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Tools For Refining the Baryonic Tully-Fisher Relation

Abstract

This paper discusses the development of tools for galactic photometry using wise, and for reduction and analysis of 21cm galaxy spectra. These Python-based tools can then be applied to selected data in order to refine the Baryonic Tully-Fisher Relation. This relation, between baryonic mass and rotational velocity of a galaxy, proves useful for extragalactic research, particularly in calculating redshift-independent distances. While these tools could be further improved, they provide a robust process for calculation of particular galaxy characteristics.

I. Introduction

Understanding the distance to and between various stellar objects is a crucial element of astronomy and astrophysics - with the distance to an object, and its brightness observed on Earth, we can begin to understand various properties. A variety of good methods exist for calculating the distances up to the local extragalactic scale. Beyond that, distances can be determined using Cepheid variables observed by Hubble, up to 10 Mpc. Further, the best methods become Type 1a supernovae, rare events with easily determined absolute magnitudes, or redshift - which is subject to the inconsistencies of peculiar velocity. Redshift-independent observations of distance remain rather lacking, as a result, which is unfortunate because a longer distance volume inherently contains more galaxies to study. New methods of determining extragalactic distances are required, ones that rely on relatively easily observed traits of galaxies. And in 1977, R. Brent Tully and J. Richard Fisher created a model which offered a potential solution.

The Tully-Fisher relation, as it is now known, relates the luminosity of a galaxy, a standin for its stellar mass, to its rotational velocity (Tully and Fisher 1977). Although this relationship only holds for spiral galaxies, it can be utilized as a new method of distance approximation. As the rotational velocity of a galaxy is relatively easy to observe, using the 21cm emission line, it could then be compared to the observed magnitude of a galaxy to find the distance between galaxies.

Atomic, neutral hydrogen atoms are the most abundant molecules in the universe. In such a simple system, there are only two relative orientations of mechanical spin between the electron and proton - parallel and antiparallel. The act of flipping from parallel to antiparallel spins emits energy in the form of light, as the change in magnetic dipoles reduces the overall energy. This

transition releases a low energy photon, with a wavelength of 21cm. This magnetic-dipole allowed transition occurs on longer timescales around 11 Myr. The sheer quantity of this most abundant molecule causes the total number of emitted radio photons to be quite high. The long wavelength of the photons and the features of the transition itself mean that the HI molecules will not block them significantly, nor will dust. The optically thin nature of this transition causes the total observed flux, the column density, to be directly relatable to an amount of molecules. Therefore, using the 21cm flux and the approximate abundance of neutral hydrogen, we can calculate the gas mass of a galaxy. Previously, Tully and Fisher used this emission line to measure the rotational velocity of a galaxy. However, their use of luminosity as a stand-in for stellar mass allowed other astronomers to relate the total baryonic mass - stellar mass and gas directly to rotational velocity.

In 2000, McGaugh et al. published a paper describing a relationship between baryonic mass and rotational velocity, which later became known as the Baryonic Tully-Fisher Relation (BTFR). Although the researchers expected dark matter, which exerts gravitational force, to randomly affect the results with a high degree of scatter, this scatter appears to be absent, allowing this direct connection to be made. The new Baryonic Tully-Fisher Relation, by capturing all the mass in a galaxy, also models lower mass, gas-dominated galaxies (McGaugh et al. 2000). Similarly to the TFR, the BTFR still fails for extremely low mass, low rotation dwarf galaxies. The broad scope and improved accuracy of the new relation allowed for focus on the specific parameters and procedures, such as those to determine the baryonic mass of a galaxy and how the rotational velocity is defined. The largest weakness is that calibration relies on other measurements of distance, which are lacking for distant galaxies. Differing measurement

methods also inevitably lead to different results, but standard candles, such as Type 1a supernovae, are well-understood, and can be used to correct errors originating from redshift-based distance calculations.

