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Warren Barrash
Boise State University

Tom Clemo
Boise State University

Michael D. Knoll
Boise State University

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BOISE HYDROGEOPHYSICAL RESEARCH SITE (BHRS): OBJECTIVES, DESIGN, INITIAL GEOSTATISTICAL RESULTS

Warren Barrash, Tom Clemo, and Michael D. Knoll
Center for Geophysical Investigation of the Shallow Subsurface
Boise State University, Boise, Idaho 83725

ABSTRACT

The Boise Hydrogeophysical Research Site (BHRS) is a wellfield developed in a shallow, coarse (cobble-and-sand), alluvial aquifer with the goal of developing cost-effective methods for quantitatively characterizing the distribution of permeability in heterogeneous aquifers using hydrologic and geophysical techniques. Responses to surface geophysical techniques (e.g., seismic, radar, transient electromagnetics) will be calibrated against a highly characterized control volume (the wellfield) with 3-D distributions of geologic, hydrologic, and geophysical properties determined from extensive field measurements. Also, these data sets will be used to investigate relationships between properties and to test petrophysical models. Well coring and construction methods, and the well arrangement in the field, are designed to provide detailed control on lithology and to support a variety of single-well, crosshole, and multiwell geophysical and hydrologic tests. Wells are screened through the cobble-and-sand aquifer to a clay that underlies the BHRS at about 20 m depth. In addition, the wellfield design optimizes well-pair distances and azimuths for determination of short-range geostatistical structure. Initial geostatistical analysis of porosity data derived from borehole geophysical logs indicates that the omnidirectional horizontal experimental variogram for porosity (possible proxy for log permeability) is best fit with a nested periodic model structure.

INTRODUCTION

Permeability is the most significant aquifer parameter for quantitatively describing or modeling groundwater flow and contaminant transport, and for designing remediation systems. A number of workers have shown the potential of supplementing sparse, expensive, direct permeability measurements with geophysical surveys that can be conducted rapidly and inexpensively (e.g., McKenna and Poeter, 1995; Hyndman and Gorelick, 1996). Recent work at a groundwater remediation site in downtown Boise, Idaho has investigated a variety of geophysical and hydrologic techniques and parameter relationships in a coarse, unconsolidated alluvial aquifer (e.g., Barrash et al., 1997a; Barrash and Morin, 1997). Currently, Boise State University is developing the Boise Hydrogeophysical Research Site (BHRS) as a 3-D control volume in a natural heterogeneous aquifer to support research on use of non-invasive geophysical techniques with hydrologic measurements to map variations in permeability in shallow alluvial aquifers (Barrash and Knoll, in press). This paper briefly introduces the BHRS and discusses site design and initial geostatistical analysis of porosity data based on geophysical logs. Companion papers at this symposium (Clement and Liberty, 1999; Clement et al., 1999a,b; Knoll et al., 1999; Liberty et al., 1999; Peretti et al., 1999; Peterson et al., 1999) present geophysical testing strategies and initial results from the BHRS.

BOISE HYDROGEOPHYSICAL RESEARCH SITE

The BHRS is located on a gravel bar adjacent to the Boise River (Fig. 1) about 15 km east of downtown Boise where the river leaves a canyon and enters the broad western Snake River Plain. Eighteen wells were emplaced in 1997 and 1998 using a coring and construction method (Fig. 2)

which resulted in >83% core recovery of the coarse, cobble-and-sand, alluvial deposits (including the sand matrix with cobbles) and which minimized disturbance to the formation next to the wells (Barrash and Knoll, in press). Coring results indicate that the cobble-and-sand deposits are 18-20 m thick at the BHRS and are underlain by a very thin layer of basalt and a clay that is >3 m thick (Fig. 2). Ground penetrating radar profiles across the site (Peretti et al., 1999) show a sequence of laterally variable (braided-stream) deposits separated by subhorizontal bounding surfaces.

Design for Geophysical and Hydrologic Testing

The wellfield includes 13 wells in a central area (~20 m diameter) for detailed testing, and five wells at some distance from the central area for information on hydraulic gradient and boundary conditions during tests (Fig. 1). The range of tests planned at the BHRS includes geophysical and hydrologic tests in individual wells (1-D), between wells (2-D and 3-D), at the surface (Fig. 3), and tests that combine excitation and sensing at the surface and in wells (Barrash and Knoll, in press; Clement et al., 1999a,b). Redundancy in methods or parameters measured (Fig. 3) is intended to investigate scale effects in the aquifer and to provide control or calibration for a given parameter that can be measured at successive scales (e.g., borehole measurements calibrate crosshole measurements which calibrate surface profile measurements along a transect between boreholes).

