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BUILDING STUDENT'S MATHEMATICAL CONNECTION ABILITY IN ABSTRACT ALGEBRA: THE COMBINATION OF ANALOGY-CONTRUCTION-ABSTRACTION STAGES

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Abstract. The objective of the study was to describe the effect of six types of mathematical connections (representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchy connections) on abstract algebraic materials through four stages, i.e., abstraction, analogy-abstraction, construction-analogy, and construction. The study employed qualitative descriptive approaches, including tests, questionnaires, and interviews. The subjects of the study were chosen based on the responses to a questionnaire regarding the employed stages. Then, two subjects who could converse and were willing to be interviewed were chosen from each stage. Data collection techniques were conducted through four stages, i.e., 1) identifying the stages used; 2) identifying the ability of six types of student mathematical connections through predictive indicators; 3) describing the capabilities of the six types of connections through interviews; and 4) conducting source triangulation and method triangulation. The results indicated that the subjects who utilized the construction stage tended to be able to construct six types of mathematical connection links in a set, as well as standard and non-standard binary operations. The subjects who utilized the construction-analogy stage likely to be able to build three forms of representation connections, structural connections, and procedural connections in a set of standard binary operations. In characterizing the symbol of a set element and the binary operation of the standard form inside the closed property of the standard form, the subjects who used the analogy-abstraction stage have the same tendency as subjects who use the abstraction-construction stage.

Keywords**:** Abstract Algebra; Mathematical Connection Ability; Abstraction; Analogy-Abstraction; Construction-Analogy

I. INTRODUCTION

Abstract algebra is a required course for those pursuing a degree in mathematics education. It is the generalization of school algebra (Findell, 2001). Furthermore, abstract algebra is defined as a collection of advanced algebraic issues connected to algebraic structures rather than ordinary number systems (Renze & Weisstein in Suominen, 2015, p. 26). Because abstract algebra is a continuation of basic algebra, it is vital to study as an advanced algebra that will be utilized in school to instruct potential teachers.

Abstract algebra 1 and abstract algebra 2 comprise the abstract algebra course. These two courses are interconnected. The content of abstract algebraic matter is made up of binary sets and operations that are linked to various definitions and theorems that bind together various notions. Students have difficulties connecting because of the multiple definitions and theorems that bind to each notion in abstract algebra. For example, students may comprehend a group definition and various examples of groups from a set with particular binary operations, but when asked to prove groups from other examples, they are too confused to begin working on them (Junarti et al, 2019a). This student's incompetence is viewed as a mathematical object that is radically different from mathematics classes taught at prior schools (Junarti et al, 2020a). Students' challenges stem from a lack of established links between university mathematics and school mathematics (Cook, 2012). The mathematical relationship between school mathematics and university mathematics is a mathematical content connection.

Connection to the content Being a link item to high school learning can imply that there is a significant relationship between advanced mathematics subject and school mathematics content, such as the relationship between groups in abstract algebra and functions in high school mathematics that do not have inverses (Wasserman & Galarza, 2018). Some of these mathematical connections between abstract algebraic content and school math curriculum include the fact that school mathematical objects, particularly groups, rings, and fields, are frequent examples of algebraic structures studied in abstract algebra (Wasserman & Galarza, 2018).

Another challenge is for students to abstract the properties of the general group concept from specific examples (Dubinsky et al, 1997). Knowledge of the concept of groups should include comprehension of the various mathematical properties and independent constructions of specific examples, as well as the fact that the group is made up of undefined elements and binary operations that match the group's axioms (Dubinsky et al, 1994). The definition includes undefined elements and binary operations that match the axioms of this group. When this definition is abstracted, the properties of its general group concept are gained through specific examples; this stage is one in the process of constructing a mathematical connection.

Suominen (2015; 2018) investigated the types of mathematical connections in several abstract algebraic books, opinions of teachers, and experts into five categories of connections, namely: (1) alternative representation, (2) comparison through general features, (3) generalizations, (4) hierarchical connections, and (5) real-world applications. Junarti et al (2020b) classified the findings of their literature review into six categories: 1) representing, 2) knowing the structure, 3) being able to carry out the process, 4) being able to imply, 5) being able to generalize, and 6) being able to generate answers based on a hierarchical order.

Mathematical connections can be constructed between pre-existing systems or networks to help students understand mathematical concepts (Suominen, 2015; Suominen, 2018). As a result, building a relationship between previously known mathematical ideas and those that are not yet known (or new mathematical ideas) for students is vital in generating connections between content in the material prerequisites of abstract algebra to support the next material. Students must engage in mathematical activities that allow them to make connections between existing knowledge and new ideas that are not yet understood in order to learn new mathematical concepts (Suominen, 2015). It is designated for knowledge with numerous links that require good conceptual comprehension.

According to the studies above, the ability to connect between materials or between content in learning abstract algebra is critical for improving knowledge. Furthermore, steps of analysis that lead to axiomatic deduction are required when learning abstract algebra. Any axiomatic deduction method necessitates an abstraction step. The concept of a general group, like property, necessitates a process of abstraction from specific examples (Dubinsky et al, 1997). According to Novotná et al (2006), there are three stages that every learner goes through when learning mathematical concepts. Oktac (2016) researched the stage of Novotná et al (2006) related to how to learn the concepts and examples that show them. Furthermore, Junarti (2020c) divided this level into four sections. The first stage begins with the extraction of known structures to form the basis of the definition, from which abstract notions in a general context are built (Novotná et al, 2006). The second stage begins by extracting attributes from known structures, which leads to generalizations and, finally, definitions (Novotná et al, 2006). The third stage begins with the creation of concepts via logical inference from their definitions (Novotná et al, 2006). The fourth step begins by logically deducing the structure of a known mathematical feature or object from the definition, and can then analogize the structure of a property or a new mathematical thing (the object is not yet known) (Junarti, 2020c).

According to Junarti (2020c) research, 35 % of 22 students used stages by repeating examples of difficulties when confronted with newly discovered problem solutions; stages like these lead to stages of construction-analogy. This indicates that pupils will need to go through this step when learning abstract algebra. This step might lead to the stage of logical deduction (Junarti, 2020c). As a result, the construction-analogy stage will be used as an alternative in this study to assist students in developing mathematical connection abilities in abstract algebraic content.

The construction-analogy stage is a process that begins with constructing the structure of known mathematical properties or objects in the form of the same or similar examples through their definitions, and then new students can learn the structure of new or unknown mathematical properties or objects by analogizing the form of these examples (Junarti, 2020c). At this stage, students are asked to construct an understanding of examples that are prefixed with definitions (or associated with understanding definitions), and then when the construction process is carried out to direct students to be able to construct familiar mathematical properties or objects through definitions, in accordance with the student's needs for which underpins abstract algebra, through basic concepts in high school mathematics such as elementary algebra as the foundation.

As a result, the objective of this research is to describe the abilities of the six types of mathematical connections (representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections) on abstract algebraic material using the stages of abstraction-construction-analogy combination. The steps of abstraction-construction-analogy combination followed the three stages adapted from Novotná et al (2006) and one stage adapted from Junarti (2020c).

II. METHODS

The descriptive qualitative research approach was employed, with test instruments, questionnaires, and interviews. The study's respondents were chosen based on the findings of a questionnaire concerning the stages used. Abstraction-construction, analogy-abstraction, construction-analogy, and construction solely are the four steps of abstraction-construction-analogy fusion. This construction stage is built on phases that begin with the definition of

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being able to construct the attributes or objects of known and unknown structures. With a total of 22 questions, the four types of Analogy-Construction-Abstraction phases included on the questionnaire are supplied with instructions and descriptions of each stage Novotná et al (2006) has three stages, i.e., abstraction-construction ($V_A \to D \to V_B$), analogy-abstraction ($V_A \to V_B \to D$), construction ($D \to V_A, V_B$), and one stage adopted from Junarti (2020c), construction-analogy $(D \to V_A \to V_B)$. The stages are depicted in the scheme shown in Fig. 1 below.