In order to fit a relationship with galaxies, one must first understand where galaxies are located. One project, the ALFALFA survey (Arecibo Legacy Fast ALFA survey), conducted a blind survey of as much of the sky as possible, in order to create a large catalog of potential HI sources. In addition to this incredibly useful catalog, ALFALFA also led to the creation of several further projects, including APPSS (the Arecibo Pisces-Perseus Supercluster Survey), which sought to measure the infall of galaxies into this supercluster. Additionally, and more importantly to this project, APPSS tested the bounds of the ALFALFA data, and how this data could be extended - including the creation of the pyAPPSS Python package, for reduction and basic analysis of galaxy HI spectra. These tools and efforts eventually led to the project proposal and current project known as GBT22A-430, which seeks to refine the Baryonic Tully-Fisher Relation using a carefully curated sample of galaxies with strong HI signals and recent Type 1a supernovae in order to reduce the characteristic scatter of this relationship. This paper discusses the creation and refinement of tools which will aid in the data reduction and analysis required to calculate the baryonic masses and rotational velocities of galaxies for this project. The pyAPPSS Python package is used to extract the total HI flux and velocity width of a galaxy. WISE data is then used, with the simple mass-to-light ratio present at WISE's smallest wavelength, estimated at $0.5\pm1 \text{ M}_{\odot}/\text{L}_{\odot}$ (McGaugh and Schombert 2014), to estimate the stellar mass of each galaxy, allowing for computation of the total baryonic mass, which can then be compared to the

rotational velocity calculated using the HI velocity width and the galaxy's ellipticity as an approximation for inclination.

II. Methodology

The Green Bank Telescope (GBT), located in Green Bank, West Virginia, is the world's largest fully steerable single dish radio telescope, with a slightly elliptical, 100 m by 110 m dish which allows for observation of fainter galaxies that are undetectable using interferometry. Among its many potential receivers is the L-band receiver, often used to take HI spectra of galaxies. Unfortunately, this frequency range shares some overlap with certain GPS satellite signals, causing the ever-present issue of pernicious radio frequency interference.

GBT22A-430 is a relatively recent project, although it has already been extended two observing semesters, due to the project not receiving its allocated time as well as mechanical issues with the telescope, which have only been recently fixed. The goal of this project is to use the GBT to obtain HI spectra of a filtered list of galaxies with recent supernovae. The original sample to be observed was pared down to 220 galaxies, although refinement and removal of unobservable galaxies has continued throughout the observations. The observations are typically taken in a pair of 5-minute scans, each also requiring a 5-minute off-source scan to determine the background. Although the receiver operates in a 8192-channel frequency space, due to redshift and the known frequency of this emission line, an easy conversion to velocity space, more useful for this project, is possible. Once the data has been collected, initial reduction occurs in GBTIDL, a Green Bank specific environment of IDL, where each scan is baselined and then added together and averaged, in order to improve signal-to-noise and minimize any other effects.

Once these initial steps have been completed, the data files are output, in order to use a more easily customizable and user friendly reduction tool - pyAPPSS.

pyAPPSS

pyAPPSS is a Python package written for data reduction of information for APPSS, and was originally developed by Anhad Gande for data from the Arecibo 305 m radio telescope. It originated as a rewriting of IDL functions used at Arecibo, in Python 2. pyAPPSS contained a successfully yet slow method of smoothing and baselining, and three methods to fit and measure the characteristics of observed galaxies: a single gaussian fit (Figure 4A), a two-peak fit based on the optimal, "double-horned" spiral galaxy HI profile (Figure 4B), and a trapezoidal fit for atypical galaxies (Figure 4C). From a simple spectrum that plots flux, in Jy, against velocity, in km/s, the total flux and velocity width, as well as their errors, could be calculated with this package.

However, the GBT data's format and various differences between Python 2 and 3 required addition or alteration of existing scripts. First, a script was needed to convert the data to a usable format. Originally, this script only used a .fits file for the spectrum, obtained from GBTIDL. The .fits file only contained a flux array, lacking full information about the relevant velocity or frequency of each channel. Enough information remained in the file header to approximate the velocity and frequency, but these approximations were inexact and their errors varied. Once this information was calculated, it was placed into a new file, along with important information about the data, such as instrument, observer, and observation time. Some of this information has to be manually set for different projects, since it does not exist in the header. Due

to the imprecision of this method, it was eventually rewritten to also require a pair of .csv files, also from GBTIDL, containing the necessary arrays.