The central area of the wellfield has a double-ring pattern with a central well surrounded by two rings of six wells each, with wells in each ring offset from each other at 60° radial angles, and with wells offset 30° between rings (Fig. 4a). This design provides a dense network for single-well and tomographic profiles, for hydrologic characterization, and for recognizing anisotropy in hydrologic and geophysical parameters. Wells are screened through the cobble-and-sand aquifer so any interval(s) can be isolated with packers to measure head changes and/or collect water samples. The wellfield also is designed to support a wide variety of pumping and tracer tests for 3-D characterization including “hydrologic tomography” (Barrash and Knoll, in press).

Design for Geostatistical Characterization

Within the constraints of having 13 wells in two rings around a central well at regular angular spacings, the exact placement of wells was determined with the additional objective of supporting geostatistical analysis of hydrologic and geostatistical parameters. We used a Monte Carlo procedure to find the best configuration of wells for defining short-range correlation structure (i.e., rising limb of horizontal variograms) for hydrologic parameters assuming a correlation length for porosity and log permeability of about 7±2 m for a cobble-and-sand aquifer (e.g., Jussel et al., 1994; Barrash and Knoll, in press). Search criteria included finding the configuration of wells that provides: (1) a smooth distribution of numbers of well pairs at 1 m intervals from 2.5 to 10.5 m separations; (2) a smooth azimuthal distribution of numbers of well pairs in each lag grouping of well pairs; and (3) a minimum of one well pair in each azimuthal octant for lag groupings in the rising limb of horizontal variograms for hydrologic parameters. The latter two criteria are included to increase the possibility of recognizing anisotropy. Figure 4 shows the resulting central wellfield design, and the distribution of well pairs at 1 m lags.

GEOSTATISTICAL STRUCTURE

Initial analysis of geostatistical structure of properties in the cobble-and-sand aquifer at the BHRS follows the common procedure of generating an experimental variogram based on measurements of a parameter of interest, and then fitting and interpreting model variograms. In this

paper we make the simplifying assumptions that the parameter population is normally distributed and that the classical formulation (Journel and Huijbregts, 1978) is a suitable estimator of the variogram. For brevity, we limit our analysis to the horizontal omnidirectional variogram. The parameter analyzed here is porosity derived from neutron geophysical log measurements. Porosity is used because it is a fundamental hydrologic parameter and because abundant data are available from logs in all 18 wells at the BHRS. Also, porosity may be a proxy parameter (Phillips and Wilson, 1989; Lahm et al., 1985) for log permeability in some sedimentary aquifers. Direct permeability measurements in wells are planned for the 1999 field season.

Porosity values were derived in the following manner. Neutron count rates at 6 cm (.2 ft) intervals were smoothed with a 5-point moving average (comparable to dimension of the tool's volume of investigation). The portion of the well logs from saturated cobble-and-sand deposits (i.e., not including basalt or clay at the base of the wells) was transformed using a common petrophysical relationship (e.g., Rider, 1986, p. 110) and assuming a porosity range of .12 to .50 based on measurements on similar sediments elsewhere (e.g., Jussel et al., 1994; Pettijohn et al., 1973; Barrash et al., 1997b). The data set consists of 4698 measurements (Fig. 5) with locations tied to surveyed reference points at wells and adjusted with data from deviation logs.

Horizontal Omnidirectional Variogram

A sequential approach was used to generate a horizontal omnidirectional experimental variogram for porosity in the cobble-and-sand aquifer at the BHRS with a minimal amount of scatter and with quantified confidence limits in order to improve interpretability and fitting of the experimental variogram with a model. We follow the standard practice of presenting and analyzing data to approximately half the distance between the greatest spacing of available data pairs (e.g., Journel and Huijbregts, 1978) across the BHRS site. The initial horizontal variogram was generated with the GAMV3 program in GSLIB (Deutsch and Journel, 1992) using 1 m lag spacing and .5 m lag tolerance (Fig. 6a). Short-range behavior appears reasonably well defined in lags to 11 m because of the large number of well pairs, but the variogram exhibits a high degree of scatter at data pair separations greater than 11 m where lags have relatively few data pairs (Figs. 3b and 6a).

