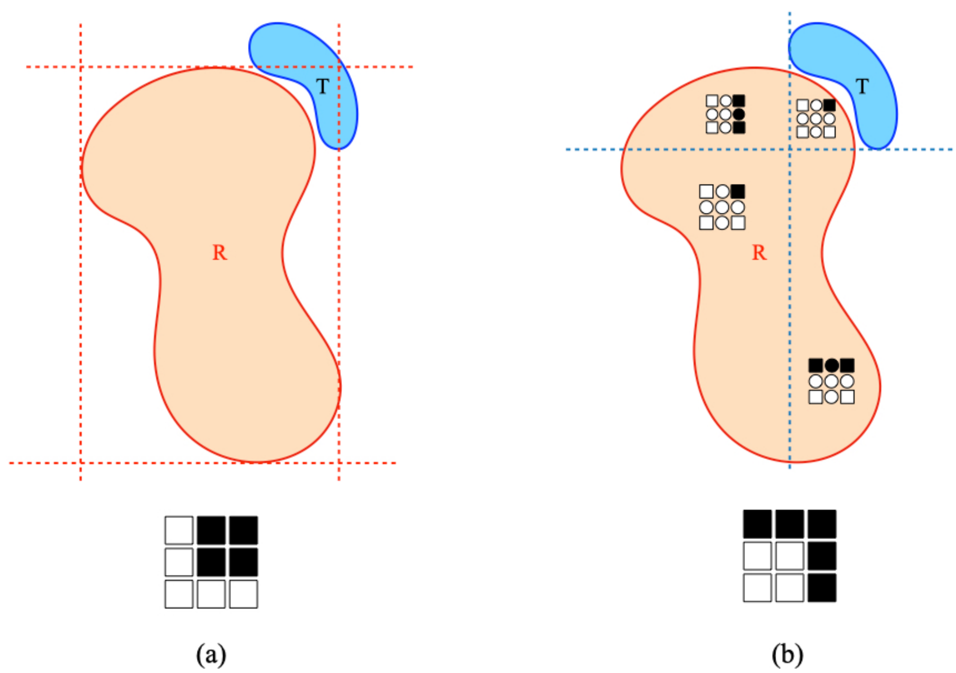


Figure 6.6 Comparison between the DRM and coarse direction relations

The OIM model is not homogeneous (Figure 6.7a), but the point-reference coarse direction relation matrix uses the fixed iconic representation to capture the direction relations from the point-reference to the target object (Figure 6.7b). The representations of the OIM and coarse direction relations are not equal in the context.

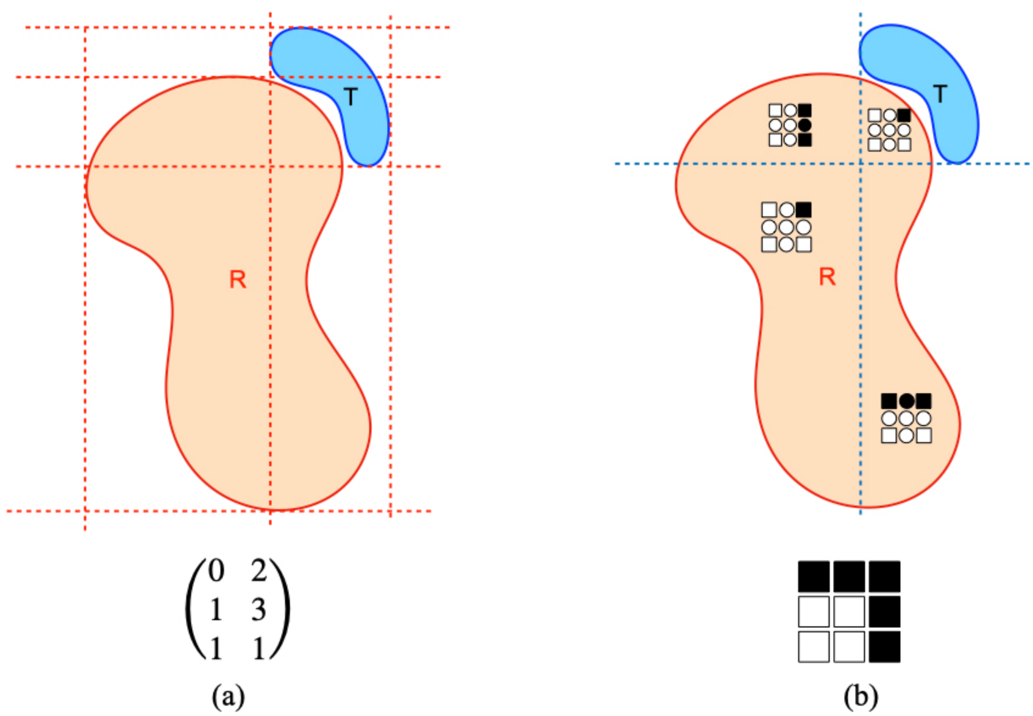


Figure 6.7 Comparison between OIM and coarse direction relation

6.5 Summary

The point-reference computational model proves its significance compared to the DRM and OIM models. The existing model of cardinal direction suffers from neutral zone and violates the first law of geography while assigning direction relations between a reference and target object within a coarse range. Those models fall short to distinguish the eight topological relations between a reference and the target object. The point-reference model proves cardinal directions as a field, such that each point on the reference region has cardinal direction relation to map the direction changes. Areas of same cardinal directions distinguish the topological relation between two simple regions. The point-reference cardinal direction model proves the hypothesis by defining mutually exclusive direction relations for eight topological relations. Such removes the ambiguities of the DRM at coarse range direction relations, DRM and OIM cannot derive the point-field.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

Direction relations between spatial entities are essential components of the representation of geographic space. Spatial entities often follow irregular shapes. Human cognition to find the direction relations between the irregular shapes uses abstract qualitative concepts without approximations, but computational models currently use approximations over them. Any method that follows human cognitive approaches to represent direction relations introduces a human-like intelligent system to represent geographic space. It allows the GIS to answer possible queries that involve direction relations. Existing models of cardinal directions approximate the spatial entities to define the direction relations. Those models do not necessary preserve human conception but argue the importance of abstract and generalized concepts, such as cardinal direction relations for spatial relations. This research introduced point-reference cardinal direction relations based on a projection-based model and algorithms for region-to-region direction relations. This chapter concludes the critical takeaways from this thesis (Section 7.1), explores significance of point-reference model for real-world applications (Section 7.2), presents the results and significant findings (Section 7.3), and highlights future research in direction relations (Section 7.4).

7.1 Summary of the Thesis

This thesis developed a symbolic representation for the projection-based model, which contains nine direction elements to capture point-reference direction relations. The 3x3 symbolic representation yields 512 possible configurations with nine direction relations. For point-to-region relations, three consistency constraints introduce 106 base relations out of 512 that normalized to 22 atomic relations considering 90^{o} rotation and mirroring. Considering the union and intersection between the relations, the order of the atomic direction relations introduces neighbors of each relation.

The 106 base relations are not closed under union and intersection. However, set of 22 normalized relations is closed under union and intersection, introducing a conceptual neighborhood graph. The union and intersection between pairs of atomic relations confirm the neighboring connections. The normalization concepts of horizontal mirroring, vertical mirroring, 90° rotation, and their combinations produce eight sets of 22 atomic point-to-region direction relations, which are closed under union and intersection. The eight sets of atomic relations construct the conceptual neighborhood graph for 106 relations. The pairs of relations that are not closed under union and intersection, indicate invalid neighboring connections in the conceptual neighborhood graph.