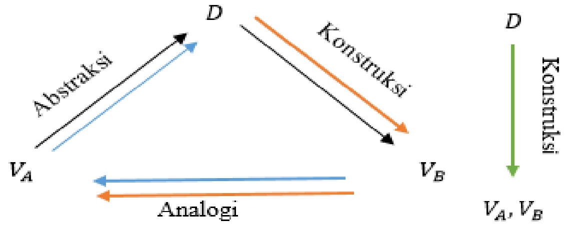

Fig. 1 Scheme of four types of stages: construction-abstraction, analogy-construction, construction-analogy, and construction.

Furthermore, the complete details of the stages and their descriptions are described in Table I below.

TABLE I
DESCRIPTION OF THE 4 TYPES OF STAGES OF ANALOGY-CONSTRUCTION-ABSTRACTION COMBINATION

Types of Approaches	Description
1. $V_A \to D \to V_B$ Construction-Abstraction	Students can abstract definitions from existing structures of properties or mathematical objects when developing new/unfamiliar structures of mathematical properties or objects.
2. $V_A \to V_B \to D$ Abstraction-Analogy	Students can use analogies to extract the structure of a known mathematical trait or object in order to construct a new or unfamiliar structure of a mathematical trait or object, and then abstract the definition.
3. $D \to V_A, V_B$ Construction	Students can build structures with known and unknown mathematical features or objects using logical inferences from definitions.
4. $D \to V_A \to V_B$ Analogy-Construction	Through logical deductions from definitions, students can create existing structures of mathematical properties or objects, and then analogize the structure of new or unknown mathematical qualities or objects.

Note:
D : Abbreviation of Definition
V_A: Stands for Property or object for a known structure
V_B: Stands for Property or object for an unknown structure

With the fixed comparison law (Creswell, 2017) in mind, the participants of the study were chosen as many as two students whose test work was the same from each level. The qualitative data of the six types of mathematical connections is limited to grouping materials based on the predictive indicators developed for each type of mathematical connection (connection representations, structural connections, procedural connections, implication connections, generalized connections, and hierarchical connections).

The phases employed in this investigation included the four listed below.

1. The first phase involves identifying the strategy employed while proving a set with a specific binary operation by completing a questionnaire of 22 questions.
2. The second phase, using predictive indicators, determines the ability of six types of mathematical connections based on student work from the results of the description form test.
3. The third phase describes six research subjects' ability to establish six different sorts of mathematical connections through interviews.
4. The fourth step involves source triangulation and method triangulation. The sources were triangulated by comparing the work of two study subjects from each of the inclinations of the phases used. The triangulation approach compares the work of the subjects based on the findings of questionnaires, tests, and interviews.

The following are the characteristics of developing the ability to make six types of mathematical connections on group material using predictive indicators:

1. Develop representational link skills if students can describe and communicate mathematical content ideas using words, graphs, symbols, tables, and diagrams. Mathematical content ideas that are described and communicated concerning presentation cohesively and exactly in exhibiting closed properties, associative properties, identity elements, and inverse elements.
2. Improve your structural connection abilities if you can recognize, complete, connect, and use set element structures, binary operation structures, associative and commutative properties structures, and identity and inverse element structures to connect mathematical content concepts. In the group concept, mathematical content ideas are known for compiling the structure of set elements so that they can connect, complete, and use them to determine the results of binary operations on closed properties, manipulate in showing associative properties, manipulate to find identity elements and their inverse elements.
3. Create procedural connection capabilities if you can utilize the specified rules, algorithms, or formulae to arrive at the description results in joining mathematical content ideas/ideas. Ideas for mathematical content regarding groups that are linked in the form of rules that apply in group axioms, algorithms that apply to systems of standard equations. The rules in the group axioms must satisfy the specification of each group condition,

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including the rules on closed properties, associative properties, identity elements, and inverse elements.

4. Create implication connection capabilities (if-then) if they can work deductively by connecting known assertions (as antecedents) to conclusions. Each demonstrates, in the demonstration of each component of the group axioms, that he or she can operate deductively by writing responses in the form of *"if..... then..."* If you can't put the implication flow in your answer, you haven't stated that you are an if-then connection (implication connection).
5. Develop generalization connection capabilities if you can draw inferences from each stage of work that is combined into a general form. Each procedure of demonstrating each supplied group requirement must be constructed in the manner of drawing general conclusions, namely writing can represent arbitrary other set elements in symbolic form or sentence form.
6. Develop hierarchical connection capabilities if you can present a work sequence with a cohesive and logical connection. The execution order of each group condition is provided logically and does not overlap or repeat.

Cresswel's (2017) procedures were followed for qualitative analysis, beginning with data reduction by selecting data matching six categories of mathematical connections. Furthermore, it evaluates the data supplied in the form of information, which is organized by connection type. The final stage involves concluding/verifying the data as a whole.

III. Results and Discussion

A. *Result*

In the first phase, the results of this study were identified by distributing the number of questionnaire responses utilizing the four types of analogy-construction-abstraction stages stated in Table II below.

TABLE III
DISTRIBUTION OF THE NUMBER OF STUDENTS USING FOUR TYPES OF ANALOGY-CONSTRUCTION-ABSTRACTION COMBINATIONS

Question Form	Types of Approaches and Number of Students				
	Type 1	Type 2	Type 3	Type 4	Abstain
(1)	6	7	1	4	8
(2)	4	7	4	5	6
(3)	4	3	4	9	6
(1)	3	6	1	10	6
(2)	1	4	3	12	6
(3)	1	4	4	11	6
(3)	-	6	2	11	7
(1)	-	6	2	11	7
(2)	2	3	1	14	6
(3)	1	1	3	14	7
(1)	-	4	1	14	7
(2)	-	6	-	13	7
(3)	-	3	3	12	8
(1)	-	3	3	13	7
(2)	-	4	2	13	7
(3)	-	6	3	14	7
(1)	2	7	3	13	7
(2)	4	8	-	10	4
(3)	2	2	3	15	5
(1)	1	2	4	14	8
(2)	2	11	-	8	6
(3)	2	2	3	16	6
	35	92	50	256	144

Based on Table II, students most often used the "analogy-construction" method, which they have done as many as 256 times. Type 2 (analogy-abstraction) was used 92 times, type 3 (construction) was used 50 times, and type 1 (abstraction-construction) was used 35 times. Type-4 was what most students use. This shows that this stage leads to logical deduction thinking that it can be done, even though in math, type-3 stages should be used (i.e., the stage of logical deduction, which is a stage that should be done in mathematics).

Based on the results of the questionnaire and the communication skills of the students as well as their willingness to be interviewed, 4 different ways were used to find 8 research subjects. Two subjects (S-4 and S-13) tended to use type-1 stages, two subjects (S-8 and S-15) tended to use type-2 stages, two subjects (S-19 and S-21) tended to use type-3 stages, and two subjects (S-4 and S-13) tended to use type-4 stages (S-2 and S-12).

Subjects M-4 and M-13 tend to use the type-1 stage when connecting binary operations in structures known to be closed properties, relating identity elements to the inverse element work of any set element written with membership conditions using standard binary operations, and can relate the identity element to the inverse element of any number set element using non-standard binary operations or in the form of a Cayley table. These two subjects tend to rely on the same examples while learning about group notions, yet they can create set forms and non-standard binary operations.