Scatter was decreased at lags greater than 11 m (Fig. 6b) by increasing the number of data pairs per lag with a routine that was added to GAMV3 to rebin adjacent lags until a user-specified minimum number of data pairs were included in a given lag. Previous workers similarly have noted the relative importance of increasing the number of samples per lag (Russo and Jury, 1987; Warrick and Myers, 1987) compared with maintaining an even spacing of lags. The experimental horizontal variogram for porosity at the BHRS (Fig. 6b) now suggests geostatistical structure with spatial correlation of 15-20 m, and perhaps also a longer wave-length periodic structure or trend. Although we analyze longer-range behavior with transitional and periodic structures only in this paper, analysis of this scale of behavior with trend analysis (e.g., Rajaram and McLaughlin, 1990; Rehfeldt et al., 1992) is in progress.

Some perspective on the significance of the apparent periodic structure or trend is gained by using the jackknife method (e.g., Shafer and Varljen, 1994; Davis et al., 1997) to add ~95% confidence limits on the variogram (Fig. 6c). The jackknife method was run 4698 times by successively leaving one data point out of the variogram analysis. With the 95% jackknife confidence limits, the experimental variogram may be interpreted either as having scatter about a constant sill at ~0.0032 (Table 1) or as having a periodic structure. Such longer-range periodic structure may be relatively common in alluvial sedimentary aquifers based on visual observation of

variograms from some well-studied aquifers and sites (e.g., Goggin et al., 1988; Rehfeldt et al., 1992; Ritzi et al., 1995; Davis et al., 1997).

Several types of geostatistical models were fit to the BHRS experimental variogram using the least squares method with the jackknife confidence limits for weights (Figs. 7-8). In particular, spherical and exponential models are better fits than a Gaussian model, but all three transition models have similar ranges. A model with a nugget and a periodic structure (decaying sine [Journel and Huijbregts, 1978, p. 168]) also is a better fit in the least squares sense compared with the transition structure models. Parameters for these models are given in Table 1. Of some concern are the relatively large ranges (12-22 m) associated with these models which are greater than ranges in: (a) the similar but more loosely constrained cobble-and-sand aquifer at the Capital Station site nearby in Boise, for porosity (Barrash and Knoll, in press); (b) a cobble-and-sand aquifer in Switzerland, for permeability (Jussel et al., 1994); and (c) other alluvial aquifers, for permeability (e.g., Gelhar, 1993, Table 6.2), where horizontal correlation lengths are in the 3-12 m range.

Closer examination of the short-range region (<12 m in Fig. 6d) suggests that the decrease in variogram values or "hole" at ~5 m may be an indication of a "correlation length" for short-range periodic structure (similar to: Prosser et al., 1995; Jensen et al., 1996) rather than part of a pattern of random scatter in a monotonically increasing (transition) structure. To examine the possibility that both short-range and longer-range structure are present in the experimental variogram, least squares fits were run for: (a) nested structures including each of three transition structures with a periodic structure (Fig. 8a), and (b) nested periodic structures (Fig. 8b). It should be noted that local least squares minimization would not occur in the combined transition structure-periodic structure models unless the initial estimate of the periodic structure was applied to the short-range behavior. Overall, the best fit visually and in the least squares sense for all the models tested is the nested periodic structures model (Fig. 8b, Table 1).

We recognize that the experimental variogram may be interpreted in several additional ways. For example, a finite-scale fractal noise model (Hewett, 1993), included in Fig. 8b for reference, is compatible with a nested periodic model if the nested periodic model is seen as a local realization of a global (fractal) structure (e.g., Molz and Boman, 1993). Alternatively the longer-range periodic portion of the variogram may represent a trend or local deterministic features. Such possibilities, as well as vertical and horizontal anisotropic geostatistical structure, are currently being investigated.

ACKNOWLEDGMENTS

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Table 1. Geostatistical Models and Parameters

<i>Model</i>	<i>Single or Short Range Structure</i>			<i>Longer-Range Structure</i>			<i>RSS</i>
	<i>Sill</i>	<i>a (m)</i>	<i>Range (m)</i>	<i>Sill</i>	<i>a (m)</i>	<i>Range (m)</i>	
Spherical	0.0032	17	17				0.00393
Exponential	0.0032	7.5	22				0.00394
Gaussian	0.0031	7.1	12				0.00455
Periodic	0.0032	4.4	34*				0.00531
Nugget+Periodic	(Nugget = 0.0011)			0.0021	4.8	38*	0.00286
Spherical+Periodic	0.00068	0.64	5.0*	0.0025	21	21	0.00359
Exponential+Periodic	0.00030	0.73	5.7*	0.0029	8.5	26	0.00390
Gaussian+Periodic	0.00099	0.73	5.7*	0.0022	10	17	0.00352
Periodic+Periodic	0.0010	0.74	5.8*	0.0021	4.8	38*	0.00273
Fractal	0.0405	3.03	NA				0.00430

where: range(spherical)=a; range(exponential)=3a; range(Gaussian)= $a\sqrt{3}$ (Deutsch and Journel, 1992); "range"(periodic)= $5\pi a/2$