Point-to-region direction relations provide evidence for cardinal direction relations as a field, by exploring continuity and location-based direction relations over the space. The 106 base direction relations define the region-to-region relations, considering the reference region is a point set. The reference region embodies, Tobler's First Law of Geography (Tobler, 1970): closer points show similar cardinal direction relations, and distant points differ in cardinal direction relations, leading to areas of same cardinal direction.

Region-to-region direction relations preserve the geometric shape of the reference and target object, considering fundamental property as a point. The areas of same cardinal direction of the reference object define by tiles and pockets of same cardinal direction. Tiles are rectangle partitions at the exterior of a target object's rectangle region, formed by its extrema and pockets appear at the interior of the rectangular region of the target. The neighboring connections between tiles and pockets follow the neighborhood graph of the point-to-region direction relations.

The point-reference cardinal direction model defines the cardinal direction relations from anywhere in the exterior of a target object, continuous throughout the 2-dimensional embedded plane space of the target object. The model provides location-based cardinal direction relations and removes DRM's ambiguities with the target object. DRM shows inconsistency whenever the target object falls into the reference minimum bounding rectangle, but the point-reference model overcomes these ambiguities of DRM.

7.2 Significance of Point-Reference Model for Real-World Applications

The existing DRM and OIM models approximate the regions into the minimum bounding rectangle. This approximation does not consider the influence of geometric shape on direction relations, because the concavity and convexity of the objects do not allow the object to have the same interior as its MBR. The DRM uses the approximation over the reference object and preserves the geometric shape of the target; however, if the target object falls into the reference MBR, which defines direction as natural, then the target object loses the direction relations. Similar problems continue in the OIM model, which approximates the reference and the target objects by their minimum bounding rectangle. The neutral zone of the models does not allow for direction relations in the course range. For instance, Hurricane Ian (target-T) moved from the west to the east of the US (reference-R). Suppose the DRM model is considered to define the direction relation between Ian and the US once the hurricane places itself in the neutral direction zone of the US, which is generated by the DRM, the model does not yield any direction relations differences.

The point-reference, projection-based model allows for direction relations between two simple regions anywhere in the exterior of the target object. The projection-based model uses the extrema of the target region (the United States) to partition the exterior space of the US. Cardinal direction relations of each tile and pocket of the target object are a subset of point-to-region base relations. The model defines the direction relation continuously over the space. It exhibits the existence of the target object through the records of direction relations. The model could track the continuous changes of Hurricane Ian (Figure 7.1).

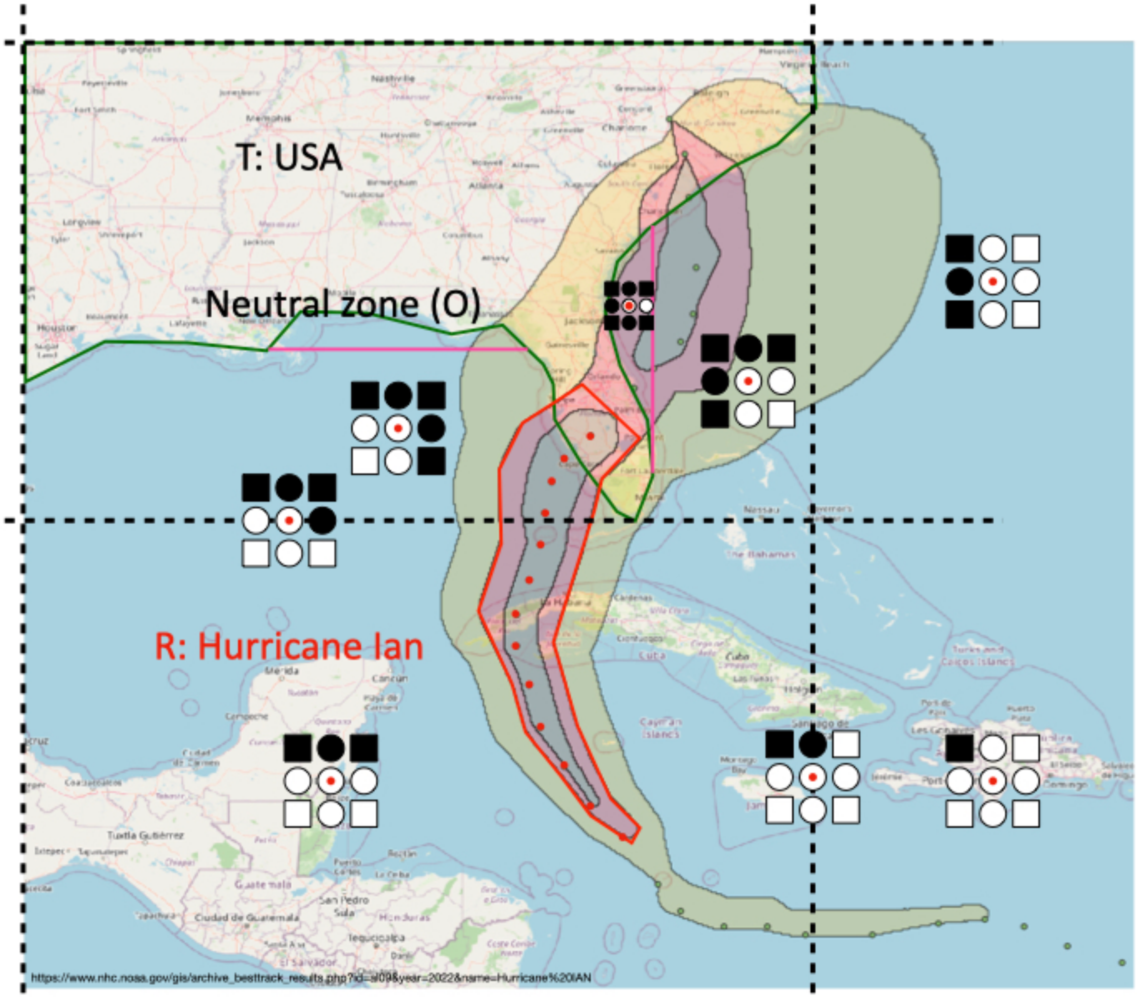


Figure 7.1 Point-reference cardinal direction model for tracking hurricane Ian.

7.3 Results and Major Findings

The major results of thesis are:

- *The point-reference cardinal direction model provides a structure to record multiple directions.*

The 3x3 symbolic representation of the projection-based model performs as a point-reference cardinal direction model. The model captures the direction relation from a point reference to a simple target region. The simple region is free from the holes and separations.

- *Point-to-region direction relations introduce 106 base relations, normalized for 22 atomic relations*.

The nine atomic point-to-point cardinal directions yield 512 possible combinations; however, three consistency constraints explore a finite set of 106 base direction relations for point-to-region. The finite set normalized, considering 90^{o} rotation and mirroring, and introduced 22 atomic relations for point-to-region direction relations. The set of atomic relations is closed under union and intersection, introducing the conceptual neighborhood graph.