Both subjects M-8 and M-15 have a preference for using the type-2 stage when linking the representative set elements of the set written with the membership conditions and using the usual binary addition operation of the two set components. These two subjects have a strong tendency to analogize the same examples, therefore they can only construct unknown mathematical things and understand their meanings.

Both subjects M-19 and M-21 tend to adopt the type-3 stage when determining a connection in a closed property, a connection with an identity element, and an inverse element. Through their definitions, both of these subjects have been able to carry out good construction in a new known form.

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The two subjects M-2 and M-12's proclivity to use the type-4 stage when describing the elements of a set of numbers that represent the set of three sets of forms, describing the results of binary operations written in the non-standard form (formula) from two elements of a set of numbers and in tabular form, relating the identity element to the inverse element work of any set element written with membership conditions with standard/nonstandard/binary operations defined in the Cayley table. These two subjects were able to create new mathematical objects based on their definitions and examples learned.

Based on test results about groups with 3 sets of forms and 3 types of binary operations, the results of this phase of research show that there are 6 types of mathematical connections. Based on the results of the mathematical connection test, especially on the group concept and the use of predictive indicators from each type of mathematical connection, the results of the students' mathematical connection skills with abstract algebra material are given. The work of each research subject on each item will be looked at based on 6 types of mathematical connections. This study used six different kinds of mathematical connections: (1) representational connections (KR), (2) structural connections (KS), (3) procedural connections (KP), (4) implication connections (KI), (5) connections generalization (KG), and (6) hierarchical connections. All of the students' work was looked at based on the signs that each type of mathematical connection made.

Summary of how the test results for each type of mathematical connection are spread out, based on what the indicators say about each type. The indicators for each type of connection are different depending on what they are or how they are described. Indicators of the type of mathematical connection: (1) there are 5 indicators for the type of connection representation; (2) there are 8 indicators for the type of connection structure; (3) there are 4 indicators for the type of connection procedure; (4) there are 4 indicators for the type of connection implication; (5) there are 5 indicators for the type of connection generalization; and (6) there are 4 indicators for the type of connection hierarchy.

1) *Exploration Results of Two Subjects using Construction Stages (Type-3):* The following is a summary of the interpretation of the work that was conducted by 2 research subjects, which was categorized using type-3 stages (construction) in the table that can be found below.

TABLE IIIII
SUMMARY OF MATHEMATICAL CONNECTION TEST WORK INTERPRETATION BASED ON INDICATOR PREDICTION RUBRIC FOR SUBYECTS M-19 AND M-21

Initials of the Research Subject	**Interpretation of Mathematical Connection Type Ability**
M-19	On the subject of item 1, the subject can establish the types of implication connections, generalizations, and hierarchies. However, the subject is less able to relate two elements and three set elements to the operation of summation of root forms in the types of representational, structural, and procedural connections, namely " and "$\sqrt{p}+\sqrt{p}=\sqrt{p}\sqrt{p}+\sqrt{p}+\sqrt{p}=\sqrt{p}$
	Because the subject cannot select set elements with exact constraints based on the scope of the set and its operations, the subject cannot establish types of representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections. However, the subject group's axiom can explain that because it does not match the closed property, the subject concludes not the group, thus the subject does not write the answer on the proof of the existence of inverse and inverse elements at all.
	Because the subject cannot make inferences in the generalization of finite set elements such as S = e, p, q, r, and the binary operation "o" defined in Cayley's table, item 3 is unable to establish the maximum type of connection of implications, connections of generalizations, and connections of hierarchy. The subject's incapacity to conclude in general, resulting in improper implications of the connection, and the outcome of the type of hierarchical connection, particularly the logicality of the conclusion, cannot be justified.
M-21	The subject can build various forms of implication connections, generalizations, and hierarchies. However, the subject is less able to relate two elements and three elements of the set to the summation operation of the root form, namely " and " and does not $\sqrt{p}+\sqrt{p}=\sqrt{p}\sqrt{p}+\sqrt{p}+\sqrt{p}=\sqrt{p}$ that $i=0$ as an element of identity obtained from the substantiation of the element of identity as a reason for the existence of a connection with the substantiation of the inverse element.
	Because the subject cannot select set elements with exact constraints based on the scope of the set and its operations, the subject cannot establish types of representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections. The subject does not write down the response to the demonstration of the existence of identity and inverse elements.
	In item 3, subject M-21 can already establish representational connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections. Although the conclusion is presented in the form of a sentence, it is logically correct.

According to the summary of the results in Table III, two subjects the M-19 and M-21 subjects tended to establish representation, structural, procedural, implication, generalization, and hierarchy connections to the proving of group axioms of the problem types of items 1 and 3. In terms of item 2, the two subjects tend to be incapable of developing the ability to make all forms of mathematical connections. The tendency of both subjects M-19 and M-21's

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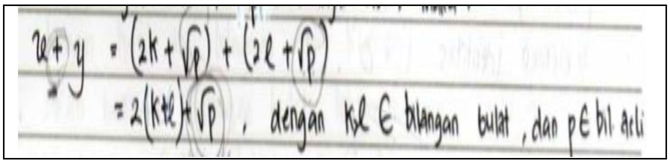
construction stages can already establish the capabilities of all types of connections for a set expressed in the form of a membership condition to the binary operation of the standard form and to a finite set with binary operations defined in Cayley's table.

a. Exploration Results of Mathematical Connections on Item 1

In addition, based on the findings of the test work and interviews with the two M-19 and M-21 research subjects, the following exposure was determined.

The following is the sample of the test results of the subject M-19 on item 1

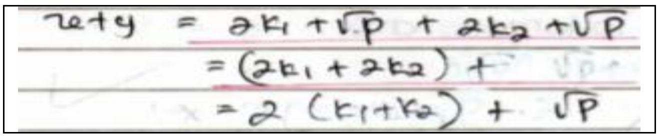

$x + y = (2k + \sqrt{p}) + (2l + \sqrt{p})$
$= 2(k+l) + \sqrt{p}$, dengan $k, l \in$ bilangan bulat , dan $p \in$ bil asli

Excerpts of the Interview with subject M-19 on item 1:

R : Researcher
M-19 : Research Subject
R : Now consider the work of item 1 on the summation of root forms, why were you doing $\sqrt{p} + \sqrt{p} = \sqrt{p}$?
M-19 : yes ma'am, is it incorrect?
R : why $\sqrt{p}$ is summed up with $\sqrt{p}$, and the result is $\sqrt{p}$?
M-19 : *(Silence.....)*
R : Is your answer correct? What if $\sqrt{2} + \sqrt{2}$?
M-19 : It is $2\sqrt{2}$ mom, so that $\sqrt{p} + \sqrt{p} = 2\sqrt{p}$.
R : Good, what about your work on this one: $\sqrt{p} + \sqrt{p} + \sqrt{p} = \sqrt{p}$?
M-19 : Oh, it's in correct, ma'am, it should be $3\sqrt{p}$
R : Good.

Based on work samples and interview results, it indicates that the M-19 subject was unable to build a type of mathematical connection associated to two or three elements of the set of root forms concerning binary operations of the root form. After being interviewed using a root form such as "$\sqrt{2}$". The respondent learns that his work is incorrect. As a result, the subject is less likely to be able to relate two or three members of the set of shapes of root into the summation operation. Meanwhile, in the presentation below, you can see the work and interview results of M-21 subjects.

