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The approximate number system cannot be the leading factor in the acquisition of the first symbolic numbers

Attila Krajcsi [a,b,*], Marta Fedele [c,d], Bert Reynvoet [c,d]

[a] *Department of Cognitive Psychology, Institute of Psychology, ELTE Eötvös Loránd University, Hungary*
[b] *Department of Cognitive Science, Central European University, Budapest, Hungary*
[c] *Brain and Cognition, KU Leuve, Belgium*
[d] *Faculty of Psychology and Educational Sciences, KU Leuven Kulak, Belgium*

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ABSTRACT

When learning the meaning of symbolic numbers, children learn the numbers up to 4 sequentially. A prominent account of this learning process proposes that the increasing sensitivity of the preverbal approximate number system (ANS) allows for reliable discrimination of increasingly large neighboring numbers, which, consequently, leads to the sequential acquisition of the first symbolic numbers. In this work, a more complete quantitative description of this account is provided. This description is based on the mathematical model of the ANS and on additional relevant parameter values reported in the literature. The quantitative description demonstrates that, in the time period during which children learn the meaning of the first number words, the improvement of ANS sensitivity cannot provide the assumed changes in the discriminability of these numbers. The present result challenges the role of the ANS as a leading factor in the acquisition of the first symbolic numbers.

Understanding symbolic numbers is a key capability in human cultures. The mechanisms behind the initial acquisition of symbolic numbers are still unknown. Here, we evaluate the possible role of the approximate number system (ANS), a preverbal representation of number, in this learning process. Because the format of the present paper is not typical, we outline here how the text is organized. In the first section, we summarize the research on the development of understanding the first number words, and the proposed role of the ANS in this developmental process. We then present the related empirical evidence to date and argue that these findings may not be conclusive about the role of the ANS in the early acquisition of symbolic numbers. In the second section, we discuss the mathematical details of the ANS model and argue that a fundamental parameter, that is the threshold for the discriminability of two values, is missing in the current description to verify the role of the ANS in symbolic number acquisition. We also reconsider the development pace of ANS sensitivity that may be mischaracterized in the literature. Finally, by applying the reasonable parameters to the model based on reasonable estimates from the available literature, we show that the pace of ANS sensitivity development cannot account for the relatively fast symbolic number acquisition that is observed in empirical results. We conclude that the ANS cannot play a dominant role in the acquisition of the first symbolic numbers.

* Corresponding author at: Department of Cognitive Psychology, Institute of Psychology, ELTE Eötvös Loránd University, Hungary.
E-mail address: krajcsi@gmail.com (A. Krajcsi).

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1. Learning the meaning of the first number words

Children initially learn the basic properties of symbolic numbers at around the age of three (Wynn, 1990, 1992). Learning the meaning of the first number words is a slow process. This process is typically measured using the give-a-number (GaN) task, in which children are asked to give a specific number of items from a larger set (Wynn, 1990, 1992). In the first phase, children cannot give even one item correctly in the GaN task, although preschoolers know the counting list, that is, a series of number words (e.g., they can recite the numbers up to 10). These children are termed pre-knowers or non-knowers. Later on, they are only able to give 1 correctly (termed one-knowers). After a few months, children become two-knowers (giving only 1 and 2 correctly), then three-knowers, and finally four-knowers. One- to four-knowers are termed subset-knowers, because they are only able to give a subset of the values from their counting list in the GaN task. In a final phase of this development trajectory, children start to correctly give all values from their counting list, and they become cardinality-principle-knowers (CP-knowers). In this final CP-knower phase, it is assumed that preschoolers understand the cardinality principle, that is, the principle that the last number word in the counting routine denotes the quantity of the set (Wynn, 1990, 1992). It is also assumed that CP-knowers understand many general properties of symbolic numbers (e.g., see Lipton & Spelke, 2006). Note that there are important debates on whether this knowledge generalizes to all numerical tasks (Davidson et al., 2012; Le Corre & Carey, 2007; Sella & Lucangeli, 2020) or whether the GaN task in its current form is the appropriate tool to measure this knowledge (Sella et al., 2021).

A key question concerning this developmental pattern is what representation may be responsible for the observed pattern. It is usually assumed that representations supporting numerical understanding in infants play a crucial role in preschoolers' initial symbolic number learning. (1) Accordingly, one such hypothesis suggests that a perceptual system that tracks visual objects through space (called the object tracking system [OTS]) is a key mechanism. This system can represent three or four objects at the same time, and it can account for several numerical information-related phenomena in infants (Feigenson et al., 2004; Spelke, 2000; Wynn, 1998). According to this hypothesis, subset-knowers rely on this representation when learning the symbolic meaning of numbers, and the limited capacity of this system accounts for the phenomenon of why children become CP-knowers after being four-knowers, not earlier or later (Carey, 2004, 2009; Carey & Barner, 2019). (2) Another often raised possibility is that an imprecise number representation, the ANS plays a critical role. The ANS is believed to be present already after birth and to account for number discrimination in infants (Feigenson et al., 2004; Izard et al., 2009; Spelke, 2000). It is a noisy representation that works according to Weber's law and offers imprecise information about values. The precision of the ANS (which is technically the Weber fraction; see more information below in the section that describes the quantitative details of the model) improves continuously in children throughout their development, which enables increasingly precise discrimination of numbers with increasing age (Feigenson et al., 2004; Halberda & Feigenson, 2008; Piazza, 2010; Piazza et al., 2010). According to this account, the improvement of ANS precision contributes to the gradual acquisition of the first number words (e.g., Mussolin et al., 2012; J. B. Wagner & Johnson, 2011).

In its most characteristic form, this account proposes that the precision improvement of the ANS alone is sufficient to account for the incremental symbolic number acquisition in subset-knowers: Preschoolers can learn the next number when the precision of the ANS allows them to discriminate that number from its larger neighbor (Piazza, 2010, p. 547.). This argument relies on the fact that, in subset-knowers, the observed development of ANS precision is in line with the observed pace of symbolic number knowledge acquisition (find more technical details below in the section that describes ANS sensitivity development). The starting point of the present work is this latter characteristic account, which we term the strong ANS hypothesis of subset-knowers' number acquisition.