- *The normalization concepts of 90^{o} rotation and horizontal and vertical mirroring and their combinations introduce the eight sets of 22 atomic relations.*

The eight sets of 22 atomic relations are closed under union and intersection, lead into the conceptual neighborhood graph for 106 elements. The neighborhood connection between each pair of relations form by the union and intersections between the pairs of relations.

- *Point-to-region direction relations and their neighborhood connections confirm cardinal direction as a field*.

The point-reference cardinal direction model captures the direction relations in the continuous space. The model captures the direction relation from anywhere in the 2-dimensional embedded space of the target object.

- *The point-reference model and its atomic point-to-region relations define the region-to-region direction relations, areas of same cardinal direction*.

The point-reference cardinal model foundation introduces algorithms to define areas of same cardinal direction that allow reference regions to have areas of same point-to-region relations.

- *Point-reference model allows for capturing multiple levels of detail from a framework*.

The model captures the point-to-point, point-to-region, and region-to-region direction relations. At a coarse level, the model indicates the existence of the target, such as direction relations that sense the target object's boundaries.

- *The proof of the hypothesis reveals that the cardinal directions as a field*.

A finite set of cardinal direction relations and its conceptual neighborhood graph confirms that cardinal direction relations continue throughout the space and are location-based.

- *The model allows interpolation in cardinal-direction relations*.

Point-to-region atomic relations and their conceptual neighborhood graph allow for interpolation over the directions.

7.4 Future Work

This thesis explored the soundness of the projection-based cardinal direction model to define point-to-region and region-to-region direction relations. Although this thesis provided significant results, a few investications remain for future work.

7.4.1 Converseness of Region-to-Region Direction Relations

This thesis only considered direct direction relations from a reference to a target region. Converse relations, from target to reference, are not tested for region-to-region relations. The converseness allows for inferring direction relations from known directions (Dube, 2012, Wang *et al.,* 2012). For example, if dir (B, A) = inv(dir(A, B)), and A is south of B, then the model must be capable of defining the direction from B to A. In the region-to-region relations, the reference region may have a set of regions of same cardinal direction.

Similarly, the converseness defines a different set of areas of same cardinal direction (Figure 7.2). The combinations between areas of same cardinal direction should be tested for converseness (Figure 7.3).

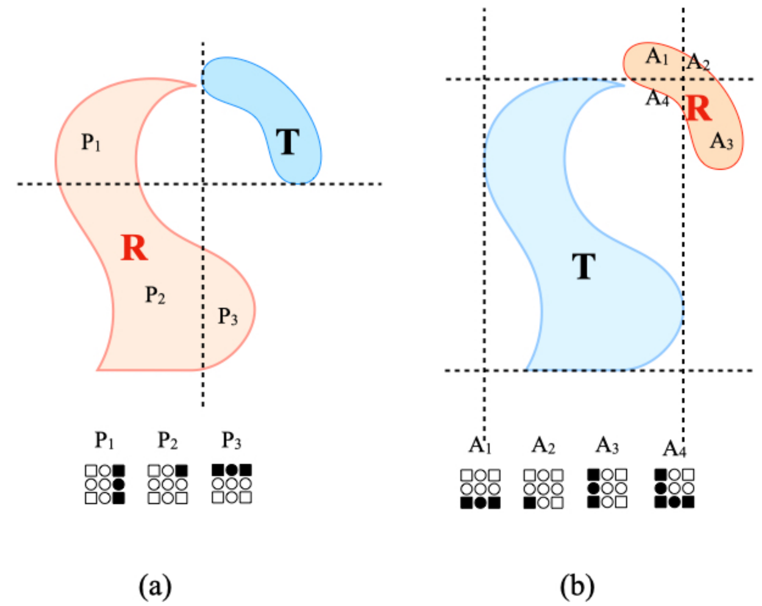


Figure 7.2 Direction relation between: (a) reference object and a target and (b) between target and reference.

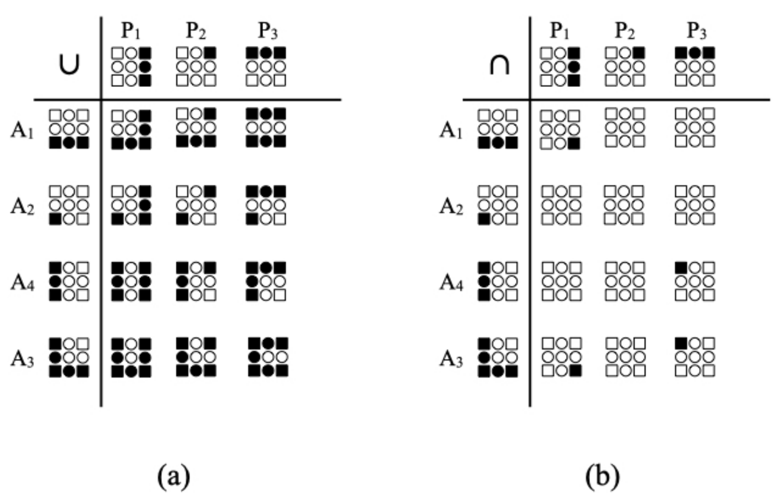


Figure 7.3 Combinations of areas of same cardinal direction.

7.4.2 Cardinal Direction on Surface of the Sphere

The volume of spatial data used to understand the Earth system desires to exploit region-to-region relations on the surface of the digital sphere (Dube and Egenhofer, 2020). The point-to-region direction relations prioritize in 2-dimensional projected space, introducing a finite set of direction relations. The method must apply to the surface of the sphere to define region-to-region direction relations.

The Earth system is typically modelled as a sphere. Direction relation between large-scale regions must consider a spherical surface to avoid the distortions of the projections (Kneissl *et al.,* 2011). A sphere often uses the meridian and parallels to define the location of the points on the surface. Meridians are great circle passes through the point location on the sphere, north pole, and south pole. Parallels are circles orthogonal to the meridians. The projection-based model for the surface of the sphere defines by a meridian and a parallel (Figure 7.4a).

In region-to-region relations at the spherical surface, extrema of the spatial entities do not create the minimum bounding rectangle, because meridians and parallels are arcs, constructing the bounding area for the spatial entities. For example, the direction relation between a reference region R and target T_1 and T_2 are southwest and south (Figure 7.4b). The point-reference method continues to define spherical areas of same cardinal direction, however, the question remains to answer what is the cardinal direction at North Pole to any disjoint region?