The following is the sample of the test results of the subject M-21 on item 1:


$x + y = 2k_1 + \sqrt{p} + 2k_2 + \sqrt{p}$
$= (2k_1 + 2k_2) + \sqrt{p}$
$= 2(k_1 + k_2) + \sqrt{p}$

According to the preceding sample from Subject M-21's work, the inability to sum on both members of the set in the form of this same root is repeated when summing the three elements of the root form to demonstrate associative characteristics. As a result, the subject cannot relate two or three pieces of the same root form.

Excerpts of interviews with Subject M-21 on item 1:

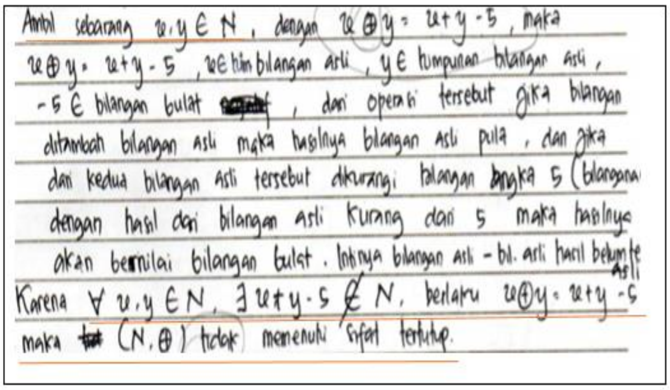
R : Now consider the work of item 1 on the summation of root forms, why were you doing $\sqrt{p} + \sqrt{p} = \sqrt{p}$?
M-21 : I'm sorry, mom, is it wrong?
R : Why $\sqrt{p}$ is summed up with $\sqrt{p}$, and the result is $\sqrt{p}$?
M-21 : *(silence)*
R : Is your answer correct?
M-21 : I think it should be $2\sqrt{p}$, mom.
R : Sure?
M-21 : Yes.
R : Now, what about this work: $\sqrt{p} + \sqrt{p} + \sqrt{p} = \sqrt{p}$?
M-21 : It should be $3\sqrt{p}$, mom.
R : Good, you know your mistake, then.

Subject M-21 demonstrated a lack of thoroughness based on the sample of the interview above, despite the fact that the subjects were completely capable of adding up the shape of the roots when the interview was conducted.

As a result, the conclusion of the work on item 1 by the two subjects M-19 and M-21 has a tendency to be able to establish almost all types of connections in item 1, namely representation connections, structural connections, implication connections, generalization connections, and hierarchy connections. It has the same incapacity to sum the shape of the roots as the type of procedural connection in exhibiting the two subjects.

b. Exploration Results of Mathematical Connections on Item 2

The following is the sample of the test results of the subject M-19 on item 2


Ambil sebarang $x, y \in N$, dengan $x \oplus y = x + y - 5$, maka
$x \oplus y = x + y - 5$, $x \in$ himpunan bilangan asli , $y \in$ himpunan bilangan asli ,
$-5 \in$ bilangan bulat , dan operasi tersebut jika bilangan
ditambah bilangan asli maka hasilnya bilangan asli pula , dan jika
dari kedua bilangan asli tersebut dikurangi bilangan angka 5 (bilangan
dengan hasil dari bilangan asli kurang dari 5 maka hasilnya
akan bernilai bilangan bulat . Intinya bilangan asli - bil. asli hasil belum tentu asli
Karena $\forall x, y \in N$, $\exists x + y - 5 \notin N$, berlaku $x \oplus y = x + y - 5$
maka $(N, \oplus)$ tidak memenuhi sifat tertutup.

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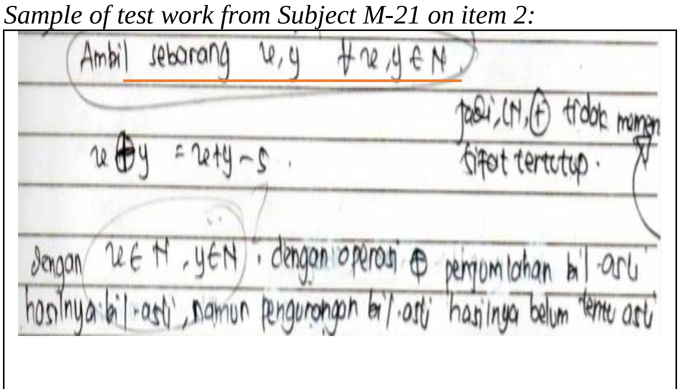
Excerpts of the Interview with subject M-19 on item 2:

R : So, what about the proof on item 2 regarding whether the set element you choose matches the concept of binary operation?
M-19 : I think it matches, mom.
R : Again, if x and y are in the set of natural numbers, then the outcome of the operation fulfills or is contained in the set of natural numbers?
M-19 : Oh, certainly, ma'am, it indicates that x and y have been chosen more than three times.
R : What about the results of the operation now?
M-19 : The result will be contained in N, ma'am.
R : What about proving its closed property?
M-19 : It's already proven, ma'am.
R : What about proving its associative property?
M-19 : It's already proven, too.
R : What about the existence of identity elements and their inverse elements?
M-19 : It means that you can find the identity element and its inverse element, ma'am?
R : How to prove with group axioms?
M-19 : Does that mean it's a group, ma'am?
R : Okay, okay, what about the next work about item 3?
M-19 : It's proven to be a group ma'am.

Based on the work excerpts and results of the M-19 subjects' interviews, the subject has not been able to establish a type of mathematical connection relating to the determination of elements of the set of natural numbers that meet the conditions of the validity of the binary operations of non-standard forms "$x \oplus y = x + y - 5$". In this scenario, the subject of M-19 can provide some evidence of associative property, but because the chosen element does not specify the specific limitations of the set element, it becomes evidence that the associative property cannot yet be realized. The subject of M-19 has a tendency to select set elements without considering the application of its binary operations, resulting in the subject M-19 becoming caught in the routine of the well-known form of binary operations. By making a set element error, the subject is unable to build all types of connections in showing the closed property, associative property, the presence of an identity element, and the inverse. As a result, the subject of M-19 was unable to recognize the structure of the set elements, which immediately link to the structure of binary operations.

Sample of test work from Subject M-21 on item 2:


Ambil sebarang x, y ∀ x, y ∈ N
jadi, (N, ⊕) tidak memenuhi sifat tertutup.
x ⊕ y = x+y−5.
dengan x ∈ N, y ∈ N, dengan operasi ⊕ penjumlahan bil. asli hasilnya bil. asli, namun pengurangan bil. asli hasilnya belum tentu asli

Based on the excerpt of the work subject M-21 above, the subject is unable to recognize the selected set element because the subject cannot relate to the definition of binary operations of non-standard forms. As a result of the M-21 subject's inability, the demonstration of the closed property could not be completed. Furthermore, the incompetence of the subject M-21 resulted in a proof of associative property, although the proof was incomplete. The subject of M-21 is likewise unable to write down the existence of elements of identity and inverse.

It was confirmed further in the following interview excerpt with the M-21 subject.

Excerpts of the Interview with Subject M-21 on item 2:

R : Okay, now try to pay attention to the work on item 2, which has to do with the set element you chose. Does it meet the definition of a binary operation?
M-21 : I think so.
R : Look again at the natural numbers x and y, and then figure out how to make the result of the operation part of the natural numbers set again.
M-21 : Oh… It*(students show a surprised look and then shut up while thinking).* It means that my work is wrong as far as the things I chose are concerned, ma'am.
R : How does the operation's result fit into the set N?
M-21 : That means x and y must be more than 3, right?
R : Are you sure about the limitations of the element?
M-21 : I think so.
R : Okay, that's right. Now, what about proving the closed property?
M-21 : Yes, it's proven.
R : What about proving its associative property?
M-21 : It's proven, too.
R : What about the existence of identity elements and their inverse elements?
M-21 : So, the identity element is 5, and the inverse element for each x is -10-x, ma'am?
R : Next, what about proving the group axiom of item 2?
M-21 : Yes, it's proven.