*NOTE: for periodic model, lag to "hole" is given as "range" value in table

Model Structure:

Spherical: $\gamma(h) = c \{ (3h/2a) - (h^3 / 2a^3) \}$

Exponential: $\gamma(h) = c \{ 1 - \exp(-h/a) \}$

Gaussian: $\gamma(h) = c \{ 1 - \exp(-[h/a]^2) \}$

Periodic: $\gamma(h) = c \{ 1 - \sin(ha^{-1})/ha^{-1} \}$

Finite-Scale Fractal Noise: $\gamma(h) = ca^{-2} \{ 1 - .5(|[h/a]+1|^{2H} - 2|[h/a]|^{2H} + |[h/a]-1|^{2H}) \}$

where: c=sill, a=model characteristic length, h=lag, H=fractal co-dimension (Hewett, 1993)

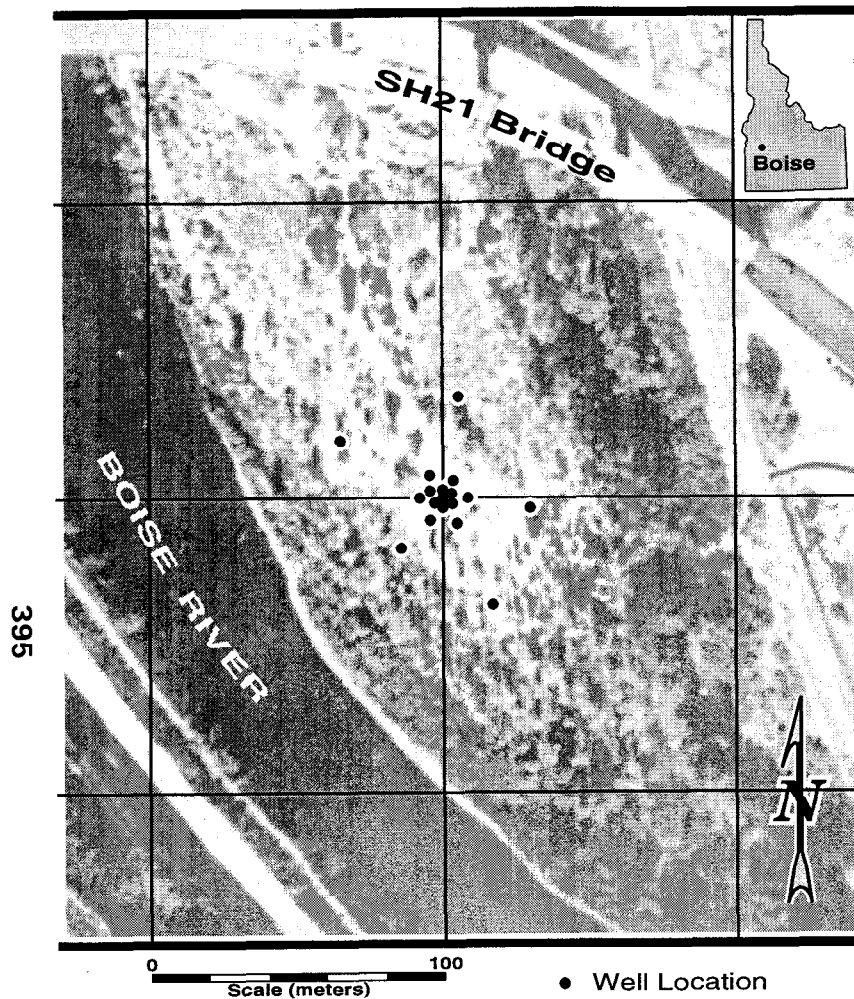


Figure 1. Photomap of the Boise Hydrogeophysical Research Site (BHRS) on a gravel bar adjacent to the Boise River ~15 km east of downtown Boise, Idaho. Wellfield includes 13 wells in the central portion of the field and five boundary wells. Flow in the Boise River at this location is to the northwest.