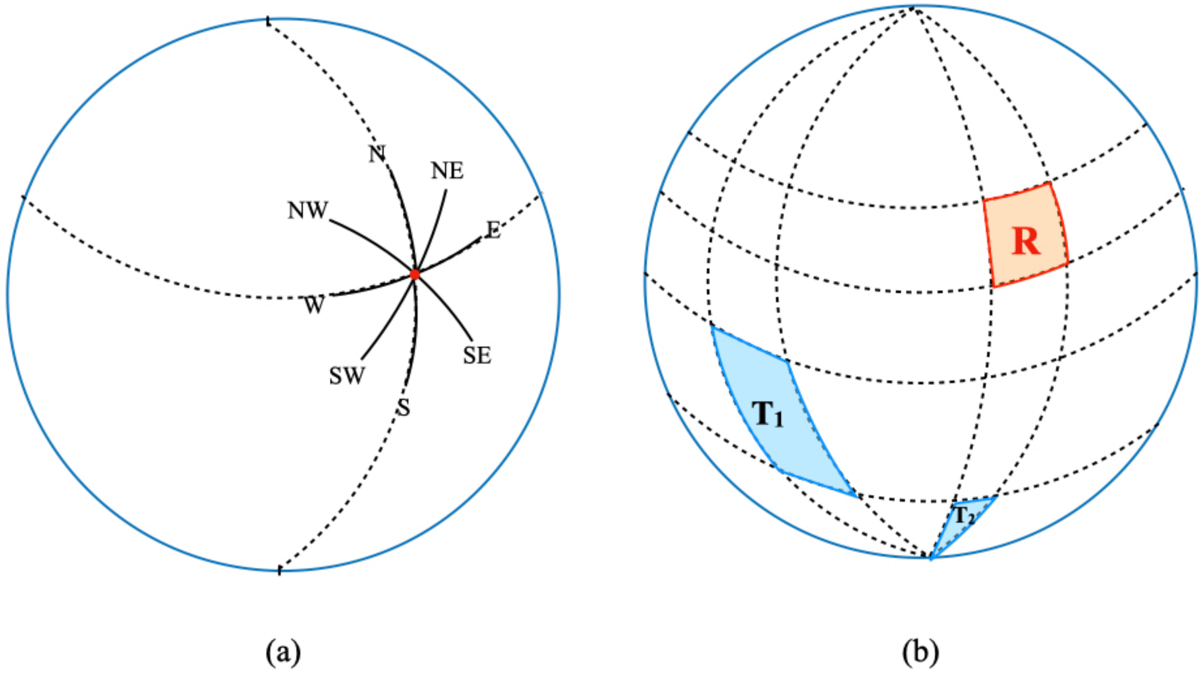


Figure 7.4 Cardinal direction relations on the surface of a sphere initiate (a) a projection-based model and (b) region-to-region relations.

The partitioning algorithms must expand to define the spherical areas of same cardinal direction. The spherical projection-based cardinal direction model at extrema points of the target region, partitions the spherical embedded surface into eight (Figure 7.5), and the partitioning algorithm must follow the spherical prosperities, such that the shortest path along the sphere is the great circle.

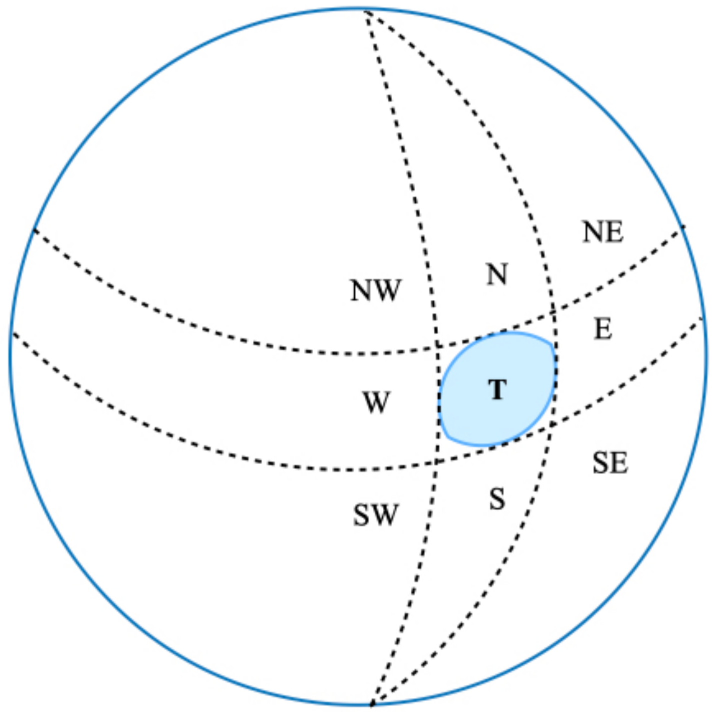


Figure 7.5: Spherical areas of same cardinal direction.

7.4.3 Extension to 3-Dimensional Space

The point-to-region and region-to-region direction relations established in 2D can be extended to 3D. The extrema of the target object partition the embedded space into cubic and volume of same cardinal direction (Figure 7.6a). The direction between the extended objects in 3D space at neutral plane could record by the 3x3 array of symbolic representations, squares, and a circle record the direction relations (Figure 7.6b). Squares represent the planes and volumes, while circle refers the point reference. The direction relation between point-reference and a volume follows the point-to-region direction relations (Figure 7.7). The model continues on its extension to define the direction between two volumes or planes, considering the point reference.

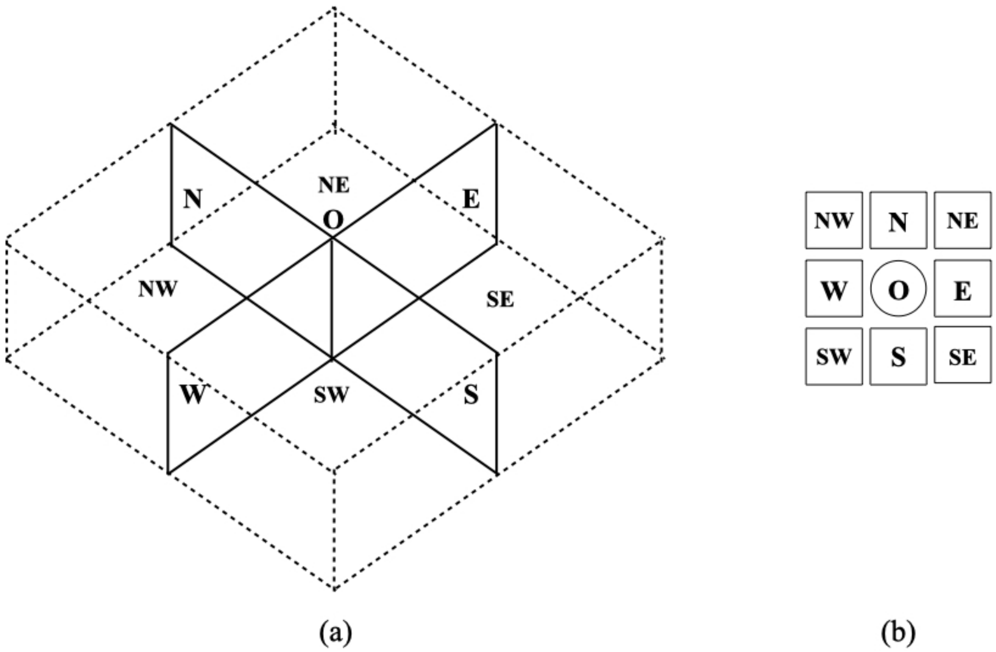


Figure 7.6 3-dimensional space (a) projection-based cardinal direction model and (b) symbolic representation of the model at neutral plane.