Based on excerpts from an interview with Subject M-21, who originally demonstrated an inability to tie the selected set element to its binary operation definition, the subject was eventually able to locate the set element's limitations. Furthermore, the subject can accurately respond orally to the associative property, the presence of elements of identity, and the inverse.

As a result, the conclusion of item 2 work by both subjects M-19 and M-21 has the same inclination to define the boundaries of set elements that fit the definition of non-standard binary operations. The failure of both subjects to build six different sorts of mathematical connections is due to an incorrect limitation of the set element. Because the subject cannot capture the representation of its binary operations and correlate the structure of its binary operation definition with the structure of its set, the subject is

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imprecise in choosing elements. Subjects M-19 and M-21 are consequently unable to make representational and structural connections, preventing them from establishing other types of connections.

c. Exploration Results of Mathematical Connections on Item 3

The following explanation is based on the work and interview results of the two participants M-19 and M-21.

Sample of test work from Subject M-19 on item 3:

Berdasarkan tabel caley di atas dapat dituliskan :
$e \circ e = e$, $p \circ e = p$, $q \circ e = q$, $r \circ e = r$
$e \circ p = p$, $p \circ p = q$, $q \circ p = r$, $r \circ p = e$
$e \circ q = q$, $p \circ q = r$, $q \circ q = e$, $r \circ q = p$
$e \circ r = r$, $p \circ r = e$, $q \circ r = p$, $r \circ r = q$
Karena $\forall e, p, q, r \in S$, $\exists e, p, q, r \in S$ dan berlaku seperti diatas S terhadap operasi biner "o" hasilnya termuat di S maka (S, o) memenuhi sifat tertutup.

Furthermore, in the following excerpt, triangulation is accomplished using interviews.

Excerpts of interviews with M-19 subjects on item 3:

R : Alright, what about your work on item 3?
M-19 : It's a group, ma'am.
R : What do you think the closed properties you're working on mean?
M-19 : Yeah, as you can see, it's like what I've done here, ma'am.
R : Ok, now look! is it logical that the form: " $\forall e, p, q, r \in S, \exists e, p, q, r \in S$ is a general conclusion?
M-19 : Oh, I see. It's wrong.
R : What is the correct conclusion?
M-19 : This means that the conclusion is "$\forall x, y \in S x o y = z,\ z \in S$", ma'am.
R : Okay, that's right. What about the proof of the associative property's conclusion?
M-19 : It's wrong, too, ma'am.
R : Now what's the conclusion?
M-19 : The conclusion should be "$\forall x, y, z \in S$so much so that $x o(y o z) = (x o y) o z$", ma'am.
R : What about your conclusion in proving the existence of identity and inverse elements?
M-19 : To conclude that there is an identity element, I think it is general, ma'am. But for the inverse conclusion, it's not general, ma'am.
R : What is the general conclusion in the conclusion of the existence of an inverse element?
M-19 : The conclusion for each element is in S there is an inverse element in S the same state that it matches the example $x o y = i$, i = identity element.
R : Okay, thank you.

The M-19 subject interview excerpt above demonstrates that the subject has not been able to build implication connections, generalization connections, or hierarchical connections. M-19 participants' incapacity to derive conclusions from the set is confined to the substantiation of closed, associative, identity, and inverse elements. M-19's subject is also unable to reach a broad conclusion. According to the conclusion of the subject M-19's work, the subject writes down all the aspects as a form of representation of his arbitrariness. Although the subject of M-19 is considered to be less capable of making inferences that represent from the four elements of the set into the evidence of closed, associative, identity, or inverse elements in general in the form of symbols. The consistency of the data revealed the tendency of the M-19 subject to be unable to create implication connections, generalization connections, and hierarchy connections based on snippets of test work and interviews. Thus, based on the method's triangulation, the qualitative data of this M-19 subject on the ability of mathematical connections in materi groups are valid.

Furthermore, the work of the M-21 subject will be deepened by a sample of the test work and the following interview snippet.

Sample of Test work from Subject M-21 on item 3:

Akan ditunjukkan : memenuhi sifat tertutup.
Berdasarkan table cayley diatas diperoleh:
$e o e = e$ $p o e = p$ $q o e = q$ $r o e = r$
$e o p = p$ $p o p = q$ $q o p = r$ $r o p = e$
$e o q = q$ $p o q = r$ $q o q = e$ $r o q = p$
$e o r = r$ $p o r = e$ $q o r = p$ $r o r = q$.
karena semua hasil operasi biner o pada tabel cayley. merupakan anggota S maka terbukti (S, o) memenuhi sifat tertutup

Furthermore, a sample of the M-21 subject's test work was triangulated using the following interview excerpt.

Excerpts of interviews with Subject M-21 on item 3:

R : Alright, what about your work on item 3?
M-19 : It's a group, ma'am.
R : How did you conclude that the closed properties that you did was representative?
M-21 : Yeah, it's like what I've done here, ma'am.
R : Try to look again at your conclusion? Is the conclusion general?
M-21 : Not yet ma'am.
R : What is a general representative conclusion?
M-21 : It means that the conclusion is "is$\forall a, b \in S$ so that it matches a closed property $a o b = c,\ c \in S,$ ", ma'am.
R : Now, what about the conclusions on the proof of associative properties?

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M-21 : The conclusion should be $\forall a, b, c \in S$ so that it applies $(aob)oc = ao(boc)$", and it's proven that it matches the associative properties.
R : Oh, I see.
M-21 : Yes, ma'am.
R : What about your conclusions on proving the existence of identity and inverse elements?
M-21 : I think the conclusion that there is an identity element and an inverse element is correct, ma'am.
R : Okay, thank you.

Based on interviews with M-19 and M-21 subjects and excerpts of their work, the subjects were unable to write symbolically, although they attempted to put the conclusions in phrases. The conclusion sentences written by the subject M-21, which were corroborated by interviews, reveal that the subject had difficulties formulating a symbolic conclusion to reflect the four known elements of the set.

As a result, both M-19 and M-21's work on item 3 has a tendency to connect everything. In both M-19 and M-21 subjects, generalizations in symbolic form are less likely, while M-21 tried to develop generalizations in the form of logical statements. The applicability of group proof on item 3 ranges from the proof of closed property, associative property, the presence of identical elements, and inverse elements based on the work of M-19 and M-21.

This means that both subjects M-19 and M-21 exhibit a tendency to be able to establish the ability of representational links and structural links and procedural links to substantiate group axioms of item 1 and item 3 problem types and hierarchical links to these links Second, the two subjects tend to fail to develop the ability to connect various sorts of mathematical concepts. Both the set represented in terms of a membership condition and the finite set with binary operations established in Cayley's table can therefore already provide all forms of connection capability for a type-3 approach employed set.

2) *Exploration Results of Two Subjects using Construction-Analogy Stages (Type-4):* Mathematical connection test results for M-2 and M-12 research subjects are summarized in Table IV below, and the following is a description of their findings.