Another important addition to the package is a new fitting method, known as "multigauss," which creates a compound gaussian model and fits it to the data array. This method is most useful for atypical or low S/N ratio galaxies - although it works well for any profile shape, the fitting usually takes a few minutes, which makes faster methods more efficient if usable. Other useful changes include user interface improvements and changes to the baselining code that lower runtime when fitting baseline polynomials.

The process of reducing and analyzing a galaxy's spectrum with pyAPPSS is straightforward. Regardless of the fitting method, all spectra must be smoothed and baselined, so that total flux calculations do not include a potentially uneven background. All data is Hanning smoothed, minimizing "ringing" of strong signals across the entire spectrum. Data can optionally be boxcar smoothed, wherein each channel is averaged with some number of adjacent channels. For baselining, the user selects an even number of lines, delineating the source- and RFI-free regions of the spectrum. Then, a number of polynomials, orders 0 to 9, are fitted to the selected regions, the RMS of these fits calculated, and the best fit selected by minimizing the RMS. The baseline fit is not final - the user can view the RMS values for each fit order, and can iterate through them to select the best option. Once the user marks the fit as finalized, it is applied to the data and passed to the various profile-fitting methods.

The first two methods, "gauss" and "twopeak", are directly derived from Arecibo IDL code. Gauss fits a single gaussian to the data, centered on the selected region (Figure 4A). This fitting method is best suited for broad profiles with a single peak, such as galaxies that are face-

on. Unfortunately, such galaxies have little use when fitting the BTFR, as rotational velocities become challenging to determine, and so the gaussian method sees little use. The twopeak method is best applied to the classic "double-horned" galaxy spectrum profiles. When using the twopeak fitting method, the user selects the region containing the outer slope of each horn, and fits a line to this slope. These lines are used to determine the x-intercepts, between which flux is calculated with a rectangular addition. The x-coordinates of the fitted lines, when their y-coordinates are equal to half of the maximum flux, are averaged to obtain a central velocity, and their difference is the velocity width. (Figure 4B). This width can then be used to calculate the rotational velocity of the galaxy, with additional information from the stellar mass calculations - using ellipticity as a stand-in for inclination. The third method from the original pyAPPSS code, "trap", functions similarly to twopeak. The user selects two points along each outer slope, which are then used to calculate the slopes as well as intercepts. These slopes and intercepts are used to plot a rough trapezoid for calculating central velocity and velocity width.

Flux is calculated in a method identical to that of twopeak.

The "multigauss" fitting method functions differently and does not rely on legacy IDL code. It first convolves the spectrum with a simple gaussian function, smoothing the array (Figure 5A). Then, the user selects a region centered on the source, optimally containing only the source signal itself. Each local maximum and minimum is assigned a gaussian centered on it. These gaussians are then summed together to create a compound model, which is fit to the selected region as well as a set of 0-flux channels artificially placed on either side of the region, in order to aid the fitting. Originally, the fitting algorithm used was allowed only 100 iterations, resulting in frequently inaccurate fits, especially for more complex models or galaxies with a

broader profile. The script now uses a default of 500 iterations, more than sufficient for most spectra, although the added iterations increase the runtime by a few minutes. Once this fitted model is produced, the needed values are calculated in a fashion similar, though not identical, to twopeak. Instead of the data, the profile slope lines are fitted to the compound model. Additionally, this script attempts to automatically select the regions for this fit, by setting the intercepts to 3σ from the outermost gaussians and using the peak values in either half of the profile to approximate the regions (Figure 5B). This automatic method often fails for complicated profiles or noisy spectra with indistinguishable peaks, but the user can manually select the region upon failure. Total flux is calculated using the total area of all gaussians, while flux error is calculated based via a combination of RMS and the deviation of the model from the data. This approximation is used because calculating error based on the residuals of the fits yields an error multiple orders of magnitude larger than the flux - due to the high number of gaussians, which often include several negative gaussians.