Other works that discuss the role of the ANS in subset-knowers are vague about whether the ANS has a dominant role or only a partial and supportive role in that development. While a number of papers (Mussolin et al., 2012; Rousselle et al., 2004; Wagner & Johnson, 2011) report only a moderate relation between ANS sensitivity and symbolic number knowledge, these works are not explicit whether this relation is moderate for theoretical reasons (i.e., the ANS may have only a supportive role) or due to unavoidable noise in the measurements and other methodological constraints (see an extensive list of possible methodological issues in those measurements in Szkudlarek & Brannon, 2017). At the same time, it is frequently assumed that number words are mapped on the ANS or that the ANS has a dominant role in number processing (Odic et al., 2015; Szkudlarek & Brannon, 2017), while usually no additional representations beyond the ANS are mentioned that could support the acquisition of number words (note that OTS-based models are usually considered to be in competition with the ANS-based models and not parallel models, e.g., see Carey & Barner, 2019; but see Spelke, 2000, 2011; vanMarle et al., 2018). These considerations hint that these empirical works implicitly assume a dominant role of ANS sensitivity in symbolic number knowledge.

In the present work, we investigate the feasibility of this ANS account for subset-knowers' number acquisition. A consensus has not been reached in the literature about the representational bases of the initial acquisition of symbolic number and various arguments have been put forth as to why competing models can or cannot account for the number knowledge of subset-knowers. While there are various perspectives on the feasibility of such models (e.g., see Carey & Barner, 2019; Shusterman et al., 2016), the present work focuses on the frequently discussed and empirically tested viewpoint whether the development pace of the Weber fraction can account for the number learning of subset-knowers (Carey & Barner, 2019; Mussolin et al., 2012, 2016; Piazza, 2010; Rousselle et al., 2004; Rousselle & Noël, 2008; Shusterman et al., 2016; vanMarle et al., 2018; Wagner & Johnson, 2011, p. 201).

1.1. Empirical works investigating the role of the ANS

Several empirical works have investigated the direct relationship between ANS sensitivity and the development of symbolic number knowledge in subset-knowers, however, their results are inconclusive. In these studies, ANS sensitivity is typically indexed by a non-symbolic numerosity comparison task in which children have to decide which one of two sets of arrays is the numerically larger one, and symbolic number knowledge is typically measured by the GaN task. In some studies (Mussolin et al., 2012; Rousselle et al., 2004; vanMarle et al., 2018; Wagner & Johnson, 2011), ANS sensitivity and number knowledge correlated, while, in others (Rousselle & Noël, 2008), no such correlation was found. Some further works have suggested that, while there is a correlation, it is not the ANS sensitivity that leads to number knowledge acquisition, but it is rather number knowledge that influences ANS sensitivity (Mussolin et al., 2014, 2016), especially when children become CP-knowers (Shusterman et al., 2016). (See a more detailed review of these empirical studies in the Supplementary materials.).

Unfortunately, it must be noted that these results should be interpreted cautiously. (1) Even if there is a correlation between ANS sensitivity and number knowledge, the direction of the effect cannot be determined: Correlational studies are uninformative about causality, crossed-lagged correlational methods are debated (Mussolin et al., 2014), and a relevant longitudinal study was not designed to determine causality (Shusterman et al., 2016). (2) The results of some of the studies suggest that many subset-knowers do not necessarily understand specific versions of the non-symbolic numerical comparison tasks, resulting in random performance and as a consequence low ANS sensitivity (see a review of the relevant literature in Negen & Sarnecka, 2015): Considering that the comparison task was understood by CP-knowers, random performance in the subset-knower group contrasted with the more appropriate performance in the CP-knower group could cause illusory correlations between ANS sensitivity and symbolic number knowledge. (See analog findings in Slusser & Sarnecka, 2011.) Note, however, that there are ANS measurements where appropriate training can ensure that young children understand the task (Negen & Sarnecka, 2015; Odic et al., 2013). (3) The strong ANS hypothesis supposes that the ANS is the main source of symbolic number learning; therefore, "ANS acuity should tightly correlate with the interindividual variability of lexical acquisition of number during the first years of life" (Piazza, 2010, p. 547.). However, the observed correlations are typically smaller (r is between 0.3 and 0.6 in Mussolin et al., 2012; 0.5 and 0.6 in Rousselle et al., 2004; and 0.4 in J. B. Wagner & Johnson, 2011) than the expected strong correlation. It is possible that the potentially modest reliability of the measurements attenuates the correlation (Spearman, 1904): There may be a strong correlation between ANS sensitivity and symbolic number knowledge, but the observed correlation may be lower because the reliability of the measured variables is not sufficiently high. Indeed, some reports hint that the reliability of non-symbolic measurements are suboptimal (e.g., Clayton, Gilmore & Inglis, 2015 found that the test–retest reliability of two versions of the non-symbolic comparison tasks were 0.57 and 0.29; and Shusterman et al., 2016 reported 0.46 reliability for a similar task), although other works find satisfying reliabilities (e.g., Gilmore et al., 2011 found larger than 0.85 split-half reliability in non-symbolic comparison and addition tasks; and Smets et al., 2014 found larger than 0.88 test–retest reliability in non-symbolic change detection, comparison, and same-different tasks). Regarding the GaN task, Marchand et al. (2022) found a relatively high overall reliability for the task (0.866 of Kappa statistic), although if the question is whether a child can be categorized to a neighboring category due to measurement noise, then the reliability is low. Still, reliability is not an objective index of a task: The reliability of the task depends on, for example, the number of trials or the variance of the to-be-measured index found in a specific sample (Lindskog et al., 2013; Rouder & Haaf, 2019). Therefore, the reliability of a task measured across different studies and samples may vary. Overall, because reliability indexes were not reported in the correlational studies mentioned above, these studies are inconclusive about whether the measured correlations reflect strong correlations that are attenuated by low reliability or whether the correlations are truly weaker than expected by the strong ANS account. (4) There is an active debate in the literature on how strongly the non-numerical features of the stimuli influence the measured Weber fraction (DeWind et al., 2015; Gebuis & Reynvoet, 2012; Piazza et al., 2018) and, relatedly, whether inhibition processes influence the measured index (Gilmore et al., 2013; Clayton & Gilmore, 2015; Fuhs & McNeil, 2013). If inhibition influences the measured indexes, then the observed correlation between assumed ANS sensitivity and symbolic number knowledge may reflect the role of inhibition in symbolic number acquisition as well. Overall, it is inconclusive whether previous empirical works support the strong hypothesis of the ANS's role in the acquisition of the first symbolic numbers: The direction of the causation is unclear in correlational studies; it is unclear whether ANS sensitivity is measured validly in subset-knowers; and, if the observed correlations are artifacts, it is unclear how strongly the correlational results are attenuated by the low reliability of the tasks.[1]