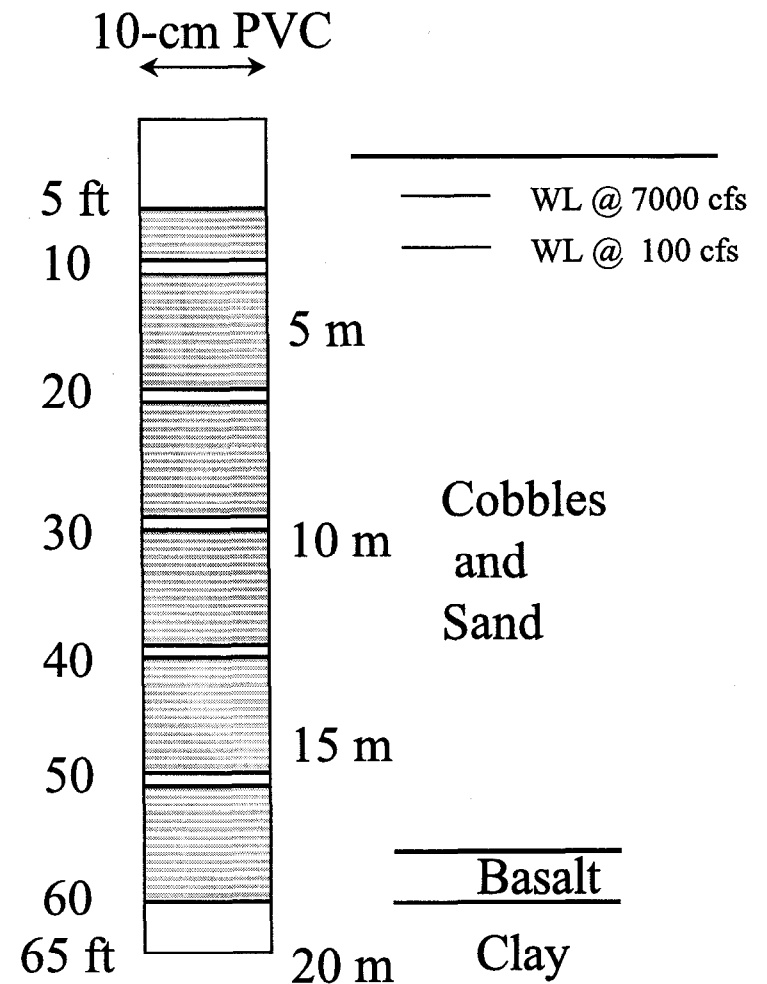


Figure 2. Wells at the BHRS were constructed with 10-cm (4-inch) ID PVC blank casing at the top and bottom, and slotted casing against the saturated portion of the cobble-and-sand aquifer. A thin layer of basalt underlies most of the site, and a tight red clay is continuous beneath the site at about 20 m BLS.

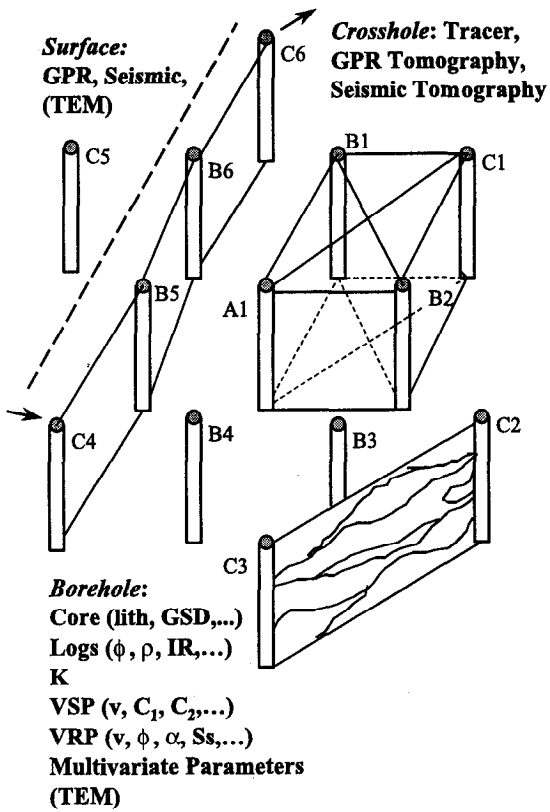


Figure 3. Schematic diagram of geophysical testing methods. For GPR and seismic methods, borehole measurements calibrate crosshole measurements, and borehole and crosshole measurements calibrate surface measurements (see also Barrash and Knoll, in press). TEM apparent resistivity soundings made prior to well coring and installation will be compared with induction resistivity logs in the wells.

