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Tracking and mathematical identity for rural adolescent students of color

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TRACKING AND MATHEMATICAL IDENTITY FOR RURAL ADOLESCENT STUDENTS OF COLOR

by
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Abstract

Tracking, or segregating students into varying levels of a course based on their mathematical ability and prior performance, is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality (Gamoran, 2016). This is even more prominent for students who are socio-economically disadvantaged or minoritized. Also, dividing students into a group by their academic ability may cause them to self-label as inferior to upper track students (Ansalone, 2004). In addition, many students who enter tracked systems end up staying there throughout the course of their academic career with no mobility to move within the system (National Council of Teachers of Mathematics, 2018). Therefore, the study on the influence of tracking is relevant and needed to provide a deeper understanding of its impact on mathematical identity.

The purpose of this study was to examine the influence of tracking on the mathematical identity of adolescent students of color at a rural high school in Southwest Georgia. This qualitative case study, framed by Bandura's Social Cognitive Theory, was conducted over one semester, and involved students in varying tracks of an Algebra 1 course, as well as members of the high school community. Multiple data sources were examined, including interviews, a focus group, classroom observations, and a mathematical identity survey.

Students and community members discussed the context of learning mathematics. Findings revealed a hierarchal system used for decision making in which the state mandated End of Course examination is at the top. Decisions regarding course pacing and rigor are made based on desired results from the exam, which then influences student course placement, and finally mathematical identity. Although teachers desired the implementation of learning tasks which fostered conceptual understanding, timing to teach the standards for the exam took precedent.

The End of Course examination influenced the pacing and rigor of the tracked courses which influenced the placement and classroom procedures for the students which in turn influenced the perceptions, motivation, and identity of the students at Bearcat High School. Additionally, student participants shared their thoughts from the results of their self-reported mathematical identity and observed mathematical identity. Their responses revealed the existence of a desired mathematical identity, also seen in the school's math teachers. The implications and limitations of the study are discussed.

Keywords: Tracking, Mathematical identity, Students of color, Rural, Adolescents, Social Cognitive Theory

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Chapter 1 – Introduction

1.1. Overview

In 2014, the American Civil Liberties Union filed a complaint against the South Orange Maplewood School District in New Jersey under Title IV of the Civil Rights Act of 1964 and Section 504 of the Rehabilitation Act of 1973 (Miller, 2018). The complaint alleged that academic tracking by the School District was a discriminatory practice whereby students of color were assigned to low-level classes and White students were assigned to high-level classes. In October of the same year, a press release from the United States Department of Education announced it had entered into an agreement with the New Jersey school district resolving a compliance review that examined whether Black students were provided an equal opportunity to access and participated in advanced and higher-level learning opportunities (Department of Education, 2014). To expand the advanced and higher learning opportunities for all its students, the district reduced tracking for many courses. Since then, the educational policy of tracking has continued to be a controversial and heated topic in educational systems across the nation. Both California and Virginia have also experienced similar heated debates on the possibility of eliminating all accelerated mathematics courses before eleventh grade as part of an equity-focused plan.

Proponents of tracking argue that the policy enables teachers to cater lessons toward the specific ability level of the students in the class. A student who struggles with mathematics will gain specialized help in a lower-level mathematics class. Conversely, a student who excels in mathematics will gain from an advanced and more rigorous mathematics class. One group supporting this claim is teachers. In a 2008 survey of American teachers, 43% agree to some extent that their classes are too mixed in terms of ability and hinders their ability to teach effectively (MetLife Inc., 2008, p. 60).

Although proponents highlight the potential benefits of tracking, there are also some disadvantages to consider. As the Department of Education has suggested, tracking systems encourage a sort of segregation within the school system. In addition, tracking may cause students to self-label themselves as inferior to upper track students when divided into groups based on their academic ability.

1.2. Problem Statement

Many students in public schools do not have access to a rigorous secondary mathematics curriculum that promotes reasoning, problem solving, meaningful mathematical discourse, or builds procedural fluency (Perna et al., 2015). As such, more students from underserved populations in the United States are falling behind in mathematics compared to others in the world (Balingit & Van Dam, 2019). This is even more prominent for students who are socio-economically disadvantaged or minoritized. The educational inequities between regions and districts divide students who can take college preparatory mathematics courses from those who cannot. As a result, more students of color graduate high school without the necessary problem-solving skills to be successful in college or competitive in a 21st century global society (Nickens et al., 2001). This is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality. Gamoran (2016) found that U.S. schools were highly segregated while noting inequalities in American public schooling. This inequity of mathematics education is further exacerbated by the policy to homogeneously assign students to classes based on ability. This homogenous grouping is also known as tracking.

Davenport (1993) indicated that homogeneous grouping, especially at the high school level, fails to increase learning and seems to widen gaps between students, particularly as female and minority students are often placed in lower tracks. Even more so, tracking can affect students' academic self-concept and personal outcomes (Ireson et al., 2001). While there has been a vast amount of research regarding ability grouping and mathematics self-concept (Davenport, 1993; Ireson et al., 2001), there exists a gap in knowledge discussing the influence of tracking on mathematical identity for students of color, especially those living in rural context. The case study reported here, conducted in a rural Georgia high school, contributes to a deep understanding of the tracking experiences of rural students of color and how these students perceive themselves as learners of mathematics. Specifically, the case of Bearcat High School (pseudonym) examines a context where rural students of color are tracked in varying levels of an Algebra 1 course, and the impact this policy has had on students' mathematical identity.

1.3. Research Questions

To help guide the study of tracking and students' perceptions of themselves as mathematics learners, the following two research questions are proposed:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

The first research question is meant to help contextualize the study from the perspectives of students and community members. Interview and observations will help solidify the context of learning while allowing each participant the opportunity of sharing their individual perspectives and opinions of mathematics learning at Bearcat High School. Understanding how each of these groups within Bearcat High School perceive learning mathematics will help guide the second research question. The aim of Research Question Two is to narrow down this perspective to help gain a deeper understanding of the influence of tracking on mathematical identity.

In the remainder of this chapter, I elaborate on the significance of the study, my personal connection to the topic, the conceptual framework informing the study, and a review of relevant terms and definitions.

1.4. Significance of Study

A case study of a rural Georgia high school has the potential to offer a deep understanding of the practices of tracking in mathematics, and its influence on students of color and their mathematical identity. Specifically, it is vital to examine a case where rural students of color are tracked into varying levels of an Algebra 1 course, and the potential this policy has had on students' mathematical identity.

Grouping students in mathematics courses by ability can influence student motivation by creating an environment like the Big-Fish-Little-Pond effect in which students compare their own self-concept with their peers (Marsh & Parker, 1984). Because Black and Brown students are often assigned to lower-level classes, this impacts the school population's perceptions of students of color, assuming they are less intellectually, or at least mathematically, capable, thus fostering inequalities. Tracking in math courses can also impact self-concept of learning mathematics by fostering social comparison (Ireson & Plewis, 2001).

The significance of this study is in the design. By utilizing a case study, the researcher can capture the context and lived reality of participants, providing a range of perspectives which in turn gave the opportunity for greater understanding of the influence of tracking on mathematical identity.

1.5. Personal Connection to the Topic / Positionality

Mathematics has always interested me, as well as a desire to help others learn mathematics. In my tenure as a public high school teacher, I have witnessed many students struggle with mathematics anxiety. I want to help those who experience this anxiety and help them enjoy mathematics as I do. I am interested in seeing what factors might influence this anxiety, particularly researching the relationship between tracking and self-concept of learning mathematics for adolescent students of color.

As a Filipino American, I am personally motivated to engage in any research which examines the perceptions and conceptions of mathematics learning for students of color. I was a perfect example of the stereotypical high school Asian who does well in mathematics. I wonder if my ethnicity influenced my conceptions and perceptions of mathematics. I wonder how tracking students of color into high-, mid-, and low-ability mathematics courses influences their perception of mathematics learning. As a student, I took top level classes and had the motivation and self-efficacy to be successful. As a professional, I observe many students take my accelerated courses simply as a social reason to be amongst their friends. I also observe students leave top level classes because they do not want their grade point averages to suffer. I also observe students develop a negative connotation towards support-level classes in mathematics. It is as if some teachers and fellow students label support classes as only for students with behavior issues. My issue with these perceptions and conceptions is that it disregards a multidimensional mathematics approach. Finally, I contend that students in any level of Algebra 1 course should learn the same set of standards. Support courses should allow students extra time and scaffolds to learn the same content as a student in an average or honors course. How students reach their learning goals is the support, ensuring student learning goals are the same across all levels is equity.

My orientation to the topic is that tracking students negatively influences the perceptions of mathematics learning for students of color (Mutodi & Ngirande, 2014). These ideas come from my personal observations in the classroom as a teacher. In a socio-economically disadvantaged high school in rural Southwest Georgia, students of color represent approximately 60% of the overall student population. However, students of color only represent 10% of the student population taking Advanced Placement or Honors-level classes.

My agenda for taking up this topic is to fundamentally change how we offer mathematics at Bearcat High School. As such, I have a transformative orientation as a researcher while conducting my study (Finnegan & Hoggan, 2023). Currently, students are placed in various levels of the same mathematics course based on performance on a standardized assessment. I believe students would have a much richer and fruitful experience if this form of segregation stopped. Students should take a course in Algebra 1, and teachers should expose all students to the same level of rigor. My agenda to change mathematics at BHS contributes positively to my research design, implementation, analysis, and findings because I am deeply passionate about the issue. However, because I am passionate and I have my own perceptions based on my personal and professional experiences, my views are likely biased and skewed. In this study, I have taken great care to keep these biases in check, discussed with the findings in Chapter Five.

I have a hunch that I will discover an influence of tracking students on their perceptions of learning mathematics and of themselves as mathematical learners. As Hemmings et al. (2011) concluded, there is a relationship between former experiences in mathematics and perceptions for learning mathematics. Tracking students can provide a negative experience for some and could influence their perception of success in future math courses.

My hope is to provide new insights into perceptions of learning mathematics for rural students of color and how current policies might influence student perceptions of mathematics learning. Regarding my worldview, I believe that all people can learn mathematics. There is an inequity of rigorous mathematics for students of color living in rural communities with extreme poverty rates. These social contexts are among many variables which affect the disparity between the students I serve and those found in suburban schools, which offer more opportunities. My beliefs not only come from my personal experience as a person of color but also from things I have observed while teaching at a low-income rural high school for the past fourteen years. I view my knowledge and these personal connections to this research project to be rooted in human experiences, a constructivist orientation to research and knowledge building (Cakir, 2008).

1.6. Conceptual Framework

Ravitch and Riggan (2017) define a conceptual framework as an argument about why the topic one wishes to study matters, and why the means proposed to study it are appropriate and rigorous. A conceptual framework is a series of sequenced, logical, propositions that ground the study and convince readers of the study's importance and rigor (2017). While the United States has made many great strides over the past two hundred years to move forward issues of social justice and equality, much work remains. Many students in public schools do not have access to a rigorous secondary mathematics curriculum that promotes reasoning, problem solving, meaningful mathematical discourse, or builds procedural fluency (Perna et al., 2015). As such, more students are falling behind and losing confidence in their mathematics skills (The Nation's

Report Card, 2022). This disparity is even more prominent for students who are socio-economically disadvantaged or minoritized. The educational inequities divide those students who can take college preparatory mathematics courses between those who cannot. This is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality.

A conceptual framework should also convincingly argue that the research questions are an outgrowth of the argument for relevance; map onto the study goals, questions, and context; provide the researcher with the raw materials needed to explore the research questions by the collected data; and allow the researcher to effectively address those questions (Ravitch & Riggan, 2017). Although there has been much research with tracking and academic self-concept, there is a gap in the literature regarding the influence ability grouping has on mathematical self-concept for rural students of color. Therefore, the guiding research questions for this study grew from the need and relevance to address this gap in research. An instrumental case study is appropriate for this research because it will provide insight into an issue, facilitate exploration of a phenomenon within its context using a variety of data sources, and help to refine a theory (Baxter & Jack, 2008). The methods to collect and analyze the data for this study will enable an exploration of and effectively address the research questions.

Figure 1 presents the elements of my conceptual framework. The first is my personal interests and goals, followed by my identity and positionality, topical research, theoretical frameworks, problem statement, and research questions. The visual concludes with my proposed research design.

Regarding my worldview, I believe that all people can learn mathematics. There is an inequity of rigorous mathematics for students of color in rural communities with high poverty rates. There are many variables which affect the disparity between the students I serve and those found in suburban schools, which offer more opportunities. My beliefs not only come from my personal experience as a person of color but also from things I have observed while teaching at a low-income rural high school. As a researcher, I tend to focus on a constructivist worldview.

Figure 1

Conceptual Framework

<table>
<tr><td colspan="2">Personal Interest and Goals
As a Filipino American, the researcher is motivated to engage in any research which examines the perceptions and conceptions of mathematics learning for students of color. My goal is to fundamentally change how students experience learning mathematics at Bearcat High School.</td></tr>
<tr><td colspan="2">Identity and Positionality (Worldview)
All people can learn mathematics. The researcher is a constructivist.</td></tr>
<tr><td colspan="2">Tracking and Mathematical Identity for Rural Adolescent Students of Color</td></tr>
<tr><td>Topical Research
• Tracking
• Tracking and Mathematics Achievement
• Identity and Intersectionality
• Identity of Rural Students
• Mathematical Identity
• Tracking and Mathematical Identity
• Mathematical Identity for Rural Students of Color</td><td>Theoretical Frameworks
• Social Cognitive Theory
    • Social Persuasion
    • Vicarious Experiences
    • Environmental Factors</td></tr>
<tr><td colspan="2">Problem Statement
It is important that a qualitative case study be conducted in a rural High School in Georgia to help reach a deep understanding of ability grouping on students of color and mathematical self-concept. Specifically, it is vital to examine a case where students of color are homogeneously grouped by ability in varying levels of an Algebra 1 course and the potential this policy has had on students’ mathematical identity.</td></tr>
<tr><td colspan="2">Research Questions
1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?</td></tr>
<tr><td colspan="2">Research Design
Case Study with 8 students and 6 community members utilizing interviews, classroom observations, and a mathematical depth survey.</td></tr>
</table>

Constructivism is a theory of knowledge that argues that humans generate knowledge and meaning from an interaction between their experiences and their ideas (Cakir, 2008). As a theory of learning, constructivism is relevant in this study as I want to establish how learners learn and teachers teach. Specifically, I want to examine how students of color perceive the learning of mathematics at Bearcat High School. My personal connection and interest with the topic include equitability and inclusivity. My goal is to fundamentally change how we offer mathematics courses at the high school level. The topical research of this study includes tracking and mathematical identity, while the theoretical framework of this study is Social Cognitive Theory.

1.7. Organization of Study

To help guide the report of the study and achieve the research objectives, the chapters are organized as follows. Chapter Two provides a thorough description of the theoretical framework supporting the study. Chapter Three includes the review of literature, consisting of research and relevant policy on tracking and its impact on both mathematics achievement and mathematical identity. In Chapter Four, the research design utilized in the study of students' perceptions of themselves as mathematics learners is elaborated while providing a rationale for specific methodology and methods. By utilizing a qualitative case study approach, the study provides a deep understanding of the influences of tracking on rural Black and Brown students' perceptions of themselves as learners of mathematics. Chapter Five contains the findings from the study while Chapter Six concludes with a discussion of those findings.

Chapter 2 - Theoretical Framework

I am just not good at math. My parents were not good at math, so I am not either. I am never going to need to know this stuff. I have always been bad at math; therefore, it is not going to get better. I am just not interested in math. Seasoned mathematics educators are familiar with the previous statements because students and adults are quite comfortable in noting their inability to do mathematics while emphasizing a deficit of mathematical ability and achievement (Beilock & Willingham, 2014). Up to 60% of school-aged students struggle with some degree of mathematics anxiety (Dowker et al., 2016). Students doubt their ability to do such academic work and even blame it on a generational issue. Their parents were not successful in mathematics; therefore, they are not successful in mathematics. As such, having a deeper understanding of how individuals perceive their learning of mathematics is relevant and vital to mathematics education.

There are multiple factors which influence the learning process. These factors can include cognitive, behavioral, and environmental. One of the major learning theories in education that explains how individuals learn is behaviorism. Behaviorists suggest that individuals learn from their environment (Ertmer & Newby, 2013). In the 1960's, Albert Bandura went a step further by proposing Social Cognitive Theory to describe how we learn behaviors. Bandura theorized that learning new behaviors occur as we observe others through social interactions, experiences, and outside media influence. Researchers apply Bandura's Theory to better understand why individuals act the way they act.

Past and present experiences also play a key role in how we learn. From these experiences, individuals develop perceptions and expectations, which influence the choices and actions that they make. Students who perceive themselves as not good at mathematics will struggle with mathematics achievement, as there is a reciprocal relationship between cognitive appraisals and mathematics achievement (Forsblom et al., 2021). Students with low perceptions of learning mathematics will also struggle with self-confidence, self-efficacy, and motivation. All these factors influence mathematical identity. Therefore, utilizing Social Cognitive Theory as a theoretical framework might help to explain student perceptions of themselves as mathematics learners. In this chapter, I will lay out the theoretical framework which supports my research study. To accomplish this, I will discuss the history and elaborate on the main components of Social Cognitive Theory while explaining how factors within Social Cognitive Theory help explain student perceptions of learning mathematics.

2.1. Social Cognitive Theory

The origins of Social Cognitive Theory stem from Bandura (1977) as he theorized that individuals learn from their environment through observations and interactions. Albert Bandura developed this theory, as well as the Bobo Doll Experiment. In this experiment, researchers studied the learning of aggression styles. Children watched a video of aggressive acts. Researchers measured how aggressive the children got after watching the video. The children observed adults hitting the doll. As a result, the kids followed the lead and behavior of the adults. Results from the Bobo Doll Experiment revealed that learning by observing others is important. Therefore, the goal of feedback in the social-cognitive learning theory is for the individual to develop positive or appropriate behaviors by observation. In a school setting, these observations occur in the school environment, specifically in classes. Tracking has been one of the predominant practices to organize students in American public schools since the early 20th century. As students are tracked into varying levels of mathematics classes and begin comparing themselves to others in their track, this experience could impact a student’s perception of learning mathematics.

Fundamental to Bandura’s Social Cognitive Theory is the assumption that an individual’s behavior is related to the relationship between cognitive, environmental, and behavioral factors. Bandura (1986) emphasized the reciprocal causation of cognitive, behavioral, and environmental factors in explaining an individual’s behavior. Cognitive factors include knowledge, expectations, and attitudes. An individual’s identity, or how an individual perceives oneself and how others perceive the individual, is also an important cognitive factor. Environmental factors include social norms, access in community, and influence on others or ability to change one’s own environment. School organizational policies are also important environmental factors because they can create varied educational pathways for students. Behavioral factors include skills, practice, and self-efficacy. Perceptions and behaviors are interlinked such that individuals behave as they perceive. Chartrand et al. (2006) suggested that mimicry, or copying another’s observables only requires the ability to perceive the behavior in the other person and the ability to form the behavior oneself, as a manifestation of this perception-behavior link. Therefore, Social Cognitive Theory helps explain perceptions of learning mathematics by better understanding the link between behaviors and perceptions. For my research, it is vital and necessary to develop a deeper understanding of behavioral and environmental factors within Social Cognitive Theory.

2.1.1. Behavior Factors

Behavior factors play a role in explaining why individuals behave or take certain actions. This domain deals with an individual’s actual ability to perform a behavior by applying knowledge and skills. One way to measure student ability to perform mathematically is by examining mathematics achievement. The relationship between perceptions of learning mathematics and mathematics achievement is reciprocal in that perceptions influence achievement while achievement can also influence perceptions. Two of the major behavior factors include self-efficacy and motivation.

2.1.1.1. Self-efficacy. Self-efficacy is one of the behavior factors important to Social Cognitive Theory because it revolves around the self and self-beliefs. Self-efficacy refers to an individual's belief in his or her ability to perform a specific behavior. According to Bandura (1977), self-efficacy reflects an individual's confidence in controlling his or her own motivation, behavior, and social environment. The influence of self-efficacy spans many fields including psychology, sociology, and education and is an accurate predictor of performance (Bandura et al., 1977). Therefore, academic self-efficacy is the cause and effect of academic achievement. Drawing from a sample of German secondary school students, Schober et al. (2018) tested for reciprocal effects between self-efficacy and achievement in mathematics. Results from the research revealed positive effects of mathematics self-efficacy on later mathematics achievement. Individuals acquire efficacy for a task from four potential sources including mastery experiences, vicarious experiences, social persuasion, and physiological reactions to the task. To better understand how students perceive mathematics learning, it is vital to develop a better understanding of how students build mathematics self-efficacy.

2.1.1.1.1. Mastery Experiences. Mastery experiences, or the positive and negative experiences associated with the completion of a task or behavior, can influence the ability of an individual. A student with negative experiences of learning mathematics is more apt to give up when they struggle with a mathematics problem. In contrast, a student who has had positive experiences learning mathematics will try harder to complete the task. The need to improve student success in mathematics continues to be a critical interest in mathematics education. Having a better understanding of motivation and beliefs of mathematics students is crucial to help cultivate student success. Zientek et al. (2019) developed an instrument to measure sources of self-efficacy. Although all four sources of self-efficacy explain a large part of mathematics skill self-efficacy, proficiency experiences were the best predictor of self-efficacy. Therefore, educators should implement lessons that allow for greater mastery experiences that provide tasks at an appropriate level of challenge while allowing for scaffolding. As students engage in appropriately and sequenced mathematical tasks, they can make new connections from prior knowledge and develop a deeper understanding overall (Simon et al., 2010).

2.1.1.1.2. Social Persuasions. Social Persuasion is another source of self-efficacy. Capa-Aydin et al. (2017), define Social Persuasion as "related to verbal encouragement received from significant people such as parents, peers, and teachers" (p. 1225). Teachers, fellow students, and parents each play a critical role in this area. Students who experience verbal affirmation in turn develop positive self-efficacy. It is critical to give verbal praises when students have success in learning mathematics. These positive comments help build self-efficacy and contribute to a better perception of learning mathematics. The importance of parental involvement and support in this area is also vital. As noted by Kiss and Vukovic (2017), "parents and teachers have their own history related to mathematics and how they communicate their attitudes to students has the potential to improve or hinder mathematics learning" (p. 37). Additionally, the comments from fellow students are also a major influence on student self-efficacy. Students in a classroom setting will evaluate their own abilities, attitudes, and traits from observing others. The policy of tracking compounds this situation through grouping students into varying classes by ability. This social comparison within a low ability tracked mathematics class can negatively influence student perceptions of learning mathematics and teacher perceptions of students. Yanisko (2016) found that tracking affected the perceptions that teachers had about their students in the courses labeled comprehensive, or low-ability, and in those labeled honors, or high-ability. Therefore, it is vital for teachers to be equipped with ways of talking to students about their ability so that students in low-tracked classes build the same mathematics self-efficacy as students in high-tracked classes.

***2.1.1.1.3. Vicarious Experiences*.** Another important source of self-efficacy comes from vicarious experiences. Observing other students successfully solving mathematics problems and persistence in the process of learning mathematics positively influences students' beliefs that they can be successful too. If we think of proficiency experiences as being internal sources of self-efficacy, then vicarious experiences would be external sources of self-efficacy. Bandura's Bobo Doll experiment is a prime example of vicarious experiences. Children observed other people doing a certain behavior, and from that observation of others, the children modeled a similar behavior. The results of this experiment also have applications in mathematics education. In a classroom, students who experience other students working through a problem will also model this behavior. Conversely, students who experience other students who easily give up when the level of rigor in a mathematics problem increase will model a similar behavior. Therefore, vicarious experiences help explain student perceptions of learning mathematics by relating negative vicarious experiences to negative perceptions of learning mathematics. In contrast, students who have had positive vicarious experiences will have positive perceptions of learning mathematics. Educators can help mediate vicarious experiences through the implementation of purposeful mathematics lessons. Lindt and Gupta (2020) examined the influence of a week-long mathematics summer camp for middle school students on motivation and self-efficacy. The results from the study showed significant positive changes in students' vicarious experiences which suggest that students felt that after the camp they were more likely to improve self-efficacy based on experiences with others (2020). Providing vicarious experiences like this allows students to work with similar students while improving their self-efficacy skills.