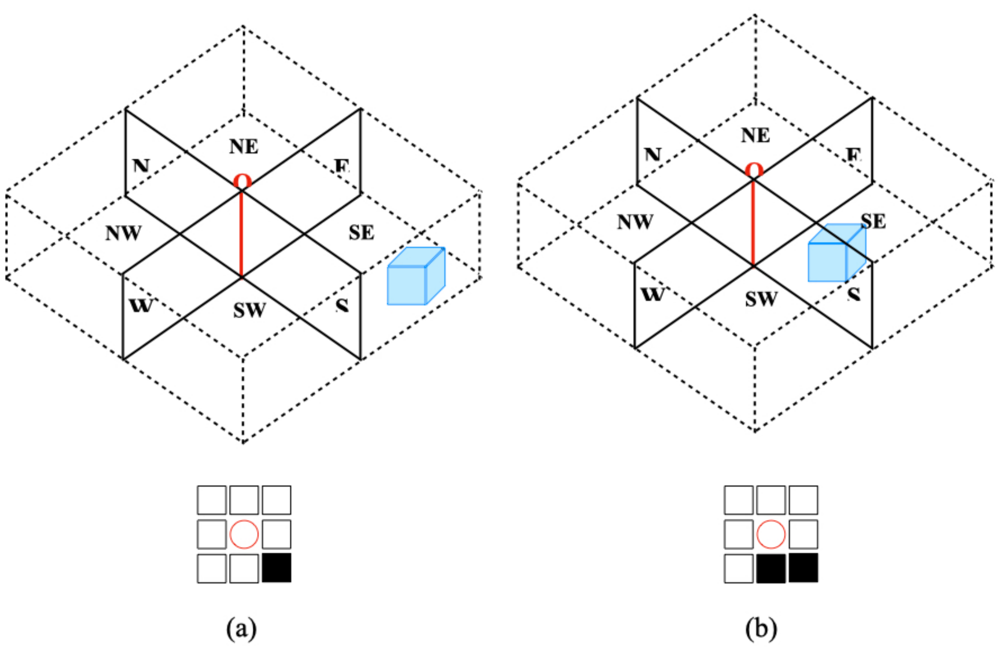


Figure 7.7 Direction relation between a point-reference and a volume: (a) volume falls into direction volume and (b) if the volume touch with south-plane.

7.4.4 Cardinal Directions on Nearest Neighbor Search Algorithm

The typical nearest neighbor search algorithm in GIS applies quantitative calculations. For example, K nearest search algorithm and R-tree (Roussopoulos *et al.*, 1995). The quantitative methods calculate the distances between each pair of points, and the shortest distance introduces the nearest point, which shows computational complexity for a large data set. The point-reference cardinal direction model reduces the computational complexity of the nearest neighbor search with its finite set of direction relations. Areas of same cardinal direction explains which object is near and far from the target object (Figure 7.8). A target object needs a one-time calculation to define its areas of same cardinal direction that applies to the search algorithm to minimize the points for calculations. In addition, this method provides the direction of the nearest points.

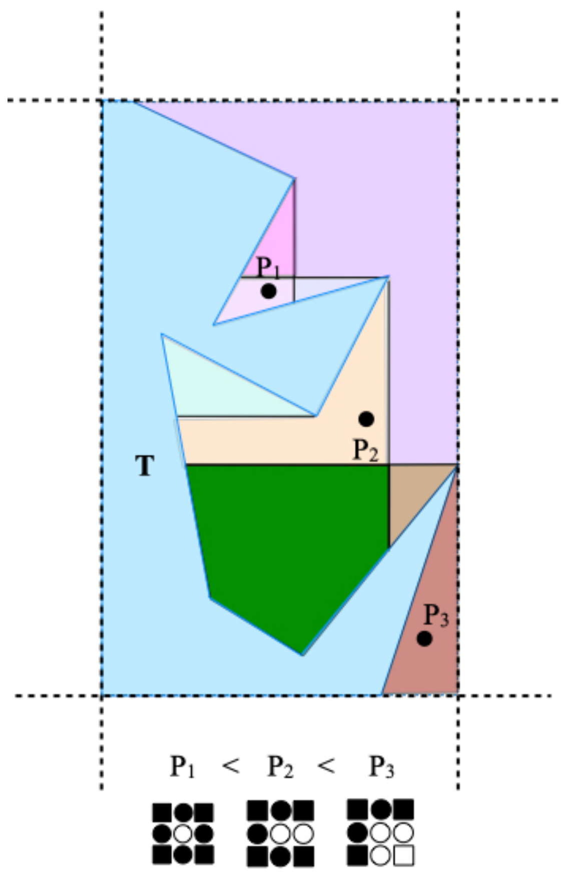


Figure 7.8 Areas of same cardinal direction for nearest neighbor search algorithm.

BIBLIOGRAPHY

Alvares, L., Bogorny, V., Kuijpers, B., de Macedo, J. A., Moelans, B., and Vaisman, A. (2007). A model for enriching trajectories with semantic geographical information. *Proceedings of the 15th Annual ACM International Symposium on Advances in Geographic Information Systems*, 1–8.

Andrienko, G., Andrienko, N., Dykes, J., Fabrikant, S., and Wachowicz, M. (2008). Geovisualization of dynamics, movement and change: key issues and developing approaches in visualization research. *Information visualization* 7(3-4): 173-180.

Anooshian, L. and Seibert, P. (1996). Diversity within spatial cognition: memory processes underlying Place Recognition. *Applied Cognitive Psychology*, *10*(4), 281–299.

Avraamides, M. and Kelly, J. (2008). Multiple systems of spatial memory and action. *Cognitive Processing*, *9*(2), 93–106.

Balbiani, P., Condotta, J., and Cerro, L. (1999). A new tractable subclass of the rectangle algebra. *International Joint Conference on Artificial Intelligence* (442-447)

Baruah, R. and Hazarika, S. (2012). Qualitative directions in egocentric spatial reference frame. *2012 12th International Conference on Intelligent Systems Design and Applications (ISDA)*, 562–567.

Bekessy, A. and Demetrovics, J. (1979). Contribution to the theory of data base relations. *Discrete Mathematics*, *27*(1), 1–10.

Belkin, M. and Niyogi, P. (2003). Laplacian eigenmaps for dimensionality reduction and data representation. *Neural Computation*, *15*(6), 1373–1396.

Bertolotto, M. (1998). Geometric modeling of spatial entities at multiple levels of resolution. Ph.D. Dissertation, University of Genova, Genova, Italy.

Brown, C. (1983). Where do cardinal direction terms come from? *Anthropological Linguistics*, *25*(2), 121– 161.

Buchin, K., Kusters, V., Speckmann, B., Staals, F., and Vasilescu, B. (2011). A splitting line model for directional relations. *Proceedings of the 19th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems - GIS '11*, 142.

Burns, P. (1999). Navigation and the mobility of older drivers. *The Journals of Gerontology Series B: Psychological Sciences and Social Sciences*, *54B* (1), S49–S55.

Burrough, P., McDonnell, R., and Lloyd, C. (2015). *Principles of Geographical Information Systems*. OUP Oxford.

Butler, D., Acquino, A., Hissong, A., and Scott, P. (1993). Wayfinding by newcomers in a complex building. *Human Factors*, *35*(1), 159–173.

Carlson, L. (1999). Selecting a reference frame. *Spatial Cognition and Computation*, *1*(4), 365–379.

Cetin, I. and Dubinsky, E. (2017). Reflective abstraction in computational thinking. *The Journal of Mathematical Behavior*, *47*, 70–80.

Chen, J., Cohn, A., Liu, D., Wang, S., Ouyang, J., and Yu, Q. (2015). A survey of qualitative spatial representations. *The Knowledge Engineering Review*, *30*(1), 106–136.

