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Bayesian Inference for the White Dwarf Initial-Final Mass Relation

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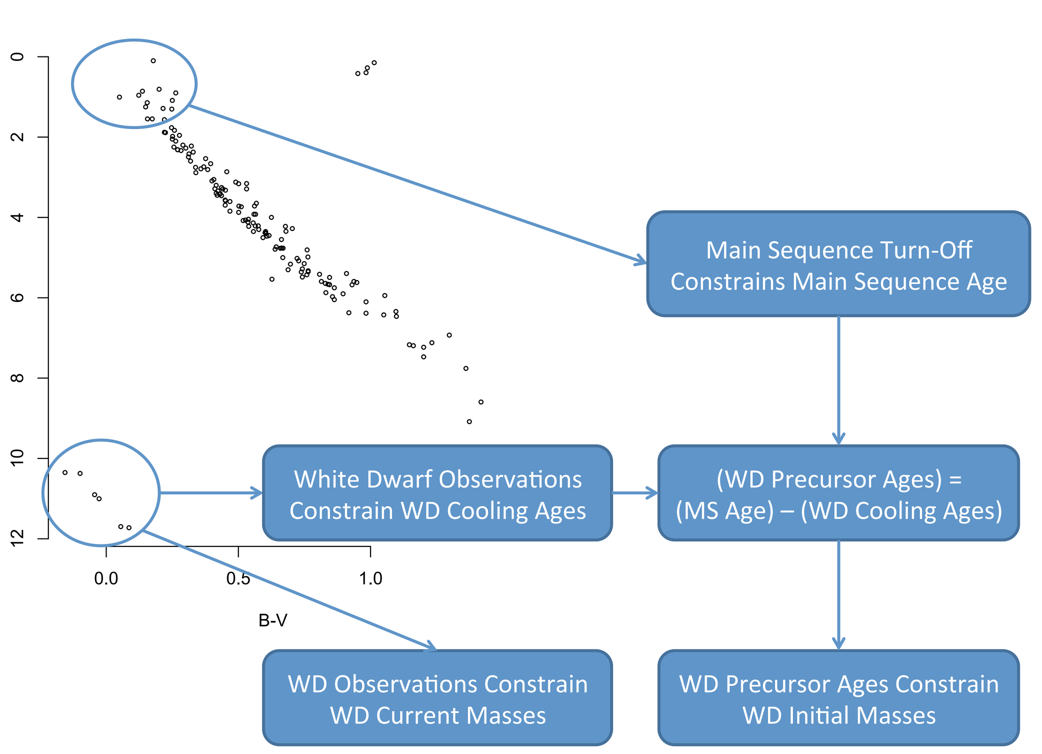
Summary

- Stars lose mass as they age, and understanding mass loss is important for understanding stellar evolution.
- The **initial-final mass relation** (IFMR) is the relationship between a white dwarf's initial mass on the main sequence and its final mass.
- We have developed a new method for fitting the IFMR based on a Bayesian analysis of photometric observations, combining deterministic models of stellar evolution in an internally coherent way. No mass data are used.
- Our method yields precise inferences (with uncertainties) for a parameterized linear IFMR. Our method can also return posterior distributions of white dwarf initial and final masses.