***2.1.1.1.4. Physiological Feedback*.** One final source of self-efficacy is physiological feedback, or the physical sensations individuals experience. Some students physically get sick at the thought of taking a mathematics test or having to solve a mathematics problem. This anxiety towards mathematics leads to physical discomforts. These consistent physical discomforts in turn influence a student's belief of efficacy while learning mathematics. Mathematics anxiety, or excessive fear or worry which interferes with the manipulation of numbers and problem solving, does not discriminate against one's race, age, socioeconomic status, or gender. This type of worry or fear exists at all levels of society. Fear, in moderation, is a necessary human emotion. It reminds us of our limitations, keeps us safe, and keeps us grounded. In contrast, persistent worry and fear is not healthy and can be detrimental to an individual's overall well-being. A study measuring anxiety levels towards mathematics in high school students from Mexico revealed students at the high school level suffer anxiety towards problem-solving which in turn causes nervousness and irritability (Escalera-Chavez et al., 2019). To better understand the relationship between perceived mathematics ability and mathematics achievement, Pérez-Fuentesa et al. (2020) analyzed the results from over two-thousand high school students on a mathematics attitudes instrument and abilities tests. The results revealed that mathematics anxiety is a mediator of the relationship between mathematics ability and mathematics achievement. Therefore, an individual's level of mathematics anxiety influences his or her perceived confidence and mathematics achievement. Mathematics anxiety does not only have an impact on academic performance and achievement, but also on an individual's physiology. Jamieson et al. (2021) examined associations among mathematics anxiety, neuroendocrine activity, and exam scores for 478 community college students. The results from the study revealed that students who perceive more demand and exhibit higher levels of mathematics anxiety had lower levels of testosterone with worse exam performance.

Self-efficacy is an integral component of Bandura's Social Cognitive Theory. Individuals who believe they can perform well are more likely to persevere when faced with difficult or challenging tasks. Individuals with high self-efficacy will seek to master a challenge versus avoid a challenge. In addition to self-efficacy, motivation is another important feature of Social Cognitive Theory.

2.1.1.2. Motivation. Self-efficacy is also closely related to motivation, as self-efficacy is a major factor of motivation (Hoffman, 2015). Schunk and DiBenedetto (2020) define motivation as processes which instigate and sustain goal-directed activities. Behaviorism and social cognitive theories of learning have had a major influence on our current understanding of motivation; however, motivation is not a one-size-fits-all term. Instead, research over the past forty years has shown that motivation is complex and consists of an array of components. In addition, research on motivation includes social cognitive theories and acknowledges that motivation is determined by beliefs and the self, cognitions, and social context (Edward et al., 2016). Goal setting and mindset are two influential components of an individual's motivation.

***2.1.1.2.1. Goal Setting*.** Goal setting affects an individual's motivation or drive to do something. For the purposes of this research, it is necessary to discuss two types of goals including performance orientation and learning orientation. An individual oriented toward performance goals is likely to lose their motivation when faced with difficulty. Additionally, these individuals take failure extremely hard because it says something especially important about them and their ability. To the individual they think they do not have the ability, which can negatively affect their self-image and self-efficacy. Hansen (2021) studied the influence of students' goal orientation on their perception of utilizing visible learning goals. Although diverse types of students are described regarding their understanding of the usefulness of visible learning goals in their mathematical learning, the goals that students have in the process of study are essential for reaching success. Talking about grades, class standings, comparisons, and competition, instills the orientation towards performance goals. In this situation, points are tied to results and children tend to choose safety over trying something new or challenging. In an environment where performance matters, individuals choose an easy path, or the path of least resistance. Children with performance goals start worrying about their ability, backing off from challenges, deteriorating when the work gets hard, and in some cases just simply stop trying. An individual's orientation towards a goal has a direct impact on his or her motivation (Usan et al., 2019).

Teachers, parents, and fellow students can also influence students' goal orientation and help build confidence and motivation to achieve. Utilizing interviews with 31 students, Aarkrog and Whalgren (2021) found that a student's relation to, and support from, teachers and guidance counselors, as well as peers and parents, are crucial to the goal orientation process. Learning-oriented students want to be challenged and learn something new. Teachers can influence students' goal orientation by talking about improvements students have made, the kind of efforts they have put forth to learn something new, and how they have developed their abilities. Teachers and parents can instill learning-oriented goals. As Vedder-Weiss and Fortus (2013) found, perceptions of the goals that parents and teachers emphasize were better predictors of students' motivation, in and out of school, than perceptions of the goals that peers and schools emphasize. Saying a child is smart and brainy when he or she does something well may be the same as saying that a child is stupid and foolish when he or she does something poor. When teachers praise their ability after success and then give them something more difficult, students who are performance goal-oriented fall into a helpless pattern of either saying they lack ability or not liking the task anymore. Praising a child's ability makes them incredibly involved in performance goals. Therefore, teachers must be more concerned with praising a student's effort. In a study of over 1,200 secondary students comparing teacher feedback and students' self-regulated learning in mathematics, Guo et al. (2019) found that teacher's praise had positive relationships with high and low-achieving students' self-regulated learning. Teachers must give compliments to the student process and effort, not to their intelligence. Teachers can specifically make comments such as "you are very careful and thoughtful about your work" or "it is great that you put so much effort into it." Praising effort instead of ability makes children more eager to learn and helps them not fear failure (Dweck, 2007).

Students find joy in learning because they view it as a process and not as a definite. Parents and teachers must recognize the worth of learning goals and emphasize this to their children by making them realize the meaningfulness of hard work and challenges and praising them in the right way in their everyday conversations. Parents and teachers need to stop asking "how did you do in school today" and start asking "what did you learn in school today." This shows that we have a value in learning. If we want our students and children to grow to be productive adults and persevere when faced with challenge and frustration, then we must impress the value of learning goals and motivation.

Additionally, competition and comparison impact children's motivation (Vallerand et al., 1986). Often, parents have a single idea about what children should be good at and do not pay attention to all the other things the child is good at. For example, a child might not be the best student in school; however, he or she is very friendly, warm, and liked by their peers. Parents need to acknowledge that distinction. Each child is an individual and learns in his or her own way. To motivate children, parents should get them interested in the material and praise them for their involvement, their effort, and their progress. This will cause the children to do better through that engagement and not lead to learned helplessness. Being obsessively anxious about studying is a direct response to a person's lack of spirit and drive. Even more so, if the expectation for a child is set too high, goals become unattainable, and this will constantly lead to failure. Parents can push a little too hard. Therefore, parents need to adjust their expectation to just a little beyond where the child is comfortable achieving. Parents need to be supportive by saying "I know you can get there."

2.1.1.2.2. Mindset. Another important driving force of motivation includes mindset. An individual can have a fixed mindset or a growth mindset. A student with a fixed mindset believes intelligence, talent, and other qualities are fixed and cannot change. In contrast, a student with a growth mindset believes that intelligence and talent can be developed over time through practice and effort. This is particularly important regarding perceptions of learning mathematics. When students arrive at a problem and have difficulty in solving it, one of two things will occur. Either the student will persist and try, or the student will shut down and quit. In either case, the student is exhibiting either a growth or fixed mindset. Samuel and Warner (2021) utilized a mixed methods study to investigate the effect of embedding a combined mindfulness and growth mindset intervention within a required first-year, two-semester developmental statistics course.

Results showed that this approach reduced math anxiety in the college student sample and increased math self-efficacy.

2.1.2. Environmental Factors

Another component of Social Cognitive Theory includes environmental factors. A student's environment consists of a variety of social factors including family, friends, and observational learning. Students must feel comfortable in sharing their thoughts and confident when justifying and explaining their solutions while respecting the opinions and thoughts of classmates. By fostering an open forum for continuous dialogue, students can add to the conversation in a welcoming environment while interacting with others.

2.1.2.1. Tracking. This topic is explored in further depth in the next chapter. While there exist varying environmental factors which could impact perceptions of learning mathematics, the educational policy of tracking students into varying levels of academic classes is very detrimental yet continues to exist in public high schools across the country. Due to the inherent nature of tracking, students of low-ability are grouped with other students of low-ability. This creates a unique academic environment in which the class consists of students who all struggle mathematically. Students become aware of the level of course they are taking and will often self-label as being in a slower or lower class. This negative perception for learning mathematics in turn influences mathematical identity and mathematics achievement. Family, friends, and teachers can compound this situation by their comments, which only reinforce the negative stereotype. As such, students who are tracked in low-ability classes typically never achieve mobility with a tracked system; therefore, their perception for learning mathematics is reaffirmed each year as they are continually in a low-ability mathematics class. Having thoroughly discussed the main tenants of Bandura's theory, the next section includes a justification for Social Cognitive Theory as a theoretical framework.

2.2. Social Cognitive Theory as a Theoretical Framework

The achievements of students in mathematics continues to be poor; therefore, having a deeper understanding of the best ways to increase students' achievement in mathematics is a critical issue in mathematics education. An individual's mathematics achievement is influenced by his or her perception of learning mathematics. As a result, it is vital and relevant to study why an individual perceives learning mathematics in a certain way. Social Cognitive Theory describes the reciprocal relationship between cognitive, behavioral, and environmental factors to understand how an individual learns. A social cognitive perspective argues that learning occurs from internal and external influences. These internal influences can include knowledge, motivation, self-efficacy, and attitudes, while external influences can include social norms and environment. Therefore, Social Cognitive Theory as a theoretical framework can help explain students' perceptions of themselves as mathematics learners.

In this chapter's conclusion, the researcher has presented the case of utilizing Social Cognitive Theory as the theoretical framework for the case study. From personal observations in the classroom as a teacher, Social Cognitive Theory aligns with this researcher's values and educational philosophy. Additionally, the main tenets of Social Cognitive Theory, which include social persuasion, vicarious experiences, and motivation, help serve in guiding the research. Tracking students into varying levels of mathematics classes can provide a negative experience for some students and could influence their perception of success in future math courses. This case study provides new insights into perceptions of learning mathematics for students, and how current policies might influence student perceptions of mathematics learning. In the next chapter, a comprehensive review of literature will further elaborate on the relevancy and necessity of examining a case involving tracked students in varying levels of an Algebra 1 course, and the potential this policy has had on their perceptions of themselves as mathematics learners.

Chapter 3 - Literature Review

Many students in public schools do not have access to a rigorous secondary mathematics curriculum that promotes reasoning, problem solving, meaningful mathematical discourse, or builds procedural fluency (Perna et al., 2015). As such, more students from underserved populations in the United States are falling behind in mathematics compared to others in the world (Balingit & Van Dam, 2019). This is even more prominent for students who are socio-economically disadvantaged or a minority. According to the Department of Education, White students scored 32 points higher than Black students and 24 points higher than Hispanic students on the National Assessment of Educational Progress for mathematics at grade 8 in 2017 (De Brey et al., 2019). The educational inequities further divide those students into two groups: those who have access to rigorous mathematics courses and those who do not.

In the 10th Annual AP Report to the Nation, the percentage of Black students leaving high school having taken an AP exam compared to the percentage of Black students in the graduating class for the state of Georgia was 25.9% to 35.6%, respectively (College Board, 2014). This is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality. Gamoran (2016) found that U.S. schools were highly segregated while noting inequalities in American public schooling. This inequity of mathematics education is further exacerbated by the policy to homogeneously assign students to classes based on ability, also known as tracking. The National Council of Teachers of Mathematics has recommended the discontinuation of tracking students in high school mathematics courses due to the practice being unjust and “insidious” (National Council of Teachers of Mathematics, 2018, p. 16). Tracking forces some students to be placed in terminal math pathways which do not prepare them for continued study in mathematics and often do not provide mobility between tracked classes once a student is placed in a particular level (2018). The call for the abolishment of tracking in education is motivated by academic reasons and social reasons. More specifically, the impact of tracking includes mathematics achievement and mathematical identity. Therefore, it is vital to examine a case where rural students of color are tracked in varying levels of an Algebra 1 course, and the potential this policy has had on students' mathematical identity. In addressing this problem, the research questions guiding this study include what is the context of learning mathematics at a rural high school in southwest Georgia and how do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

This chapter provides a rationale for the importance and significance of the study. To accomplish this, the researcher provides a review of literature, consisting of research and relevant policy, on tracking and its impact on both mathematics achievement and mathematical identity. Specifically, the literature will include a comprehensive definition of identity, an intersectional approach to identity, the mathematical identity of rural black and Hispanic adolescents, and the impact of tracking on mathematical identity.

3.1. Relevant Research and Policy

Research which examines the perceptions and conceptions of mathematics learning for students of color is relevant and needed. In a tracked system, some students can take top level mathematics classes and have the motivation and self-efficacy to be successful. Some students take accelerated mathematics courses simply as a social reason to be amongst their friends. Additionally, some students leave top level classes because they do not want their grade point averages to suffer. For low-tracked classes, students sometimes develop a negative connotation towards support classes in mathematics. It is as if some teachers and fellow students label support classes as only for students who struggle academically or have behavior issues. The issue with these perceptions and conceptions is that it disregards student potential and assumes student learning is fixed and does not change. This is a stark contrast to the researcher's educational philosophy in which all students can learn mathematics. Therefore, studying how tracking students of color into high-, mid-, and low-ability mathematics courses influences their mathematical identity is relevant and needed. Although educators should strive to ensure all students receive access to high-quality mathematics instruction and rigorous grade-level content, the policy of tracking within secondary schools does not allow for such opportunities.

To synthesize the current breadth of literature associated with my research topic, this chapter presents themes in terms of topical research. As Ravitch and Riggan (2017) noted, topical research surveys what is known and has been investigated. The topical research will include a deep understanding of tracking, mathematical identity as it may be conceived as academic self-concept, perceptions of learning math, and self-efficacy, the impact on both mathematics achievement and mathematical identity, and Social Cognitive Theory as a Theoretical Framework.

3.1.1. Tracking

Tracking has been one of the predominant practices to organize students in American public schools since the early 20th century. Oakes (1987) defined tracking as being one of two forms, either curriculum tracking or ability grouping. For the purposes of the research, this study utilizes Oakes' definition of ability grouping as being "the division of academic subjects into classes at different 'levels' for students of different abilities" (p. 131). Historically, Georgia's Department of Education utilized curriculum tracking for many decades in public high schools across the state. High school students were tracked into either a college-preparatory pathway which led to a college preparatory diploma or a vocational pathway which led to a vocational diploma. Also, there was a third option to earn a dual diploma by completing prerequisites in the college-preparatory and vocational pathways. In 2007, Georgia became one of many other states to join the American Diploma Project with the mission for all students to complete high school with the skills necessary to be successful in college and the workplace (Talley et al., 2017). With this decision made to only offer one diploma track, curriculum tracking was no longer existent in Georgia public schools. Although this was an accomplishment in the fight for educational equality in Georgia, tracking of students into varying levels of academic classes persists in high schools across the state.

Although there has been substantial research regarding the negative consequences associated to tracking, proponents argue that tracking enables teachers to provide better differentiation toward the specific ability level of the students in the tracked classes, specifically benefiting high-achieving students in high-ability tracks (Card & Guiliano, 2010; Fu & Mehta, 2018). Theoretically, tracking reduces the disparity in student ability levels and allows teachers to provide instruction that is neither too easy nor too hard. However, tracking also encourages a demographic and academic segregation within the school system (Francis & Darity, 2021). In addition, tracking causes some students to self-label themselves as inferior to upper track students. Furthermore, tracking limits mobility within the different tracks as many students stay on the same track throughout high school (Ware, 2021). Finally, tracking of students typically creates tracking of teachers, in which teachers having less experience than their colleagues are placed in classes to teach low achieving students (Bush, 2019).

With so many negative consequences associated with tracking, it is perplexing for such a policy and practice to continue existing in public schools today. Oakes (1987) proposed that schooling contexts, which include curriculum organization and how a particular track influences student experiences, and societal contexts, which asserts that the historical need to certify people for various post-secondary opportunities is greater than the potential educational benefits of common curricular experiences, explain why tracking persists within our secondary schools. For the research, the researcher intended to develop a deeper understanding of the influence of tracking and its impact on mathematics achievement and identity.

3.1.2. Tracking and Mathematics Achievement

When discussing mathematics achievement within a tracked system, the literature can be a little cloudy. Schools typically form communities of practice by homogeneously grouping students by ability or tracking to organize them. Theoretically, tracking reduces the disparity in student ability levels and allows teachers to provide instruction that is neither too easy nor too hard. Due to the inherent nature of tracking, grouping students by ability while not teaching each track with the same level of rigor presents issues regarding mathematics achievement. From one educational viewpoint, there are two ways to measure mathematics achievement through formative assessments. There are external assessments created by state, national, and international institutions and there are internal assessments developed by teachers, math coaches, and lead teachers. Both types of assessments are used to measure mathematics achievement; however, there is one major difference in a tracked educational environment. The external assessments do not differentiate the level of rigor in the questioning based on tracking. Students in an Algebra 1 Honors course will take the same state assessment as a student in an Algebra 1 Support course. In contrast, teachers from an Algebra 1 Honors course typically will create a more rigorous unit assessment than a teacher from an Algebra 1 Support course. Students in an Algebra 1 Honors course might have more free response questions that require deep analytical thought, whereas an Algebra 1 Support course might have basic multiple-choice procedural problems such as solving a quadratic equation using the Quadratic Formula. This is not only inequitable, but it is also potentially problematic if students decide to flee high-ability classes in favor of a lower tracked class simply to have less rigor while improving their grade point average. Tracking allows for an inflation in grades (Mayer et al., 2018). Instead of all Algebra 1 students being grouped in one course, which would produce one standard normal curve, creating three tracks would allow for the creation of three different standard normal curves, providing one for each distinct track. An Algebra 1 Support student could have a numerical class average of 100 in the class, whereas an Algebra 1 Honors student might have an 89. According to numerical class averages, the Algebra 1 Support student is achieving more in mathematics than the Algebra 1 Honors student. This is not equitable for the Algebra 1 Honors student since this student is having to learn from a more rigorous curriculum and take more rigorous assessments yet, from a numerical standpoint, is doing worse than the Algebra 1 Support student. This depression in student achievement can, in turn, impact the student's motivation, self-efficacy, and even students' academic self-concept.

Venkatakrishnan and William (2003) reported the findings of a retrospective study of tracked groupings in a mathematics department in a co-educational comprehensive school in Greater London. Fast-track students were not significantly advantaged by their placement on these tracks. Regarding mathematics achievement, high track students did not benefit by being in a high track mathematics class. However, placement in the mixed ability grouping shows a significant advantage on lower attaining students. Teaching an Algebra 1 course in which all students receive the same rigorous curriculum and instruction empowers those classified as low track to rise to the level of expectations. Although the low track students will struggle with the level of rigor in a non-tracked mathematics course, these students will have the opportunity to receive the same quality of education as their high track counterparts.

3.1.3. Identity and Intersectionality

Every individual has an identity, or way in which one defines oneself and how one is defined by others. One's identity includes perceptions of experiences and relationships with others. An identity can be malleable and can change over time. One's identity also includes one's self-concept and self-efficacy. Identity is not one-dimensional. Instead, a person's identity is multifaceted and can include multiple domains. According to Delgado (2001), everyone has potentially conflicting, overlapping identities, loyalties, and allegiances. Introduced by Kimberlé Crenshaw in 1989, intersectionality is an analytical tool for understanding how aspects of a person's social and political identities combine to create different modes of discrimination and privilege. This intersectionality of identities is unique to everyone. The researcher personally identifies as a Christian, man, Filipino American, educator, husband, and father with a strong work ethic and sociable disposition. The intersection, or summation, of each of these identities make the researcher the person he is. Since this research will investigate the influence concerning tracking on mathematical identity of rural students of color, it is important to understand the role in which identity and intersectionality has on rural black students and rural Hispanic students.

3.1.4. Identity of Rural Students

For research purposes and to better understand students and their experiences in educational contexts, it is important to examine rural students' identity. As discussed in the previous section, an individual's identity is complex and created from the intersectionality of multiple social and political identities. In a study exploring the identities of three college students, Cain and Willis (2022) found that while students' race and ethnicities were prominent social identities, it was their rural background that shaped their perceptions and experiences. Therefore, the rurality of this case study makes the research unique and distinct. Unfortunately, one major misconception associated with rural America is that rural is synonymous with whiteness and white, uneducated, working-class farmers. In contrast, rural America is increasingly diverse with more than one in five rural residents being people of color (Lichter, 2012). For that reason, having a better understanding of the intersectionality of rural identity and racial identity is vital to developing a rich and meaningful grasp of students' mathematical identity.

3.1.4.1. Identity of Rural Students of Color. In a study examining differences in school characteristics and experiences for students of color in rural high schools, researchers found that African American and Hispanic/Latino youth more often attend rural schools characterized by extreme poverty rates that are often segregated by race/ethnicity as well as socioeconomic background (Irvin et al., 2016). Therefore, African American, and Hispanic/Latino youth in rural schools may have less access to resources that could help them prepare for postsecondary education. Regarding this case study, it is significant to further discuss in detail the identity of African American and Hispanic rural students.

Williams-Johnson and Cain (2020) found that family, school, and community were prominent factors that influence rural African Americans' educational aspirations. Parents and extended family members provide emotional support by encouraging the students to not only complete high school but also pursue some postsecondary education or training. Additionally, school personnel including school counselors, teachers, and coaches of sports teams influence students' aspirations to attend college. School environment and curriculum also influence the educational aspirations of rural African American students. Thus, the educational policy to track students, especially rural students of color, has the potential of influencing students' mathematical identity. Finally, the context and people within a community are influencers. Religion and religious communities are an integral part of the identity of rural African American by providing hope for the future and support for educational aspirations.

Since this case study examined the mathematical identity of students of color, it is necessary to also elaborate on the identity of rural Hispanic/Latino students. Irvin et al. (2016) demonstrated that there are differences in the school characteristics and experiences that may have important implications as African American and Hispanic/Latino youth in rural high schools. Hispanic/Latino youth indicated that their parents had significantly less education and their families experienced significantly more economic hardship than White and African American youth. For this case study, many Hispanic/Latino students are migrant students. While some were born in the United States, there remains a substantial portion of students who emigrated from other countries and have parents who do not primarily speak English. Additionally, Hispanic/Latino youth were significantly less often enrolled in a college preparation program and had a lower academic self-concept than that of White and African American youth. These unique educational experiences help shape the identity of rural Hispanic/Latino students.

3.1.5. Mathematical Identity

Academic self-concept is foundational to how students perceive their academic achievement. Shavelson originally defined academic self-concept as a single higher-order facet and comprises at least two higher-order academic facets, verbal, and math (Marsh & Shavelson, 1985). More specifically, mathematical self-concept, or how a student rates his or her skills, ability, enjoyment, and interest in mathematics, is a key factor because it influences students' mathematical achievement. Liou (2014) examined the relationships between fourth & eighth grade students' self-concept and achievement in math and science in Taiwan and concluded that self-concept is positively associated with individual achievement in math and science, and the correlation between student self-concept and school mean achievement was stronger for eighth grade students.

Mathematical identity is an extension of academic self-concept, specifically dealing within the discipline of mathematics. For this research, the researcher used two distinct definitions for mathematical identity. The first is from Kaspersen et al. (2017), which defines mathematical identity as "where one positions oneself relative to the social structure of being mathematical within the activity in which they participate and contribute." This definition closely aligns to Social Cognitive Theory. The second definition is from Gweshe and Brodie (2019), which defines mathematical identity as "stories about a person's confidence, beliefs, and persistence with mathematics" (p. 255). This closely aligns to the definition of academic self-confidence.

Although a new concept, mathematical identity is multifaceted and can include fixed or growth mindsets and robust or fragile identities (Gweshe & Brodie, 2019). Students who hold these two different mindsets have many unique characteristics of each. One of the most common features of children who have a fixed mindset is that they give up more quickly on difficult tasks. In contrast, those with growth mindsets have the attitude to choose the task that is more interesting than a problem they have already solved. Those who have a fixed mindset tend to avoid difficult tasks. Another characteristic common to individuals having a fixed mindset is their tendency to lose their confidence and self-efficacy when they fail. In contrast, having a growth mindset allows individuals to maintain their confidence and optimism when something goes wrong. Learning is a process, and there will be successes and failures along the way. This causes issues with students who have a fixed mindset because they are more likely to attribute their failure to a lack of ability versus a lack of effort.