To summarize, although the strong ANS account of subset-knowers' symbolic number learning is a characteristic model of numerical cognition development, empirical results on the relationship between ANS sensitivity and symbolic number knowledge are inconclusive about how strongly related ANS sensitivity and the number knowledge of subset-knowers are and whether the ANS does indeed lead the development of symbolic number knowledge.

2. Towards a more complete quantitative description of the strong ANS account for subset-knowers

It is still unclear what drives the symbolic number knowledge acquisition in subset-knowers, and the empirical works that measure

[1] Because the present work focuses on the question of whether ANS sensitivity directs symbolic number knowledge acquisition in subset-knowers, only empirical works that directly investigate this relationship are mentioned in this section. Note, however, that there are additional empirical results and arguments on the role of the ANS in subset-knowers and on its role in CP-knowers. For example, a critical test is whether or not scalar variability (the constant ratio of the mean and the standard deviation of the responses, which is a signature of the ANS representation) can be found in the responses for larger unknown numbers (Sarnecka & Lee, 2009; Wagner & Johnson, 2011), while methodological arguments suggest that this approach is not appropriate to test the role of the ANS in early number word meanings (Wagner et al., 2019). Additional related critical reviews of empirical works and arguments can be found in Mussolin et al. (2016), Shusterman et al. (2016), Wagner et al. (2019) and Carey and Barner (2019), however, these works investigate the role of the ANS in acquiring the cardinality principle, not the role of the ANS in the numerical development of subset-knowers.

the relation of ANS sensitivity and number knowledge are inconclusive. Here, we follow an alternative approach to construct a more comprehensive description of the ANS account for subset-knowers and to evaluate its feasibility. Because the ANS model has a specific mathematical description, this can be used to create a more extensive and more specific description of the ANS account for subset-knowers and investigate the conditions under which the ANS could be responsible for subset-knowers' number knowledge. First, we briefly describe the basic mathematical model of the ANS and its sensitivity. Then we highlight essential missing or mis-characterized features of the strong ANS account of subset-knowers' development. Finally, we provide a more complete mathematical description of this account and, in so doing, demonstrate that the model is not in line with the empirical observations.

2.1. Basic quantitative description of the development of ANS sensitivity

In the ANS model, numbers are processed by a noisy representation following a Gaussian distribution (Dehaene, 2007). The standard deviation and mean of the distribution are directly proportional; therefore, larger values are noisier (see Fig. 1). The standard deviation of a distribution is also related to the sensitivity of the system, which is expressed as the Weber fraction. Technically, the standard deviation of a distribution is the product of the to-be-represented value and the Weber fraction. This also means that a larger Weber fraction equals lower ANS sensitivity.

This simple model can predict performance (e.g., error rate or reaction time) in various tasks, such as the comparison task. The larger the overlap between the noisy representations of two values is, the harder the task is (Dehaene, 2007; Halberda & Odic, 2014). Therefore, the overlap of two number representations can also specify how easily two values can be differentiated.

The sensitivity of the ANS improves with age, that is, the Weber fraction becomes smaller in older children (Halberda & Feigenson, 2008; Piazza et al., 2010, but see Piazza et al., 2018). As the Weber fraction decreases, the overlap between the distributions of neighboring numbers also decreases (see Figs. 1 and 2 A). In the ANS account for subset-knowers, it is supposed that the meaning of a number word can be learned when that number can be distinguished reliably from its neighbors (Piazza, 2010), which depends on the overlap of relevant neighboring number representations. According to this account, a number is learned, as shown in Fig. 1, when the overlap between the representation of that number and its next larger neighbor is sufficiently low or, as shown in Fig. 2A, when the function of the appropriate number pair takes a lower value than a specific threshold (y value). This explanation therefore suggests that the learning pace of symbolic numbers in subset-knowers is directly and exclusively controlled by the Weber fraction's improvement, which determines whether two numbers can be discriminated reliably and enables the acquisition of specific symbolic numbers. In other words, when a sufficiently small Weber fraction (i.e., sufficiently precise ANS) is reached, the meaning of the next number word can be learned.

This explanation is supported by the empirical results summarized by Piazza (2010). Her summary reveals that the Weber fraction shows considerable improvement in infants and preschoolers, where specific Weber fractions improve with age according to a power function (see Fig. 1 in Piazza, 2010). By relying on the observed Weber fractions and their estimated developmental trajectory, that work proposes that, in the subset-knower age range, the Weber fraction improves in a way that ensures that the first few numbers will become discriminable; therefore, the ANS could be the main source of symbolic number knowledge acquisition in subset-knowers. Note that this argument does not rely on the reported correlations between ANS sensitivity and the number knowledge of subset-knowers (as discussed in the previous section), but rather it makes an inference based on the empirical data about feasible Weber fraction development.

2.2. Extending the quantitative description of the development of ANS sensitivity

A neglected problem with this strong ANS account is that some facets of the account are underspecified and partly mis-characterized. Here, we focus on two specific issues: the lack of a specific threshold for reliable number discrimination and the pace of ANS sensitivity improvement. After considering these limitations, we reevaluate the model.