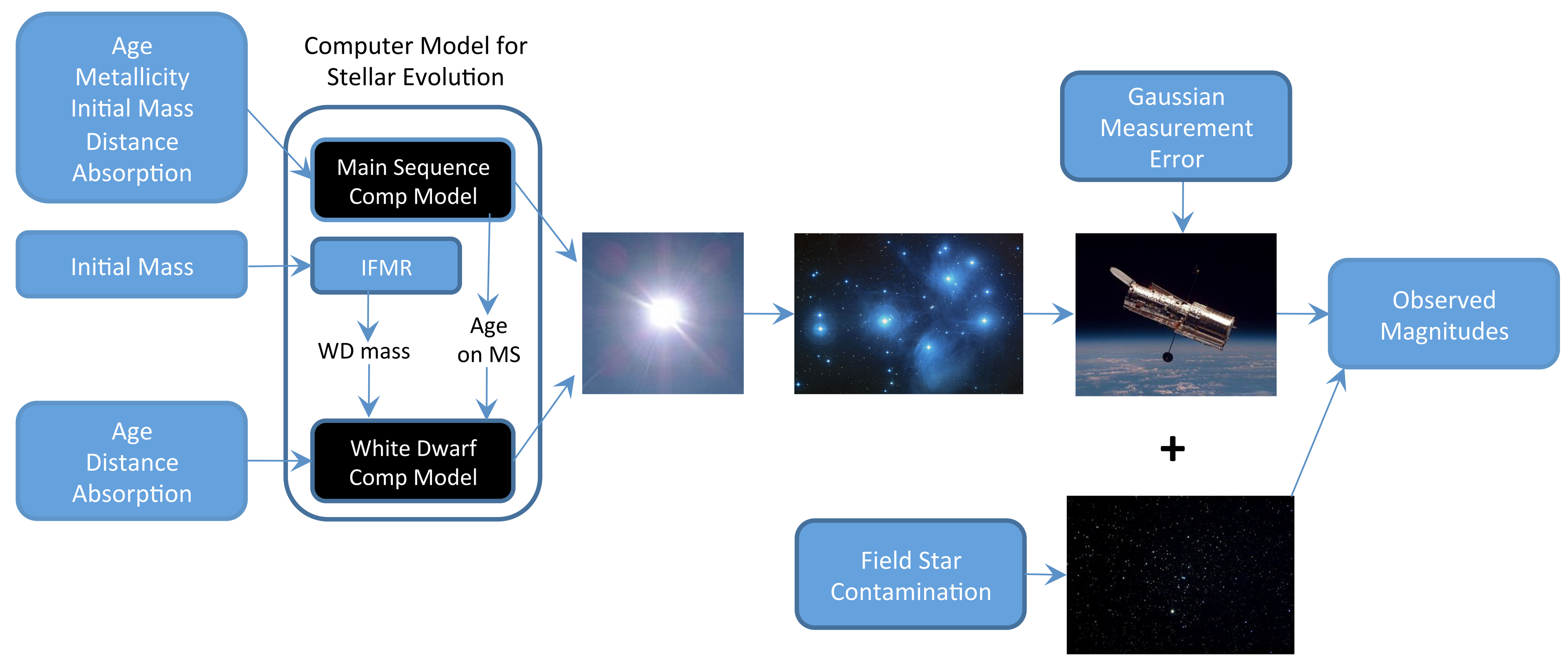
Background: Color-Magnitude Diagrams

- Observe stars' luminosities through different filters
- For the star clusters we study, several parameters are common to all stars:
  - Chemical composition (metallicity)
  - Age
  - Distance
  - Absorption
- Initial masses vary star to star
- **Color-magnitude diagrams** show the temperature (horizontal axis) and brightness (vertical axis) of stars in different evolutionary states
- For single-age clusters, these different evolutionary states are determined by stars' initial masses

Fitting the IFMR



Statistical Model



Cluster Star Likelihood

- Gaussian errors:

$$\boldsymbol{Y}_i \mid M_i, \boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}_i \overset{indep}{\sim} N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$$

- $\boldsymbol{Y}_i$ = vector of observations of magnitudes through different filters
- $M_i = (M_{i1}, M_{i2})$ = the primary and secondary mass of star i
- $\boldsymbol{\theta}$ = vector of cluster parameters, including age, metallicity, distance, and absorption
- Observational uncertainties $\boldsymbol{\Sigma}_i$ are assumed known
- Means $\boldsymbol{\mu}_i$ are functions of unknown parameters and depend on deterministic stellar evolution models $\boldsymbol{G}_{\rm ms}$ and $\boldsymbol{G}_{\rm wd}$
- If star i is a main sequence star, we model it as a binary system:

$$\mu_{ij} = -2.5 \log_{10}\left(10^{-G_{{\rm ms},j}(M_{i1},\boldsymbol{\theta})/2.5} + 10^{-G_{{\rm ms},j}(M_{i2},\boldsymbol{\theta})/2.5}\right)$$

  Single star systems will have small M_{i2}, with negligible effect on modeled luminosity
- If star i is a white dwarf, then $\boldsymbol{\mu}_i$ depends on the IFMR:

$$\mu_{ij} = G_{{\rm wd},j}(M_i, \boldsymbol{\theta}, f, \boldsymbol{\alpha})$$

- f is the IFMR, which we parameterize as a linear relationship:

$$f(M_i, \boldsymbol{\alpha}) = \alpha_0 + \alpha_1(M_{i1} - M^*)$$

  where M^* is a fixed value for centering the white dwarf initial masses.
- $M_{i2} = 0$ for all white dwarfs (we do not model binary systems involving white dwarfs)