Many studies have utilized The Mathematical Depth Instrument to measure mathematical identity (Kaspersen et al., 2017; Kaspersen & Ytterhaug, 2020). Using this instrument, research has shown that mathematical identity can be influenced by one's sense of belonging within communities of practice, past and present experiences, teachers, peer labeling, and parents (Gweshe & Brodie, 2019; Kafoussi et al., 2020). This is why social context is important when discussing mathematical identity. A student having a negative experience in a mathematics class will carry that memory to future mathematics coursework. Additionally, Barba (2020) proposed six characterizations of mathematical identity including: superiority, authority/power, spectator, inferiority, instructor/solidarity, and relative indifference. Some students have a superiority identity in which they assert their elevated proficiency in mathematics, while others are more of a spectator and remain neutral in any mathematical discourse or dialogue. Students with low self-confidence will exhibit inferiority or assert their low ability in comparison to others. In contrast, those who identify as an instructor will encourage learning in others while also remaining quiet in their understanding of how others failed. These six characterizations of mathematical identity are useful analytic tools and will be helpful to code qualitative data relevant to my research proposal.

Another component of mathematical identity includes self-efficacy. Self-efficacy refers to an individual's belief in one's ability to complete a task through control of one's own motivation, behavior, and social environment (Bandura, 1986, 1989). Mathematics self-efficacy is an extension of this definition to include self-belief in the ability to overcome difficulties with solving problems in mathematics. Having mathematics self-efficacy is important because it leads to higher levels of motivation and confidence which are both important in mathematics achievement.

3.1.6. Tracking and Mathematical Identity

Mathematical Identity is positively correlated with mathematics achievement (Gonzalez et al., 2020). Tracking creates vastly different learning and schooling opportunities for students. As students are tracked into varying levels of mathematics classes and begin comparing themselves to others in their track, this experience has the potential of impacting a student's mathematical identity. Legette and Kurtz-Costes (2021) found that a student's academic identity is related to mathematics track placement at the middle school level. Some adolescents might hold negative perceptions about students enrolled in lower tracked classes, while those enrolled in higher tracked classes might have more positive perceptions about themselves. Many teachers, as well as students, share a common negative perception that students in lower track mathematics classes are reserved for students with behavioral issues and problems (Geven et al., 2018). Additionally, many parents typically request their students be tracked in higher mathematics classes to ensure social cliches remain. In both instances, these scenarios create the opportunity of a negative impact on a student's mathematical identity. Students in the lower track, who may have the ability, perceive themselves to not be doers of mathematics simply because of the title of the mathematics course in which they are enrolled. Whereas students in the higher track can experience a rigorous mathematics course to maintain a social status.

Preckel and Frenzel (2010) found that Austrian students in gifted ninth-grade classes reported a decrease in mathematics academic self-concept at the beginning of the school year due to exposure to a high-ability reference group. Self-concept changes were most pronounced early in the school year, as students must quickly adjust to the rigorous curriculum. Academic self-concept of the gifted also decreased when attending more selective classes such as honors and Advanced Placement. Studying mathematics does not interest everyone, nor does everyone want to learn it. Many students struggle with mathematics due to having a low self-concept of their ability to do mathematics (Mutodi & Ngirande, 2014). Tracking students in mathematics courses can influence student motivation by creating an environment like the Big-Fish-Little-Pond effect, a frame of reference introduced by Marsh and Parker (1984), in which students compare their own self-concept with their peers. Tracking in mathematics courses can also impact mathematical identity by fostering social comparison between races and ethnicities (Morales-Chicas & Graham, 2021). Effective mathematics teachers strive to help students with a low academic self-concept develop positive mathematical identity traits and help them enjoy and appreciate the study of mathematics to better understand the world around us.

Chiu et al. (2008) found that math track placement and gender affected seventh-grade student’s self-esteem, self-concepts, and social comparisons. Higher track students had significantly higher ability self-concepts in math and school; however, math track did not influence students’ self-esteem. Legette and Kurtz-Costes (2021) examined adolescents’ beliefs about their math teachers’ perceptions of students and math track placement for sixth graders in the Southeast United States and revealed that honors math placement predicted increases in math self-concept over the course of the academic school year.

3.1.7. Mathematical Identity of Rural Students of Color

In addition, tracking negatively influences the mathematical identity for students of color (Mutodi & Ngirande, 2014). Interviews with 20 socioeconomically diverse Black seventh graders regarding school tracking and youth self-perceptions revealed that students in lower tracked classes view the negative perceptions of their classmates as “reflections of themselves as Black people and as students” (Legette, 2018, p. 1311). Comparatively, students in higher tracked classes viewed themselves as “positive representatives of Blackness” (p. 1311). Ozer and Perc (2020) found that tracking can have catastrophic consequences by punishing students at the start of the school year who struggle academically while rewarding those who are successful in the beginning. Tracking widens gaps between students deemed to be able. Bottiani et al. (2016) explored how students’ racial backgrounds interact with the school context to shape their perceptions of school support. Black students perceived less caring & equity relative to White students. Equity and elevated expectations were lower in diverse schools for both Black and White students. With such a controversial and heated topic in educational systems across the nation, the legal case against tracking is ever increasing. The American Civil Liberties Union filed a complaint against the South Orange Maplewood School District in New Jersey under Title IV of the Civil Rights Act of 1964 and Section 504 of the Rehabilitation Act of 1973 (Miller, 2018). The complaint alleges that academic tracking by the school district is a discriminatory practice whereby students of color are assigned to low-level classes and White students are assigned to high-level classes.

Tracking is the most detrimental to socio-economic disadvantaged students of color (Legette, 2018). This is because it reinforces the perception of low-ranking students that mobility potential is low. NCTM (2018) found that tracking limits a student’s mobility within a mathematics pathway. The next course in a sequence starting with Algebra 1 Support will typically be another math support class. These tracked classes continually label students as being *support*. Hemmings et al. (2011) concluded there is a relationship between former experiences in mathematics and perceptions for learning mathematics. Tracking students can provide a negative experience for some and could influence their perception of success in future math courses. The student would lack confidence to take a more rigorous math class or be uninterested in continuing study in mathematics because of a lack of relevance or interest. More insights into perceptions of learning mathematics for students of color and how current policies might influence student perceptions of mathematics learning is needed because there exists an inequity of rigorous mathematics for students of color in rural communities with extreme poverty rates.

In the United States, the implications of tracking are often easy to discern as some mathematics classes are identified as either college prep or not. In many schools, students are grouped according to ability, either heterogeneously (students of all ability levels in one class) or homogeneously (stratified by ability level, so low-ability students are in one classroom, high-ability students in another). This homogeneously grouping, or tracking, can influence a student's self-concept (Chiu et al., 2008). Self-concept is a person's perceptions of him or herself. These perceptions are formed through experience with and interpretations of one's environment. It is multifaceted in that people categorize the vast amount of information they have about themselves and relate these categories to one another. It is hierarchically organized, with perceptions of behavior at the base moving to inferences about self in subareas, then to inferences about self in general. There remains a gap in the literature regarding the influence tracking has on perceptions of learning mathematics for adolescent students of color. While there has been a vast amount of research regarding mathematics tracking and mathematics self-concept (Chiu et al., 2008; Lawrence, 2019), it is important that a more specific qualitative case study be conducted to reach a deep understanding of the influence of tracking on students of color and mathematical identity. It is vital to examine a case where students of color are tracked in varying levels of an Algebra 1 course at a rural high school in Southwest Georgia, and the impact this policy has had on students' mathematical identity.

3.2. Moving Forward

The ability to solve problems analytically and critically is a fundamental skill needed in a 21st century global society. In response to the question of how to best maximize the wide range of students with varied interest, attainment, and potential, many schools in the United States have utilized the practice of tracking, or separating students by perceived ability into different classes, in secondary mathematics education. Unfortunately, tracking often produces one or more of the following negative results: a lack of understanding when the pace of lessons was too fast; boredom when the pace of the lesson was too slow; anxiety created by the competition and pressure of grouped environments; disaffection related to the restricted opportunities students faced; and perceived discrimination in grouping decisions. Anxiety, disaffection, and perception each influence a person's self-concept. Marsh and Shavelson (1985) define self-concept as being multifaceted in that people categorize the vast amount of information, they have about themselves and relate these categories to one another. Therefore, tracking has the potential of influencing an individual's self-concept of learning mathematics. More specifically, tracking can have a negative impact on a person's mathematical identity, or one's beliefs, self-concept, perceptions, and engagement with learning mathematics.

The macro and micro socio-political circumstances in which people make meaning and choices in their lives influence each other. These circumstances influence individual perceptions, even regarding mathematics learning. On the macro level, it is the state which formally adopts the standards for each mathematics course and the federal government which provides funding and support to school systems who serve low-income schools and students. On the micro level, it includes preconceived notions of mathematics from family, friends, and prior experiences.

Research regarding the topic of tracking and mathematical identity for adolescent students of color is relevant and needed. My research will move forward and expand our current understanding of this problem by specifically addressing a case of rural students of color and the intersectionality of their identities. While the United States has made great strides over the past two hundred years to move forward the issues of social justice and equality, much work remains. Students in public schools need to have access to a rigorous secondary mathematics curriculum that promotes reasoning, problem-solving, meaningful mathematical discourse, or builds procedural fluency, especially students who are socio-economically disadvantaged or a minority. More students of color need to graduate high school with the necessary problem-solving skills to be successful in college or competitive in a 21st century global society. The practice of tracking is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality.

The researcher’s agenda for taking up this topic is to fundamentally change how the high school at which he teaches offers mathematics. As such, the researcher had a transformative orientation while conducting the study. Currently, students are placed in various levels of the same mathematics course based on performance on a standardized assessment. Students would have a much richer and fruitful experience if this form of segregation stopped. Students should take a course in Algebra 1, and teachers should expose all students to the same level of rigor. Therefore, the agenda contributed positively to the research design, implementation, analysis, and findings because the researcher is deeply passionate about the issue. However, because of this passion and perceptions based on personal and professional experiences, the researcher's views could be biased and skewed.

The call to action and change comes from my personal observations in the classroom as a teacher. In a socio-economically disadvantaged high school in rural Southwest Georgia, students of color represent approximately 60% of the overall student population. However, students of color only represent 5% of the student population taking Advanced Placement or Honors-level classes. At Bearcat high school, ninth graders are tracked in either Algebra 1 with support, Algebra 1 regular, or Algebra 1 Honors. There is also an additional track for students who need additional remediation in the Foundations of Algebra course. In the three distinct Algebra 1 tracks, students learn the same set of state standards; however, the level of rigor with the curriculum, instruction, and formative assessments is not equal. Students in any level of Algebra 1 course should have access to the same level of rigorous curriculum and instruction. Support courses should allow students extra time and scaffolds to learn the same content as their average and top-level students. How students reach their learning goals is the support, ensuring student learning goals are the same across all levels is equity.

It is important that a qualitative case study be conducted in a rural High School in Georgia to help reach a deep understanding of the influence of ability grouping on students of color and mathematical self-concept. Specifically, it is vital to examine a case where students of color are tracked in varying levels of an Algebra 1 course in a rural high school in Southwest Georgia, and the potential impact this policy has had on students' mathematical identity.

Chapter 4 - Methodology

In response to the question of how to best maximize the wide range of students with varied interest, attainment, and potential, many schools in the United States have utilized homogenous grouping of students in mathematics classes by ability, also known as tracking. Boaler (2002) noted that tracking often produces one or more of the following results: a lack of understanding when the pace of lessons was too fast; boredom when the pace of the lesson was too slow; anxiety created by the competition and pressure of grouped environments; disaffection related to the restricted opportunities students faced; and perceived discrimination in grouping decisions. Anxiety, disaffection, and perception each influence an individual's identity. In addition, tracking often indirectly sorts students by demographics, particularly ethnicity and socioeconomic status. As a result, the separation of high-poverty and minority students in low-ability groups between students in middle- or high-ability groups continues to widen. Gamoran and Borman (2006) found that schools in the United States are highly segregated and the Black-White achievement gaps remain substantial today. The effect of this achievement gap perpetuates the inequities that exist in mathematics education in the United States regarding standardized-test scores, graduation rates, college matriculation, and course selections. Therefore, tracking has the potential of influencing students' perceptions of themselves as learners of mathematics. Additionally, the National Council of Teachers of Mathematics has recommended the discontinuation of tracking students in high school mathematics courses due to the practice being unjust and "insidious" (National Council of Teachers of Mathematics, 2018, p. 16). This research brought a deeper and unique understanding of the influence of tracking on mathematical identity at the secondary level. Specifically, the research explored tracking's influence on the mathematical identities of high school students of color in rural areas. The following two research questions helped guide the study:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

This chapter will elaborate on the research design utilized in the study of students' perceptions of themselves as mathematics learners while providing a rationale for the identified methodology and methods.

4.1. Research Design

Many educational research methods are descriptive in that they describe and interpret the context of a problem. To detail, compare, contrast, classify, analyze, and interpret the structure and circumstances that form the various fields of inquiry researchers have, studies look at individuals, groups, institutions, methods, and materials. The methodology in this study encompassed the research design, context and participants, data gathering methods, and data analysis. Within the past two decades, research regarding identity in mathematics education has continued to grow. Graven and Heyd-Metzuyanim (2019) reviewed the literature on identity in the top twenty mathematics education journals to describe the extent to which the field is evolving and highlight key absences in the research. The majority of the 47 studies focused on qualitative methods with eight or less participants with 76% of the data collected being interview data. To supplement this data, many of the studies also included observational data.

The research design for the study was a qualitative approach. More specifically, it was a case study examining an Algebra 1 course at a high school in southwest rural Georgia. The participants for the case study included students and teachers of an Algebra 1 course. Data gathering methods consisted of surveys, interviews, observations, and documents. Finally, a variety of techniques, hardware, and software were utilized for data analysis. The next section provides a rationale for choosing a qualitative approach.

4.1.1. Choosing an Appropriate Approach

This research is about the influence of tracking and its impact on mathematical identity. The goal was to develop a deeper understanding of the mathematical experiences of Black and Brown children affected by tracking and other deficit-oriented views on adolescent mathematics students, namely the impact on mathematical identity of Black and Brown students. Qualitative research is used to understand beliefs, experiences, attitudes, behaviors, and interactions. Research Question One aimed to understand the context of learning mathematics at Bearcat High School. To provide a deep description of the interactions between students, teachers, and other community members was the goal. Understanding the context of learning would not have been beneficial with strictly numerical data but relied on categorical data to vividly describe the story of the mathematical learning experience through the lens of each participant. Additionally, the context of learning is within itself comprised of multiple factors which interact and influence each other (Tessmer & Richey, 1997). While a Likert scale would have provided a more efficient method of measuring attitudes and opinions, qualitative data such as individual interviews and focus groups allowed participants to describe their perspectives on the environment, culture, social structure, and their emotional experiences while learning mathematics at Bearcat High School. Also, observations ensured a rich picture of the participants and their interaction within their learning environment. Specifically, the classroom observations revealed how students and teachers interacted through social persuasion and vicarious experiences.

Research Question Two furthered this narrative by aiming to describe the influence of tracking on students' mathematical identity. While collecting numerical data from a survey measuring students' mathematical identity was utilized, the purpose of Research Question Two was not to label each student with a number. Instead, the aim was to use their self-reported identity and observed identity to initiate a dialogue on the influence of their learning experiences in a tracked system and how they viewed themselves as learners of mathematics. Grasping and describing this experience had to be produced through multiple interviews that when combined, provided a rich and detailed description, unique to each student. While the initial individual interviews allowed students to describe in detail their individual opinions and attitudes, the use of a town hall focus group interview allowed teachers and other community members to add their own experiences while students elaborated on other participants' responses. Students and community members discussed the pros and cons of the tracked system, described the perception of being tracked into a support level mathematics class, and the learning environment of mathematics classes at Bearcat High School. This melting pot of unique qualitative data components produced a thick, cohesive, and comprehensive story which helped students solidify and better describe their identity in the individual follow up interviews.

Therefore, a qualitative study worked better than either a quantitative or mixed method for this research interest because the qualitative approach best explored the influence of tracking on students of colors' mathematical identity, specifically as it may be conceived as academic self-concept, perceptions of learning math, and self-efficacy. Although mixed methods were not the intended approach with the study, quantitative data was included through descriptive statistics to help describe the context and participants. In the following section, a justification for the utilization of a case study is explored.

4.1.2. Case Study

As made evident in the previous section, qualitative research methods were appropriate by providing a way to develop a deeper understanding of the context of learning mathematics at Bearcat High School while allowing participants to descriptively communicate their attitudes, opinions, and mathematical identity within a tracked system. Within the realm of qualitive research, there also exists several types of designs including ethnography, phenomenology, narrative research, and case studies. For this research, a case study was most appropriate because the goal was to develop an in-depth, detailed examination of a particular case. Specifically, the case being tracked students of color taking Algebra 1 at Bearcat High School. Since the goal of the research was to better understand the context of learning and how students' mathematical identities had been influenced by tracking, a case study captured a range of perspectives from students and community members. Each of the two research questions also addressed this goal by looking at the overall context of mathematics learning and the individual stories, unique to each student, of the influence of the learning environment on mathematical identity. In contrast to the singular view of an interview or survey, a case study allowed all stakeholders to voice their opinions and attitudes in the context of learning mathematics at Bearcat High School. While participants shared their unique and individual experiences, utilizing multiple interviews and a group interview allowed a greater understanding of the case. The overall picture of the mathematics learning experience of tracked students of color taking Algebra 1 at Bearcat High School was pieced together from the combination of numerous data sources and multiple participants contributing their own distinct perspectives. Therefore, the structure and experience of learning mathematics at Bearcat High School provided a unique context which made a case study the most appropriate qualitative design for studying students' perceptions of themselves as mathematics learners.

This research applied Sharan Merriam's Qualitative Research and Case Study Applications in Education definition. Merriam (2002) described qualitative research as a means of better understanding through the study of individuals as they interact within their socially constructed environments. The socially constructed environment for this qualitative research was the context of learning mathematics at Bearcat High School. The case under study was the collective of adolescent rural students of color taking Algebra 1 at Bearcat High School. The case study was most interested in understanding the students' perspectives and perceptions and how their unique experiences have affected them as mathematics students. An instrumental case study was appropriate for this research because it provided insight into an issue, notably the influence of tracking on mathematical identity (Baxter & Jack, 2008). To help explore this case, Research Question One addressed the overall context of learning mathematics while Research Question Two delved into each students' unique story on the influence of tracking on mathematical identity. Interviews and classroom observations provided the best means to collect relevant data for this research. Although surveys of mathematical identity could allow for quantifiable data; the goal was not to show some form of correlation or causation between these two variables, but instead, explore how these two variables have shaped and defined mathematics learning for students of color. The research was particularistic by specifically looking at the policy of tracking, richly descriptive of this situation, and heuristic as to better understand the perception of learning mathematics for tracked students of color. Having justified the reasoning behind the research design, the next section will provide more specific details regarding the context and participants.

4.1.3. Context and Participants

The case study was an in-depth investigation of a particular group of students at Bearcat High School, namely adolescent rural students of color taking Algebra 1. The research involved this high school because it provided a means to utilize a convenience sample of students and community members for the study. To gain a greater understanding of the case, it was also vital to give a detailed background of the school and elaborate on issues unique to Bearcat High School. The focus of the following sections will be on the case and an introduction to the participants. A more elaborate description of the individual participants will follow in the next chapter.

4.1.3.1. Background. Bearcat High School was a Title I public high school with 89 teachers, 1597 students, and served grades 9–12. The school was in the city of Bearcat, a rural community in Division County of rural southwest Georgia. While the high school is in the city of Bearcat, it serves students from the entire county. As of the 2020 Census, Division County had a population of 29,367. With a median household income of $41,481, 25.6% of the population lived in poverty. In addition to the county seat of Bearcat, Division County also has three smaller cities, seven unincorporated communities, and a town. Division County also had two post-secondary school options available which include a small community college and regional technical college.

Bearcat High School was not the only high school within the Division County School System. In 2008, the Division County School System created an alternative learning center as a secondary option for high school students not finding success in a traditional school setting. Students who struggle academically or are referred based on behavior issues will often enroll in the alternative school. Upon graduation, students from both schools share the same graduation but students from the alternative school do not receive the same diploma as students who graduate from Bearcat High School. In addition, there were two other high schools within the county including a private Christian academy and public charter academy. Both academies were smaller in size and enrollment than the high school.

At Bearcat High School, the student population consisted of 48% Black, 38% White, 11% Hispanic, and 3% Other. Although diverse, there is an intense sense of school spirit and pride within the community, which may be attributed to all students in the district attending the same middle school. Unlike other school systems comprised of multiple middle schools, students entering Bearcat High School have already shared many years and experiences together as a cohort. This strong school spirit was even more evident from the community support at the football stadium on Friday nights as sold-out bleachers of fans cheered on the high school's Junior Reserve Officer Training Corps, marching band, cheerleading, and football programs.

Bearcat High School was ranked 183rd in the state of Georgia. According to the 2019 Georgia Department of Education College and Career Readiness Performance Index system, the district high school score was 70.9. 100% of the student population identified as economically disadvantaged and 3.6% identified as English learners. Due to its size, Bearcat High School provided multiple academic and extracurricular activities, clubs, and sports for its students. In addition to a commitment for excellence in academics, Bearcat High School had a flourishing fine arts program and strong Career, Technical, and Adult Education program. Students could take Advanced Placement® coursework and exams. The AP® participation rate at Bearcat High School was 15%.

The case of study for this qualitative research was students of color taking Algebra 1 at Bearcat High School. In mathematics, students entered Bearcat High School under a tracked system. Upon entering ninth grade, students are either tracked into a Foundations of Algebra, Algebra 1 Support, Algebra 1 Regular, or Algebra 1 Honors class. Bearcat High School is on a semester schedule with four blocks of classes each day; therefore, students in the Algebra 1 honors, Algebra 1 regular, or Foundations of Algebra class take the course either in the fall or spring of their first year. Students in the Algebra 1 Support track earn two credits. Provided they pass the first semester, students receive an elective credit for a math support class. Second semester, students can earn a mathematics credit and take the state mandated End of Course Assessment for Algebra 1. Due to the block scheduling, each mathematics class is approximately 90 minutes long. Providing a deep and detailed description of the case was critical to the research as it addressed the aim of Research Question One and the context of learning mathematics at Bearcat High School. In the next section, a brief description of the participants of the case study is provided with a more elaborate description following in the next chapter.

4.1.3.2. Participants. The participants of this case study consisted of eight students and six community members. The community members consisted of the math coach, ninth-grade guidance counselor, ninth-grade administrator, parent, and two teachers. The eight students were divided between the four different Algebra 1 courses. Therefore, there were two students from Foundations of Algebra, two students from Algebra 1 Support, two students from the regular Algebra 1 course, and two students from the Algebra 1 Honors course. Of the eight students, six identified as African American and two identified as Hispanic. The research specifically chose a sample of students of color to address the problem statement and research question. Both teachers identified as Caucasian female and were selected for the study because they both taught Algebra 1.

Purposeful sampling was appropriate because the research selected students to optimize data sources for answering research questions for the study. From a populated list of all students enrolled in Algebra 1, each student received a number, and a random number generator was utilized to randomly select the eight students. To recruit the students, a classroom visit allowed each prospective participant to be spoken to individually. In terms of the community members, each of the six participants worked at Bearcat High School. To obtain consent from each community member, a letter was sent out which thoroughly explained the risks, benefits, and alternatives (See Appendix A). To obtain consent and assent for each student participant, a letter was sent out to all parents and guardians (See Appendix B). Contact was completed with all participants via telephone. After reviewing the study's details, the participants and their guardians signed two copies of the consent and assent forms. Each participant retained a copy of the consent and assent forms. A fully elaborate description of each of the participants will follow in the next chapter.

4.1.3.3. Issues. There were also issues which made Bearcat High School a unique context in which to do a case study. At the middle school level, students in each grade were tracked into two different teams depending upon academic achievement. Students had mobility within the tracked system. As students transitioned to high school, students were tracked into one of four courses including Foundations of Algebra, Algebra 1 Support, Algebra 1 Regular, and Algebra 1 Honors. Although not publicly advertised within the community, students did have mobility within the tracked system as parents could specifically ask for the school’s guidance counselors to place the student in any of the courses.

Starting in the 2021–2022 academic school year, a selected group of eighth-grade students participated in an acceleration program. Selection of these students was solely based on prior academic achievement. These students were bused from middle school to high school and allowed to take ninth-grade honors literature in Fall 2021 and Algebra 1 Honors in Spring 2022 versus eighth-grade literature and math. Therefore, the Algebra 1 Honors class in Spring 2022 consisted of both eighth and ninth-grade students. In contrast, the Algebra 1 Honors class in Fall 2021 consisted of only ninth-grade students.