Once the total flux and other characteristics are recorded for a galaxy, calculating the HI mass is simple, with the known distances:

 $M_{HI}(M_{\odot}) = (2.36 \times 10^5) (D(Mpc))^2 \int \left(\frac{S_{\nu}}{Jy}\right) \left(\frac{dV}{km/s}\right)$

From there, total gas mass is estimated at 1.3 times this HI mass, accounting for other gas. All the attributes that can be calculated using pyAPPSS are recorded and written out - Total Flux, Flux Error, W50, and W50 error. Once these values are known, the remaining step is using aperture photometry to calculate the stellar mass and ellipticity.

Stellar Mass Calculations - WISE data reduction

In order to determine the stellar mass of a galaxy, this project uses WISE data, which has total sky coverage. Unfortunately, due to the sensitivity and lack of selectivity in the WISE data, contaminants in the form of Milky Way stars or background galaxies are common. These contaminants need to be excised before aperture photometry can be performed, so that the total flux at $\lambda = 3.4$ micron can be calculated. The original procedure was written in a .ipynb notebook, which allows use of Python in an interface, rather than the command line, and allows sections of code to be run independently. While originally created by Professor John Cannon, this notebook was heavily modified by another research student, Joshua Marine, for use with primarily Spitzer data. Certain conversions and calculations needed to be altered for this notebook to work with WISE data, as the attributes of the Spitzer Space Telescope are very different - for example, Spitzer pixel values are flux density, MegaJanskys per Steradian, while WISE pixel values correspond to D/N, ie a detector's inherent units, which have a known conversion to flux. Additionally, the photometric zero points are vastly different.

In order to remove the contaminants, the user originally had to use a program known as DS9 in order to view the image and create a set of regions containing each contaminant, which would then be imported and used to mask all contaminants to NAN values. However, this manual selection is not quick - there are typically many contaminants in any given image. The largest change made, therefore, was to reduce the burden on the user by automating the beginning of this process. The photutils Python package contains a function that automatically calculates star positions on an image using an algorithm based on the point spread function, star brightness, and star roundness, which is used in the WISE data reduction procedure in order to

determine the location of each star on the image (Figure 6A). Then, an approximate radius for each star is calculated (Figure 6B), based on the point of optimal signal-to-noise ratio. These radii and positions are used to calculate a region file, which the user must then tailor in DS9 modifying some radii and occasionally removing or adding regions, but this process is now much faster and typically requires little alteration. Using the region file, a cropped version of the image then has these contaminants masked to NAN values - which are then interpolated over, filling the now-gaps with one of two algorithms - a proper interpolation algorithm which uses a gaussian kernel to approximate the background, and a convolution algorithm, which consequently smooths the entire image and fills in the gaps (Figure 6C).

Once contaminants are removed and final images produced, aperture photometry can be performed. The galaxy's shape and size are approximated using elliptical isophotes, a process that typically takes a few minutes as the algorithm iterates through many potential elliptical fits. The user selects an optimal ellipticity and ellipse angle (Figure 6D), which are used for the final aperture photometry. This photometry is conducted using a series of concentric ellipses, at incrementally increasing radii. The optimal radius is where the average flux in the annulus, a region around the aperture used to determine the background of an aperture, falls below the average background flux (Figure 7C) - in order to capture the entire flux of the galaxy of interest. Once the optimal radius is determined (Figure 7A), the characteristics of the galaxy that can be determined using this infrared stellar image are calculated and output - the 3.4 micron flux, magnitudes, stellar mass, and ellipticity. Due to the high error in the near-IR mass-to-light ratio, which could range from 0.4 solar masses per solar luminosity to 0.6, while the value used here is

0.5, the error in mass due to error in measuring the flux becomes insignificant, so this error is not calculated here.

Once both baryonic mass components have been calculated, along with other necessary characteristics of this galaxy, the total mass and actual velocity width at 50% of the peak can be fitted against each other, in order to create a model for the Baryonic Tully-Fisher Relationship.