Cicerone, S. and Di Felice, P. (2000). Cardinal relations between regions with a broad boundary. *Proceedings of the Eighth ACM International Symposium on Advances in Geographic Information Systems- GIS '00*, 15–20.

Cicerone, S. and Di Felice, P. (2004). Cardinal directions between spatial objects: The pairwise-consistency problem. *Information Sciences*, *164*(1–4), 165–188.

Clementini, E. (2013). Directional relations and frames of reference. *GeoInformatica*, *17*(2), 235–255.

Clementini, E., Di Felice, P., and Hernández, D. (1997). Qualitative representation of positional information. *Artificial Intelligence*, *95*(2), 317–356.

Cohn, A. and Renz, J. (2008). Qualitative spatial representation and reasoning. In F. van Harmelen, V. Lifschitz, and B. Porter (Eds.), *Foundations of Artificial Intelligence*. Elsevier.

Cohn, A., Li, S., Liu, W., and Renz, J. (2014). Reasoning about topological and cardinal direction relations between 2-dimensional spatial objects. *Journal of Artificial Intelligence Research*, *51*, 493–532.

Colombo, D., Serino, S., Tuena, C., Pedroli, E., Dakanalis, A., Cipresso, P., and Riva, G. (2017). Egocentric and allocentric spatial reference frames in aging: A systematic review. *Neuroscience and Biobehavioral Reviews*, *80*, 605–621.

Couclelis, H. (1992). People manipulate objects (but cultivate fields): Beyond the raster-vector debate in GIS. In A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space* (pp. 65–77). Springer.

Couclelis, H. (2012). Climbing on a milestone for a better view: Goodchild's 'Geographical information science' paper as vantage point and ground for reflection. *International Journal of Geographical Information Science*, *26*(12), 2291–2300.

Couclelis, H. and Gale, N. (1986). Space and spaces. *Geografiska Annaler: Series B, Human Geography*, *68*(1), 1–12.

de Floriani, L., Magillo, P., and Puppo, E. (2000). Chapter 7—Applications of computational geometry to geographic information systems. In J.-R. Sack and J. Urrutia (Eds.), *Handbook of Computational Geometry* (pp. 333–388). North-Holland.

Deng, M. and Li, Z. (2008). A statistical model for directional relations between spatial objects. *GeoInformatica*, *12*(2), 193–217.

Devogele, T., Parent, C., and Spaccapietra, S. (1998). On spatial database integration. *International Journal of Geographical Information Science*, *12*(4), 335–352.

Du, S., Feng, C., and Guo, L. (2015). Integrative representation and inference of qualitative locations about points, lines, and polygons. *International Journal of Geographical Information Science*, *29*(6), 980–1006.

Du, S. and Guo, L. (2010). Modeling and querying approximate direction relations. *ISPRS Journal of Photogrammetry and Remote Sensing*, *65*(4), 328–340.

Dube, M. (2016). Algebraic refinements of direction relations through topological augmentation. Ph.D. Dissertation. The University of Maine.

Dube, M. (2021). Deriving topological relations from topologically augmented direction relation matrices. *Journal of Spatial Information Science*, 23, Article 23.

Dube, M. and Egenhofer, M. (2020). Binary topological relations on the digital sphere. *International Journal of Approximate Reasoning*, *116*, 62–84.

Dube, M., Barrett, J., and Egenhofer, M. (2015). From metric to topology: determining relations in discrete pace. In S. Fabrikant, M. Raubal, M. Bertolotto, C. Davies, S. Freundschuh, and S. Bell (Eds.), *Spatial Information Theory* (Vol. 9368, pp. 151–171). Springer International Publishing.

Dutton, G. (2006). Part 4: Mathematical, algorithmic and data structure issues: geodesic modelling of planetary relief. *Cartographica: The International Journal for Geographic Information and Geovisualization*.

Egenhofer, M. (1989). A formal definition of binary topological relationships. In W. Litwin and H.-J. Schek (Eds.), *Foundations of Data Organization and Algorithms* (pp. 457–472). Springer.

Egenhofer, M. (1993). What's special about spatial? database requirements for vehicle navigation in geographic space. *Proceedings of the 1993 ACM SIGMOD International Conference on Management of Data*, 398–402.

Egenhofer, M. (1997). Query processing in spatial-query-by-sketch. *Journal of Visual Languages and Computing*, *8*(4), 403–424.

Egenhofer, M. and Dube, M. (2009). Topological relations from metric refinements. Proceedings of the 17th *ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, 158–167.

Egenhofer, M. and Franzosa, R. (1991). Point-set topological spatial relations. *International Journal of Geographical Information Systems*, *5*(2), 161–174.

Egenhofer, M. and Mark, D. (1995). Naive geography. *Spatial Information Theory A Theoretical Basis for GIS: International Conference COSIT'95 Semmering, Austria, September 21–23, 1995 2*, 1–15

Elkind, D. (1967). Egocentrism in adolescence. *Child Development*, *38*(4), 1025.

Escrig, M. and Toledo, F. (1998). Cardinal directions on extended objects for qualitative navigation. In J. Mira, A. del Pobil, and M. Ali (Eds.), *Methodology and Tools in Knowledge-Based Systems* (Vol. 1415, pp. 291–301). Springer Berlin Heidelberg.

Freksa, C. (1992). Temporal reasoning based on semi-intervals. *Artificial Intelligence*, 54(1), 199–227.

Fonseca, F. and Egenhofer, M. (1999). Ontology-driven geographic information systems. *Proceedings of the 7th ACM International Symposium on Advances in Geographic Information Systems*, 14–19.

Foreman, N. and Gillett, R. (1997). *Handbook of spatial research paradigms and methodologies: spatial cognition in the child and adult*. Psychology Press.

Frank, A. (1991). Qualitative spatial reasoning with cardinal directions. *7. Österreichische Artificial-Intelligence-Tagung/Seventh Austrian Conference on Artificial Intelligence: Wien, Austria, 24.–27. September 1991 Proceedings*, 157–167.

Frank, A. (1992a). Spatial concepts, geometric data models, and geometric data structures. *Computers and Geosciences*, *18*(4), 409–417.

Frank, A. (1992b). Qualitative spatial reasoning about distances and directions in geographic space. *Journal of Visual Languages and Computing*, *3*(4), 343–371.

Frank, A. (1996). Qualitative spatial reasoning: cardinal directions as an example. *International Journal of Geographical Information Systems*, *10*(3), 269–290.

Freeman, H. (1974). Computer processing of line-drawing images. *ACM Computing Surveys*, *6*(1), 57–97.

Freksa, C. (1991). Qualitative spatial reasoning. In D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space* (pp. 361–372). Springer Netherlands.

Freksa, C. (1992). Using orientation information for qualitative spatial reasoning. In A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space* (Vol. 639, pp. 162–178). Springer Berlin Heidelberg.

Freundschuh, S. and Egenhofer, M. (1997). Human conceptions of spaces: Implications for GIS. *Transactions in GIS*, *2*(4), 361–375.

Golledge, R. (2008). Behavioral geography and the theoretical/quantitative revolution. *Geographical Analysis*, *40*(3), 239–257.

Goodchild, M. (1992). Geographical data modeling. Computers & Geosciences, 18(4), 401–408.

Goodchild, M. and Haining, R. (2004). GIS and spatial data analysis: Converging perspectives. *Papers in Regional Science*, *83*(1), 363–385.