Figure 5. Histogram of porosity (based on 4698 neutron geophysical log data points) from 18 wells at the BHRS.

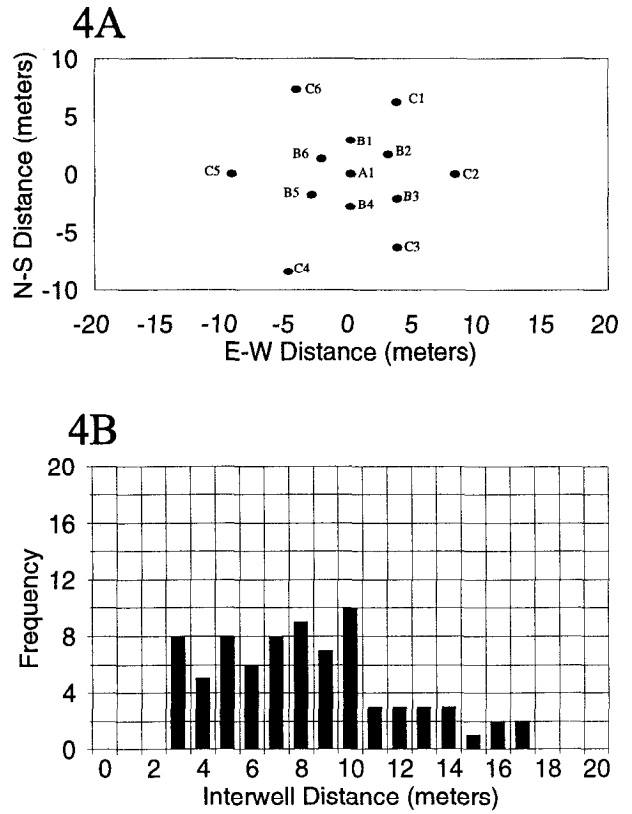
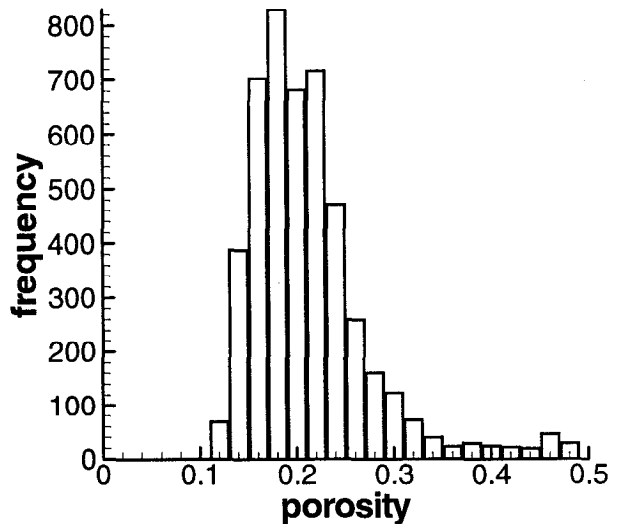


Figure 4. (A) Design of central well area based on a Monte Carlo search for a two-ring configuration. (B) Histogram of well-pair distances at 1-m lags shows even distribution through the expected range (i.e., rising limb) of the horizontal variogram for porosity and, perhaps, log permeability.