TABLE IV
SUMMARY OF MATHEMATICAL CONNECTION TEST WORK INTERPRETATION BASED ON INDICATOR PREDICTION RUBRIC FOR SUBJECTS M-2 AND M-12

Initials of the Research Subject	Interpretation of Mathematical Connection Type Ability
M-2	The subject cannot function by implication in the withdrawal of the conclusion to the closed property proof. Indirectly, the proof is not hierarchical since the subject cannot use the technique of the first statement to prove the existence of identity and inverse elements according to logical norms of proof. As evidence of the subject's associative character, representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections can be demonstrated.
	Subject M-2 failed to exhibit the ability to construct representational connections, structural connections, procedural connections, implication connections, generalization connections, and connections hierarchy in the demonstration of item 2. This is because the subject cannot connect the structure of the set element's existence to the non-standard structure of the definition of binary operations. This shortcoming allowed M-2 subjects to create alternative types of connections. The inability to understand or comprehend the structure of the set element and the structure of binary operations impedes the capacity to connect them to other mathematical concepts or ideas.
	Based on the demonstration of closed property, associative properties, the presence of identity elements, and inverses, the subject of M-2 can demonstrate representational connections, structural connections, implication connections, and hierarchical connections. Although the procedural link in demonstrating the proof of a closed property, associative characteristics, elements of identity, and inverse are written illogically, it is nonetheless a closed property. In addition, the subject has not been able to generalize to all of the conclusions of the proof of closed, associative, and identity characteristics, but in the proof of the inverse element, the subject may already generalize with the correct and logical. The work of Subjek is extremely coherent, but it lacks a consistent use of symbolism to demonstrate the existence of identity parts.
M-12	The subject is capable of establishing representational, structural, and procedural links in the concept of set elements, the result of binary operations, and associative connections. However, the subject is less capable of establishing the ability of implication, generalization, and hierarchical connections to closed and associative qualities. At the time of identifying identity elements and inverse elements, the consequences of connection connections, generalization connections, and hierarchy connections are considered while deciding on identity elements and inverse elements (especially for inverse elements the subject does not work at all)
	The subject is sufficient to demonstrate the existence of identity and inverse elements, but it cannot demonstrate the closed property and associative qualities. Due to a lack of acquaintance with the structure of set members and the structure of non-standard binary operations, the subject of M-12 is unable to pass. This is because the M-12 subject concentrates on the form of equations that apply in establishing identity elements and inverses, but does not recognize the limitations of the set elements that are chosen to satisfy the definition of binary operations. Thus, the subject of M-12 is adequate for establishing a structural connection between the existence of an identity element and an inverse, as it is able to determine the existence of both elements.

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	Tracing the structure of the work reveals that the M-12 student has not been able to effectively establish a variety of connections.
	Based on the demonstration of closed property, associative properties, the presence of identity elements, and inverses, the M-12 subject is able to demonstrate representational connections, structural connections, implication connections, and hierarchical connections. However, the ability of procedural connections at the beginning of the demonstration of closed property, associative property, the presence of identity elements, and inverses has not been established. In addition, M-12 was unable to construct a generalization connection for all the evidentiary conclusions of closed property, associative property, the presence of identical elements, and inverses.

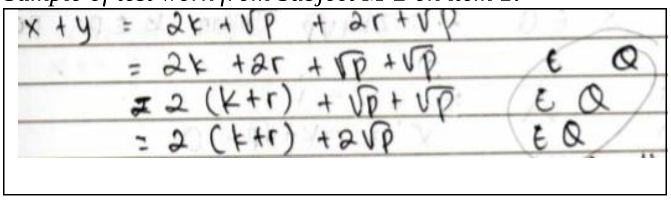
Based on the prediction of connection type indicators rubric, both M-2 and M-12 demonstrated a comparable inability to make representational, structural, and procedural connections after the demonstration of a closed property. Both subjects are less capable of establishing the ability of procedural links to demonstrate that identity and inverse elements exist. Due to the incapacity of the two subjects to form a procedural connection, the types of hierarchical connections collide. As for associative proof, subjects M-2 can establish all types of connections, however, subjects M-12 are unable to establish connection implications, generalizations, or hierarchies. Because M-12 does not record the answer when establishing the existence of an inverse element for each known element, M-12 cannot build six distinct types of connections.

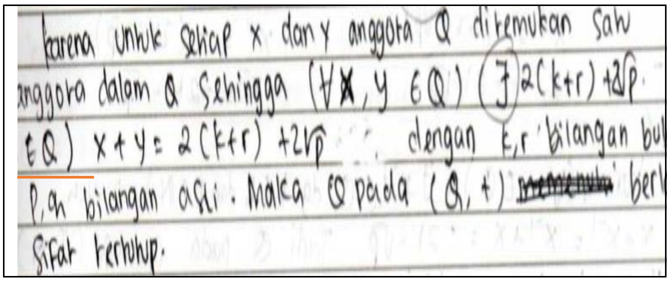
The failure of M-2 and M-12 subjects to exhibit the same tendency in specific instances, notably representational connections in closed property conclusions and procedural connections and hierarchical connections, is evidence of the existence of an element of identity.

a. *Exploration Results of Mathematical Connections on Item 1*

In the next parts, it will also be examined based on samples of test work results and interview data from the two research subjects.

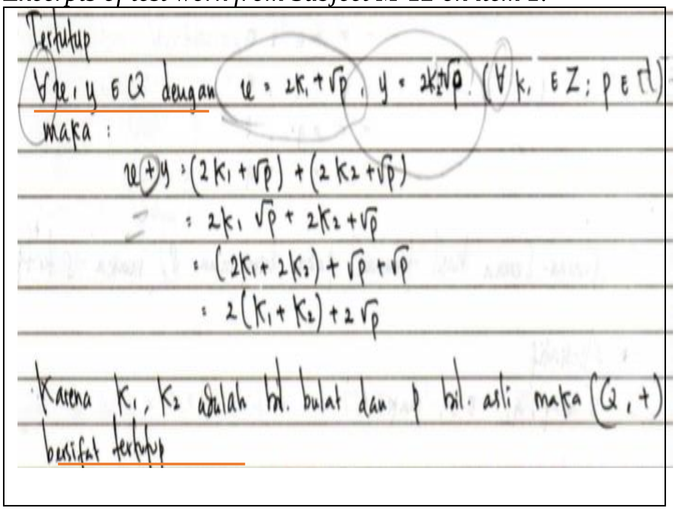
Sample of test work from Subject M-2 on item 1:


$x + y = 2k + \sqrt{p} + 2r + \sqrt{p}$
$= 2k + 2r + \sqrt{p} + \sqrt{p} \in Q$
$= 2(k+r) + \sqrt{p} + \sqrt{p} \in Q$
$= 2(k+r) + 2\sqrt{p} \in Q$


karena untuk setiap x dan y anggota Q ditemukan satu anggota dalam Q sehingga $(\forall x, y \in Q)\ (\exists\ 2(k+r) + 2\sqrt{p} \in Q)\ x + y = 2(k+r) + 2\sqrt{p}$ ∴ dengan k, r bilangan bulat p, q bilangan asli. Maka Q pada (Q, +) ~~memenuhi~~ berlaku sifat tertutup.

In addition, a piece of the subject M-12's work related to item 1 is described in the following section.