2.2.1. Threshold for reliable number discrimination

While the description above is reasonable, one specific point is underspecified: The description does not define how small the overlap should be to reliably discriminate two numbers (i.e., the overlap of the distributions in Fig. 1 or the threshold value on the y-axis in Fig. 2A). In terms of a comparison task, moreover, it is not clarified what it means to be reliably discriminable: How precisely should a child discriminate between two neighboring non-symbolic numerosities (e.g., what minimum correct response rate should be reached in a comparison task) to provide a firm basis for precise symbolic number acquisition (i.e., the threshold value on the y-axis in Fig. 2B)? For example, do children only have to be able to differentiate two neighboring numbers above the random chance level, or do they have to differentiate them with almost 100 % accuracy, or somewhere in between? To the best of our knowledge, the literature has not discussed this factor explicitly, even though it is critical in the ANS account of symbolic number acquisition.

In the literature, there are a few related considerations about the required precision, which implicitly suggest some specific discrimination thresholds, although these considerations show an extremely wide range of required precision values: from 75 % mean correct responses for comparisons (Halberda & Feigenson, 2008), to somewhat lower than 75 % mean correct responses (Halberda & Feigenson, 2008; Piazza et al., 2004), to a reliable mean response rate between 77 % and 82 % (Pica et al., 2004), and even a correct response rate as high as 97.5 % (Piazza et al., 2004). (See a more detailed description of these implied number discrimination thresholds in the Supplementary materials.) In sum, although specific criteria for the discriminability of two non-symbolic values are needed for an appropriate quantitative description of the ANS explanation for several phenomena, to the best of our knowledge, there is no consensus in the literature on what the comparison performance should be or how the internal Weber fraction should be qualified

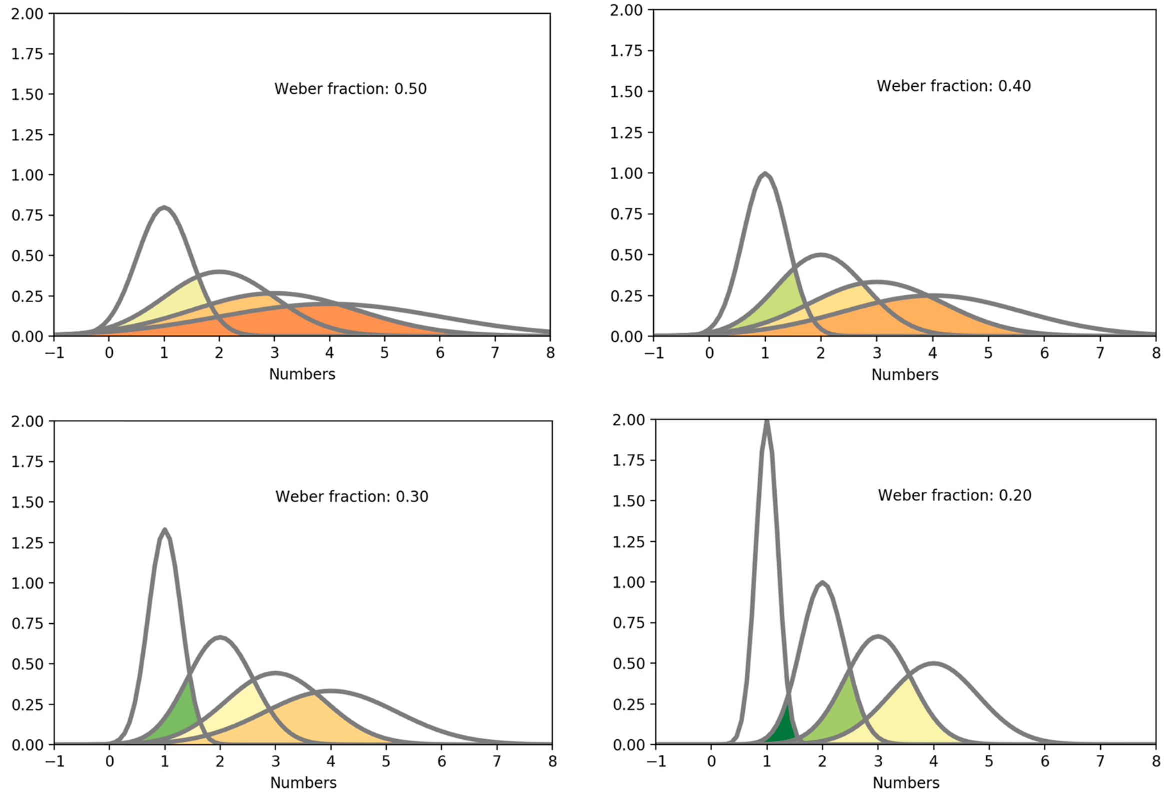

Fig. 1. Representation of the numbers 1, 2, 3, and 4 in the ANS model as a function of the Weber fraction (panels). Overlaps of the neighboring values are colored: Larger overlaps are red, and smaller overlaps are green. Find an animated version of the development at https://osf.io/rmubz/. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