4.1.4. Instruments

Qualitative studies, specifically a case study, consists of various forms of data gathering methods ranging from participant observations, interviews, surveys, focus groups, and document analysis. The purpose of utilizing various research tools in a case study is to increase validity. While using both qualitative and qualitative approaches, a researcher can employ different data collection instruments such as surveys, interviews, documentation review, and collecting artifacts. By doing so, researchers can triangulate the research instruments or techniques to provide different views of the case.

In his Social Cognitive Theory, Bandura (1986) emphasized the reciprocal causation of cognitive, behavioral, and environmental variables in explaining an individual's behavior. Cognitive variables include knowledge, expectations, and attitudes. An individual's identity, or how an individual perceives oneself and how others perceive the individual, is also an important cognitive variable. Environmental variables include social norms, access in community, and influence on others or ability to change one's own environment. School organizational policies are also important environmental variables because they can create varied educational pathways for students. Behavioral variables include skills, practice, and self-efficacy. To better understand the implications of these variables and address the two research questions guiding the study, the case study required more than one tool for data collection and many evidentiary sources.

While the research blended numerical and qualitative data into the case study, the intent was not to approach the research from the view of mixed methods. For the purposes of studying students' perceptions of themselves as mathematics learners and understanding the context of learning mathematics at Bearcat High School, this research utilized interviews, surveys, and observations. Table 1 helps summarize the data collection used in the case study while addressing the two research questions. In the following sections, a detailed description of each technique used to collect and analyze the data is included.

Table 1

Data Collection Summary

	Address Research Question	Student	Teacher	Counselor	Administrator	Math Coach	Parent
Initial Interview	1, 2	X	X	X	X	X	X
Focus Group Interview	1	X	X	X	X	X	X
Closing Interview	1, 2	X	X	N/A	N/A	N/A	N/A
Overt Observations	1	X	X	N/A	N/A	N/A	N/A
Surveys	2	X	N/A	N/A	N/A	N/A	N/A

4.1.4.1. Interviews. Interviewing was the primary source of instrumentation in this study. Gweshe and Brodie (2019) investigated mathematical identities for 10th grade students, with a focus on influences on the learners' identities. Researchers gave a mathematical identity questionnaire to students and assessed their test scores. Gweshe and Brodie purposefully selected four students for interviews based on interesting relationships between their questionnaire responses and test scores. Researchers analyzed the interview data for stories of the students' confidence, beliefs, and persistence with mathematics, as indicators of mathematical identity. This case study took a similar path.

For the purposes of studying students' perception of themselves as mathematics learners, this study utilized a three-phase interview process. To address Research Question One regarding the context of learning mathematics at Bearcat High School it was necessary to interview all eight students and all six community members. The interviews with each student also provided insight into the influence of tracking on mathematical identity. Each interview was recorded using audio visual equipment. Additionally, observer memos taken while transcribing interviews, watching videos, or during the overall process served as instrumentation. Each interview was face to face and occurred in a reserved study room at the Bearcat High School campus. The study room was a familiar location for most students. Each interview was limited to under an hour and was recorded. After the interview, the researcher made memos as to the physical characteristics and mannerisms of the participant. It was an important observation whether the participant appeared at ease with the content.

An initial interview in the fall allowed for a starting point in the data collection process. The research utilized a Student Initial Interview Protocol (See Appendix C). Student interviews lasted approximately 15 minutes and focused on three major topics. The first set of questions was about previous math classes each student took. To help ease any initial discomfort, students were asked to recall some of the things they remembered about the math classes they have taken. Additionally, students were asked to share their thoughts on whether people are born good at math. This question helped address Research Question Two by allowing students to share their own beliefs and opinions regarding the mathematical identity of others. The second round of questions revolved around tracking. Each student identified which tracked Algebra 1 course he or she was currently enrolled in. Additionally, each student described how the name of the math class made him or her feel, whether the math class challenged him or her, and on mobility for students to move up or down in the tracked courses. This set of questions addressed Research Question One and helped illuminate the context of learning mathematics at Bearcat High School. The final round of questions focused on mathematical identity. Students were asked to describe how they view themselves as a learner of mathematics and described how others might describe them as a learner of mathematics. Students were also asked to discuss how important it was to them to receive verbal praise from their teachers and to see other students successfully solving math problems on the board. This was a needed and relevant area of discussion that addressed both research questions. Students shared classroom procedures, the overall learning environment for their mathematics class, and their own reflections as learners of mathematics. However, only interviewing students was not sufficient to develop a clear picture of the overall case and context. Interviews with all six community members were also needed.

For the community member interviews, each session lasted approximately 30 minutes. The research utilized a Community Member Initial Interview Protocol (See Appendix D). To start the conversation and help ease any discomfort, each interviewee was asked to describe their role at the high school and how long they served in that role. The interview consisted of two major topics. The first round of questions revolved around tracking. Community members read statements from the Department of Education, American Civil Liberties Union, and NCTM on tracking. Each community member was asked to describe their thoughts on each statement. To help supplement this round of questions, community members were asked to discuss the pros and cons of tracking students in Algebra 1 at Bearcat High School. This helped address Research Question One and gave a description of learning mathematics at Bearcat High School from an adult perspective. The second set of questions focused on mathematical identity. Each community member was asked to discuss his or her assumptions of students tracked into each of the various Algebra 1 courses offered at the high school. The community members also discussed the advantages and disadvantages of the tracked system at the high school. Finally, each interviewee described the importance of teachers giving verbal praise and affirmation to students in math class and students observing other students successfully solving or not giving up when working on a math problem.

The second interview was conducted as a focus group. In this phase, the focus group was filmed and limited to 60 minutes. The key role of the focus group was to facilitate conversations in a social setting, while encouraging every participant to engage in the forum. Before beginning, the participants were told that all opinions were welcome, every perspective was notable, and all voices were significant. After studying responses in the individual interviews, questions were posed to the focus group. Participants were able to hear their peers' responses and elaborate on perceptions and compare experiences among the participants. Interviewing as a social behavior is supported by Patton (2015). The talking points for this forum came directly from memos and notes recorded after interviewing all participants and after watching the interview videos for physical observations. To better understand the context of learning mathematics at Bearcat High School and address Research Question One, participants were asked to share their thoughts on the issues of misplacement in tracking, and the rigor and pacing within each tracked Algebra 1 class. Participants were also asked to discuss the importance of motivation through social persuasion and vicarious experiences and the perceptions of the different tracked Algebra 1 classes.

The third portion of data collection was a final interview with the individuals. The final interview was an individual follow-up interview to clarify or extend any ideas that emerge. This interview provided an opportunity for the participants to also elaborate on experiences or perceptions. Additionally, it gave the participants an opportunity to reflect on the process and offer any new perspective on the case study. The final interview was based on themes emerging from the focus group or on individual perceptions that need elaboration. The teachers were asked to further discuss their views on the context of learning at Bearcat High school and the students were asked to discuss the results from their self-reported mathematical identity and observed identity. Both sets of interviews revealed a startling finding which will be further discussed in the next chapter.

All three rounds of interviews were useful in exploring the experiences, opinions, views, and beliefs for tracked students of color in an Algebra 1 course at a rural and socio-economically disadvantaged secondary public school. Instead of utilizing a structured interview, the research utilized a semi-structured interview. To help guide the interview process, the research implemented an interview protocol to help guide the conversation and included flexibility to probe the participants for additional details. In addition to the interviews, the case study also utilized observations.

4.1.4.2. Observations. The research included two overt classroom observations (one per teacher). For this case study, these observations helped better understand the specific case by studying students' and teachers' accounts and actions in a classroom context. Kawulich (2005) proposed three types of participant observations including descriptive, focused, and selective. The research utilized focused observations in which insights guided the decisions about what to observe. The research developed an observation protocol before starting data collection, and then noted observations in appearance, verbal behavior and interactions, physical behavior and interactions, and students who stand out.

Figure 2

Classroom Observation Protocol

Teacher ___________ Date/Time ___________ Class Name __________	**Social Persuasion** Hearing others being successful	**Vicarious Experiences** Observing others being successful
Teacher ← → Student		
Student ← → Student		

Finally, the research included relevant documents to better understand the functioning of the case under study. Supporting documents consisted of four Algebra 1 Curriculum Maps (one per Foundations, Support, Average, Honors); one set of Algebra 1 Georgia State Standards; four sets of Lesson Plans (one per Foundations, Support, Average, Honors). Through analysis of these documents, the research sought convergence and corroboration to triangulate the data. Additionally, the research gave a voice and meaning around mathematical identity for tracked students of color.

4.1.4.3. Surveys. Finally, the case study included a survey as an instrument in the data collection to study students' perceptions of themselves as mathematics learners. Researchers utilize surveys to gather data at a particular point in time with the intention of describing the nature of existing conditions, or identifying standards against which existing conditions can be compared, or determining the relationships that exist between specific events. Surveys may vary in their levels of complexity, from those which provide simple descriptive statistics to those which present correlational analysis. Researchers may further differentiate surveys in terms of their scope and complexity. To better understand students' perceptions of themselves as mathematics learners, the case study utilized The Mathematical Dept Instrument (see Appendix E).

Many studies have utilized The Mathematical Depth Instrument to measure mathematical identity (Kaspersen et al., 2017; Kaspersen & Ytterhaug, 2020). Using this instrument, research has shown that mathematical identity can be influenced by one's sense of belonging within communities of practice, past and present experiences, teachers, peer labeling, and parents (Gweshe & Brodie, 2019; Kafoussi et al., 2020). This is why social context is important when discussing mathematical identity. A student having a negative experience in a mathematics class will carry that memory to future mathematics coursework.

Kaspersen and Ytterhaug (2020) examined whether researchers can use the same instrument for measuring and comparing the mathematical identities of lower secondary school students and those of university students in science, technology, engineering, and mathematics (STEM). The results indicated that the same instrument could measure mathematical identity in STEM contexts and lower secondary school contexts. Also, evidence is provided that the instrument is unchanging.

4.1.5. Data Analysis

In qualitative research, data analysis is a concurrent process with data collection. Ary et al. (2019) describe data analysis as an ongoing process that shapes and refines research design. The observer must take thorough notes and continually analyze and categorize data in search of recurring themes or codes. To aid in transcribing and coding the data collected from interviews, focus group transcripts, observations, and memos, Microsoft Word was utilized to transcribe and code all data sources. For analysis purposes, Microsoft Word software was utilized because of its user friendliness and availability. For hardware, the use of voice and video recorders was necessary. In this section a brief overview of the data analysis is presented and then a more comprehensive description follows in the next chapter.

There are three levels in the analysis process including open, focused, and axial coding (Ravitch & Riggan, 2017). The first level is initial or open coding. This preliminary level allows the researcher to focus and label substantial amounts of raw data. In this phase, the research examined the tentative grandiose ideas that recurred by using the interviewee's words to chunk data initially. In the second phase of focused coding, recurring labels and ideas were generated by looking at similar words used by the participants and common perspectives and behaviors. In the third phase of axial coding, connections among the codes were uncovered which allowed the creation of labels that described relationships among the open codes created in phase one. Through this process themes emerged to describe the experiences and perceptions as mathematics learners for students of color tracked in an Algebra 1 course. However, the research also remained open to potential emergent codes and themes as there was little research on the influence of tracking on mathematical identity for students of color. Open coding was utilized after the first round of initial interviews. Following this process, questions were developed for the semi-structured focus group interview and subsequent final follow-up interviews. Table 2 summarizes coded comments and their characterizations during the open coding process.

An advantage to qualitative research is in the validity of studies. The authenticity of the findings relies on the merits of the researcher and participants. Ensuring validity in qualitative research is accomplished by implementing specific procedures in the research design including rich descriptions, member checking, and triangulation (Creswell & Creswell, 2018). Credibility, transferability, dependability, and confirmability were utilized for assessing the research quality and rigor of this case study.

Table 2

Coded comments and their characterizations determined during the open coding process

Coded Comment	Characterization
Placement	• My math teacher from last year put me in this class. • I would have liked to take Honors in 8th grade but was not given the opportunity. • I think I could have handled regular Algebra 1, but my teacher put me in Foundations.
Rigor	• Honors is harder than support. • Those problems are just at a higher level. • The rigor in the support classes is lower than honors. • Teachers in support classes spoon feed the students with like basic math concepts.
Pacing	• We learn the same stuff. Support just learns everything over a year instead of a semester. • The Honors class goes fast.
Motivation	• It means a lot to me for my teacher to let me know I am doing a good job. • It makes me happy that my teacher acknowledges me on doing my math work.
Perceptions	• Support is a class that helps people that need it more. • Honors classes have better opportunities. • Being in honors makes me feel a lot more confident in my math ability. • Support kids could be good at math, they just need a little bit more help along the way. • Honors kids are smart. Support kids are becoming smart.
Classroom Procedures	• I would like to work in a group. • I would like more group work because somebody in your group might need help or someone in your group can help you if you need help. • She teaches us everything we must do. We take down notes and work out problems.
Mathematical Identity	• I am ok, but I just need more help in math. • I catch on to math fast. • Math is something simple for me personally. • I am not the smartest in the world. I just want to do the work, learn, and keep going. • I struggle with math. • I am usually the first one finished with my work, so I get to help other students who might be struggling.

To ensure the credibility of the study, member checking was utilized by returning the interviews to each participant for checking and confirming the results (Birt et al, 2016). This was also accomplished with the third round of follow-up interviews where teachers and students were asked to share their thoughts on statements made in the focus group interview regarding tracking and mathematical identity. Students and teachers were also asked to clarify comments made in the initial round of interviews. Triangulation strategies were also utilized to ensure confirmability and the convergence of information from various sources (Santos et al., 2020). Data collected from the initial interviews were supported from notes made from the classroom observations, which were confirmed in discussions from the focus group and solidified in conclusions from the follow-up interviews. Purposeful sampling of the eight student participants using a random number generator ensured the transferability of the case study. Finally, dependability was ensured through the creation of an audit trail. A transparent description of the research steps taken from the start of the research project to the development and reporting of the findings was maintained throughout this qualitative case study.

A case study gleans understanding of participants' impressions within their culture or environment. Continual assessments of the research's quality and rigor were conducted throughout the study. Using multiple methods of data collection ensured the triangulation of the data to build consistent findings and themes. Validity was secured by allowing participants to check the researcher's findings and discuss emerging ideas within the established focus group. This allowed participants to argue misrepresentation or contradict the perceptions of the interviewer. The use of rich descriptions was also used to share insights and acknowledge any biases possessed when approaching the study. While the goal throughout the research was to uphold an elevated level of rigor and quality, the following section addresses some of the limitations unique to the case under study.

4.2. Limitations

Limitations represent areas of weakness within a research design which could potentially influence the validity, outcomes, and conclusion of the research. For example, the researchers' own subjective feelings, or researcher bias, may influence the case study. As with any qualitative case study, the research context and participants are difficult to replicate. As a result, we can never be sure if the case study investigated is representative of the wider body of similar instances because a case study deals with only one person/event/group. This means the conclusions drawn from a particular case may not be transferable to other settings. Finally, a lot depends on the interpretation the researcher places on the information acquired because case studies are based on the analysis of qualitative data. For this case study, the research identified two such limitations.

4.2.1. COVID-19

The COVID-19 pandemic crippled the country and caused major disruption for the public educational systems worldwide. Teachers and students at Bearcat High School and all the other schools in the system had to transition to virtual learning during the spring of 2020. While the fall of 2020 did bring a return to in person learning, students and teachers have had to adjust to virtual, traditional, and hybrid teaching on numerous occasions since then. There are also gaps of learning for students due to the past few years of disrupted school. Additionally, the ongoing implications of the COVID-19 pandemic is a loss of intrinsic and extrinsic motivation for academic achievement.

4.2.2. Administration and Leadership

In addition to the influence of the COVID-19 pandemic, the entire school system experienced major changes in administration and leadership. At the beginning of the 2021–2022 academic school year, the high school principal contracted the COVID-19 virus and passed away from complications due to the virus. Until April 2022, the high school had no principal and the morale of the faculty and staff declined. In addition, the Assistant Superintendent of Curriculum of Instruction announced her resignation from the school system to pursue a job at the State Board of Education. Instead of replacing this position, the school system decided to rebrand the title to Director of Curriculum of Instruction and had not announced a replacement at the time of the case study. Finally, two assistant principals also resigned their positions from the high school. While one retired, the other took the position of Headmaster at the private school in Division County. With so much change within the administration and questions regarding the future of leadership at the high school, teacher and student morale had declined.

4.3. Conclusion

The role of context in mathematical cognition and learning continues to be relevant within mathematics education research. In addition to understanding how people learn, more research must include how the role of culture and race influences students' mathematical engagement and learning. Regarding the school and district in this case study, the area is a rural and low-income community. More research needs to address how the sociopolitical contexts of schooling interact with how students experience and learn mathematics, how our students view mathematics as relevant to their futures, and how do policies affect how educators' structure and teach mathematics.

With a setting at a low socio-economic and rural high school, this case study explored the interplay between tracking and mathematics identity, with a special focus on rural African American and Hispanic learners. A qualitative case study guided by two research questions was utilized to develop a deeper understanding of the case under study. Interviews, a focus group interview, observations, and a survey were instrumental for the data collection to understand the context of learning mathematics in a tracked Algebra 1 class at Bearcat High School and allow students to describe their mathematical identity. The research was vested in seeing how tracking students in an Algebra 1 course influenced their mathematics identities. These students were placed in various levels of the same mathematics course based on performance on a standardized assessment. Students would have a much richer and fruitful experience if this form of segregation stopped. Students should take a course in Algebra 1, and teachers should expose all students to the same level of rigor. Students should learn the same set of standards no matter when they take the Algebra 1 class. It was important that a qualitative case study be conducted in a rural high school in Georgia to help reach a deep understanding of the influence of tracking on students of color and mathematical identity. Specifically, it was vital to examine a case where students of color are tracked in varying levels of an Algebra 1 course, and the potential this policy has had on students' mathematical identity. By utilizing a qualitative case study approach, this study provided a deeper understanding of how tracking influences Black and Brown students' perceptions of themselves as learners of mathematics. Moving forward to the next chapter, the findings and implications of the research are further explored.

Chapter 5 – Findings

This chapter contains the findings from data collected in this case study. It includes the data descriptions and data analysis. The data analysis includes results for the total sample and initial research questions. The research questions guiding this case study include:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

This chapter's organization will begin with a brief description of the case under study, detailed description of the participants, followed by a summary of the collected data. Next, the chapter will delve into the data analysis by detailing coding techniques and utilizing specific quotes. Additionally, a summary and an analysis of the results will follow. Finally, the chapter will conclude with a bridge to chapter six.

5.1. Collection of Data

The sample for this case study consisted of eight students, two teachers, one guidance counselor, one math coach, one parent, and one assistant principal from a Title 1 rural public high school in southwest Georgia. The high school has an enrollment of 1,597 with 100% of the students economically disadvantaged. Of the eight students in this study, two were in a Foundations of Algebra course, two were in an Algebra 1 Support course, two were in a regular Algebra 1 course, and two were in an Algebra 1 Honors course. The two teachers both taught Algebra 1, with one teaching the Algebra 1 Honors course and the other teaching Algebra 1 Support. For all participants, an initial interview took place with students responding to a Mathematical Depth Instrument to measure each of their mathematical identities. A focus group interview for all participants followed, and a final interview for all students and both teachers concluded the data collection.

5.1.1. Bearcat High School

Classes for the newly constructed district high school began in the 2009–2010 academic school year. Situated approximately five miles from the previous facility, the city limits expanded to include the 150-acre campus and provide municipal services to the building. The grandeur of the facility is highlighted by a central two-story atrium with massive Doric columns gracing the main entrance. First-year students might experience intimidation due to the grand size of the campus, but one can simply look above the hallways in the atrium and quickly determine each of the four wings representing a different academic pathway, with the Mathematics Department being on the second floor of the Social Studies wing.

Due to state curriculum changes, mathematics education at the high school has seen many adjustments over the past fifteen years with the transitions from Georgia's Quality Core Curriculum to Integrated Mathematics to the anticipated modifications of the current Georgia Standards of Excellence for the 2023–2024 academic school year. Over that time, the high school mathematics department has instituted many policies to advance mathematics learning and achievement for all students. Most notably was the creation of common summative assessments for each math course at the high school. This improved consistency between teachers and allowed for deep discussions on assessment analytics and differentiated instruction. Another positive policy was the creation of the retest policy. Originally designed to offer students a retest opportunity on each summative assessment, the policy shifted to only at the midterm and final, to its current state of the final examination being a test replacement opportunity for the lowest test grade a student earned in the semester. In addition to this policy, the mathematics department also offered Saturday School, a time for students to receive extra help each Saturday during a four–hour window before lunch from a certified mathematics teacher, and a tutoring program during school hours by a retired certified mathematics teacher. Due to declining participation, these programs ended in 2018. Finally, the partnership created between the high school and local community and technical college has given many students the opportunity of taking College Algebra while still in high school.

Although there have been multiple positive changes for mathematics learning at high school, there remains much room for improvement. While the high school offers Advanced Placement, honors, and college coursework, the registrar utilizes a non-weighted Grade Point Average (GPA) to calculate class ranks. This has had a negative effect on students taking rigorous coursework at high school. To help maintain a higher GPA, students typically will forgo enrolling in Advanced Placement courses. The tracked system at the high school, which includes Support, Regular, and Honors classes for Algebra 1 and Geometry have also exacerbated this as students will choose to leave an honors class for an easier, lower-leveled course. While the high school has celebrated increased high school graduation rates over the past ten years and some of the highest Algebra 1 End of Course Assessment scores in the region, the number of students earning the prestigious Zell Miller Scholarship, early admittance to Division 1 schools, and enrollment in rigorous college-preparatory mathematics has experienced a sharp decline over the past ten years.

All these policies directly impacted on the student’s experience of learning mathematics at Bearcat High School. The extent of this impact is unknown. Therefore, it is relevant and necessary to study the influence tracking has had on the mathematical identity of students at Bearcat High School, specifically students of color in an Algebra 1 class. The two research questions guiding this case study are to provide a deeper understanding of the context of learning at Bearcat High School and the influence the tracked system has had on its students' mathematical identity. To accomplish this, the next section will explore a detailed description of each sample participant in the case study.

5.1.2. Student Participants

Selection of the eight students through a simple random sample helped minimize biases while giving each member of the population an equal probability of being chosen. Working with the school's registrar, the researcher populated a list of all ninth-grade students currently in math class who identified as a student of color. The list was further segregated into each of the four possible Algebra 1 courses, which include Foundations of Algebra, Algebra 1 Support, (regular) Algebra 1, and Algebra 1 Honors. Assigning each student with a number and utilizing a random number generator to select numbers at random, the researcher randomly selected three students from each Algebra course, with two students being the initial selections and allowing for one additional backup. None of the backups were required to complete the data collection in this case study. Table 3 provides a brief overview of each student participant.

5.1.2.1. Basketball mentor. A first-year student at the high school, this student identified as an African American male. Average in stature, this student's style includes blue jeans, a neutral t-shirt, and a pair of sneakers. His hair is trimmed and kept short. While he speaks in soft whispers, the quality of his conversations and thoughts are heavy and deep. Although he finds being at high school a cool new endeavor, he is also cautious in no longer being the big dog on campus but instead being the little dog.

As a student in the Foundations of Algebra course, he believes the class helps students who might need more time in learning mathematics. He attests to his own need for extra help learning mathematics, even at the middle school level. Even with his own struggles, he affirms anyone can be good at math. It is simply a matter of how much effort and time they put into learning mathematics. To accomplish this, a student must be responsible and want to do the work. He verbalizes his self-reported limitations by not wanting to take an Algebra 1 Honors class as he believes he would be lost. He supports the need for collaborative group work and the importance of his teachers letting him know he is doing an excellent job in the class because he wants to know that he is doing good.

Table 3

Student Demographics

Algebra class	Title	Demographics	Description
Foundations	Basketball mentor	African American Male 15 years old	Basketball Coach Soft spoken, mentor
Foundations	Teacher's aide	African American Female 14 years old	Teacher's Assistant Extrovert, cheerful, festive
Algebra 1 Support	Caregiver	Hispanic Male 14 years old	Sibling Caregiver Quiet, reserved
Algebra 1 Support	Team player	African American Male 15 years old	Phone a Friend Gesticulate, team player
Algebra 1	Opinionated student	African American Female 14 years old	Favoritism in the Classroom Opinionated, outspoken
Algebra 1	Cheerleader	African American Female 14 years old	Basketball Cheerleader Positive, self-confident
Algebra 1 Honors	Band member	African American Female 13 years old	Marching Band Introvert, hard-working
Algebra 1 Honors	Soccer star	Hispanic Male 14 years old	Soccer Star Independent, aims for success

Of the four classes currently taking at the high school, his favorite is physical education. Not only is this a fun time to work on his basketball skills, but it allows him to move around and burn off some energy. With his love for sports, it is no surprise he aspires to one day be a coach, specifically any career involving basketball. Although he is not involved in extracurricular activities at the high school, he gives back to his community by taking time outside of school to help mentor younger children in his neighborhood on the basics of basketball. He not only enjoys doing this to keep his skill level high, but because he knows the importance of being a positive role model for others.