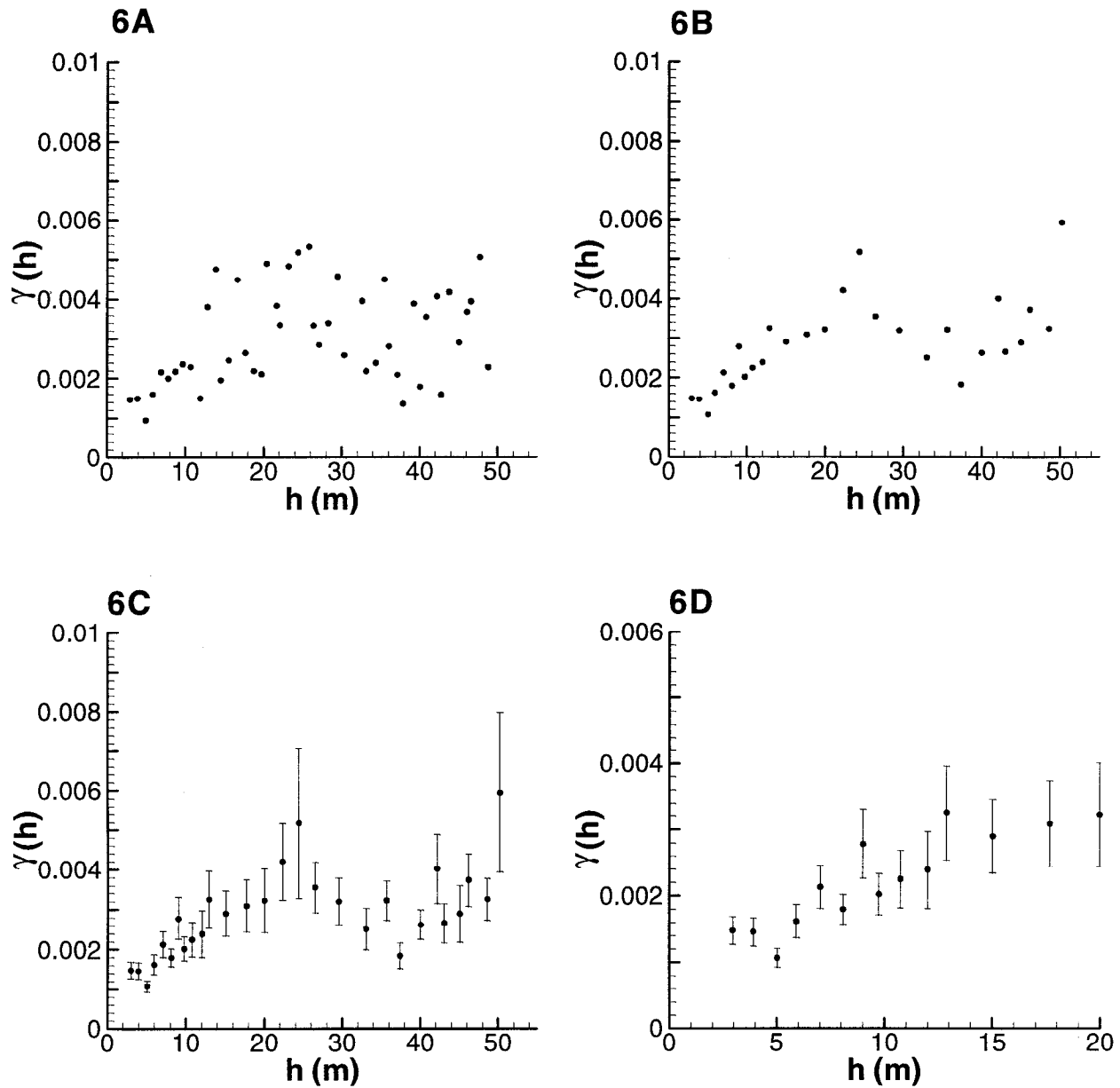


Figure 6. Experimental horizontal omnidirectional variogram for porosity at the BHRS. (A) Variogram for data at 1-m lags (.5-m tolerance). (B) Variogram for data at 1-m lags (.5-m tolerance) and with rebinning of adjacent lags when a given lag has fewer than 14,000 data pairs. (C) Variogram for data at 1-m lags (.5-m tolerance) with rebinning and 95% jackknife confidence intervals. (D) Variogram for data at 1-m lags (.5-m tolerance) with rebinning and 95% jackknife confidence intervals for short-range data.