III. Results and Discussion

Limitations of the Code:

Neither of the Python-based reduction procedures are necessarily perfect, and could see improvement. pyAPPSS can fit a wide variety of spectra shapes using one of the four measuring methods, but each has their own limitations. The "gauss" model (Figure 4A) is generally an inappropriate model for galaxies useful for this process - a lack of distinct peaks indicates that the galaxy is highly face-on, and therefore not useful for determining the rotational velocity. The velocity, for this method, is the full width at half maximum of the gaussian, rather than properly fitting profile edges. The "twopeak" model is strongest for classic, double-horned profiles, and struggles with atypical galaxy profiles, as it relies on two distinct peaks with consistent outside edges. Additionally, low signal-to-noise galaxies are challenging to fit as the noise can alter the outside edges and cause a high-error fit, heavily impacting the velocity width. Finally, archival galaxies, with accessible data from previous studies, cause issues because their resolution is low enough that there aren't enough valid points for a width error to be calculated - as is visible in Table 1, with certain galaxies having their velocity error as N/A. The "trap" fit is easily the most versatile of the three original methods, after certain issues were corrected, although it has a tendency to moderately overestimate the velocity width when compared to twopeak

While the "multigauss" fit is nominally able to create a model for any spectrum and provide the characteristics of the observed galaxy, regardless of low S/N or irregular profile, it still suffers from some aspects of twopeak's struggle to calculate velocity widths for extremely low S/N galaxies. Due to the methods which it currently uses to fit the edges for this calculation, it additionally cannot function properly for archival data, again due to the low resolution.

When compared to the pyAPPSS package, the WISE data reduction notebook is much more consistently successful, however the approximations of star radii of the contaminants could be improved in order to further reduce the user's need to manually edit the regions.

Sample Data:

The sample data, after being properly reduced, shows a variety of features - although many of the velocity width approximations have extremely low error, there are a few galaxies, which had low signal-to-noise ratios, whose velocity errors are significantly higher. This difference is likely due to the galaxy's weak signal, and does not reflect on pyAPPSS itself. Overall, the total baryonic mass of these galaxies ranges from 9.83 dex to 11.17 dex, and they are mostly stellar mass-dominated, which is characteristic of high mass galaxies. The three galaxies which are gas mass-dominated, AGC 442, 5066, and 9814, are generally at the lower mass end of this sample. A comparison to a recently produced fit of the BTFR, by Ball et al. (Figure 8), yields interesting results. Although some of the galaxies fall nicely on this relation, there is either a systematic overestimation of velocity width, or underestimation of baryonic mass, comparatively. Unfortunately, without the complete set of data, further conclusions cannot be drawn from this difference.

IV. Summary and Conclusions

pyAPPSS and the WISE reduction notebook offer versatile tools useful for working with the Baryonic Tully-Fisher Relation, as well as other purposes. These pieces of code can still undergo improvement, in order to further refine their accuracy. To this end, a direct comparison of calculated mass and velocity width values to those yielded by other methods may aid in highlighting the discrepancies and sources of error. This discrepancy can be seen when compared to another, recent template relation, wherein our galaxies mostly fall below this BTFR. Once our observations with the Green Bank Telescope resume, the rest of the data can be reduced and hopefully yield a more distinct and better aligned fit for the Baryonic Tully-Fisher Relation. A successful refinement would highlight the effectiveness of these two tools, but a failure would not preclude their use elsewhere, and could indicate avenues of improvement. Regardless of the success of this project in reducing the scatter of the BTFR, the Python-based tools described here can be used in a wide variety of future research projects.