Excerpts of test work from Subject M-12 on item 1:


Tertutup
$\forall x, y \in Q$ dengan $x = 2k_1 + \sqrt{p}$, $y = 2k_2\sqrt{p}$ $(\forall k, \in Z; p \in N)$
maka :
$x + y = (2k_1 + \sqrt{p}) + (2k_2 + \sqrt{p})$
$= 2k_1\ \sqrt{p} + 2k_2 + \sqrt{p}$
$= (2k_1 + 2k_2) + \sqrt{p} + \sqrt{p}$
$= 2(k_1 + k_2) + 2\sqrt{p}$
Karena k_1, k_2 adalah bil. bulat dan p bil. asli maka (Q, +) bersifat tertutup

In these samples, subjects M-8 and M-12 appear to be able to demonstrate the closed property and associative property comprehensively; nonetheless, the conclusion is incomplete. In addition, in order to demonstrate the identity property, the subject can document the existence of a portion of identity, although this is insufficient. Regarding the substantiation of the inverse element, subject M-12 does not record any response. Based on the work of the two subjects, M-2 and M-12 tend to be less capable of establishing the ability of representational connections, structural cones, and procedural connections after closed property proof. Consequently, both subjects are less able to demonstrate the ability of procedural connections while showing the existence of identity and inverse elements. Incompetence in this type of hierarchical connection stems from the incapacity of both subjects to form a procedural link. The two subjects can then construct a connection between implications, generalizations, and hierarchies after being interviewed. Thus, both subjects are capable of establishing any form of connection.

In contrast, based on interviews with both M-2 and M-1 2 subjects, item 1 was unable to make a connection between the results of binary operations and the obedience of the set element in the set Q. In conclusion to the evidence of closed property, subject M-2 misrepresents symbols. As a result, the subject is less able to connect by implication, which might lead to a diminished ability to communicate broad

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conclusions. M-2 people are less capable of connecting in a representational, structural, and procedural manner while showing the existence of identity elements and inverses. The subject's inability to build a procedural connection is evidence that it is not a flow hierarchy; therefore, subject M-2 has also been unable to establish a hierarchical connection. Regarding the conclusion of establishing the identity and inverse of elements, the subject can already employ the connection of implications and generalization.

b. Exploration Results of Mathematical Connections on Item 2

Based on work samples, M-2 participants were unable to build six types of mathematical connections. However, after an interview, M-2 subjects showed a predisposition to be able to establish representational, structural, procedural, implication, generalization, and hierarchical connections. While Subject M-12's work demonstrates that the subject is marginally capable of proving the presence of an element of identity and an element of inverse, the subject cannot prove the existence of a closed set and associative property.

Based on interviews with both M-2 and M-12 subjects, the ability to establish all types of mathematical connections tends to be less prevalent than the ability to establish any type of mathematical connection when setting the constraints of set elements that satisfy the definition of non-standard binary operations. In addition, the incapacity of the two subjects to build procedural sorts of connections, implication connections, generalization connections, and hierarchy connections is a result of their inability to establish representation and structural connections.

c. Exploration Results of Mathematical Connections on Item 3

Based on subject 2's work on the proof of a closed property, it is evident that subject M-2 is unable to describe the beginning of the proof and the end. In addition, M-2 demonstrated difficulty to draw general inferences regarding the existence of an element of identity in the form of closed property and associative property. However, the subject can draw a generalization from the demonstration of the inverse element. The previous work of subject M-12 demonstrates that the subject's ability to establish representational connections, structural connections, implication connections, and hierarchical connections are quite good. However, if observed, subject M-12 demonstrates that it is less capable of connecting the beginning of the proof to its conclusions procedurally. The unfinished property of the work from the introduction to the conclusion. It is evident from the phrase "because all results are contained in the set S, then S is closed" that an explanation should be given following the word "result": "the result of the binary operation of every two picked items..." The subject's inability to recognize the type of procedural link and the generalization connection is also evident in the lack of evidence for associative characteristics, identity elements, and inverses.

The M-2 subject demonstrates the appropriateness of the responses based on the results of the test work and the results of the interview. The subject of M-2 is fairly capable of establishing any type of generalization connection but is less capable of establishing procedural connections at the beginning of the proof of a finite set with its definition of binary operations expressed in Cayley's table. Work is appropriate based on the interview with the M-12 subject and a fragment of the test question. In this instance, the subject of M-12 in terms of procedural connections and generalizations cannot relate to the existence of a set whose elements are finite and whose binary operations are defined in Cayley's table. As a result, the subject feels less familiar, and the procedure that should have become easier to write procedurally and generally becomes more difficult to write. This M-12 subject's ineptitude is increasingly supported by the associative property, the presence of identity elements, and the inverse.

The subjects of M-2 and M-12 have a sufficient propensity to be able to establish the ability of representation connections, structural connections, implication connections, and hierarchical connections to the proving of group axioms for issue types 1 and 3. Regarding criterion 2, the two subjects exhibit a propensity for being unable to develop the capacity for all forms of mathematical connections. Thus, the two subjects utilizing the type-4 approach are sufficiently capable of establishing all types of connections for a set expressed in the form of membership conditions against binary operations of the default form and for a finite set with binary operations defined in Cayley's table, but it still requires practice and a deeper understanding of the Varied question form.

3) *Exploration Results of Two Subjects Using Analogy-Abstraction Stage (Type-2):* Following is a summary of the work's interpretation of the results of the mathematical connection test for M-8 and M-15 research participants.

TABLE V
A SUMMARY OF HOW THE QUESTIONS ON ITEMS 1, 2, AND 3 SHOULD BE INTERPRETED BASED ON THE RUBRIC FOR PREDICTING INDICATORS ON SUBJECTS M-8 AND M-15

Initials of the Research Subject	Interpretation of Mathematical Connection Type Ability
M-8	The subject of M-8 is incapable of establishing a representational connection since it is unable to identify the description of a known set and cannot relate to its defined binary operations. The inability to represent the set's elements rendered the subject incapable of relating to the element's structure and the defining structure of its operations. Additionally, it pertains to incompetence in procedural connections, generalization connections, and hierarchical connections, as there are connections in the selected set of components that have not met the definition of binary operations.
	The subject does not provide the same response as in

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	item 2, hence the subject of M-8 cannot identify the categories of representation connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections.
	The subject of M-8 is capable of establishing a form of representational connection between all elements S of the set and exhibiting associative properties, but cannot infer that it can represent anything that satisfies a closed property. Based on the evidence for the presence of identity elements and inverse elements, the subject cannot construct all possible connections.
M-15	The subject of M-15 is able to establish a representational connection because it can recognize the description of the set to the item that it can relate two or three elements of the set to its binary operating definition, but it cannot establish a representational connection in the presence of identity elements and inverse elements. In addition, the subject of M-15 demonstrated an inability to establish structural connections in the structure of the guarantee of the applicability of any element of the set to the conclusion of a closed and associative property, as well as an inability to establish structural connections in the structure of proof of the elements of identity and inverse. The patient also has a failure to make procedural, generalization, and hierarchical connections when making judgments about the existence of closed, associative, and evidential qualities of identity and inverse elements.
	The subject cannot form representational connections in the proof of a closed property due to its inability to recognize the constituents of the set "N = the set of Native numbers on binary operations $x \oplus y = x + y - 5 \; \forall x, y \in \mathbb{N}$. In proving associative properties, identity elements, and inverses, sub-objects can relate elements to their set, but the subject cannot impose limitations on the chosen element; hence, the conclusion does not represent arbitrariness members of a known set. The subject cannot build representational, structural, procedural, implication, generalization, or hierarchical connections.
	The subject of M-15 is less able to establish a type of representational connection, a structural connection between the elements of the set and the result of binary operations of two or three elements because the proof is less able to represent the arbitrariness of a set element against a binary operation on Cayley's table. In addition, the subject cannot build all types of representational, structural, implication, generalization, and hierarchical connections based on the confirmation of the existence of element identities and the presence of inverse elements.

According to the summary of the preceding interpretations, both subjects demonstrate incompetence in procedural connections, generalization connections, and hierarchical connections when concluding the closed, associative, and evidentiary property of the existence of identity elements and vice versa. As shown by the order of proof of the closed property, proof of the existence of elements of identity, inverse, then proof of the closed property, and proof of associative, the proof of the subject of M-15 is not hierarchical in demonstrating groups. Indicates that the subject was unable to establish a hierarchical connection. Subject M-8 composes conclusions regarding the proof of closure, associative property, the presence of identity elements, and inverse. The approach for showing the existence of identity and inverse elements has not been supported by logical proofs. However, it is sufficient to symbolically describe the demonstration of the group's four requirements.