5.1.2.2. Teacher's aide. A first-year student at the high school, this student identified as an African American female. Her voice is as strong as her personality. With long braids below her shoulders, a pink blouse, and colorful accessories, her fashion style is complimented by her huge and welcoming smile. Although this is only her first semester at high school, she has already established her favorite class to be musical theater. With a personality as big as her imagination and passion for helping and working with others, she excels at the arts.

This student does not view math class as an enemy. Instead, she feels her first-year math class is not a challenge. She attests most of the topics she is learning in the Foundations of Algebra course are the same as she studied last year in middle school. This view has not always been the case in previous years. She recalls a point in middle school where she did not like math, but now she is at a point where she does. She credits this change in attitude to her putting her mind to learning math and working hard to get her math work done.

She tries her best to stay positive and help other people in class. Since she is usually the first one done with her work, she often steps in to help her classmates who are struggling because she knows this is helpful to the teacher. It makes her happy when her teacher acknowledges she is doing her math and helping her classmates. She is also a supporter of group work because it allows everyone in the group the opportunity to ask for or give help to others.

She is quick to say she did not get to choose being in the Foundations of Algebra course. Her middle school math teacher put her in the class. She would have liked the chance to take a support class instead of the Foundations course because she knows she can advance to the next level. Additionally, students taking honors math courses have better opportunities to get better at math, be on top of everything, and learn new stuff. She would like the same opportunities because she knows that if she puts her mind to something in math, she can get it done.

5.1.2.3. Caregiver. A first-year student at the high school, this student identified as a Hispanic male. With dark black hair, parted to the left, this student is comfortable in a simple pair of shorts and a plain white t-shirt. With a slight grin on his face, he speaks in a quiet and soft tone. Of his four courses this semester at the high school, his favorite is communication skills. Having emigrated with his parents from Mexico, he has had to make major strides in learning the English language for his coursework. Learning how to communicate in English is not only a necessity for his academics but also a necessity in being a translator for his parents who only speak Spanish. Unfortunately, this student does not participate in any extracurricular activities at the high school. This is due to him having to quickly go home after school to help look after his younger siblings. He ensures each of them completes their homework assignments and has a proper meal before bed, while their parents work in the fields at the farm.

Being an English learner has not been his only struggle throughout school. Sometimes, he is forgetful in the methods of mathematics. Although he likes learning new stuff in math class, he is also vocal on his limitations. Given the option, he prefers to stay on an Algebra 1 Support course where he believes he gets the extra time and support he needs to do mathematics. The name of Support makes him feel like he is getting "help on like new stuff." In contrast, the name of Honors makes him feel "like harder things." For him, the Support class is not too fast and not too slow. When the class is too fast, he and others get left behind.

He would prefer his teacher to be more proactive in telling him when he was doing the math correctly in class as this would help build his confidence. He would also appreciate the ability for him and other classmates to work math problems on the board during class. Watching other students being successful at math is a good thing to him because it reassures him that he is not the only one doing the problem. Instead, we are all doing it together.

5.1.2.4. Team player. A first-year student at the high school, this student identified as an African American male. With a low taper faded haircut, average stature, blue jeans, Nike t-shirt, and friendly disposition, this student is casual not only in his style but also in his personality. While he likes to talk with his hands, he sometimes gets excited in his conversations which causes him to talk faster. He feels good being a first-year student at the high school.gh school. For him, some classes such as health and social studies are easy. However, some things at high school, such as algebra class, are not always easy.

His Algebra Support class is unique in that there are two teachers for the course. He finds this helpful because one of those teachers can work with him specifically. While he perceives the Honors Algebra class as being “smart,” his perception of students in the Algebra 1 Support class as “becoming smart.” He is quick to assert his proficiency in mathematics and has his hopes set on taking a Regular level math course next school year. For him, the Honors class just sounds too hard. As a learner of math, this student knows some things in algebra, but is also still learning things as he goes on. He does not like to give up. Instead, he wants to keep striving and see where he can go and what he can accomplish. But he is also willing to ask for help when he needs it.

He finds working with his other classmates to be the best remedy for times he needs help. If he gets stuck on a problem, he can work with his group to do the math. Consequently, when others of his group members struggle, he is sometimes able to provide a little guidance and direction to help them. He also enjoys his math class because both of his teachers have created an environment where students not only hear verbal praises from their teachers, but students are also allowed to share their work on the board with the class. This collaboration extends beyond the classroom, as he also calls his classmates in the evenings to discuss homework problems. Whereas one of his friends in class might not understand how to approach the problem, he has a desire to help the student better understand and produce a solution.

5.1.2.5. Opinionated student. A first-year student at the high school, this student identified as an African American female. Petite in stature, the glow of her smile is complemented by the glasses she wears. Blue jeans, a colorful t-shirt, and a pair of fresh Jordans complete her unique style. She is not shy to voice her opinions or concerns with the tone of her voice. She has referenced her struggle with mathematics on many occasions; however, she is also quick to reach out for help when she is struggling.

Of the four classes she is taking this semester at Bearcat High School, her favorite class so far is technology. During this class, she can be creative and build things with her hands. For extracurricular activities, this student is a member of the Book Club. This fuels her passion for reading and allows her to use her voice while participating in group discussions on themes found in various literary works.

Although she has struggled in previous math courses, she looks forward to the opportunity to redefine herself with the fresh start at the high school. Struggles in her current math course are present, as she states the introduction of letters inside of the math is confusing. As a result, sometimes she does give up when she gets lost. To her, struggling with math problems is not fun. Instead, it is very frustrating. To offset this, she is a firm supporter in collaboration and group work. The value of working with others and learning from others means a great deal to her. Sometimes, she might forget how to do steps for a math problem. Having someone in your group who might be a little better at it helps her to persevere when she wants to give up. She finds this beneficial, especially as the teacher cannot always help each student during class. She also voices a concern for suggested favoritism within the classroom by affirming her opinion on how teachers can favor helping some students while neglecting others. Although hearing verbal praises from her teacher does not influence how she learns mathematics, watching her classmates being successful and trying to solve math problems is especially important to her.

5.1.2.6. Cheerleader. As a first-year student at the high school, this student identified as an African American female. In contrast to her petite stature, her personality is larger than life. With an ever-present smile on her face, she is quick to laugh and chuckle while maintaining a positive disposition. Her hair is draped in braids well below her shoulders and bright pink, blue jeans with a dark blouse compliment her unique fashion style. She also lends her voice every game night as a member of the high school basketball cheerleaders.

Her experience at the high school has been positive for the beginning of the first semester. She appreciates the new concepts and challenges she is experiencing as a first-year student. She is aware of the different tracked classes of Algebra at the high school and believes the support classes are for students struggling a little and need the extra help to get better. Some people struggle with math, and some struggle, and no one is born good at it. It depends on if the student focuses, pays attention, and communicates with the teacher. While she does profess to struggle some in Algebra, she does communicate with others if she needs help. Additionally, she will also reach out to her classmates who may also be struggling in math class. Working together is important to her. She loves her math class because the people in her class help her out, especially when the teacher is busy doing things and working with other students. Additionally, it is especially important for her to see other classmates having problems on the board because they might do it one way and she might do it another way. This helps her learn multiple ways of approaching the same problem, and in the end, they all produce the same answer.

She is also quick to say she did not get to pick to be in her tracked math course. Instead, her math teacher from eighth grade placed her in the course. She would have liked to be challenged in an honors class and even wants to be. By working harder, paying attention more in class, and getting her grades up, she will be able to have that opportunity. She likes problems and other things that are challenging for her, and she tries to work harder on the problems and things on her own.

5.1.2.7. Band member. As a first-year student at the high school, this student identified as an African American female. Petite in stature, she enjoys having her braids wrapped in a bun except two single strands, one on each side of her face. With a grin on her face, her voice is quiet and soft spoken, on the verge of being a whisper. A vibrant yellow blouse with frills and a pair of khaki slacks completes her fashion style. Although this is her first year at the high school, it is not her first high school experience. As a member of the high school marching band for one year in middle school, she has been able to experience the social aspect of high school for some time. It is no wonder why her favorite class is the advanced band where she plays the clarinet in the symphonic band. Her time after school is spent with rehearsals and practice, with the hopes of being a section leader one day.

Personally, math is something simple for her. She likes being challenged and trying to face different math problems that are harder. She believes she can catch on to math easily and does not think it is important or necessary for her math teachers to verbally praise her for answering questions in class. On a scale from one to five, she rates the rigor of the Honors class as being a four and the rigor of a support class as being a two or three. She does not believe that teachers should make the support classes harder for students because that might sometimes overwhelm the students and cause them to break down and not want to work as well. Support kids could be good at math, they just need a little bit more help along the way. Some people just learn a little slower than others, and she believes that is ok. Saying support classes need more help is not a terrible thing, it is just because we are at a different pace from each other.

She is aware that the algebra support classes learn the same content and take the same End of Course Assessment at the end of the course, however, support classes are given a year to complete the course whereas the average and honors level classes will complete the course in a semester. Algebra support is simply a class for people who need more help. In contrast, being in honors makes her feel a lot more confident in her math ability since learning math is something for her.

5.1.2.8. Soccer star. As a first-year student at the high school, this student identified as a Hispanic male. With dark black hair medium in length, a big smile with braces, blue jeans, a simple t-shirt, and a sports jacket, one could quickly mix up this student with his twin brother who shares the same first and last name. But beyond the physical characteristics, the similarities end. His speech is very precise, and he makes sure to choose the words he wishes to use very carefully. He attributes this to having to learn to be a translator for his non-English speaking parents. While he was born in the United States, his parents emigrated from Mexico. On many occasions, he has acted as a mediator for his parents. After school hours, you can find him on the soccer field. This is his favorite pastime. He is a member of the high school varsity soccer team, with the hopes of earning a state title one day.

He has an extraordinarily strong view that people are not born good at math. To him, success at learning mathematics depends on the type of person, more specifically if the person possesses a desire to learn. For him, he is the type of person who is really engaged and likes to learn new things. He does not profess to be the smartest person in the world, and he knows that to be successful, he must keep working, learning, and going. He does not focus on other people, and where they are. Instead, he focuses on himself and what he must do. Group work does not interest him. Instead, he likes working by himself so he can have a lot more space and time. He also feels privileged to be in the Honors class and proud of himself. Although he does not know a lot about the Algebra 1 Support class, he thinks the class does allow for more time to practice the concepts and skills. While the support classes offer the same amount of rigor as the regular classes, he believes the teacher would have to kind of spoon feed those students basic math concepts.

5.1.3. Community Members

In addition to the eight student participants, the case study included six community members. The selection of the two teachers through a convenience sample allowed a representation from all four Algebra 1 courses offered at the high school. One of the teachers had an Algebra 1 Honors class and a regular Algebra 1 class. The other teacher had a Foundations of Algebra class and an Algebra 1 Support class. Both teachers are seasoned educators and have been in the classroom for at least six years. The researcher included the guidance counselor and administrator in the study as they were both a part of the ninth-grade cohort. Adding the math coach allowed for a perspective across all grade levels throughout the school district. Finally, the researcher asked the senior guidance counselor to serve as the voice of a parent for the study. She provided a unique perspective on the tracked system as her three children had all experienced the Algebra 1 tracked system at Bearcat High School.

5.1.3.1. Track coach. Although her first year teaching at the high school, this teacher has spent 17 years in the classroom within the school district. She identified as a Caucasian female and has a close connection to the high school as a former graduate herself. Having taught at the elementary, middle, and secondary levels with an undergraduate degree in early childhood education and a master's degree in middle grades education, this teacher brings a wide range of experience to high school. In addition to her teaching duties and responsibilities, she also coaches cross country in her spare time. Tall with long hair below her shoulders, she has the personality of an experienced sage with a deep and heavy southern accent. She loves hunting and anything to do with the outdoors.

Although her tenure at the high school is new, she thinks the mathematics department does an adequate job of providing rigorous math courses to all students. As a supporter of the tracked system at high school, she concurs with the policy of placing students in courses in which they can function and learn but are at their personal levels. It is not that kids who are not doing well in math cannot do well in math, it is that you have a group of learners that need a whole lot more practice than others. These students think they cannot get it, that they are not smart enough, but they just need more practice than they would get in the regular math class.

Unfortunately, she also believes there are some circumstances in which a student is misplaced within the tracked system and believes there should be more clarity and transparency on the different math courses offered at the high school. When the middle school chose the first group of kids to come to the high school from the eighth grade and take Algebra 1, it was not made available to all. This is regrettable, especially for other kids who could have very much handled the course but were not afforded the opportunity to participate.

5.1.3.2. Youth leader. Having taught five years at the high school and six years at the elementary and middle school level, this teacher has a combined experience of 11 years in the classroom. She identified as a Caucasian female. Holding an undergraduate degree in early childhood education, a master's degree in education with a certificate in teacher leadership, and a specialist degree in education with an emphasis on instructional technology, this teacher brings a wealth of knowledge to the high school. She is gifted certified and currently working on her Tier 1 certification. She has also been the Algebra 1 team's course leader for four years. Petite in stature with long dark hair below her shoulders, she always possesses a warm smile to compliment her bubbly personality. A perfectionist in and outside of the classroom, she aims to stay organized and design lesson plans to the exact day. She strives to develop a meaningful relationship with each of her students, and often refers to them as her "babies." In addition to her teaching responsibilities, she also is a sponsor of the high school's Fellowship of Christian Athletes organization. As a volunteer for the youth at one of the local churches, she has a connection with many of her students beyond just being their math teacher.

Pacing and trying to cover all the standards before the state mandated End of Course Examination continually influences her classroom work. As she attests, there are so many concepts these students must learn in a day and a teacher cannot always expect a kid to go to the board when they have just been introduced to the concept. To address this, tracking students into groups of similar ability helps. She believes a group of support kids that have been adequately grouped together do well together. She also believes some students are misplaced. To help alleviate this, providing more clarity on the different math courses offered would be beneficial for students and parents especially when they start choosing some of the higher math courses.

5.1.3.3. Math coach. Serving in the role as math coach for the first year, this individual identified as a Caucasian female. With over 20 years of experience at the elementary, middle, and secondary levels, she has a unique perspective of seeing mathematics education from a vertical standpoint. She holds an undergraduate degree in middle grades education, a Master's in education, and a Specialist in education with an emphasis in teacher leadership. Petite in stature with short wispy hair, she always enters the room with a commanding presence. In her spare time, she also serves as the high school's tennis coach where she has led the boys and girls to many regional and state titles over her ten–year career.

She fundamentally believes there is a kink in the system at the high school. If the exact same materials are being used, then a teacher is not differentiating the content and if the same methods are being used, the teacher is not differentiating instruction, it sounds like the teacher is just teaching at an average level and not at a more rigorous level for Honors. Rigorous does not necessarily mean harder to her, but rigorous does mean incorporating different modalities within the lesson and causing students to think and apply what they have learned. She also thinks a student can sometimes learn more from another student than from a teacher. It is all about the environment created in the classroom.

Additionally, she notes that tracking students in math did not start at Bearcat High School. As early as elementary school here in the county, students are tracked into support classes. And some of the students do need support and this is why the support class exists. She believes that academically, tracking makes sense and is the best option, but she also thinks for social, emotional, well-being, tracking sometimes can be detrimental to some students.

Unfortunately, educators usually do not push AP or college prep math courses to lower-level students, and there is room for improvement in this area.

5.1.3.4. Guidance counselor. With 20 years in public education, this individual identified as a Caucasian female. She has served as a guidance counselor at the high school for six years and has also taught at the elementary school level. She has an undergraduate degree in early childhood education and a Master's in education. She currently serves as the ninth-grade cohort guidance counselor and will follow this group of students through their high school career until they graduate in 2026. Of average stature with long blond hair, one can instantly recognize her elementary school background with the calmness in her voice. An outdoor person, she enjoys fishing and being a support of the high school's Bass Cats club.

Her experience has shown her that the different tracks of Algebra courses at the high school have a negative connotation. Students taking a foundation class do not think they are smart enough or they think that they are in what they call "the slow class." There are always going to be some negative connotations with support courses from some people; however, as a counselor it is necessary to explain the reasoning behind placement regarding time and how support is spread over the year versus being crammed into one semester at a fast pace. She is a strong proponent for the tracking system because it enables teachers to find a student's strength and move them forward based on that strength or students' ability.

5.1.3.5. Administrator. Although in his first year as zone administrator for the mathematics department at the high school, this individual has spent over 25 years in education. He identified as a Caucasian male. He has served various roles in the school system as a business teacher, baseball coach, and assistant principal. He holds an undergraduate degree in education and a master's degree in educational leadership. Tall and lengthy with a short crew cut, he brings the expertise of a baseball coach to the classroom to do whatever he can to help his teachers and department be the winning team.

He agrees that low level math classes alone give the stereotype that these are low struggling students and believes support and foundations classes may be taught at a slower pace for each student to grasp the foundations with proficiency. Not all students need a support class, but those who need it should take it before Algebra 1. From a teaching standpoint, to be able to reach that group of students with similar abilities would be an advantage. A student in support will be getting some extra support while taking that algebra course. He relies heavily on the expertise of the teachers to place those kids at the appropriate level. And, that kid's performance in each math class from year to year should really determine where that kid is placed. He has a firm belief that more communication and transparency with incoming ninth graders as to what path they will take as it relates to post high school will help determine what academic classes, and specifically mathematics, to take.

5.1.3.6. Parent. Not only a parent of three students in the school system, this individual also serves as a guidance counselor at the high school. This individual identified as a Caucasian female. She has spent over 25 years in education and holds degrees in early childhood education and counseling. Tall with short blond hair, she sometimes looks over her glasses when in deep conversation. Softly speaking, she takes time to consider how she responds when asked a question. There is a sweet nurturing tone in her voice most probably influenced by her many years at the elementary school level. Although she is not a club sponsor at the high school, she keeps up to date on all things related to college admissions and passes the information to all students at the high school.

In her firsthand experiences, her child was nervous about the honors algebra course. She said that her child had to work at it. Her child had to put study time into it and did not get mathematics as easily as some of her smarter friends. There was even discussion about moving her from the honors class to a regular algebra class. They eventually decided against it. While she is aware placement of students in the different tracked algebra courses is based on teacher recommendations, she also knows students can move up or down depending on their success and parent recommendations. The parents have the final say. So, if a kid can perform well, that student could move up within the tracked system at the high school.

5.2. Analysis of Data

The following section provides a presentation and analysis of the data collected. Analyzing across all data types, results are presented in a particular order in response to each of the research questions. A comprehensive codebook is also included (See Appendix F). A response to Research Question One follows, namely the context of learning mathematics at Bearcat High School. The chapter concludes with a response to Research Question Two, the influence of the tracked system on the mathematical identity of students at Bearcat High School.

5.2.1. Initial Interviews

Initial interviews with all participants took place during the fall semester. This included eight students, two teachers, the math coach, assistant principal, first-year guidance counselor, and a parent. The initial interviews addressed Research Question One by allowing each participant to share their perceptions and opinions of the context of learning mathematics at Bearcat High School. Research Question Two was also addressed as students were asked to describe themselves as learners of mathematics and complete a survey measuring their mathematical identity. Five key themes emerged from the initial interviews about the context of learning mathematics at Bearcat High School. These were a lack of vicarious experiences, the pacing for honors and regular versus support was not equal, differences in rigor between honor and regular versus support, no voice in placement, and limited mobility.

Through the initial interviews, one theme which emerged involved the lack of vicarious experiences. Each student participant described the daily flow of the class period. One commonality between all responses was the use of technology, specifically the online graphing calculator Desmos. A few students noted a need for more vicarious experiences and opportunities to work together in groups in the classroom. Caregiver added, “it would be helpful to see my other classmates doing problems on the board so I’m not the only one like doing the problem so that like we’re all doing it together.” Teacher's aide agreed with the response, “group work would be better because it’s like you never know somebody in your group might need help, or like other people needed help, or like somebody that sees you need help.”

In addition to limited vicarious experiences, participants also highlighted the pacing of the classes. The theme which emerged was the inequality of pacing for honors and regular classes versus support. The students talked about honors and regular algebra being at a faster pace than support algebra. Opinionated student suggested, “taking a support class would be an advantage to some students because if someone is struggling in an honor class, they should have the ability to be removed then placed into a class more on their level because they go a lot slower than regular algebra.” The teachers and math coach explained why this pacing occurs. Youth leader said, “The support class is the average class stretched out over two semesters. The whole idea behind support is to slow the process down because there are a lot of standards to learn. Support kids have clearly shown that they struggle learning at the same speed as the other ones, which is why they are in support to being with. We have those that learn at different rates, and they can go deeper, faster, and they may understand how to get there faster." The guidance counselor, math coach, and administrator also agreed on the necessity for the pacing of support to be over two semesters versus one.

Having no voice in placement was another theme which surfaced from the initial interviews. Many students, Team player, Teacher's aide, Soccer star, and Cheerleader said their previous teacher placed them in their math course. Each of them said they would have liked to have picked to be in a different tracked class. Opinionated student was unaware there was an honors, support, and regular algebra course. Teacher's aide continued by saying, "I would have liked the opportunity to take a support class instead because I know that I know what I am capable of. Like, I know that if I really put my mind to something in math, I can get it done." Band member and Caregiver went on further and suggested some students are misplaced and pressed the need for more clarity on the different math courses offered at the high school. Track coach furthered the need for clarity in that, "when they chose the first group of kids to come over here from the eighth grade, it was not really made available to all. It was, they went on one standardized test. But I know that there were other kids that I had that when they were in the eight, they could very much have handled it, but they were not allowed." Math coach furthered the discussion, "we usually push non-AP or college prep math courses to our lower-level students. Last year, in a class of almost 300, we only had eight students who took precalculus. So, there's room for improvement."

Finally, some of the statements from the initial interviews referenced the theme of limited mobility within the tracked system at the high school. While Youth leader agreed on kids being misplaced, she argued that mobility still exists, and students can move up or down as needed throughout the varying levels of math courses. Team player noted limited mobility in that he did not know there was an option to be in a different algebra class. While Teacher's aide was optimistic, there are still more opportunities, and she must move to a different level math class next year. Track coach noted "one negative of the tracked system at BHS is some kids may feel stuck once they get in the track. But let us say they were in this level this year, I could see what they were doing, and so next year, I recommend them for average instead of the support class" Youth leader agreed that students do have the opportunity to change their trajectory and take advanced math courses, but "there's got to be a lot of self-determination and motivation."

The disparity of rigor between honors, regular, and support was also an important theme discussed by the participants. Caregiver noted, "when you are applying what the concepts are in order to solve the problem, that is rigorous." Beyond that, there was little agreement on the level of rigor in the tracked courses at the high school. Students not in the honors class noted the problems in the honors algebra class are harder than the problems they do in their classes. Team player argued that "Honors got different kind of problems than what algebra support has." While Soccer star disagreed, "the support classes allow for more time to practice the concepts and skills. The support classes offer the same amount of rigor as the regular classes." Band member concurred and cited her experience in her honors class, "my teacher shows us where some of the support classes are after we are done with our class, and they are just a little behind us. They are taught the same content and given the same EOC at the end of the course, however, support classes are just given a year to complete the course whereas the average and honors level classes will complete the course in a semester."

In addition to the student interviews, the argument of a disparity in rigor also emerged as a theme for the community members. As Youth leader states,

> *All three levels of Algebra 1 learn the same content, the difference with honors, it is at a higher rigor level. In honors algebra, we set up the problem, so it is not like walking through step by step as we do with support. And if you are trying to raise the rigor for the upper level, the lower level is overwhelming.*

Guidance counselor agreed of the existence of rigors for honors, “what teachers of Honors Algebra are trying to specifically make that course a bit harder than the regular course. If you do not, what is the point of having the name honors attached to it? There’s gotta be something different, something a bit more rigorous.” In contrast, Math coach disagreed and noted a kink in the system.