Goyal, R. (2000). *Similarity assessment for cardinal directions between extended spatial objects*. Ph.D., Dissertation. The University of Maine.

Goyal, R. and Egenhofer, M. (2001). Similarity of cardinal directions. In C. S. Jensen, M. Schneider, B. Seeger, and V. J. Tsotras (Eds.), *Advances in Spatial and Temporal Databases* (pp. 36–55). Springer.

Guesgen, H. (1989). *Spatial reasoning based on Allen's temporal logic*. International Computer Science Institute Berkeley, CA.

Gustafson, E. (1998). Quantifying landscape spatial pattern: what is the state of the art? *Ecosystems*, *1*(2), 143–156.

Haar, R. (1976). Computational models of spatial relations. Federal Report. TR-478, Computer Science, University of Maryland, College Park, MD.

Harvey, D. (1990). Between space and time: Reflections on the geographical imagination1. *Annals of the Association of American Geographers*, *80*(3), 418–434.

Hernández, D. (1994). Qualitative representation of positions in 2-D. *Qualitative Representation of Spatial Knowledge*. *Lecture Notes in Computer Science*, vol 804. Springer, Berlin, Heidelberg.

Hong, J., Egenhofer, M., and Frank, A. (1995). On the robustness of qualitative distance-and direction-reasoning. *Autocarto-Conference*, 301–310.

Hu, X., Wang, L., and Yuan, B. (2012). Querying representative points from a pool based on synthesized queries. *The 2012 International Joint Conference on Neural Networks*, 1–6.

Hua, H., Renz, J., and Ge, X. (2018). Qualitative representation and reasoning over direction relations across different frames of reference. *Sixteenth International Conference on Principles of Knowledge Representation and Reasoning*, 551-560.

Huttenlocher, J. (1984). The child's representation of information about location. *The Origin of Cognitive Skills*, 81–111.

Izmirlioglu, Y. and Erdem, E. (2020). Reasoning about cardinal directions between 3-dimensional extended objects using answer set programming. *Theory and Practice of Logic Programming*, *20*(6), 942–957.

Kitchin, R. and Blades, M. (2002). The cognition of geographic space. I. B. Tauris.

Kneissl, T., van Gasselt, S., and Neukum, G. (2011). Map-projection-independent crater size-frequency determination in GIS environments—New software tool for ArcGIS. *Planetary and Space Science*, *59*(11–12), 1243–1254.

Kor, A. and Bennett, B. (2010). Reasoning mechanism for cardinal direction relations. In D. Dicheva and D. Dochev (Eds.), *Artificial Intelligence: Methodology, Systems, and Applications* (Vol. 6304, pp. 32–41). Springer Berlin Heidelberg.

Kritzman, G. and Hahmann, T. (2018). Using commonsensical cardinal directions to describe bordering objects. *Proceedings of the 26th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, 412–415.

Kulik, L. and Klippel, A. (1999). Reasoning about cardinal directions using grids as qualitative geographic coordinates. In C. Freksa and D. Mark (Eds.), *Spatial Information Theory. Cognitive and Computational Foundations of Geographic Information Science* (Vol. 1661, pp. 205–220). Springer Berlin Heidelberg.

Kurata, Y. and Shi, H. (2009). Toward heterogeneous cardinal direction calculus. In B. Mertsching, M. Hund, and Z. Aziz (Eds.), *KI 2009: Advances in Artificial Intelligence* (pp. 452–459). Springer.

Laurini, R. and Thompson, D. (1992). *Fundamentals of Spatial Information Systems*. Academic Press.

Li, B. and Fonseca, F. (2006). TDD: A comprehensive model for qualitative spatial similarity assessment. *Spatial Cognition and Computation*, *6*(1), 31–62.

Li, S. and Liu, W. (2015). Cardinal directions: A comparison of direction relation matrix and objects interaction matrix. *International Journal of Geographical Information Science*, *29*(2), 194–216.

Li, S., Liu, W. and Wang, S. (2013). Qualitative constraint satisfaction problems: An extended framework with landmarks. *Artificial Intelligence*, *201*, 32–58.

Ligozat, G. (1998). Reasoning about cardinal directions. *Journal of Visual Languages and Computing*, *9*(1), 23–44.

Liu, W. and Li, S. (2011). Reasoning about cardinal directions between extended objects: The NP-hardness result. *Artificial Intelligence*, *175*(18), 2155–2169.

Liu, W., Zhang, X., Li, S., and Ying, M. (2010). Reasoning about cardinal directions between extended objects. *Artificial Intelligence*, *174*(12–13), 951–983.

Liu, Y., Wang, X., Jin, X., and Wu, L. (2005). On internal cardinal direction relations. In A. G. Cohn and D. Mark (Eds.), *Spatial Information Theory* (pp. 283–299). Springer.

Lu, F. and Milios, E. (1997). Globally consistent range scan alignment for environment mapping. *Autonomous Robots*, *4*(4), 333–349.

Maguire, D. (1991). An overview and definition of GIS. *Geographical Information Systems: Principles and Applications*, *1*(1), 9–20.

Mark, D. (1999). Spatial representation: A cognitive view. *Geographical Information Systems: Principles and Applications*, *1*, 81–89.

Mark, D. and Frank, A. (1996). Experiential and formal models of geographic space. *Environment and Planning B: Planning and Design*, *23*(1), 3–24.

Mark, D., Freksa, C., Hirtle, S. C., Lloyd, R., and Tversky, B. (1999). Cognitive models of geographical space. *International Journal of Geographical Information Science*, *13*(8), 747–774.

McNamara, T., Rump, B., and Werner, S. (2003). Egocentric and geocentric frames of reference in memory of large-scale space. *Psychonomic Bulletin and Review*, *10*(3), 589–595.

Miller, H. (2000). Geographic representation in spatial analysis. *Journal of Geographical Systems*, *2*(1), 55–60.

Miller, H. and Wentz, E. (2003). Representation and spatial analysis in geographic information systems. *Annals of the Association of American Geographers*, *93*(3), 574–594.

Montanari, A., Puppis, G., and Sala, P. (2009). A decidable spatial logic with cone-shaped cardinal directions. In E. Grädel and R. Kahle (Eds.), *Computer Science Logic* (pp. 394–408). Springer.

Montanari, A., Puppis, G., and Sala, P. (2015). A decidable weakening of compass logic based on cone-shaped cardinal directions. *Logical Methods in Computer Science*, 11(4), 1-32.

Moratz, R. and Ragni, M. (2008). Qualitative spatial reasoning about relative point position. *Journal of Visual Languages and Computing*, *19*(1), 75–98.

Moratz, R., Tenbrink, T., Bateman, J., and Fischer, K. (2003). Spatial knowledge representation for human-robot interaction. In C. Freksa, W. Brauer, C. Habel, and K. Wender (Eds.), *Spatial Cognition III* (pp. 263–286). Springer.

Mossakowski, T. and Moratz, R. (2012). Qualitative reasoning about relative direction of oriented points. *Artificial Intelligence*, *180–181*, 34–45.

Nunes, J. (1991). Geographic space as a set of concrete geographical entities. In D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space* (pp. 9–33). Springer Netherlands.