Mixture Model for Field Stars

- Field stars appear in observational field of view, but are not part of the cluster.
- For simplicity, field stars are assumed uniformly distributed in magnitude space.
- Mixture model

$$Z_i \sim \text{Bernoulli}(\pi_i)$$

$$(M_{i1}, R_i) \mid Z_i \sim p(M_{i1}, R_i \mid Z_i)$$

$$\boldsymbol{Y}_i \mid M_{i1}, R_i, Z_i \sim \begin{cases} p_1(\boldsymbol{Y}_i \mid M_{i1}, R_i, \boldsymbol{\theta}, \boldsymbol{\alpha}) & \text{if } Z_i = 1 \\ p_0(\boldsymbol{Y}_i) = \text{constant} & \text{if } Z_i = 0 \end{cases}$$

  - $Z_i = 1$ if star i is a cluster member, $Z_i = 0$ otherwise
  - π_i = prior probability of cluster membership for star i
  - $R_i = M_{i2}/M_{i1}$ = ratio of secondary to primary mass
  - p_1 = cluster star likelihood
  - p_0 = field star likelihood

Prior Distributions

- Primary mass:

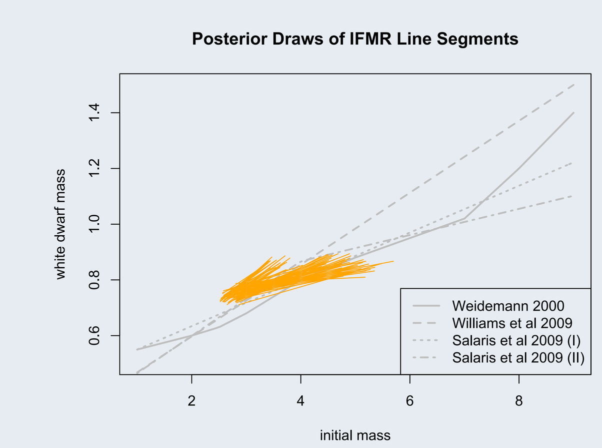
  $\log_{10}(M_{i1}) \sim N(-1.02, 0.677^2), \ 0.1M_\odot < M_{i1} < 8.0M_\odot$

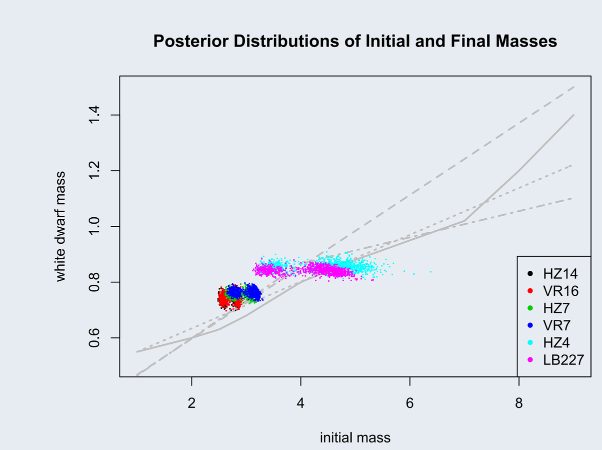
  based on Miller and Scalo's initial mass function
- Cluster membership prior probabilities come from external information when available
- Uniform on R_i, $\log_{10}$(age), and $\boldsymbol{\alpha}$, with appropriate boundaries
- Gaussian prior distributions on metallicity and distance
- Truncated Gaussian prior distribution on absorption (absorption is positive)

Model Fitting

- Unknown parameters: $\boldsymbol{M}_1, \boldsymbol{R}, \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\alpha}$
- MCMC (Metropolis algorithm) on lower-dimensional marginal distribution $p(\boldsymbol{\theta}, \boldsymbol{\alpha} \mid \boldsymbol{Y})$, where it is more reliable
- Numerical integration to marginalize over $(\boldsymbol{M}_1, \boldsymbol{R})$
- Because of conditional independence, $2N$-dimensional integral factors into N 2-dimensional integrals that can be evaluated in parallel within each MCMC iteration

Results: Hyades





- Analyzed Hyades data after adjusting for different distances to individual cluster members.
- Inferences agree with IFMRs from the literature, without using white dwarf mass data.
- Bimodality due to two possible age solutions, at approximately 525 Myr and 665 Myr.

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We wish to thank Kurtis Williams for helpful discussions. This material is based upon work supported by the National Aeronautics and Space Administration under Grant No. 10-ADAP10-0076 issued through the Astrophysics Data Analysis Program, and by NSF grants DMS 09-07522 and 09-07185.

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