> *The only thing that differentiates honors from regular and support is the assessment. If the exact same materials are being used, then you are not differentiating the content and if the same methods are being used, and you are not differentiating instruction, it sounds like they are just doing an average. When I think about a course being rigorous, I think about the course being challenged and one that is going to push the limits of what a student already can do and expand upon their base of knowledge. Rigorous does not necessarily mean harder to me, but rigorous does mean that you are incorporating different modalities within the lesson and that you are really causing students to think and apply what they have learned. I think you can add that critical thinking into any course that you teach so that any course can be rigorous.*

The five key themes of rigor disparity, limited mobility, no voice in placement, unequal pacing, and lack of vicarious experiences emerged from the initial interviews conducted with all the participants. These themes were helpful in addressing Research Question One and better understanding the context of learning mathematics at Bearcat High School. Additionally, three key themes emerged from the interviews about students' mathematical identity. These included the perception of support students needing more time, a desire for social persuasion through verbal encouragement, and the uniqueness and complexity of each student's mathematical identity.

Many students discussed perceptions of the tracked algebra courses at the high school. Team player said, "the name of support makes me think of 'becoming smart.' Support does need more time, they are becoming smart, and some need help. The honors students also go through the problems the other kids do just at a different pace." Basketball mentor shared this opinion by saying "Foundations is a class that helps people with math help that need it more." Cheerleader agreed by suggesting support classes are for students struggling a little bit and want to get help and get a little better." Band member further elaborated on perceptions of the tracked courses by suggesting "being in honors makes me think that, like math is something simple for me personally, and it makes me feel a lot more confident in my math ability. I think an honors teacher would say that students in the class are good at math too. Support kids could be good at math, they just need a little bit more help along the way. Some people just learn a bit slower than others, and that is ok." Soccer star talked about his own personal perceptions of being in the honors class, "I feel really privileged to be in the honors class. I feel proud of myself. I really do not know anything about the Algebra 1 Support class. But I feel like the teacher would have to kind of spoon feed them like basic math concepts."

A desire for social persuasion through verbal encouragement was also a theme which surfaced from the initial interviews. All but two of the student participants, namely Band member and Soccer star, discussed the importance of their teacher letting them know they are doing an excellent job in class, watching other students being successful in the class, and working with other classmates if they got stuck on a problem. Teacher's aide went further to say "it makes me happy when my teacher acknowledges me on doing my math. When I am done with my work and my teacher sees other people not catching on to it fast, she lets me help them because I am usually the first person done with my work." Track coach agreed that:

> *Some teachers can give students a lot of small opportunities for success to build them up about math and so that the students think they can be a good math student. And when going around the room, if somebody gets something wrong, instead of saying, oh, that is wrong, we should say, could your partner help you get it right. By watching others, and if we see other people doing well, and even though they may have to struggle a bit, they kept on until they finished, well, then you put yourself in that position and you think, well if I see them doing that, then its ok to struggle sometimes too, and so that I still can get the right answer.*

Students articulated an array of responses unique to their own experiences. These individual experiences and comments helped develop a better understanding of each student's unique and complex mathematical identity and helped address Research Question Two. Caregiver commented, "sometimes I forget the methods in math class, so I'd like to stay in support, but that's okay because my friends help me out when I get stuck, and I enjoy helping my friends when they don't get it." Team player described himself as knowing some things in algebra but really struggling in the class. He does not like to give up right away, and strives to see what he can do, and asks for help whenever it is needed. Basketball mentor said he was ok at math but struggled a lot and needed more help. He also said people could be good at math if they put in the time. In contrast, Teacher's aide was at a point where she did not like math but now, she does like it, tries to stay positive, and helps others because she knows if she puts her mind to something, she can get it done. Band member commented, “I catch on to things very easily and math is extremely easy for me. I like being challenged, trying to face different math problems that are harder.” Soccer star described himself as really engaged, likes math, good at math, and just wants to keep on working and learning. Opinionated student said she was barely doing good in math class, and when she is struggling with a math problem, she gets frustrated, gives up, and does nothing. Cheerleader sometimes “struggles with math, but I communicate with others when I need help. If I can help my friends, I will help them. I like problems that are challenging for me, and I try to work harder on the problems and think for myself.”

Initial analysis of the first interviews using open coding revealed many themes including no voice in placement, limited mobility, rigor disparity, unequal pacing, a desire for social persuasion through verbal encouragement, perceptions of support students needing more time, lack of vicarious experiences, and the uniqueness and complexity of students’ mathematical identity. From these themes, a set of questions for the second round of interviews was created to be used in the focus group. Also, responses from the initial student interviews were used to create an observed mathematical identity for each student participant, as shown in Table 4.

Barba’s (2020) proposed six characterizations of mathematical identity including: superiority, authority/power, spectator, inferiority, instructor/solidarity, and relative indifference were utilized to assign each student with a unique mathematical identity. The four identities which surfaced through the initial interviews were superiority, inferiority, spectator, and instructor. Some of the students, such as Soccer star and Band member, displayed a superiority identity because they asserted their elevated proficiency in mathematics. While Opinionated student was more of a spectator by choosing to remain neutral and do nothing in any mathematical discourse or dialogue. Students such as Basketball mentor and Team player expressed low self-confidence or asserted their low ability in comparison to others and exhibited an inferiority observed identity. Finally, Cheerleader and Caregiver identified as an instructor due to their passion of encouraging learning in others.

Table 4

Observed Mathematical Identity

Title	Observed Identity	Justification
Basketball mentor	Inferiority	Struggles at math, needs a lot more help in math
Teacher's aide	Instructor	Likes math, stays positive, helps others
Caregiver	Instructor	Wants to stay in support, helps friends with they do not understand
Team player	Inferiority	Really struggling at math, does not like to give up, asks for help if needed
Opinionated student	Spectator	Barely doing good, frustrated, gives up, does nothing in math
Cheerleader	Instructor	Struggles with math, communicates when she needs helps, helps others when they need help
Band member	Superiority	Math is easy for me, catches on very easily, likes being challenged
Soccer star	Superiority	Really engaged, likes math, good at math, wants to keep learning

While Barba (2020) proposed six characterizations of mathematical identity, for this case study authority/power and relative indifference were not present. An individual with an authority/power identity would have asserted his or her superior proficiency in mathematics while simultaneously executing their authority to exert control over the narrative. While Soccer star and Band member did assert their proficiency in mathematics, they did not assert an authority or control of being number one. Therefore, it was most appropriate to label each of their identities as superiority over authority. In addition, an individual with an indifferent identity would be disinterested in engaging further in discourse. Opinionated student asserted being frustrated and giving up in math; however, she also stated she was doing good in the class and would help others if she could. Therefore, she did engage in some discussion and was not completely indifferent. Some of the students had hints of multiple identities also. Cheerleader professed her low ability in mathematics while also her desire to help her friends who were struggling. Overall, the more evident characterization which surfaced through comments during the interview took precedent and made the final determination in the classification of each student's observed mathematical identity.

Classifying each student with an observed mathematical identity was needed in addressing Research Question Two. The observed mathematical identities also provided a means for comparison with the self-reported mathematical identity generated from the student survey. A detailed description of the mathematical identity instrument follows in the next section.

5.2.2. Mathematical Identity Instrument

In addition to the initial interviews with all participants, each student was asked to complete a mathematical identity survey. From the previous table, an observed mathematical identity for each student was created from interviews describing themselves as learners of mathematics. It was also vital to have students' self-report their mathematical identity. To accomplish this, a mathematical identity instrument was included after the initial interviews. The survey consisted of questions regarding motivation, perseverance, making connections, and mathematical modeling. Dr. Elvind Kaspersen, the creator of the survey, used a Rasch Measurement. Technique with the raw scores to obtain the measures for each student. With these scores, a student with a higher measure would have a stronger mathematical identity than a person with a lower measure. Table 5 contains the converted measure of each student from the self-reported mathematical identity survey.

Table 5

Results of Mathematical Depth Instrument

Title	Score
Basketball mentor	-0.71
Teacher's aide	0.98
Caregiver	1.20
Team player	0.87
Opinionated student	0.66
Cheerleader	2.53
Band member	0.66
Soccer star	0.14

Having each student's self-reported mathematical identity allowed for a comparison with the student's observed mathematical identity. The lowest measure of self-reported mathematical identity was from a student in the lowest track algebra class and had an inferiority observed mathematical identity. The strongest measure of self-reported mathematical identity was not from a student in the honors track algebra class and did not have a superiority observed mathematical identity. Instead, this student identified as an instructor and was in a regular algebra class. Interestingly, the three students with the strongest self-reported mathematical identities had an instructor observed mathematical identity. The results from the self-reported and observed mathematical identities helped guide the questing in the follow-up interviews and will be further discussed in the chapter.

5.2.3. Classroom Observations

To help answer the two research questions guiding the study, classroom observations were necessary to develop a deeper understanding of the high school mathematics context and address Research Question One. Bandura's Social Cognitive Theory was the theoretical framework used to support this research study. One of the key components of Bandura's Social Cognitive Theory is self-efficacy, or the belief an individual has in his or her capacity to be successful. Two sources of self-efficacy include social persuasion and vicarious experiences. While the initial interviews were helpful and allowed for students to share their perceptions of the classroom, it was critical to also observe the learning environment for evidence of social persuasion and vicarious experiences. Social persuasion, or students receiving verbal encouragement from their peers or teachers, helps students overcome self-doubt and persuade students to believe they have the skills necessary to be successful. Observing vicarious experiences, when a student sees another student accomplish a task, was also vital. While both sources of self-efficacy help paint a better picture of the context of learning mathematics at Bearcat High School, the importance of self-efficacy is also deeply related to an individual's mathematical identity. Therefore, data collected from the classroom observations were also beneficial and relevant for responding to Research Question Two. To maintain a record of times social persuasion and vicarious experiences occurred in the classroom, a protocol sheet was developed to include detailed notes on the classroom procedures and structure of the class.

The first classroom observation was of Youth leader from 7:45 a.m. to 9:15 a.m. on 10/11/22. The course was Algebra 1 Honors with 28 students consisting of 12-eight graders and 16-ninth graders. Of the 28 students, 14 identified as students of color. At the beginning of the class period, all students took out their Chromebooks. The teacher instructed them to open two tabs, one for the lesson and the other for Desmos. The teacher passed out worksheets and instructed the class that they were going to review and learn two new concepts today. The teacher also told the students they would be practicing together and then dividing into partners or groups. The review was on the topic of Quadratic roots. The teacher asked the class, "how do we feel about yesterday's topic?" One student responded, "some concepts are super easy." Before officially starting the review, the teacher started a video recording app to record the lesson. The first 60 minutes of the class period was devoted to teacher led instruction. The teacher went over problems on the Analyzing Quadratic Graphs Worksheet which reviewed topics including axis of symmetry, vertex, maximum, minimum, extrema, domain, and range. The teacher would also stop and allow students to answer at will. She often responded, “do y’all agree with her?” Then, the teacher asked students to complete a few problems independently. The teacher gave students approximately five minutes to answer a question on characteristics of quadratic functions. Once time had expired, the teacher called on a volunteer to show their answers on the board. Next, the teacher introduced two new concepts to the class, intervals of increasing and decreasing and end behavior. The teacher instructed the class how to describe each of those characteristics. For the final portion of the teacher led instruction, the teacher distributed a worksheet on quadratic roots, specifically introducing the discriminant. The teacher showed students how to calculate the discriminant and how to interpret the value of the discriminant, namely the types of roots a quadratic would have based on the value of the discriminant. The final ten minutes of class, the teacher instructed the students to form partners with a group maximum of three. The students completed the worksheet while the teacher circled the room as a facilitator. The teacher could not review the answers to the worksheet as she had to dismiss the eighth-grade students to board the bus and go back to middle school.

During the first classroom observation, there were many key elements of the classroom experience which surfaced in response to Research Question One. The first major theme was teacher-led instruction. Except for the last ten minutes of class, instruction during the first classroom observation came from only one source, the teacher. No tasks promoting reasoning and problem solving were implemented. Instead, students worked on multiple problems of similar format on graphical characteristics of quadratic functions. While the teacher posed multiple questions to the class, responses from students were limited and did not facilitate meaningful mathematical discourse. Another major theme was the lack of social persuasion and vicarious experiences. Over the 90-minute class period, only one student worked out a problem on the board in front of the class. Also, there was limited verbal encouragement between students and the teacher. A final theme was the pacing of the class. The teacher covered three different concepts during the class period. While most of the time was devoted to graphical characteristics of quadratic functions, the teacher also introduced two new concepts and only allotted 15 minutes for covering the discriminant of a quadratic. Many students verbalized a desire to spend more time on the new concepts.

The second classroom observation was of Track coach from 9:20 a.m. to 10:50 a.m. on 10/11/22. The course was Algebra 1 Support with 13 students, of which seven identified as students of color. The first 15 minutes of class were devoted to a warmup problem. The teacher instructed students to solve the following systems of equations by graphing, $\{-3x + 3y = 4, -x + y = 3\}$. After ten minutes of time to allow students to work on the problem individually, the teacher led the class and went over how to arrive at the solution. The next five minutes of class was devoted to a homework check. The teacher circled the classroom and checked to ensure students had or had not completed their homework assignment for the previous night. Afterwards, the teacher spent the next 20 minutes of class on a systems of equations word problem worksheet. The teacher assigned the worksheet as individual work. The teacher assigned a problem to each row of students in the class. The teacher walked around and helped students individually. The teacher verbally reassured the students and spoke on the difficulty of these types of word problems. Next, the teacher spent 30 minutes going over each of the assigned problems. In one problem, a student suggested one method of arriving at a solution. The system of equations was $\{x - 2y = 7, x + y = 25\}$. The student suggested multiplying the bottom equation by 2. The teacher corrected the student and told him to multiply the top equation by –1. Finally, during the last 20 minutes of the class, the teacher showed the students how to solve each system of equations using the Desmos graphing calculator.

For the second classroom observation, there were also many similar themes presented in response to Research Question One. As in the first classroom observation, the second classroom observation revealed a learning environment with a teacher-led focus and limited social persuasion and vicarious experiences. Also, the pacing of the class emerged as a theme but not in the same manner as the Algebra 1 Honors class. Whereas the pace of the Algebra 1 Honors class was fast, the pace of the Algebra Support class moved at a glacial speed. Both classroom observations revealed two distinct characteristics of the mathematics learning environment at Bearcat High School. First, group work and collaboration are either scarce or non-existent in daily class procedures. Second, while students did experience some positive social persuasion of teacher to student, the number of vicarious experiences and social persuasion between student and student was limited in the classroom. Track coach gave six verbal affirmations during the support algebra class and Youth leader gave 13 verbal affirmations during the honors algebra class. Further clarification on these findings was needed for discussion during the focus group interview to better address Research Question One, and in the follow-up interviews with the students and teachers to respond to Research Question Two.

5.2.4. Focus Group

The second round of the interview process included a focus group, or town hall format. All participants met in a classroom and were asked guiding questions developed from the first round of interviews. Before the participants entered the classroom, a video camera was set up in the front and middle. There was no assigned seating, and it was intended to be an informal setting. Many of the students sat close to each other, while the community members sat together as a group. To ensure participation of all students and community members, a Desmos activity was created so participants could type their responses in addition to verbally responding during the focus group interview. Desmos activities are interactive and allow participants to represent and share their ideas with sketches, text responses, card sorts, number responses, multiple choice, and more. Desmos also has an anonymity feature which allows the presenter to assign randomly generated names of famous mathematics to each participant. Once the activity was created, a code was generated. The invitation link and six-letter code for participants to join on student.desmos.com was posted on the classroom whiteboard. As participants entered the classroom at about 11:00 a.m., they received instructions on how to join the Desmos activity using a laptop with internet capabilities. Each slide of the Desmos activity transitioned to coincide with the discussion topics. While some participants shared their response verbally, all participants shared their responses electronically with the text response and anonymity features engaged.

This focus group revolved around eight major questions with each slide in the Desmos activity corresponding to those topics. These eight questions were on the topics of placement, rigor, pacing, motivation, perceptions, class procedures, conceptual understanding, and mathematical identity. Table 6 provides a list of the eight topics, the exact questions, and how each question addressed the two research questions.

Table 6

Desmos activity

Topic	Questions Asked	Addressed Research Question
Placement	Are some students misplaced? Do we need more clarity on the different math courses offered at BHS?	RQ1 - Describes the context of learning mathematics by asking how students are tracked.
Rigor	What do we define as rigor? Do we provide rigorous math courses to all students at Bearcat High School?	RQ1 - Describes the context of learning mathematics by asking what rigor looks like in tracked algebra classes.
Pacing	Some words associated with the pacing of honors and regular were fast, and with the pacing of support were okay and slow. What are your thoughts on these associations?	RQ1 - Describes the context of learning mathematics by asking about the pace of study in tracked algebra classes.
Motivation	Students want to hear verbal praise and see other kids successfully solving problems. Do we see this in the classroom of all tracks?	RQ1 - Describes the context of the learning environment and the interaction between students and teacher. RQ2 - Helpful in better understanding the mathematical identity of some students.
Perceptions	Words associated with honors algebra from the interviews were privilege, top, and smart. Needing help, more time, and becoming smart were associated with support algebra. What are your thoughts on these perceptions?	RQ2 - Helpful in better understanding the mathematical identity of students in tracked algebra classes.
Class Procedures	Many students voiced a desire for more opportunities to work together in groups to solve problems versus individually. What are your thoughts?	RQ1 - Describes the context of the learning environment by asking how students' complete problems in class.
Conceptual Understanding	What is the difference between conceptual understanding and procedural fluency? When working problems in class, do you see more of one type?	RQ1 - Describes the context of the learning environment by asking what types of questions students answer in class.
Mathematical Identity	Looking at the self-reported scores and observed mathematical identities, what do you notice and wonder?	RQ2 - Helpful in better understanding the mathematical identity of students.

Regarding placement, all participants believe some students are misplaced. To help alleviate this, some suggested providing more clarity and detailed descriptions on the different math courses offered would be beneficial to students, counselors, and parents. The next topic of rigor revealed mixed opinions. Some believed that support courses are not as hard. They are taught the same content and given the same End of Course examination at the end of the course, however, Support classes are just given a year to complete the course whereas the average and honors level classes will complete the course in a semester. One participant suggested the high school does provide rigorous math courses to all students because students are placed in courses in which they can function and learn but are at their personal levels. Pacing was the next topic on the agenda. While the participants toyed with the notion of offering support as a corequisite course, the consensus was the need for the support and foundations classes must be taught at a slower pace, so each student grasp the foundations with understanding as the goal. Not all students need a support class, but those who need it should take it before Algebra 1. Finally, for the topic of motivation, some of the student participants mentioned teachers giving verbal affirmation in class and stressed the importance of it to help build confidence and preserve.

The second half of the focus group interview discussed the topics of perceptions, class procedures, and mathematical identity. For perceptions, most participants agreed on a negative connotation for support classes. The guidance counselor added, “there are always going to be some negative connotation with support courses from some people and students; however, when explained to the students in regard to time frame and how support is spread over the years versus being crammed into one semester at a fast pace, students tend to be extremely accepting of the platform.” One student responded, “saying that support classes need more help isn’t a bad thing, but it could make students feel less.” Classroom procedures was the next topic, and there was unanimous student opinion on the importance of group work. One student commented, “I’d prefer group work because I’d be able to get help from other students who might be able to understand the concept better than me or the other way around, I might be able to help someone else if they don’t understand the math.” In contrast, the community members cautioned the use of group work. The guidance counselor responded, "I have had some students complain about group work. Those complaints usually come from the higher performing students, who pull the weight of the group work and are looked upon as being the leader. I cannot recall complaints coming from students who tend to be off-track, unfocussed, and who have poor work ethics. The biggest complaint is when a grade is assigned that could have been higher if everyone in the group pulled their weight. Students do not feel comfortable 'addressing' the work habits of peers and sometimes lack the skills needed to direct the group to work as a whole."

The final two topics included conceptual understanding versus procedural fluency and mathematical identity. The participants agreed that the current curriculum uses more procedural tasks versus conceptual tasks in Algebra 1 and noted the importance of both when learning the content. As the participants viewed the results from the self-reported and observed mathematical identity labels, one student noted, "no matter what level of math the student is in, they can still identify with a strong efficiency in math." Also, one community member looking at the 2.53 score replied, "I would question if this student were placed correctly or may need encouragement to take more rigorous courses."

5.2.5. Student Follow-up Interviews

The final stage in the data collection process was follow-up interviews with all eight students and both teachers. A Final Interview Protocol was designed for the students and teachers (See Appendix G). For the students, the interview focused on three groups of questions. The first group revolved around mathematical identity. Each student was provided results from the Mathematical Depth Instrument and all self-reported scores were arranged in the following diagram.

Figure 3

Results of Mathematical Depth Instrument

{–0.71, 0.14, 0.66, 0.66, 0.87, 0.98, 1.20, 2.53}

Students discussed whether they felt the score aligned with how they viewed themselves. All but one of the participants agreed with his or her score relative to the other scores. Opinionated student said, "I would have honestly thought my score would have been lower."

Each student was informed of the mathematical identity label given based on responses from the first interview. Figure 4 shows a description of those mathematical identity labels.

Figure 4

Mathematical Identity Labels

Mathematical Identity: Determined by written language indicators relative to positioning acts	**Position of Superiority:** • Asserted their elevated proficiency in mathematics and desired to maintain their high standing. **Position of Authority/Power:** • Asserted their superior proficiency in mathematics while simultaneously executing their authority to exert control over the narrative. **Position of Spectator:** • Neutral bystanders to a mathematical debate. **Position of Inferiority:** • Asserted their low normative comparison to others and desired to maintain it. **Position of Instructor/Solidarity:** • Exhibited an intent to encourage learning in other users and solidarity in their understanding of how others failed. **Position of Relative Indifference:** • Disinterested in engaging further in discourse.

A brief description of each label was given and then each student was told their unique observed mathematical identity. The student described whether this label aligned with how they would name him or herself. Then, the student was asked if he or she would rather have a different identity, if so, which identity and why? Each student agreed with the observed mathematical identity assigned to him or her. But when asked which identity they would desire to have, those with a label of inferiority, superiority, or spectator quickly selected the position of instructor. Of the three already with the label of instructor, they each desired no change in label. When asked why, each participant spoke on the importance of learning math so he or she could help their other classmates learn it also.

The second round of questions focused on curriculum. Two tasks were provided for the students. The first was on characteristics of exponential functions (See Appendix H). The second was a Marvel of Medicine task (See Appendix I). Both tasks were on exponential functions. Students described what they noticed on each worksheet. Then, the student was asked to identify which type of activity was most common in his or her math class. All participants recognized an inherent difference between the two tasks. They also unanimously stated they did more of the first task in their classes.

The final group of questions in the students' follow-up interviews revolved around mathematical discourse. The students were asked if they could talk in math class while working on problems. The students then described who they were talking to and what they were talking about during these times. All the participants stated they were able to talk with their classmates during class while working on problems; however, articulated a desire for more opportunities to work in groups and teams.

5.2.6. Teacher Follow-up Interviews

In addition to the student follow-up interviews, there were teacher follow-up interviews. The first question involved NCTM’s Mathematics Teaching Framework. Each teacher was asked to identify one of items from the framework that she would work hard to implement in her classroom. Figure 5 shows the NCTM’s Mathematics Teaching Framework.

Both teachers chose “pose purposeful questions.” Youth leader elaborated on why she made this selection, “I mean I see a need for all of it, but that would be the one I would work the hardest on. I think the way to have successful instruction is in posing your question so I would think it would be the most useful.” Track coach defended her selection by saying, “you know that question that they always ask. ’Well, why do I need to know this?’ and so if your questions are relatable, then maybe it would answer their question for them.”

Figure 5

NCTM Mathematics Teaching Framework

Mathematics Teaching Framework

<table>
<tr><td colspan="2">Establish mathematics goals to focus learning</td></tr>
<tr><td>Implement tasks that promote reasoning and problem solving</td><td>Build procedural fluency from conceptual understanding</td></tr>
<tr><td colspan="2">Facilitate meaningful mathematical discourse</td></tr>
<tr><td>Pose purposeful questions</td><td>Use and connect mathematical representations</td></tr>
<tr><td>Elicit and use evidence of student thinking</td><td>Support productive struggle in learning mathematics</td></tr>
</table>

Expanding on the first question, each teacher was asked to describe her ideal classroom if all constraints, including state mandated testing, were no longer present. Then, each teacher described how much of this she would be able to implement at the high school and what she wished she could do more of. Track coach’s wish list included more opportunities for group work. Although she believes she has a lot of freedom in her classroom, she has not “done a lot of that.” She went on to explain her reluctance, “I must get to the point where I know that they have that foundation before I try to just let there be questions that they work through and try to figure out. Although, some people would say they are learning their concepts when they are doing that but it is just such a change from the way I was taught and the way I have been teaching."