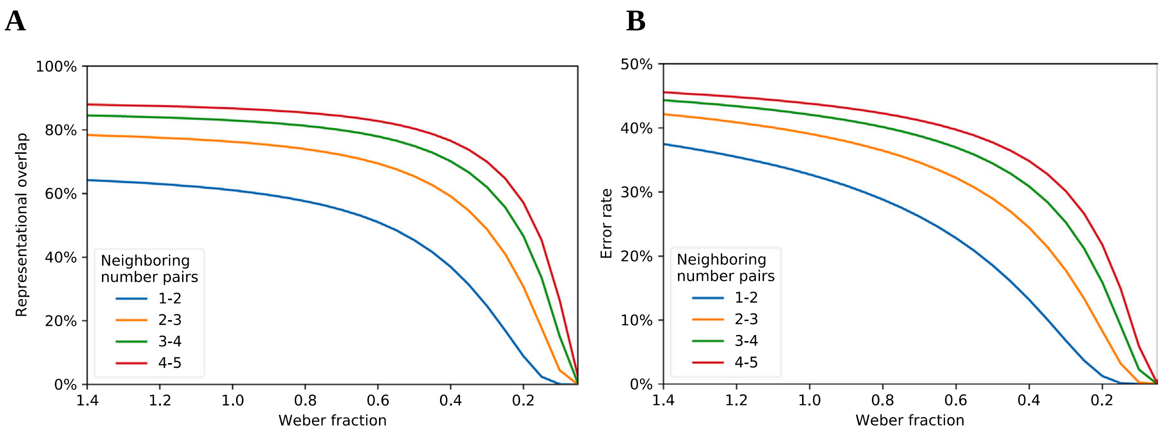

Fig. 2. A. Representational overlap as a function of the Weber fraction (x-axis) and neighboring number pairs (solid lines). Note that the values of the x-axis decrease from left to right, so that more sensitive ANS (and older children) can be found on the right. Fig. 1 and the present figure display the same information: In Fig. 1, the Weber fraction is represented by the different panels, while, here, it is shown on the x-axis; in Fig. 1, representational overlap is represented as the area intersecting two distributions (representations), while, here, it is represented on the y-axis. The representations are a Gaussian distribution, where the mean is the to-be-represented value and the standard deviation is the product of the to-be-represented value and the Weber fraction. Overlaps can be between 0 % and 100 %. B. Comparison task error rate as a function of the Weber fraction (x-axis) and neighboring number pairs (lines). The estimated error rate in a comparison task was calculated based on psychophysical models (Pica et al., 2004). Error rates can be between 0 % and 100 %, where 50 % is random guessing.[21] The calculations were implemented in the Python language in Jupyter Notebook, which is available at https://osf.io/rmubz/.

for reliable discriminability.

2.2.2. Weber fraction's increase in subset-knowers

When the feasibility of the ANS account for subset-knowers is evaluated, a critical parameter is the developmental decrease of the Weber fraction over the period of time when children are subset-knowers. Seemingly, ANS sensitivity development is well described and it is often cited (Halberda & Feigenson, 2008; Odic et al., 2013; Piazza, 2010). Nonetheless, it may still include critical issues, which we discuss below.

Importantly, it has been argued that the observed Weber fraction's development trajectory supports the idea that the development of number knowledge in subset-knowers is caused by the improvement of ANS sensitivity (Piazza, 2010). However, there are several problems with this argument. One major issue is that, in fact, the referred data do not support the proposed hypothesis. According to Fig. 1a(ii) in Piazza (2010), infants can already differentiate 1:2 and 2:3 ratios in their first year. This would mean that the youngest children who understand the GaN task and give relevant responses and who know the counting list should already be able to give both 1 and 2 sets correctly because they (a) understand the task, (b) know the first few items of the counting list, and (c) can discriminate 1 from 2, and 2 from 3, which, according to the strong ANS account, is the only necessary semantic source for understanding numbers. Consequently, one-knowers should not be observed because even the youngest children who can give adequate responses to the task should be able to give 1 and 2 correctly; therefore, they should at least be two-knowers. Even so, the literature coherently reports one-knowers (e.g., see the extensive sample reported in Sarnecka et al., 2015). A second major issue is that while the observed Weber fraction's developmental trajectory is presented, including data from infants to early teenagers (Halberda & Feigenson, 2008; Odic et al., 2013; Piazza, 2010), the data of infants may not be comparable to the data of older children in terms of the Weber fraction, and, therefore, the inclusion of infant data may bias the description of the Weber fraction developmental trajectory in older children. For example, empirical studies have demonstrated that the change detection task (used in infant studies) is harder than the comparison task (used in preschooler studies) (Gebuis & Van der Smagt, 2011; Smets et al., 2014); therefore, the Weber fraction of infants may be underestimated compared to the Weber fraction estimations of preschoolers. As another example, looking time paradigms (such as habituation tasks) that rely on hypothesis tests (used in infant studies) are not trivial to transform into forced choice paradigms that rely on effect sizes (used in preschooler studies) because effect sizes and hypothesis tests cannot be transformed into each other without knowing additional details. As a further example, different Weber fraction calculation methods lead to different values, and the difference between these methods is larger in younger children (Halberda & Feigenson, 2008). (See a more detailed description of these issues in the Supplementary materials.) These factors may contribute to a biased description of the Weber fraction's development in children. This potentially biased description may contribute to the fact that the referred developmental data (see Fig. 1a(ii) in Piazza, 2010) seemingly do not support the strong ANS account; it is possible that a revised Weber fraction development pace may provide evidence for the ANS model. However, it is also possible that a revised development pace may provide even stronger argument against the ANS model, as we argue below.

In light of this background, to evaluate the ANS account for subset-knowers, it is essential to specify the Weber fraction range that is typical in subset-knowers. However, specific Weber fraction values of subset-knowers are rarely reported in the literature (and these reports may include invalid measures, see Negen & Sarnecka, 2015). Here, with an indirect method we intend to provide a more appropriate Weber fraction range for subset-knowers based on previous reports in the literature. To find a reasonable Weber fraction range, we first specify the typical length of the period and age range when children are subset-knowers. Then, in the second step, we look for the typical Weber fraction range in that age range in the existing literature. Here, we primarily consider the typical development of children raised in Western cultures and who are from socio-economic backgrounds available in most previous studies.

To foreshadow our findings below, while the typical age range of subset-knowers can be identified, the Weber fraction measurements in that age range are more difficult to find. Still, it is possible to indirectly provide a reasonable range that safely includes the Weber fraction values of subset-knowers. We argue that, typically, the Weber fraction is not larger than 1 when children become one-knowers and not smaller than 0.3 when they are no longer four-knowers.