Núñez, R. and Cooperrider, K. (2013). The tangle of space and time in human cognition. *Trends in Cognitive Sciences*, *17*(5), 220–229.

Osada, R., Funkhouser, T., Chazelle, B., and Dobkin, D. (2002). Shape distributions. *ACM Transactions on Graphics*, *21*(4), 807–832.

Pacione, M. (1982). The Use of objective and subjective measures of life quality in human geography. *Progress in Human Geography*, *6*(4), 495–514.

Pederson, E., Danziger, E., Wilkins, D., Levinson, S., Kita, S., and Senft, G. (1998). Semantic typology and spatial conceptualization. *Language*, *74*(3), 557–589.

Peters, D. (2017). Recognizing multidimensional Euclidean preferences. *Proceedings of the AAAI Conference on Artificial Intelligence*, *31*(1), 642-648.

Peuquet, D. (1988). Representations of geographic space: Toward a conceptual synthesis. *Annals of the Association of American Geographers*, *78*(3), 375–394.

Peuquet, D. (1994). It's about time: A conceptual framework for the representation of temporal dynamics in geographic information systems. *Annals of the Association of American Geographers*, *84*(3), 441–461.

Peuquet, D. (1999). Making space for time: Issues in space-time data representation. *Proceedings. Tenth International Workshop on Database and Expert Systems Applications. DEXA 99*, 404–408.

Peuquet, D. and Ci-Xiang, Z. (1987). An algorithm to determine the directional relationship between arbitrarily-shaped polygons in the plane. *Pattern Recognition*, *20*(1), 65–74.

Pincock, C. (2004). A new perspective on the problem of applying mathematics. *Philosophia Mathematica*, *12*(2), 135–161.

Preparata, F. and Shamos, M. (1985). *Computational geometry: An introduction*. Springer-Verlag.

Qi, C., Shu, H., and Xu, A. (2006). Formal properties of cognitive distance in geographical space. *16th International Conference on Artificial Reality and Telexistence–Workshops (ICAT'06)*, 408–412.

Renz, J. (2002). *Qualitative Spatial Reasoning with Topological Information*. Springer.

Renz, J. and Mitra, D. (2004). Qualitative direction calculi with arbitrary granularity. *PRICAI*, *3157*, 65– 74.

Retz-Schmidt, G. (1988). Various views on spatial prepositions. *AI Magazine*, *9*(2), 95-105.

Rigaux, P., Scholl, M., and Voisard, A. (2002). *Spatial databases: With application to GIS*. Morgan Kaufmann.

Robinson, A. and The Committee on Map Projections. (2017). Matching the map projection to the need. In M. Lapaine and E. Usery (Eds.), *Choosing a Map Projection* (pp. 49–115). Springer International Publishing.

Robinson, V., Frank, A., and Blaze, M.(1986). Expert systems applied to problems in geographic information systems: Introduction, review and prospects. *Computers, Environment and Urban Systems*, *11*(4), 161–173.

Roussopoulos, N., Kelley, S., and Vincent, F. (1995). Nearest neighbor queries. *Proceedings of the 1995 ACM SIGMOD International Conference on Management of Data*, 71–79.

Schmidtke, H. (2003). A geometry for places: Representing extension and extended objects. In W. Kuhn, M. Worboys, and S. Timpf (Eds.), *Spatial Information Theory. Foundations of Geographic Information Science* (Vol. 2825, pp. 221–238). Springer Berlin Heidelberg.

Schneider, M., Chen, T., Viswanathan, G., and Yuan, W. (2012). Cardinal directions between complex regions. *ACM Transactions on Database Systems*, *37*(2), 1–40.

Schockaert, S., De Cock, M., and Kerre, E. (2008). Modelling nearness and cardinal directions between fuzzy regions. *2008 IEEE International Conference on Fuzzy Systems (IEEE World Congress on Computational Intelligence)*, 1548–1555.

Schultheis, H., Bertel, S., and Barkowsky, T. (2014). Modeling mental spatial reasoning about cardinal directions. *Cognitive Science*, *38*(8), 1521–1561.

Sharma, J., Flewelling, D., and Egenhofer, M. (1994). A qualitative spatial reasoner. *Sixth International Symposium on Spatial Data Handling*, *2*, 665–681.

Shekhar, S., Liu, X., and Chawla, S. (1999). An object model of direction and its implications. *GeoInformatica*, *3*(4), 357–379.

Siegel, A., Herman, J., Allen, G., and Kirasic, K. (1979). The development of cognitive maps of large- and small-scale spaces. *Child Development*, *50*(2), 582-585.

Skiadopoulos, S. (2015). Directional relations. In S. Shekhar, H. Xiong, and X. Zhou (Eds.), *Encyclopedia of GIS* (pp. 1–7). Springer International Publishing.

Skiadopoulos, S. and Koubarakis, M. (2004). Composing cardinal direction relations. *Artificial Intelligence*, *152*(2), 143–171.

Smith, R., Self, M., and Cheeseman, P. (1990). Estimating uncertain spatial relationships in robotics. *Autonomous Robot Vehicles*, 167–193.

Sun, H. (2008). Computational models for computing fuzzy cardinal directional relations between regions. *Knowledge-Based Systems*, *21*(7), 599–603.

Tarski, A. (1959). What is elementary geometry? In L. Henkin, P. Suppes, and A. Tarski (Eds.), *Studies in Logic and the Foundations of Mathematics* (Vol. 27, pp. 16–29). Elsevier.

Tolman, E. (1948). Cognitive maps in rats and men. *Psychological Review*, *55*(4), 189–208.

Vickery, P., Masters, F., Powell, M., and Wadhera, D. (2009). Hurricane hazard modeling: The past, present, and future. *Journal of Wind Engineering and Industrial Aerodynamics*, *97*(7), 392–405.

Vince, A. and Little, C. (1989). Discrete Jordan curve theorems. *Journal of Combinatorial Theory, Series B*, *47*(3), 251–261.

Walsh, S. (2007). Geographic information systems an instructional tool for earth science educators. *Journal of Geography, 87(1), 17-25*.

Yan, H., Chu, Y., Li, Z., and Guo, R. (2006). A quantitative description model for direction relations based on direction groups. *GeoInformatica*, *10*(2), 177–196.

Yuan, M. (2001). Representing complex geographic phenomena in GIS. *Cartography and Geographic Information Science*, *28*(2), 83–96.

Zhang, Z. and Siekmann, J. (2007). *Knowledge science, engineering and management: Second international conference, KSEM 2007, Melbourne, Australia, November 28-30, 2007*. Springer.

Zierer, H. (1991). Relation algebraic domain constructions. *Theoretical Computer Science*, *87*(1), 163–188.

BIOGRAPHY OF THE AUTHOR

Thunendran Periyandy was born July 20, 1993, in a small rural town in Sri Lanka. He attended the Sabaragamuwa University of Sri Lanka, Faculty of Geomatics, and received a Bachelor of Science in Surveying Sciences (Photogrammetry and Remote Sensing) in May 2018. He joined to Faculty of Geomatics, Department of Surveying and Geodesy as a lecturer in 2021. He is a Fulbright Scholar and received a Master's Grant for 2021-2023 from the USA. He enrolled in the Master of Science program in Spatial Information Science and Engineering in August 2021. Thunendran is a candidate for the Master. of Science in Spatial Information Science and Engineering in May 2023.