Youth leader shared a similar response, "I do love and see the need for certain things for direct instruction. But I also love station work. I love independent activities. I love challenges. So, for me, without the pressure of the state and state testing, getting through all the standards by a certain period, I would love to just constantly mix up the classroom every day. They would not know what they were walking into one day. It would be this one day. It would be that." When asked could she see her wish list being implemented at Bearcat High School, she quickly and regrettably answered, "with the way it is now, no."

The final round of questions for the teachers was their reactions to various statements made by other participants in the interview process. The capstone question is to get their reaction to the following statement: The tracked system at the high school for Algebra 1 follows a design where the End of Course Assessment is at the top. The EOC influences the pacing and rigor of the math courses at the high school, which in turn influences the placement and class procedures of those students in those courses, which in turn influences the perceptions, motivation, and identity of the students at the high school. Track coach noted, "Well, I mean if you are required for students to pass a test, you do need to make sure that you're doing what is necessary for them to be able to pass the test because you want them to be able to get a good foundation of it." Although she did not directly answer yes, Youth leader directly said, "Unfortunately, 100% agree. I mean, I really do feel like that is exactly how it holds out, yep."

At the end of the final interviews, an analysis of all data types followed. Looking for the interconnectedness between all the collected data, the next sections present four major findings which addressed the two research questions guiding this study:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

Addressing Research Question One and the context of learning mathematics at Bearcat High School, findings included: the need for more group work and collaboration, and the hierarchal design where the state mandated end of course assessment trumps everything.

5.3. Research Question 1 Finding: Group Work and Collaboration

Across all data types, the topic of group work and collaboration was evident. In the initial interviews, students talked about the importance and desire of working with other classmates to make them feel good and persevere. If they did not know how to answer or start a problem, they could ask their classmate. Equally important, if one of their classmates was struggling, they could lend a helping hand to them too. While the students asked for more group work and collaboration, the teachers in the focus group interview or follow up interviews noted the limited availability in the classroom. The classroom observations also reiterated few opportunities for students to see other students successfully solving or preserving through problems on the board. Although there were a few verbal affirmations from teacher to student, the consensus is there is not dedicated time for peers to work together to solve problems collaboratively in Algebra 1 at Bearcat High School.

Fundamental to Social Cognitive Theory, social persuasion and vicarious experiences are crucial sources of self-efficacy. Capa-Aydin et al. (2017), define social persuasion as “related to verbal encouragement received from significant people such as parents, peers, and teachers” (p. 1225). Teachers, fellow students, and parents each play a critical role in this area. Students who experience verbal affirmation in turn develop positive self-efficacy. It is critical to give verbal praises when students have success in learning mathematics. These positive comments help build self-efficacy and contribute to a better perception of learning mathematics. Although there are verbal affirmations directed from the teacher to student, it is evident there is room for improvement and group work would allow for positive collaboration and mathematical discourse between students.

Just as important, vicarious experiences are another important source of self-efficacy. Observing other students successfully solving mathematics problems and persistence in the process of learning mathematics positively influences students' beliefs that they can be successful too. Educators can help mediate vicarious experiences through the implementation of purposeful mathematics lessons. By doing so, teachers at Bearcat High School will allow students an opportunity to develop a positive perception of learning mathematics by emphasizing positive vicarious experiences supplemented with positive social persuasion best implemented through collaborative groups and class discussions.

5.4. Research Question 1 Finding: A Hierarchical Design

The End of Course Assessment is the driving force of instruction for Algebra 1 at Bearcat High School. In the final interview, both teachers confirmed the importance and necessity of providing instruction for preparation of the EOC. Bearcat High School has one of the highest content mastery scores for Algebra 1 in the region. This statistic is a great bragging right for the district and a necessary component for its CCRPI score. For students who struggle learning the content of Algebra 1, altering the pace of the course from one semester to two semesters or even three semesters would give teachers more time to cover the material and students more time to absorb the material. Therefore, offering an Algebra 1 course over an entire year or a Foundations of Algebra as a prerequisite for a yearlong Algebra 1 course would ensure adequate time to be prepared for the state mandated test. It is also interesting to note that no other high school in the region offers the Support class as Bearcat High School does.

With the courses now designed to maximize pacing relevant to a student's academic achievement, it follows naturally to simply place students into an appropriate track based on their previous achievement on standardized assessments. This in turn creates the perception that Support classes are meant for students who need more time to learn the content, which was articulated by the student participants and confirmed by the community members. The need for clarification on the tracked courses at the high school was common across all data types. Finally, where a student is placed within the tracked system at Bearcat High School, influences his or her mathematical identity. More specifically, as reported in the next section of findings, the student's desired mathematical identity.

In addition to the two findings addressing Research Question One, there were also two pivotal findings which address Research Question Two and the influence of tracking on mathematical identity. These findings included: the perception of support needing more time and desired mathematical identities for students and teachers.

5.5. Research Question 2 Finding: The Perception of Support

Across all data types, students and community members identified the purpose of support classes as a means of providing students who struggle mathematically with an extended period to learn the concepts. This was made evident in the initial interviews with all participants. None of the students associated the algebra support class as being slow or dumb. Instead, they continually asserted algebra support as being a path for students to learn the content over one year instead of one semester. This claim was corroborated during the classroom observations. While the honors algebra class learned three distinct mathematical topics in one class period, the support algebra class worked on about five problems on solving systems of equations. Looking at the structure of the algebra support class could potentially result in effective changes for efficiency and time on task; however, interview data and observational data conclude algebra support does go at a significantly slower pace than the other tracks.

Participants also indicated a need for better clarification and descriptions of all math courses offered at the high school. As noted, some students were unaware of the different tracked Algebra 1 classes. Additionally, some would have liked to have chosen a different track than the one on which they are currently. Students want more input in their educational journey, and this can all occur if they are properly informed of the choices. Unfortunately, most students do not have mobility once tracked in a low-level math course. While many of the participants did mention the possibility of moving up or down in level as needed, the reality is for the students taking the algebra foundations class during 9th grade year, their probability of taking a rigorous college-preparatory class their senior year is diminished. Students must become a participant instead of merely a spectator in the classes they take at Bearcat High School, and doing so will only occur with an informed stakeholder.

5.6. Research Question 2 Finding: Desired Mathematical Identities

In pursuit of answering Research Question Two and understanding the influence of tracking on student’s mathematical identity, it was necessary to assign each student with an observed mathematical identity. It was also vital to allow students the opportunity of self-reporting their identity. Barba’s (2020) six characterizations addressed the observed identity, and the survey addressed the self-reported identity. During the follow-up interviews, students were asked to discuss if these two labels accurately described their mathematical identity. Unintended during the follow-up interviews, students were then asked if there was any identity which they wished they had, and teachers were asked if there was any effective mathematics teaching practice they wanted to implement in their classrooms. Most striking was the revelation of a desired mathematical identity for each student participant, and a desired classroom environment for each teacher.

The inclination of a desired mathematical identity aligns with the concept of a growth mindset. An individual is not locked into a singular identity indefinitely. Instead, one’s identity will evolve and change over time. Even more so, the preferred identity selected by all participants as being an instructor is eye-opening. Yet, not entirely surprising given the demographics of the participants. Family closeness, community bonding, and solidarity are foundational to African American and Hispanic culture. Grouping students by achievement into either low, average, or top tracks at the high school has created “families of courses.” All students needing extra support are in a support class. They realize their limitations and know success will only be achieved through collaboration and teamwork. It is not just about me being successful, but we all are succeeding, together. These “families of courses” have created community bonding. Students feel a sense of responsibility to their classmates and will reach them inside and outside of the classroom. Finally, students do not celebrate when they see their fellow classmates struggling. Instead, within their own ability, they help freely because they anticipate the help being reciprocated when the need arises. It is important to note the unique structure of the school system in which Bearcat High School is located could have also influenced a sense of family and community for the students. While many school systems have multiple middle schools that eventually combine to form a high school, Bearcat High School’s district has only one middle school in which all students attend. Therefore, students developed close friendships and family bonds for three years before starting their educational journey at Bearcat High School.

Students were not the only participants who desired something more out of their learning experience. Teachers had desired identities also. Specifically, both teachers professed a desire to improve the educational experience in their own classrooms. Track coach and Youth leader both asserted a desire to spend more time posing purposeful questions and implementing tasks that promote reasoning and problem solving. Unfortunately, the teachers also recognized the limitations of their desires. Because of the state mandated end of course assessment taking precedent, both teachers felt pressured to teach in a way which prepared students for the assessment. In particular, the honors algebra class dedicated three weeks of the semester to reviewing for the exam. This perceived need to cover all the standards limited the availability of time to do engaging tasks and activities.

While the revelation of desired mathematical identities was a groundbreaking finding for this case study, it also leads to a need for moving beyond the data. Students deserve a learning environment in which they can pursue and reach their potential and desired mathematical identity. Additionally, teachers deserve the autonomy to create desired learning environments which foster mathematics learning for all without the pressure of simply teaching for the state exam. As this chapter concludes in the next section, the next chapter will further discuss these concerns and provide appropriate recommendations.

5.7. Moving Beyond the Data

After the follow-up interviews and subsequent data analysis, the data collection process was finalized. To help address each of the research questions proposed in this study, the conduction of multiple interviews, observations, and surveys were necessary. Guiding this qualitative case study were two research questions:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

The interviews and observations allowed a deeper understanding of the context of learning mathematics at Bearcat High School and addressed Research Question One. The interviews and surveys allowed for a deeper understanding of the mathematical identity of tracked Algebra 1 students and addressed Research Question Two. In the next chapter, a discussion of the analysis will provide a starting point to initiate positive changes to help improve the learning of mathematics at Bearcat High School for all students.

Chapter 6 – Discussion

Tracking continues to be a contested topic within mathematics education. Although there has been substantial research regarding the negative implications of the policy, some individuals in education support the continuation of tracking. This study delved into tracking's influence on the mathematical identity of adolescent rural students of color. To accomplish this, the researcher proposed two guiding questions:

1. What is the context of learning mathematics at Bearcat High School?
2. How do students of color perceive themselves as learners of mathematics when tracked into varying levels of an Algebra 1 course?

The researcher utilized multiple data types to answer the two research questions. Individual interviews, a focus group interview, follow up interviews, surveys, and classroom observations allowed for a deep description and understanding of not only the context of mathematics learning at Bearcat High School but also uncovered many telling findings related to mathematical identity, group work, perceptions, and student input. Students want more opportunities to collaborate and work together to solve problems in their math classes. Students also want more of an input in the mathematics course they take at the high school. Students have a desired mathematical identity, which may or may not be the same as their observed identity. The students at Bearcat High School deserve the opportunity of a rigorous mathematical learning experience. As the National Council of Teachers of Mathematics suggests, these practices should include tasks that promote reasoning and problem solving, facilitate meaningful mathematical discourse, and pose purposeful questions. Findings from the data revealed a deficiency in these practices. To improve the context of mathematics learning at Bearcat High School, there needs to be a strategic plan which addresses these issues. Figure 6 shows the pamphlet containing an organized description of the problem, issues, and viable solutions.

Figure 6

Pamphlet

What is the problem? The influence of tracked Algebra 1 courses at Bearcat High School on students' mathematical identity.	
Issue	**Solution**
Limited Group work and Collaboration	Experience First, Formalize Later • Working collaboratively to think, discuss, construct their own understanding • Georgia Department of Education is already on board with Engage, Explore, Apply, Reflect • Flexible thinking • Success for all students • Engaging context with Academic rigor • Repurposing old lessons
The Perception of Support	Course sequencing with Decision Points • Georgia Department of Education is already on board with decision points at 7th, 9th, and 10th grade • Students, parents, community members see the end goal and the journey from start to finish • All stakeholders have a voice in the process and not simple based off achievement scores or one teacher's opinion • Students are invested and take on the responsibility
Desired Mathematical Identity	Peer Tutoring Program. • Students learn from other students • Tutors strengthen their conceptual understanding • Tutors learn effective communication and work-based learning skills • Create a pipeline for future mathematics teachers for our distinct and surrounding regions • $15 an hour per tutor < Cost of remediation software

Change can sometimes occur at an opportune moment. This is never more apparent than the 2023–2024 academic school year as the state of Georgia officially implemented the updated Georgia Standards of Excellence for mathematics. Therefore, implementing these proposed solutions is critical for improving mathematics learning at Bearcat High School. The next section will give a more detailed description of each proposed solution.

6.1. Implications for Practice: Short Term Solutions

Change is very rarely easy, yet necessary for the continued improvement of an institution. This is evident in the classroom. Teachers plan and develop lesson plans to be engaging and relevant. These plans might require adjustments from year to year or even disposal considering an activity or strategy which is more effective. Professional development is at the heart of any career, and most needed in mathematics education. The case study revealed four major findings which are a hindrance to mathematics learning at Bearcat High School. The following sections go into a more descriptive solution which addresses each of these issues. It is crucial to note that these solutions are simple, short-term solutions. Although they do alleviate each of the specific issues with majority teacher-led instruction in the classroom, limited stakeholder voice in course choice, and the desired mathematical identities of students, together, they still do not address the overarching problem in mathematics learning at Bearcat High School.

6.1.1. Course sequencing with Decision Points

To provide district leaders, school administrators, counselors, and teacher leaders with information related to personalized mathematics pathways for all students, the Georgia Department of Education released a document which discusses student access to opportunities to advance in mathematics based on the student's post-secondary goals and aspirations. While the state's minimum core mathematics course requirements to earn a high school diploma include Algebra, Geometry, Advanced Algebra, and a fourth course mathematics course, the Department of Education also created a course catalog of secondary courses to help students meet their graduation requirements. This list includes support, enrichment, enhanced, advanced placement, and International Baccalaureate options. The Department of Education encourages unrestricted access for all students with multiple on-ramps and off-ramps as they progress through the secondary grade levels. These personalized, student-centered decision points are the solution to the issue of perceptions of support at Bearcat High School.

The first decision point occurs at the end of seventh grade. Here students can take Grade 8 Mathematics or Enhanced Grade 8 Mathematics and Algebra: Concepts and Connections. The second decision point occurs at the end of ninth grade. Students can take Advanced Algebra: Connections or Enhanced Advanced Algebra and Precalculus: Concepts and Connections. Finally, students who did not take the Enhanced option during 10th grade will have the opportunity to take Enhanced courses in 11th grade with a final decision point at the end of their sophomore year. While the Department of Education gives local districts the flexibility to create additional pathways that support student success based on the needs in the district, there should be unrestricted access for any interested student.

In line with the state of Georgia's recommendation for a concurrent Algebra Support class, Bearcat High School would also need to provide students necessary intervention support in real time as the students are working toward understanding of the grade-level standards. The state's suggested co-requisite support would not be suitable for the unique environment at the high school due to block scheduling. However, better preparation and planning is necessary so as students learn new concepts, they have adequate time to practice. A situation where students learn a new concept in one day, followed by opportunities to explore, apply, and reflect with varying tasks is most appropriate.

6.1.1. Peer Tutoring Program

In the past, Bearcat High School offered a robust tutoring program. While students have always worked with their teachers before and after school, the high school once had a retired former mathematics teacher who served as a dedicated tutor. Her tutoring classroom was the first classroom in the math hallway. She provided mathematics tutoring for all students throughout the school day. Once she decided to completely retire, the position remained vacant. While any introduction of an academic tutor would be beneficial, a specific peer tutoring program would provide many benefits.

First, peer tutoring would benefit the students who need help. Students often perceive peer tutors as less threatening by taking away the stigma of asking for help. Peer tutors can also help their tutees become better learners and provide positive motivation. Bearcat High School has a unique situation to help develop a robust peer tutoring program. Known as Golden Guides, students can enroll in a course at the high school and serve in many capacities as a staff assistant. The high school could use the Golden Guides to provide a dedicated block for peer tutoring in mathematics. Teacher recommendations for peer tutors would be foundational to the program. With a teacher recommendation, students who have passed Algebra 1 would be able to tutor Algebra 1. The same would be true for each respective math course at the high school. The Academic Resource Center, or ARC for short, would be a formal dedicated space for the peer tutoring program. The guidance department at the high school currently has a guidance classroom which can be renovated and repurposed to serve this need. Guidance counselors could also act as facilitators of the program; however, a dedicated teaching assistant or paraprofessional would be best to serve as an administrator to the program. Equally important to the program's success is providing compensation for peer tutors. Instead of relying on costly, impersonal software, offering competitive compensation of $15 per hour to each Golden Guide who is a peer tutor would provide motivation to build a robust peer tutoring program.

6.1.3. Experience First, Formalize Later

Critical to the improvement of mathematics education at Bearcat High School is the need for a change in thinking in mathematics instruction. A fundamental change from direct teacher-led procedural-based instruction to collaborative learning tasks which foster conceptual understanding is possible with the implementation of *Experience First*, *Formalize Later* lessons, or EFFL for short. Luke Wilcox and Lindsey Gallas created EFFL lessons as a means for students to work collaboratively by thinking, discussing, and constructing their own understanding of new concepts before the teacher helps students to arrive at formal definitions and formulas.

At Bearcat High School, students struggle to make connections in the mathematics they are learning which in turn influences long-term retention. While students will study every type of problem for a test and memorize an algorithm to solve a problem, they struggle when they encounter a non-routine problem because they are not flexible in their thinking. At Bearcat High School, the general flow of a mathematics class includes the introduction of new concepts through a lecture with substantial practice afterwards. While this approach works quite well for some students, other students disengage and quickly fall behind. The goal of mathematics learning at Bearcat High School should be to provide a rigorous and engaging learning experience for all students regardless of ability level.

The Georgia Department of Education has also recognized the importance of creating engaging student-centered lessons. With the implementation of the new standards update for the 2022–2023 school year, the Georgia Department of Education website states,

> *The Georgia Mathematics standards are designed to help learners achieve a balance among concepts, skills, and problem solving. They provide clear expectations for*

curriculum, instruction, assessment, and student work. The standards stress rigorous concept development and real-world applications while maintaining a strong emphasis on computational and procedural skills. At all grades, the standards encourage students to reason mathematically, to evaluate mathematical arguments both formally and informally, to use the language of mathematics to communicate ideas and information precisely, and to make connections among mathematical topics and to other disciplines.

Each lesson is broken into four distinct subsections. The first is Engage where the learning experiences include evidence-based instructional strategies that can be used as an introduction that mentally engages students to capture their interest, provides an opportunity to communicate what they know, and allows them to connect what they know to new concepts. The second is Explore where the learning experiences include evidence-based instructional strategies that allow students to engage in hands-on activities to explore the new concept/big idea at a deep level. The third section is Apply where the learning experiences include evidence-based instructional strategies that allow students to apply what they have learned in a new situation to develop a deeper understanding of the big idea. The last section is Reflect where the learning experiences include evidence-based instructional strategies that allow students the opportunity to review and reflect on their own learning and new understandings.

While the Georgia Department of Education has created lessons for the updated mathematics standards, teachers can also update their own lessons to align with EFFL. One way to accomplish this is by taking each bit of information and figuring out how to ask students a question so that they will be able to communicate their information. While designing a new EFFL lesson, the teacher must work backwards from the intended learning targets. Additionally, the teacher must identify the parts used for direct instruction before and think about how to get students to discover these ideas on their own. While this can be a daunting task requiring a great deal of front work, the benefits of collaborating with co teachers to create an accessible and rigorous learning task are beneficial. As a reference, the researcher included a sample lesson repurposed under the EFFL and Engage, Explore, Apply, Reflect design (see Appendix J).

6.2. A Call to De-track

While the recommendations would provide a quick repair to the status of mathematics learning at Bearcat High School, the root of the problem stems from the policy of tracking students into varying levels of math courses based on ability. Tracking must end at Bearcat High School. With the implementation of the updated mathematics standards, Honors Algebra no longer exists. Instead, students can take either Grade 8 Math or Enhanced Grade 8 Math with Algebra. While we will still provide a Foundations of Algebra course and support for students to be successful in their grade level math course, there should only be Algebra, Geometry, Advanced Algebra, and Precalculus courses offered at Bearcat High School. Fundamental to this proposal is the need to ensure the level of expectations for each respective course is maintained. Students struggling with mathematics should not be placed in a non- rigorous math class. These students should have appropriate support in place but experience the challenge and rigor the Georgia Mathematics standards offers. Additionally, there should not be an Honors track. Instead, students receive the same level of instruction and then can take Advanced Placement, dual enrollment, or rigorous fourth-year level mathematics to prepare them for their post-secondary goals. Only then, with such a bold move, will Bearcat High School offer all students a mathematical learning experience which makes connections between ideas and produces mathematics learning that will last long-term.

6.3. Implications for Future Research

One critical implication from the results of the case study was the discovery of student desired mathematical identities. Every individual has an identity, or way in which one defines oneself and how one is defined by others. One's identity includes perceptions of experiences and relationships with others. An identity can be malleable and can change over time. While we can use a survey to measure one's mathematical identity and through interviews and observations label one's mathematical identity, there needs to be further research on students' desired mathematical identity. Specifically, exploring potential changes in a student's desired identity throughout their mathematical learning experience and the influence from tracking on students of color.

6.4. A Voice for Change

The only thing constant is change. While change can be positive, negative, or indifferent, change is a necessity. The time for change has come at Bearcat High School. Many students at Bearcat High School do not have access to a rigorous secondary mathematics curriculum that promotes reasoning, problem solving, meaningful mathematical discourse, or builds procedural fluency because of the policy of tracking. As a result, Bearcat High School students are falling behind in mathematics learning. This is even more prominent for students who are socio-economically disadvantaged or a student of color. The educational inequities divide the students into two groups, either being able or not being able to take college preparatory mathematics courses. Therefore, more students of color graduate without the necessary 21st century problem solving skills to be successful in college or their post-secondary goals. This is truly a social justice issue as it perpetuates racial injustice, widens the income gap, and does not foster equality. The time to end tracking in mathematics is now. All students deserve access to a challenging mathematics curriculum, taught by skilled and effective teachers who differentiate instruction as needed. To strive for excellence, engagement, and accountability for all in the mathematics department is more than a mission statement, it is a goal worth attaining.

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Appendix A: Consent Form

COMMUNITY MEMBER CONSENT FORM

Title of Research Study: Tracking and Mathematical Identity for Rural Adolescent Students of Color

Researcher's Contact Information: Mr. Joseph Kelly, 229-726-8300, jkell108@students.kennesaw.edu. Dr. Brian R. Lawler, 470-578-4235, blawer4@kennesaw.edu.

You are being invited to take part in a research study conducted by Mr. Joseph Kelly of Kennesaw State University. Before you decide to participate in this study, you should read this form and ask questions if you do not understand.

Description of Project
The study's purpose is to understand the context of learning mathematics at Bainbridge High School and develop a deep understanding of tracking and mathematical identity for rural adolescent students of color.

Explanation of Procedures

You will be asked to participate in an individual interview, followed by a town hall-style focus group with other members of the Bainbridge HS community. I may request a second one-on-one conversation, as needed. For each of the members who are teachers of the Bainbridge HS community, Mr. Joseph Kelly will conduct one classroom observation.

During all the interviews and focus groups, I will record audio to create a record of the conversations for analysis.

Time Required
The entire study period is six months, starting in July and concluding in December 2022.

Risks or Discomforts
There are no known risks or anticipated discomforts in this study.

Benefits
It is our hope that the students and our community will develop a better understanding of the policy of tracking and mathematical identity.

Confidentiality
The results of this participation will be confidential. Your name (or other identifiable information) will never be documented or used in any report.

Inclusion Criteria for Participation
You have been selected to participate in this study due to your association as a student or community member of Bearcat High School.

Volunteerism
Participation in this study is completely voluntary. You may decide to not participate or withdraw during participation without worry of retribution. Your relationship with the school or the instructor will not be harmed or benefited by the decision to participate.

Signed Consent. I agree and give my consent to participate in this research project. I understand that participation is voluntary and that I may withdraw my consent at any time without penalty.