In the first step, based on previously published data, we specify the age range when children are subset-knowers. The largest GaN aggregated dataset of which we are aware, which includes the data of 641 children (Sarnecka et al., 2015), shows that the youngest subset-knowers are two years and three months old and the oldest subset-knowers are four years and three months old, in which dataset outliers were excluded. While there could be factors, for example, linguistic environment, that can influence the pace of symbolic number learning (Almoammer et al., 2013; Wagner et al., 2015), the data that we rely on in our estimation, are in line with the data of other smaller reported samples. Another key question is how long it takes children to develop from a one-knower to a four-knower. To our knowledge, there are only two longitudinal studies that have investigated the development of number knowledge in subset-knowers (Shusterman et al., 2016; Wynn, 1992). These studies suggest that it takes one year or less for children to advance through all the subset-knower phases. (See more detailed presentation of these studies and their relevant data in the Supplementary materials.) In summary, there is a two-year period (from soon after the second birthday to soon after the fourth birthday) when

[2] Representational overlap is a neutral index of number discrimination ability, which is independent of the task: While performance predictions depend on the representational overlap and the task-specific solution strategy (Dehaene, 2007), overlap is independent of task-specific information. On the other hand, comparison performance index is more comparable with previous findings of comparison tasks in the literature. Comparison performance can be described as an optimal maximum-likelihood-based choice that relies on the difference between two Gaussian random variables (Pica et al., 2004), applying the formula shown in footnote 4. Note that, while there is a functional relationship between representational overlap and comparison performance, it is not linear.

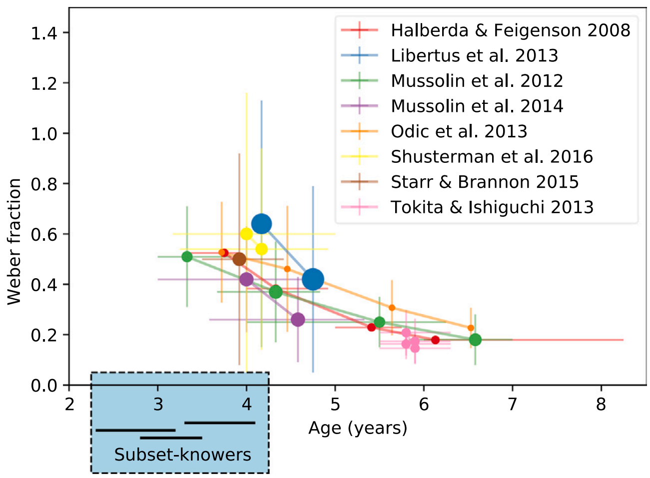

Fig. 3. Mean Weber fraction (y-axis) as a function of mean age (x-axis) in several former measurements. Vertical error bars represent the standard deviation of the Weber fraction; horizontal error bars represent the age range of the group. The size of the points represents the size of the group, with the largest sign denoting 144 participants. Connected points are from the same study. The blue rectangle on the x-axis represents the typical age range when children are subset-knowers. Horizontal lines within the blue rectangle represent possible example age ranges of subset-knowers. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

children may be subset-knowers (see the blue area displayed on the x-axis in Fig. 3), but the whole subset-knower phase takes less than a year for an individual child, at least in the cultures and from the socio-economic background available in the reviewed data (see the horizontal lines inside the blue area in Fig. 3 as possible examples).

To find a reasonable range of Weber fractions when children are subset-knowers, we collected Weber fraction values from the literature. A summary of the Weber fraction data from these papers as a function of age is shown in Fig. 3. (See the methods of the literature search in the Supplementary materials.) Fig. 3 shows that, based on the available data in the literature, it is hard to specify a feasible Weber fraction range for subset-knowers. One critical issue is that the Weber fraction of two-year-old children is not available in the literature. Second, the available data show considerable variability. Third, the Weber fraction values could be biased either because subset-knowers do not always understand the comparison task (Negen & Sarnecka, 2015; Slusser & Sarnecka, 2011) or because the Weber fraction calculation methods include strong bias in young children (Halberda & Feigenson, 2008). Still, based on the available data and taking the mentioned constraints into account, a reasonable estimate of the Weber fraction range for subset-knowers can be provided. We argue that while an estimation of the mean Weber fraction around the children's fourth birthday would be 0.4 based on the present summary, the data from Halberda and Feigenson (2008) suggest that the more appropriately estimated value is around 0.3. Second, we argue that, based on the infant data, it is safe to assume that two-year-old children's Weber

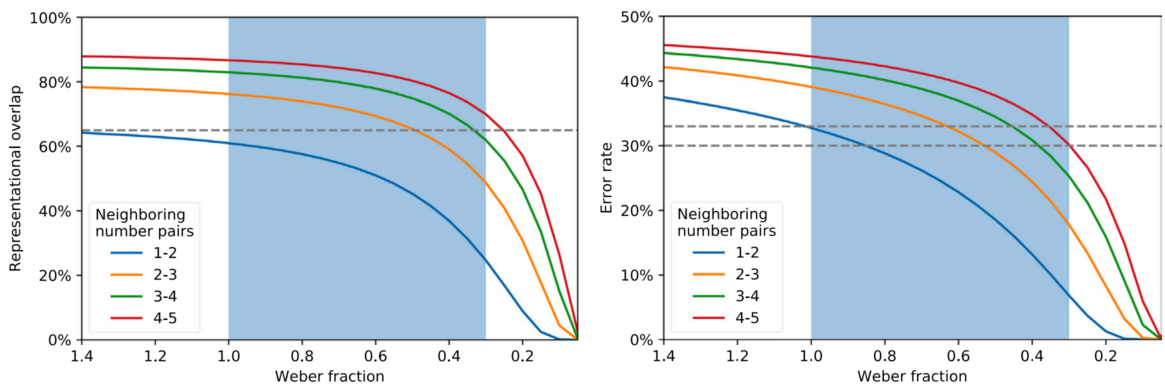

Fig. 4. This figure is an extended version of Fig. 2, where the new components are the probable Weber fraction range when children are subset-knowers (blue rectangle in the background) and possible discriminability thresholds (dashed gray horizontal lines) **A.** Representational overlap as a function of the Weber fraction (x-axis) and neighboring number pairs (lines). Note that the values of the x-axis decrease from left to right, so that more sensitive ANS (and older children) can be found on the right. The blue area denotes the probable Weber fraction range for subset-knowers. The gray dashed horizontal line denotes an example of the discriminability criterion where neighboring numbers become reliably discriminable, allowing for the meaning of smaller number words to be learned. **B.** Comparison task error rate as a function of the Weber fraction (x-axis) and neighboring number pairs (lines). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.).

fraction cannot be larger than 1. (See a more detailed reasoning about the potential issues of the measured Weber fraction data and how the potential biases can be either corrected or avoided in the Supplementary materials.).