References

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Tables

AGCnr	Log(Gas Mass)	Log(Stellar Mass)	Log (Total Mass)	W50 P (km/s)
	(logM⊙)	(logM \odot)	(logM⊙)	
40	9.8827	10.678	10.7424	237±4
442	9.8713	9.741	10.1122	154±2
1027	10.1140	10.407	10.5858	263±12
1259	10.1345	10.58	10.7133	231±6
1324	10.2831	11.11	11.1705	624±12
1993	10.5314	10.955	11.0937	259±5
3122	9.8248	10.359	10.4707	199±N/A
3236	10.2761	10.588	10.7602	229±N/A
3238	10.0137	10.871	10.9272	355±N/A
3375	10.4533	11.08	11.1723	267±4
3376	10.0972	10.392	10.5699	93±3
3782	10.0833	10.158	10.4233	155±1
4195	9.9554	10.544	10.6438	192±N/A
4455	9.9338	10.806	10.8609	290±37
4481	9.7883	11.154	11.1724	298±10
4510	9.1095	10.398	10.4196	208±17
5066	10.0406	9.979	10.3120	148±3
5753	9.6485	10.69	10.7281	234±N/A
5884	10.2327	10.565	10.7306	183±7
6015	10.4077	11.063	11.1497	375±73
6960	9.8447	10.396	10.5033	201±N/A
8686	10.0837	10.8	10.8761	273±6
9247	10.1331	10.82	10.9009	269±7
9419	9.7299	10.518	10.5832	184±7
9814	9.5436	9.511	9.8284	113±3

100219.36289.91810.0249180±1Table 1: A table containing the final values which would be used to fit a prospective BTFR. Thevelocity width at 50% of the peak flux lacks an error for certain galaxies - these were thegalaxies with issues related to low resolution preventing calculation of this error.

Figure Captions

Figure 1: Examples of the data reduction process conducted in GBTIDL for this project, including an example, smoothed galaxy. A) The spectrum of AGC 1993, based on a single 5-minute scan, with no smoothing or other reduction applied. B) The same spectrum, with user indicated regions (the boxes), as well as GBTIDL's polynomial fit of the baseline. C) A baselined, coadded and averaged spectrum, produced from two separate 5-minute scans, which were each baselined, summed, and then averaged, in order to reduce the overall signal-to-noise ratio. The three output files for conversion are output at this stage. D) An example coadded and averaged spectrum, of AGC 26163, which has been smoothed in order to use GBTIDL measuring tasks. This galaxy spectrum serves as an example of profiles which GBTIDL can easily fit and measure - strong, classical double-horned profiles.

Figure 2: An example of the pyAPPSS baselining procedure. A) The user-selected, signal and RFI free regions over which to calculate the baseline. Note the exclusion of the very edges, so the characteristics of the bandpass do not influence the baseline fit. B) An example overlaid baseline, in this instance a 7th-order polynomial, on the spectrum of AGC 1993. This order was chosen due to its minimized RMS without overfitting the baseline. C) A baselined spectrum of AGC 1993. The baseline has been zeroed out, to a nearly linear background.

Figure 3: Examples of the measuring routines in the original version of pyAPPSS, using AGC 1993. A) The "gauss" measuring fit, which fits a single gaussian to the center of the galaxy's signal.

Figure 4: Two examples of the "multigauss" measuring fit for pyAPPSS, highlighting two different kinds of galaxy spectra - AGC 1993, with a high S/N, classic double-horned profile, and AGC 4455, a galaxy whose signal is challenging to distinguish from the noise, yet can still be fit. A) The spectrum of AGC 1993, overlaid in orange with the data convolved by a gaussian, which acts as a smoothing method. The vertical, green, dashed lines are the user selected region to which the model is fit. B) The spectrum of AGC 1993, overlaid in orange with the smoothed spectrum, and in green with the fit, compound gaussian model. The two edge fits are in red, and the dotted, crossing lines indicate the center velocity (the vertical line) and the width at 50% of the peak flux (the horizontal line). C) The same as figure 5A, but instead for the galaxy AGC 4455. D) The same as figure 5B, but for AGC 4455. Of note, this image lacks the edge fits and velocity indicators - the low S/N did lead to a high-error fit, which is not displayed here.

Figure 5: The automated WISE photometry procedure. A) words. B) words. C) even more words. D) last words.

Figure 6: WISE elliptical aperture photometry. A) things. B) things. C) more.

Figure 7: Data compared to Ball et al. V50 P fit, i.e. velocity width at 50% of the peak value.

Figures



Figure 1



Figure 2



Figure 3



Figure 4



Figure 5



Figure 6



Figure 7