____

Signature of Research Participant, Date

____

Signature of Investigator, Date

PLEASE SIGN BOTH COPIES OF THIS FORM, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

Research at Kennesaw State University involving human participants is done under the oversight of an Institutional Review Board. Address questions or problems regarding these activities to the Institutional Review Board, Kennesaw State University, 585 Cobb Avenue, KH3403, Kennesaw, GA 30144-5591, (470) 578-2268.

Approved IRB-FY22-504. The influence of tracking on mathematical identity for rural students of color.

Appendix B: Parental Consent and Assent Form

PARENTAL CONSENT FORM WITH CHILD ASSENT STATEMENT

Title of Research Study: Tracking and Mathematical Identity for Rural Adolescent Students of Color

Researcher's Contact Information: Mr. Joseph Kelly, 229-726-8300, jkell108@students.kennesaw.edu. Dr. Brian R. Lawler, 470-578-4235, blawer4@kennesaw.edu.

Your child is being invited to take part in a research study conducted by Mr. Joseph Kelly of Kennesaw State University. Before you decide to allow your child to participate in this study, you should read this form and ask questions if you do not understand.

Description of Project
The study's purpose is to understand the context of learning mathematics at Bainbridge High School and develop a deep understanding of tracking and mathematical identity for rural adolescent students of color.

Explanation of Procedures
Your student will be asked to participate in an individual interview, followed by a town hall focus group with all participants, and an individual follow-up interview as needed. In addition, your student will be asked to complete a brief survey (less than 15 minutes) that is intended to capture a person's mathematical identity.

During all interviews, I will utilize an audio recording device to create a record of the conversations for analysis purposes.

Classroom Observations
The researcher will also conduct a classroom observation (maximum of 60 minutes) of your student's algebra class. The researcher will not directly interview your student or any other student during these observations.

Time Required
The entire study period is six months, July through December 2022.

Risks or Discomforts
There are no known risks or anticipated discomforts in this study.

Benefits
It is our hope that the students and our community will develop a better understanding of the policy of tracking and mathematical identity.

Confidentiality
The results of this participation will be anonymous. Your student's name (or other identifiable information) will never be documented.

Inclusion Criteria for Participation
Your student has been selected to participate in this study due to his or her current enrollment in the 9th grade at Bainbridge High School.

Volunteerism
Participation in this study is completely voluntary. You may decide to not participate or withdraw during participation without worry of retribution. Your and your student's relationship with the school or the instructor will not be harmed or benefited by the decision to participate.

Parental Consent to Participate
I give my consent for my child, ____, to participate in the research project described above. I understand that this participation is voluntary and that I may withdraw my consent at any time without penalty. I also understand that my child may withdraw his/her assent at any time without penalty.

____
Signature of Parent or Authorized Representative, Date

____
Signature of Investigator, Date

PLEASE SIGN BOTH COPIES OF THIS FORM, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

Research at Kennesaw State University involving human participants is done under the oversight of an Institutional Review Board. Address questions or problems regarding these activities to the Institutional Review Board, Kennesaw State University, 585 Cobb Avenue, KH3403, Kennesaw, GA 30144-5591, (470) 578-2268.

Approved IRB-FY22-504. The influence of tracking on mathematical identity for rural students of color.

Child Assent to Participate

My name is Joseph Kelly. I am inviting you to be in a research study regarding tracking and mathematical identity. Your parents have given permission for you to be in this study, but you get to make the final choice. It is up to you whether you participate.

If you decide to be in the study, I will ask you to answer a few questions about the math classes at your high school and how you identify as a learner of mathematics. During some of the interviews, we may record the audio. Anytime we are recording the audio, if you do not want to be heard in the recording, we will make sure that you are not heard on the microphone.

You do not have to answer any question you do not want to answer or do anything that you do not want to do. Everything you say and do will be private, and your parents will not be told what you say or do while you are taking part in the study. When I tell other people what I learned in the study, I will not tell them your name or the name of anyone else who took part in the research study.

If anything in the study worries you or makes you uncomfortable, let me know and you can stop. No one will be upset with you if you change your mind and decide not to participate. You are free to ask questions at any time and you can talk to your parents any time you want. If you want to be in the study, sign or print your name on the line below:

____

Child’s Name and Signature, Date

Check which of the following applies

- Child can read and understand the assent form and has signed above as documentation of assent to take part in this study.
- Child is not capable of reading the assent form, but the information was verbally explained to him/her. The child signed above as documentation of assent to take part in this study.

____

Signature of Person Obtaining Assent, Date

Appendix C: Student Initial Interview Protocol

Project: Tracking and Mathematical Identity for Rural Adolescent Students of Color

Date____________________________

Time ____________________________

Location _________________________

Interviewer ______________________

Interviewee ______________________

Release form signed? ____

Notes to interviewee:

Thank you for your participation. I believe your input will be valuable to this research and help us better understand learning mathematics at Bearcat High School.

Confidentiality of responses is guaranteed

Approximate length of interview: 15 minutes, three major questions

1. You have taken a lot of math classes over the years.

- What are some of the things you remember about the math classes you have taken?
- Are people born good at math? Why do you think that?

2. Tracking is a term we use in education to describe segregating students into various levels of a math class based upon their ability. For example, students are tracked into support, average, or honors level classes based on previous mathematics achievement.

- In what math class are you currently enrolled?
- How does the name of the math class make you feel about math?
- Do you feel that your math class challenges you?
- Is there mobility for a student to move up or down in our tracked classes?

3. A person’s mathematical identity can be defined as how a person views themselves as a learner of mathematics and how others view the person as a learner of mathematics.

- How would you describe yourself as a learner of mathematics?
- How do you think others (teachers, parents, guidance counselor, administrators) describe you as a learner of mathematics?
- How important is it for your math teacher to verbally praise you for answering questions in class?

- How important is it for you to see other students successfully solving problems in math class or not giving up?
- Would it benefit students who struggle with math to see other students being successful in a math class? How would it benefit them?

Closure

- Thank you to interviewee
- reassure confidentiality
- ask permission to follow-up ______

Appendix D: Community Member Initial Interview Protocol

Community Member Interview Protocol Form

Project: Tracking and Mathematical Identity for Rural Adolescent Students of Color

Date___________________________

Time ___________________________

Location ________________________

Interviewer ______________________

Interviewee ______________________

Release form signed? ____

Notes to interviewee:

Thank you for your participation. I believe your input will be valuable to this research and in helping grow all our professional practice.

Confidentiality of responses is guaranteed

Approximate length of interview: 30 minutes, two major questions

1. Tracking is a term we use in education to describe segregating students into various levels of a math class based upon their ability. For example, students are tracked into support, average, or honors level classes based on previous mathematics achievement.

- Are you familiar with the Department of Education's statement in 2014 regarding tracking? (If not, read the statement)
- What are your thoughts?
- Are you familiar with the ACLU's complaint against a New Jersey school district in 2014 regarding tracking? (If not, read the statement)
- What are your thoughts?
- Are you familiar with the NCTM's statement in 2018 regarding tracking? (If not, read the statement)
- What are your thoughts?

2. A person's mathematical identity can be defined as how a person views themselves as a learner of mathematics and how others view the person as a learner of mathematics.

- What assumptions would you make about a student who was in a math course labeled as support?

- What assumptions would you make about a student who was in a math course labeled as honors?
- What assumptions would you make about a student who was in a math course labeled as average?
- How important is it for students to receive verbal praise and affirmation from their teacher and other students in a math class?
- How important is it for students to observe other students successfully solving or not giving up when working on a math problem?
- What courses of math do you teach?
- What do you consider the differences between the different tracked courses?
- What do you consider the advantages of tracking?
- What do you consider the disadvantages of tracking?

Closure

- Thank you to interviewee
- reassure confidentiality
- ask permission to follow-up _______

Appendix E: The Mathematical Depth Instrument

The Mathematical Depth Instrument (English Version)

Never/almost never (1), Sometimes (2), Often (3), Always, almost always (4), Don't know (9)

1. I take the initiative to learn more about math than what is required at school/work.	1	2	3	4	9
2. When I learn a new method, I take time to find out if I can find a better method.	1	2	3	4	9
3. When I learn a new method, I try to think of situations when it wouldn't work.	1	2	3	4	9
4. I struggle with putting math problems aside.	1	2	3	4	9
5. If I forget a formula or method, I try to derive it myself.	1	2	3	4	9
6. I get engaged when someone starts a mathematical discussion.	1	2	3	4	9
7. When I learn something new, I make my own problems.	1	2	3	4	9
8. Math ideas that I hear or learn about help me inspire new trains of thoughts.	1	2	3	4	9
9. When I learn a new method, I like to be told exactly what to do.	1	2	3	4	9
10. When I try to use a method that doesn't work, I spend time to find out why it did not work.	1	2	3	4	9
11. When I learn a new formula/algorithm, I try to understand why it works.	1	2	3	4	9
12. When I face a proof, I study it until it becomes meaningful.	1	2	3	4	9
13. When I face a math problem, I consider different possible ways I can solve it.	1	2	3	4	9
14. When I work with a math problem, I move back and forth between various strategies.	1	2	3	4	9

15. When I learn something new, it makes me want to learn more things.	1	2	3	4	9
16. When I work with a problem, I pause along the way to reflect on what I am doing.	1	2	3	4	9
17. If I get stuck on a problem, I try to visualize it.	1	2	3	4	9
18. I can explain why my solutions are correct.	1	2	3	4	9
19. I try to connect new things I learn to what I already know.	1	2	3	4	9
20. If I immediately do not understand what to do, I keep trying.	1	2	3	4	9

Appendix F: Codebook

Coded Comment	Characterization
Placement	• My math teacher from last year put me in this class. • I would have liked to take honors in 8th grade but was not given the opportunity. • I think I could have handled regular Algebra 1, but my teacher put me in Foundations.
Rigor	• Honors is harder than support. • Those problems are just at a higher level. • The rigor in the support classes is lower than honors. • Teachers in support classes spoon feed the students with like basic math concepts.
Pacing	• We learn the same stuff. Support just learns everything over a year instead of a semester. • The Honors class goes fast.
Motivation	• It means a lot to me for my teacher to let me know I am doing a good job. • It makes me happy that my teacher acknowledges me on doing my math work.
Perceptions	• Support is a class that helps people that need it more. • Honors classes have better opportunities. • Being in honors makes me feel a lot more confident in my math ability. • Support kids could be good at math, they just need a little bit more help along the way. • Honors kids are smart. Support kids are becoming smart.
Classroom Procedures	• I would like to work in a group. • I would like more group work because somebody in your group might need help or someone in your group can help you if you need help. • She teaches us everything we must do. We take down notes and work out problems.
Mathematical Identity	• I am ok, but I just need more help in math. • I catch on to math fast. • Math is something simple for me personally. • I am not the smartest in the world. I just want to do the work, learn, and keep going. • I struggle with math. • I am usually the first one finished with my work, so I get to help other students who might be struggling.

Appendix G: Final Interview Protocol

Project: Tracking and Mathematical Identity for Rural Adolescent Students of Color

Date ___________________________

Time ___________________________

Location ________________________

Interviewer ______________________

Interviewee ______________________

Release form signed? ____

Notes to interviewee:

Thank you for your participation. I believe your input will be valuable to this research and in helping grow all our professional practice.

Confidentiality of responses is guaranteed

Approximate length of interview: 30 minutes, one major question

Students

- It has been a while since our first interview, and you have been doing a lot of math since then too. How would you rate yourself as a math learner?
- The survey you took during the first interview measured your math identity. Here is the list of scores from everyone who took it. {-0.71, 0.14, 0.66, 0.66, 0.87, 0.98, 1.20, 2.53} Your score was _____. Do you think the score aligns with how you view yourself?
- I also utilized your initial interview to give you a specific math identity label. Your label was ______. Do you think this label aligns with how you would name yourself? Would you rather have a different identity? What identity would you like to have? Which identity do you think is best?
- Notice the following two tasks on exponential functions. What do you notice about each? Which is most common in your math class? (Show examples)
- Do you get to talk in math class while working on problems? When you do, who are you talking with? What about?
- Teachers
- The NCTM posted a Mathematics Teaching Framework as shown here. Which of these would you work hard to implement in your classroom?
- Describe what would be the ideal mathematics classroom, if you could be freed of all constraints, state testing included.
- How much of that are you able or unable to implement here at BHS and which do you wish you could do more of?

- One interviewee said, “the end of course exam seems to be the driving force behind instruction and assessment choices.” Given the Regular and Honors track devoted approximately 10 days of instructional time on block to EOC review, what is your reaction to this statement?
- One interviewee said “support is the same amount of information being put into an entire year…. Because these students need to slow it down a bit, and they need extra time to absorb the information so that they can acquire it and maintain it.” What are the advantages for these students to spend one year taking a Foundations class and then another year of Algebra Support before taking the EOC? Are there any disadvantages?
- One interviewee said “a student in support will be getting some extra support while taking that algebra course. If kids need to be given more support, that is where their ability level is from a foundational standpoint, and that is where they fall.” Another interviewee said, “it's that you have a group of learners that need a whole lot more practice than others… in actuality they just need more practice than they would get in a regular math class.” Given these statements, placement is influenced by pacing. A student who needs more support, needs the pacing of the algebra slowed down. What is your reaction to this statement?
- One interviewee said, “I think the tracks have a negative connotation because students who think that they come into a foundations class, they don’t think they’re smart enough or they think that they are in what they call the slow class.” Given that statement, what is your reaction to the influence of placement on perceptions and or math identity?
- What is your reaction to the following statement: the tracked system at BHS for Algebra 1 follows a design where the EOC is at the top. The EOC influences the pacing and rigor of the math courses at BHS, which in turn influences the placement and class procedures of those students in those courses, which in turn influences the perceptions, motivation, and identity of the students at BHS.

Closure

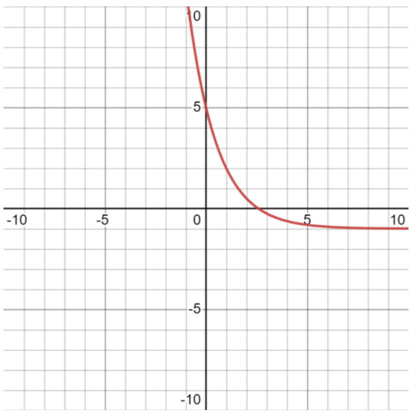
- Thank you to interviewee
- reassure confidentiality

ask permission to follow-up ______

Appendix H: Final Interview Student Handout 1

Characteristics of Exponential Functions

1. $y = 3 \cdot (\frac{1}{2})^{x+1} - 1$



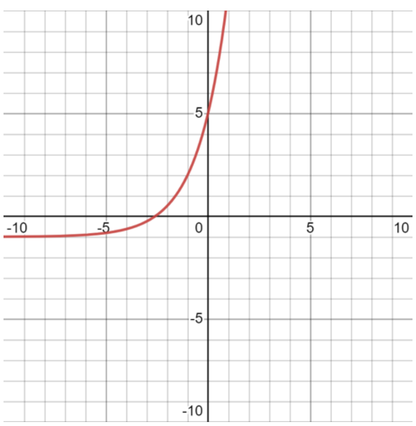
- Growth or Decay
- Domain ____________________
- Range ____________________
- Asymptote: ____________________
- Increase or Decrease
- X-int ____________________
- Y-int ____________________
- End Behaviors

As $x \to -\infty$, $y \to$ ___________

As $x \to \infty$, $y \to$ _________

- Transformations _________________

2. $y = 3 \cdot 2^{x+1} - 1$



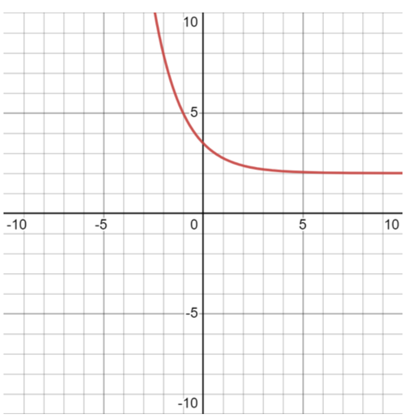
- Growth or Decay
- Domain ____________________
- Range ____________________
- Asymptote: ____________________
- Increase or Decrease
- X-int ____________________
- Y-int ____________________
- End Behaviors

As $x \to -\infty$, $y \to$ ___________

As $x \to \infty$, $y \to$ _________

- Transformations _________________

3. $y = 3 \cdot (\frac{1}{2})^{x+1} + 2$



- Growth or Decay
- Domain ____________________
- Range ____________________
- Asymptote: ____________________
- Increase or Decrease
- X-int ____________________
- Y-int ____________________
- End Behaviors

As $x \to -\infty$, $y \to$ ___________

As $x \to \infty$, $y \to$ _________

- Transformations _________________

Appendix I: Final Interview Student Handout 2

The Marvel of Medicine (Constructing Task)

Name _________________________________ Date _____________

Part 1. A doctor prescribes 400 milligrams of medicine to treat an infection. Each hour following the initial dose, 85% of the concentration remains in the body from the **preceding hour**.

Complete the table showing the amount of medicine remaining after each hour.

Number of Hours	Process	Number of Milligrams Remaining in the Body
0	400	400
1	400(0.85)	340
2	400(0.85) (0.85)	
3		
4		
5		

Appendix J: Experience First, Formalize Later Repurposed Lesson

Engage: The Mystery Function - We have been working with a lot of exponential functions, but today we are going to explore a different kind of function. Can you figure out how this new function works?

1) Given $y = 2^x$, what type of function is this? (Circle your response)

Linear or Quadratic or Exponential

2) Complete the table for $y = 2^x$ below

x	-3	-2	-1	0	1	2	3
y							

3) Recall what an INVERSE function is. Given the table above for $y = 2^x$, complete the table below that represents the INVERSE of this function.

x							
y^{-1}							

4) Now that you have completed the table of the INVERSE function, rewrite $y = 2^x$ as its INVERSE.

The equation you have created above is the INVERSE of the exponential function. We have a special name for this new function, we call it a LOGARITHM, or log for short.

In general, we can convert between the two forms:

(Exponential Form) $b^x = y$ ← to → $\log_b \quad y = x$ (Logarithmic Form)

5) Write each of the following exponential equations as logarithmic equations.

(a) $2^5 = 32$ (b) $3^3 = 27$ (c) $4^0 = 1$ (d) $7^{-2} = \frac{1}{49}$ (e) $10^{-3} = 0.001$

6) Write each of the following logarithmic equations as exponential equations.

(a) $\log_4 16 = 2$ (b) $\log_9 1 = 0$ (c) $\log_8 \frac{1}{64} = -2$ (d) $\log_5 125 = 3$

7) Without a calculator, try to evaluate each of the following logarithms

(a) $\log_5 25$ (b) $\log_2 128$ (c) $\log_3 81$ (d) $\log_{13} 1$ (e) $\log_7 \frac{1}{49}$ (f) $\log_8 8$

Explore: Acidity of Solutions

"pH" stands for "potential" of "hydrogen." The "H" in pH is capitalized because the symbol for hydrogen is H. The concentration of hydrogen ions, $[H^+]$, in a substance determines whether the substance is more acidic or alkaline. $[H^+]$, however, is usually an exceptionally large or exceedingly small number so we use logarithms to convert $[H^+]$ to pH. pH provides a more convenient way to express how acidic or alkaline a substance is.

The pH scale ranges from 0 to 14. A solution with a pH of 7 means it is a neutral solution. Pure water has a pH of 7. A pH less than 7 means the solution is acidic. A pH greater than 7 means the solution is alkaline (basic). The smaller the pH, the more acidic the solution is; the greater the pH, the more alkaline the solution is.

pH is the negative logarithm of the concentration of free hydrogen ions, measured in moles per liter (moles/L). The formula to convert the concentration of hydrogen ions to pH is

$$pH = -\log [H^+]$$

Notice that for this logarithm there is not a base. When the base is not listed, it is understood to be a base of 10. This is called a COMMON Logarithm.

1) Consider the general common logarithmic function, *f(x) = log x*. Using Desmos, how will the graph of the pH conversion function, *g(x) = - log x*, differ from the graph of *f(x)*?

a. Is the Domain different between f(x) and g(x)?

b. Is the Range different between f(x) and g(x)?

c. Are the x-intercepts different for f(x) and g(x)?

d. What kind of graphical transformation has occurred?

2) If a water sample has a pH of 5, use the conversion formula to determine the concentration of hydrogen ions in the sample. *(Hint: Set up the equation using the conversion formula, divide both sides by negative 1, rewrite as an exponential equation)*

3) Suppose another water sample has a pH of 6.

a. Determine the concentration of hydrogen ions in the sample.

b. How does the concentration of hydrogen ions in this sample compare to the concentration of hydrogen ions in the sample with pH of 5?

4) The $[H^+]$ in drinking water should range between approximately 3.16×10^{-9} and 3.16×10^{-6}. Determine the approximate range for the pH of drinking water. *(Hint: Substitute given values for $[H^+]$ into conversion formula, use calculator, round answers to nearest tenth)*

5) The concentration of hydrogen ions of a solution is measured and found to be 10^{-4}. Is this solution more acidic or less acidic than drinking water? Explain. *(Hint: Use conversion formula to find pH value and then compare)*

Apply- The acidity or alkalinity of a solution is determined by the concentration of hydrogen ions, the percentage of hydrogen ions contained in the solution. Here are some abbreviations we will use:

- H^+ will stand for hydrogen ions.
- $[H^+]$ will stand for the concentration of hydrogen ions in a solution. In other words, the percentage of H^+ ions divided by all total ions in the solution.

Consider the following solutions. Notice the solution on the right has more volume than the other solutions.

OH^- H^+ OH^- H^+ H^+ H^+ H^+ H^+ H^+ H^+	OH^- H^+ OH^- H^+ OH^- H^+ OH^- OH^- H^+ H^+	OH^- H^+ OH^- OH^- OH^- OH^- H^+ OH^- OH^- H^+ OH^- H^+ OH^- OH^- H^+ OH^- OH^- OH^- H^+ OH^- OH^- H^+ OH^- OH^- OH^- OH^- OH^- H^+ OH^- H^+
Solution A	Solution B	Solution C

For solution A, we see there the number of hydrogen ions in the solution is 8. So, H^+= 8

For solution A, we see there the concentration of hydrogen ions in the solution is 8/10 because there are 10 ions total. So $[H^+]$ = 0.8

1) In solution B, H^+= 2) In solution B, $[H^+]$ =

3) In solution C, H^+= 4) In solution C, $[H^+]$ =

When we want to determine how acidic or basic a solution is, it is important to examine the concentration better than examining the total number of ions. The chart below shows the hydrogen ion concentration $[H^+]$ and pH value for assorted items.

[H+]	**pH**	**Examples**
10^{-13}	13	Household bleach
10^{-8}	8	Eggs
10^{-7}	7	Pure water
10^{-3}	3	Soda pop
10^{-2}	2	Lemon Juice

Using the previous table, notice the relationship between the hydrogen ion concentration [H+] and the exact pH value is given by the following conversion formula: $[H^+] = 10^{-pH}$

5. Rewrite the conversion formula as a logarithmic equation.

6. Which pH has a greater hydrogen ion concentration? pH = 3.6 or pH = 7.6? Why?

7. Suppose a solution has an [H+] of $2.17*10^{-3}$, what is its pH level? *(Hint: Use the conversion formula and round to the nearest tenth).*

8. Suppose a solution has a pH level of 1.4, what is its [H+]?

Reflect: Discuss with a partner to determine the answers to the following questions:

1) Describe the relationship among exponential functions and logarithmic functions. Explain why logarithms are needed to solve exponential equations.

2) Give numerical, graphical, and symbolic representations of a base-5 logarithmic function.

3) Compare and contrast the graph of $y=3^x$ and $y=\log_3 x$.

Evidence of Student Success

1. Using the conversion formula pH=-log[H+], calculate the [H+] for each pH level. Round answers to the first non-zero digit.

(a) 1.7 (b) 2.9 (c) 3.1 (d) 4.7

2. For what value of x would make the following two solutions have the same pH level?

$$\log(2x+1) = \log(0.02)$$

3. Without using a calculator, evaluate each logarithmic expression.

(a) $\log_2 32$ (b) $\log_{10} 10$ (c) $\log_7 \frac{1}{343}$ (d) $\log_{123} 1$

4. Solve each equation.

(a) $\log_2 x = 5$ (b) $\log_7(n+2) = \log_7 8$ (c) $\log_6(4x+4) = \log_6 64$

5. The following logarithmic equation models the number of roaches, x, at Spring Creek Charter Academy t days into the school year. Determine how many roaches are in the school after 9 days. *(Hint: Rewrite as an exponential equation and solve)*

$$\log_2(x+7) = t$$