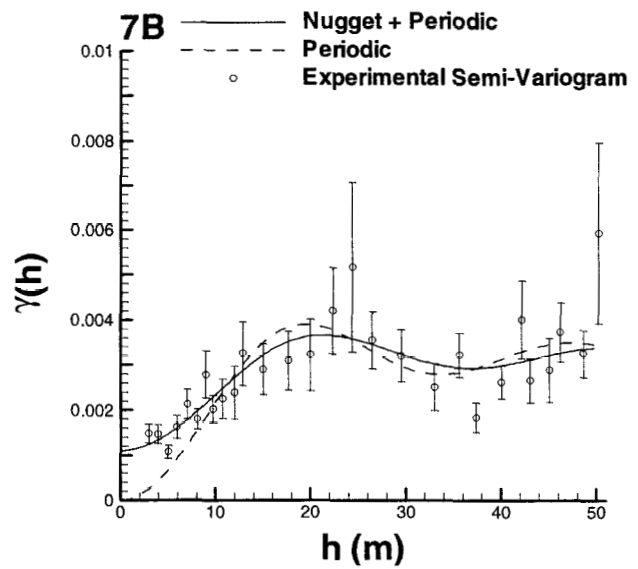
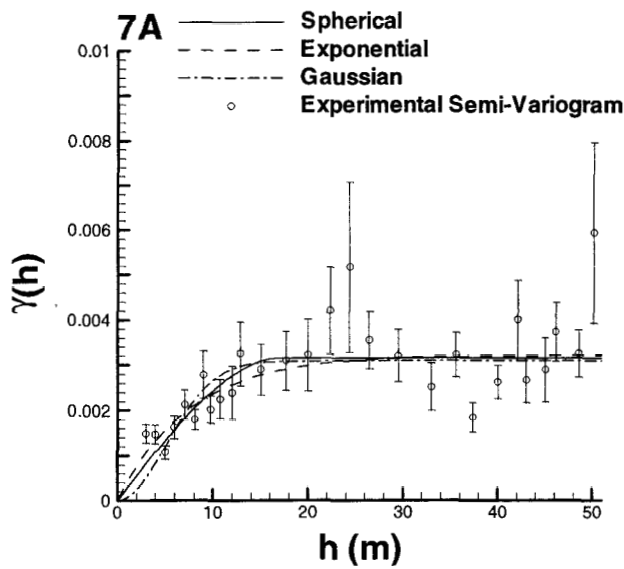


Figure 7. Least squares fits of single-structure models to the experimental variogram. Experimental variogram was generated using rebinning and shows 95% jackknife confidence intervals (see Fig. 6). (A) Transition models. (B) Periodic models with and without a nugget.

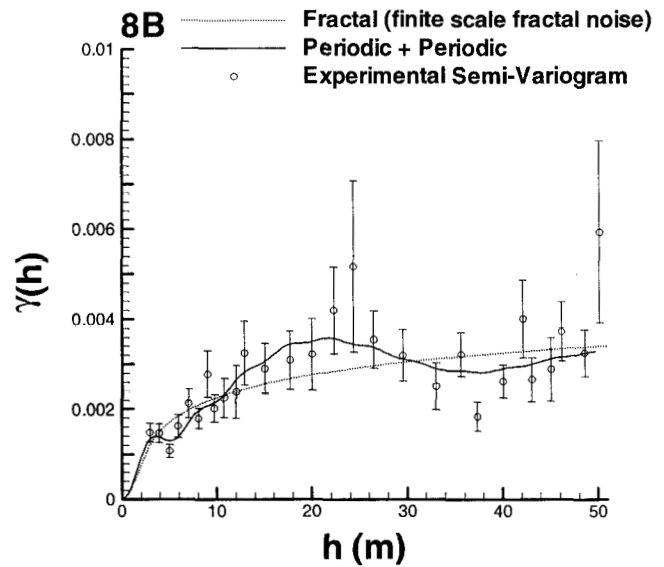
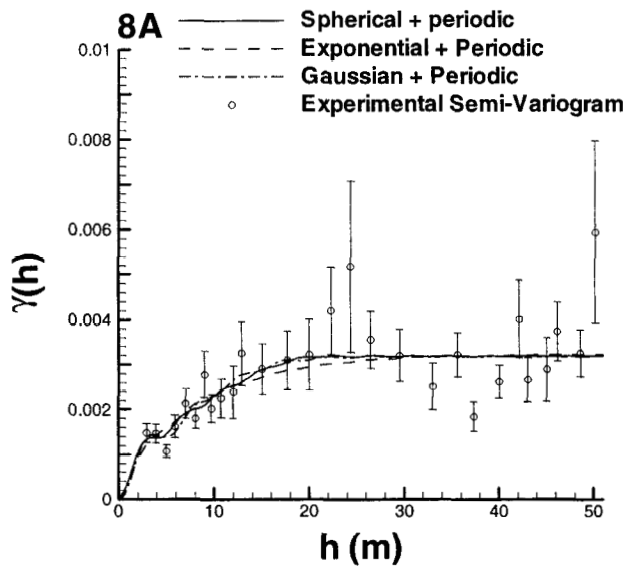


Figure 8. Least squares fits of nested-structure models to the experimental variogram. Experimental variogram was generated using rebinning and shows 95% jackknife confidence intervals (see Fig. 6). (A) Combined transition and periodic models. (B) Nested periodic model and fractal ("continuously nested") model.