2.2.3. Applying a reliable discrimination threshold and a feasible Weber fraction range to the ANS sensitivity development curves

In a final step, we estimate the feasibility of the strong ANS account of subset-knowers by adding the probable Weber fraction range of subset-knowers and the possible threshold for discriminability to the mathematical description of the account.

To demonstrate the change in the ability to discriminate neighboring numbers throughout subset-knowers' development, Fig. 4 shows the representational overlap (Fig. 4A) and the comparison error rate (Fig. 4B) as a function of Weber fractions and for the various value pairs. Fig. 4 is an extended version of Fig. 2, to which a reasonable Weber fraction range for subset-knowers and some possible thresholds are added.

Compared to Fig. 2, Fig. 4 includes a blue rectangle in the background, which denotes the 1–0.3 Weber-fraction range when children are subset-knowers. Note again that this range is an extensive group-based range, taking into account considerable individual differences and uncertainties due to measurement. Individual children show a much narrower Weber fraction change over the time period when they are subset-knowers.

Fig. 4 also includes horizontal dashed lines to represent the discriminability threshold. The ANS account proposes that a new symbolic number is learned when the appropriate overlap value (e.g., 1 vs. 2 overlap for understanding number 1) or the appropriate comparison error rate falls below a certain threshold (on the chart, from left to right, the point is where the function – denoted with solid color lines – goes below the dashed gray horizontal line). In Fig. 4A, an example of a 65 % overlap criterion is shown as a horizontal dashed gray line, but this line could be anywhere because it is not specified by current theories or descriptions, and it can be considered as a free parameter here.

Regarding the strong ANS hypothesis of subset-knowers' number acquisition, the question is what thresholds are feasible to have the numbers 1–4 become discriminable in the Weber fraction range for children when they are subset-knowers. In the chart, this question can be formed as follows: What is the range of representational overlaps (range along the y-axis) where a dashed gray horizontal line would cross *all* four solid color lines within the blue area?

Regarding the overlap values, Fig. 4A shows that there is no such value: It is impossible to find any specific overlap threshold that would cross all colored lines reflecting the discrimination abilities (i.e., representational overlap) of the numbers 1–4 within the Weber fraction range that is typical in that age range. In other words, within the probable Weber fraction range when children are subset-knowers, the Weber fraction does not decrease sufficiently to make all four numbers distinguishable with the same criterion. Thus, the improvement of ANS sensitivity cannot be the leading factor of symbolic number learning in subset-knowers.

Regarding the comparison task error rates, similar results can be found (see Fig. 4B).[3] Importantly, in Fig. 4B, there is a possible range of error rates where the error rate discriminability criterion may cross the performance of all number comparisons of neighboring number pairs in the probable Weber fraction range: Error rates between 30 % and 33 % (see these values displayed as two dashed gray horizontal lines). Note again, however, that the given Weber fraction range for subset-knowers is based on group data, and individual children show a much smaller Weber fraction range when they are subset-knowers: While it takes approximately two years for most children in the population to transition from 1-knowers to CP-knowers, for an individual child it takes only approximately one year. If we assume that the development of the Weber fraction can be approximated with a linear function in that age range, then the available Weber fraction range for an individual child is approximately half of the range that is estimated based on the whole group. In other words, in the chart, the width of the blue area for an individual child may be half that of the group-based blue area. Consequently, it is very unlikely that, in these much narrower development ranges for the Weber fraction, there is any criterion value for which performance in all neighboring number pair comparisons becomes lower. Therefore, we conclude that, similar to the representational overlap analysis, the error rate analysis shows that the improvement of ANS sensitivity alone is unlikely to account for the learning of the symbolic numbers 1–4 in subset-knowers.

An additional interesting property of the quantitative description can be observed in Fig. 4, which has not been discussed in the literature to the best of our knowledge: The pace of learning larger numbers should accelerate. In terms of the horizontal dashed line, while the distance between the points crossing the 1–2 and 2–3 lines is relatively long, the distances between the larger number pair lines become shorter. In other words, supposing a close-to-linear development pace of the Weber fraction in that age range, a subset-knower should need less and less time to learn the next number in their counting list. We are unaware of any empirical studies that have investigated this property, and currently available longitudinal data may be limited in terms of sample size to investigate this question reliably. This property is observable measured with either representational overlap or comparison error rate.

3. General discussion

The present work has provided an extensive discussion and a more complete quantitative description of the strong ANS hypothesis of numerical development in subset-knowers, which proposes that the ANS is the main driving factor in the acquisition of the first symbolic numbers. Unlike former accounts that propose a simpler qualitative description of how, with the improvement of ANS

[3] Note again that representational overlap is a neutral index of the discriminability of two numbers independent of the task, while comparison task performance is a functional consequence of the overlap. Although there is a functional relationship between representational overlap and comparison performance, it is not linear; therefore, the present analysis should not necessarily provide entirely similar results compared to the results of the representational overlap.

sensitivity, new numbers can be learned, the present description utilizes the mathematical description of the ANS model and applies feasible parameters to the model. Our analysis presents the discriminability (implemented as either a representational overlap or an error rate in a comparison task) of neighboring number pairs in a feasible Weber fraction range for subset-knowers.

We found that not a single discriminability criterion can be found that would allow the acquisition of all four first numbers in the given Weber fraction (and age) range. In other words, the strong ANS hypothesis requires a larger Weber fraction change in subset-knowers than what has been empirically observed to make all four first numbers distinguishable. To put it another way, we found that, when children are subset-knowers, the Weber fraction does not decrease sufficiently to make all four numbers distinguishable with the same criterion. This means that, in its current form, it is unlikely that the ANS can be the leading factor for the numerical development of subset-knowers. This result is in line with former arguments (e.g., Carey & Barner, 2019) that question the leading role of the ANS in subset-knowers' acquisition of symbolic numbers (note, however, that some of these arguments only apply to understanding the cardinality principle and not subset-knower development, which is what is addressed here).

Although the current form of the strong ANS account cannot support subset-knower development, there may be modifications that would make the model compatible with the data or make the data compatible with the model. (1) While several former studies have reported the Weber fraction in children, there is an active debate on how the Weber fraction should be measured, such as how the perceptual correlates of numerical information can be handled, how measuring inhibition components can be avoided, which psychophysical fitting method is appropriate, how it can be ensured that young children understand the task, and when slope measurements are appropriate (DeWind et al., 2015; Dietrich et al., 2015; Gebuis & Reynvoet, 2012; Halberda & Feigenson, 2008; Krajcsi, 2020; Leibovich & Ansari, 2016; Negen & Sarnecka, 2015; Smets et al., 2014). It is possible that some previous works measured the Weber fraction incorrectly; and that future measurements and results will offer a different Weber fraction range, modifying the present conclusion. Relatedly, when measuring number knowledge, several works using different versions of the GaN task may have provided biased results or even the use of the GaN task to measure children's number knowledge can be questioned in general (Sella et al., 2021), which, in turn, could affect the feasible Weber fraction range provided here. (However, to be consistent with the literature, here, the development pattern measured with the GaN task was used. At the same time, alternative extensive data measured with other tasks are not available in the literature.) In terms of Fig. 4, these modifications would mean a modification of the blue area. (2) The present description supposes that a single threshold is applied to all values. However, it is possible that the threshold can be increased for larger numbers, that is, in Fig. 4, the dashed gray lines would not be horizontal but would instead show a positive slope, or the lines might not be even linear. This version could be justified with an argument that, after the first numbers have been learned, less strict criteria could be used to learn the next numbers. With an appropriately high slope, it is possible that all four first numbers could be learned within the given Weber fraction range. Note, however, that this version is not supported by any evidence at the moment, and it is entirely speculative; this possibility is only considered here to make the ANS account compatible with the existing parameters. Overall, future research may change the relevant Weber fraction range for subset-knowers (in the figures, the blue area) or the role of the threshold (the dashed gray line). However, because the psychophysical model is the defining feature of the ANS (Dehaene, 2007), the relationship between the Weber fraction and the discriminability of the number pairs (in the figures, the solid color lines) cannot be modified without substantial changes to the ANS model itself. (3) Theoretically, it is possible that pre-knowers cannot give any set appropriately because they do not understand the task, not because they do not understand 1. It is possible that pre-knowers already know 1 (i.e., they can discriminate 1 and 2; therefore, in this sense, they are one-knowers) while not understanding the task, so the GaN task categorizes them as pre-knowers. Later on, when they understand the GaN task, their performance improves, and they seemingly become one-knowers, not because they learned 1, but because they now understand the task. In this scenario, the knowledge of 1 should be removed from the ANS description (i.e., in Fig. 4, the color line 1 vs. 2 should be ignored) because this seeming understanding of 1, as measured using the GaN task, depends on understanding the GaN task itself, not on discriminating 1 vs. 2. Importantly, this is a theoretical possibility at the moment, and empirical data is needed to justify this potential modification of the ANS account.

However, on the other hand, it is also possible that some other consideration will make it even less probable that the ANS model accounts for the subset-knowers' development. For example, Piazza et al. (2018) demonstrated that, in preschoolers, number comparison mainly improves because children start to ignore nonnumerical features of the stimuli and, instead, start to focus on numerical information, that is, not because their Weber fraction improves. If this finding is correct, then the Weber fraction change should be even smaller than estimated in the present and previous works. Therefore, it is even less likely that the ANS can be the leading factor in the development of subset-knowers. Another example is that, in some languages, number words are limited only to the first few values; and, in such cases, number knowledge, when measured with the GaN task, can be delayed (Piantadosi et al., 2014). However, the older a child is, the less Weber fraction change can be expected (see Fig. 3). Therefore, if, in those cultures, the ANS development is not delayed similar to the number knowledge delay, then the ANS sensitivity change can be even smaller during the time period when children are subset-knowers, which would question the role of the ANS in the initial acquisition of the first number words.

Overall, while the specific details of the present description could be modified by future research, and, consequently, the results of the future works might modify the present conclusions, an evaluation of the ANS account of subset-knowers should be at least as specific as demonstrated in the present work.

While the present analysis and argument has focused on the strong ANS hypothesis of subset-knowers' number acquisition, this analysis cannot exclude weak ANS hypotheses. If the ANS is hypothesized to have only a limited role (if it has any) in learning the meaning of number words, then the analysis we provided here cannot be conclusive about the model. It must also be highlighted that a weak ANS hypothesis means that, if the ANS is not central to the acquisition of symbolic numbers, then some other representation should take this role. While the ANS model is often mentioned as a core and essential mechanism in understanding numbers, a weak ANS hypothesis means that it may only have a supplementary role in subset-knowers. For example, ANS has been proposed as an error-detection system to find grossly incorrect results in symbolic arithmetic (Feigenson et al., 2013; Wong & Odic, 2021).

To conclude, based on the present mathematical description of the ANS model, extended with the use of reasonable, relevant parameters of subset-knowers that could be derived from previous works, we found that the improvement of ANS sensitivity alone cannot explain the learning of the first four symbolic numbers in subset-knowers. While it is possible that the initial parameters in our analysis are imprecise or biased and that a model with updated parameters based on future works will lead to a different conclusion about the possible role of ANS development in subset-knowers, the current data available in the literature do not support the strong hypothesis that the ANS is the leading factor in learning the meaning of the first number words.

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Conflict of interest disclosure

There is no conflict of interest to report.

Data Availability

No data was used for the research described in the article.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cogdev.2022.101285.

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