Therefore, both subjects M-8 and M-15 tend to establish the ability of representational connections on issues of items 1 and 3 by associating two or three proofs of closed property and associative property. Concerning item 2, M-8 and M-15 students have a propensity to be incapable of establishing all types of mathematical connections. The following conclusion is that the two subjects are fairly capable of establishing a representational connection, but have not been able to build structural, procedural, implicative, generalization, or hierarchical connections.

a. Exploration Results of Mathematical Connections on Item 1

Based on the sample of the work of the subject M-8 on proving the closed property, it shows that the subject has not been able to recognize the elements of the set according to its definition, but in operating the two selected elements the results have been fulfilled. The inability of the subject M-8, because the selected elements, namely $x = 2k + \sqrt{p}$ and $y = 2a + \sqrt{b}$, already meet the definition of the set, if operated $+y = \left(2k + \sqrt{p}\right) + (2a + \sqrt{b})$. These results indicate that the subject does not think that the result of the operation must meet the known set or not. The inability of the subject to add up the elements of the set selected by representation, the students did not think that the results of the operation of the two elements were contained in the set Q.

Subject M-8 showed the fulfillment of the associative property by taking the three selected elements, namely $x = 2k + \sqrt{p}$, $y = 2a + \sqrt{b}$, and $z = 2c + \sqrt{d}$ have the same representation as the representation in the closed trait proof, so that when added together we get the result that the element structure is long even though it fulfills the element structure of the set Q. It already meets the definition of the set, if operated $+y = \left(2k + \sqrt{p}\right) + (2a + \sqrt{b})$. Then the subject representing the proof of identity element shows the answer: "there is an identity element, for example $(\exists i \in Q)(\forall x \in Q)$, then $i + x = i + x = x$ …..etc, so we get $i = 0$". This answer shows that the subject did not write down the element x that satisfies the set Q and did not pay attention to the procedure for obtaining it and the structure of the identity element i.

This shows that the subject M-8 does not show the representation of x and i of the elements of the set Q, and does not show the structure of the elements x and i of the

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identity elements. Procedurally from the work of proving the existence of an identity element shows "$i + x = i + x = x$", the subject writes the same structure between the left and right sides without using the right procedure, namely by linking the commutative property. Furthermore, based on proving the existence of an identity element, will affect the proof of the inverse element. This is shown from the subject's work because the identity element has been obtained, namely 0, then the subject finds the inverse element of x, namely –x, the reason for the subject $x + (-x) = 0$ because 0 is an identity element. Based on three types of representational, structural, and procedural connections of this subject's work will affect the next connection, namely connections when implication, generalization, and hierarchy do not fulfill them.

Based on the work and interviews indicate the suitability of the answers. Subject M-8 cannot describe the symbols $x, y \in Q$ and these symbols are $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, where $k, m \in \mathbb{Z}, p \in \mathbb{N}$ is the representative of the known members of the set. The subject cannot describe the symbol $x + y = (2k + \sqrt{p}) + (2m + \sqrt{p}) = 2(k + m) + \sqrt{p}$, such that 2 $2(k + m) + \sqrt{p}$ is the result of a binary operation of two elements x and The y chosen in the set Q and $2(k + m) + \sqrt{p}$ is contained in the set Q, because $m \in \mathbb{Z}, p \in \mathbb{N}$ then $(2(k + m)) \in \mathbb{Z}$. Symbol $2(k + m) + \sqrt{p} \in Q$ can show closed properties. Subject M-8 also cannot recognize the structure of the selected elements $x, y \in Q$ in the form of $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, where $k, m \in \mathbb{Z}, p \in \mathbb{N}$ of the set Q are written symbolically. Subject M-8 cannot describe the use of procedures or rules that $\forall x, y \in Q$ applies to $x + y \in Q$. Subject M-8 cannot work by implication in proving the closed property of the equation: if $\forall x, y \in Q$ with $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, with $k, m \in \mathbb{Z}$, $p \in \mathbb{N}$, then $x + y = (2k + \sqrt{p}) + (2m + \sqrt{p}) = 2(k + m) + \sqrt{p}$.

Furthermore, subject M-8 cannot conclude the process of working on closed properties which are arranged in such a way that it becomes a conclusion that $\forall x, y \in Q$ with $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, $k, m \in \mathbb{Z}$, $p \in \mathbb{N}$, such that $x + y = (2k + \sqrt{p}) + (2m + \sqrt{p}) = 2(k + m) + 2\sqrt{p}$, and $[2(k + m) + 2\sqrt{p}] \in Q$ which matches the closed property rule. Subject M-8 cannot present the order of execution: i) Take any $x, y \in Q$ with $x = 2k + \sqrt{p}$ and $y = 2m + \sqrt{p}$, with $k, m \in \mathbb{Z}$, $p \in \mathbb{N}$; ii) then $x + y = (2k + \sqrt{p}) + (2m + \sqrt{p}) = 2(k + m) + \sqrt{p}$; iii) Since $m \in \mathbb{Z}, p \in \mathbb{N}$, then $(2(k + m)) \in \mathbb{Z}$; iv) so that $[2(k + m) + \sqrt{p}] \in Q$. Subject M-8 cannot show a coherent and logical connection from evidence i) to ii), from evidence ii) to iii), and from evidence i), ii), iii) to iv).

Based on the description above, it shows that subject M-8 can only describe the representation of elements of the set Q in the form of $x = 2k + \sqrt{p}, y = 2a + \sqrt{b}$, and $z = 2c + \sqrt{d}$, thus the subject can only build connections to the representation of elements of the set Q.

Based on the following sample of the subject M-15's work, it demonstrates the ability to represent set elements and the results of binary operations but is unable to make general inferences. The inability of the subject M-15 in item 1, the proof for each group requirement is not written hierarchically. Subject M-15 writes the presence of identity elements, inverse elements, commutative properties, and associative properties following the demonstration of closed properties. The arbitrary elements that are acted on when establishing the existence of identity elements and inverses are not expressed in a form that corresponds to the properties of the set. The M-15 subject is only able to express the existence of a set element and the outcome of a binary operation, whereas the other connections in the proof of item 1 have not been constructed.

Excerpts of interviews with Subject M-15 on item 1:

R : How can you identify the elements of the set Q in item 1?
M-15 : I knew it form its terms, ma'am.
R : Okay, now what about the sum of the two elements of Q?
M-15 : I grouped the same symbols and root forms, ma'am.
R : Next, what about the conclusion, is it representative?
M-15 : I don't know, ma'am

Based on interview excerpts from the work on item 1 by both subjects M-8 and M-15, the answer tends to be the same as when using the analogy-abstraction approach; it is less able to conclude by proving closed property and associative property, and thus the conclusion is not general. In showing the existence of an element of identity, it is demonstrated that the two subjects cannot be connected in a representational, structural, procedural, implicational, generalizable, or hierarchical manner.

b. Exploration Results of Mathematical Connections on Item 2

Because the M-8 subjects' work samples and interview excerpts show uniformity, the data from the M-8 subjects is valid. Because the subject of M-8 could not recognize the structure of the set elements that satisfy the structure of its operations, the subject of M-8 cannot establish all types of representation connections, structural connections, procedural connections, implication connections, generalization connections, and connections hierarchy in this item 2.

Subject M-15's incapacity to recognize items of the selected set that fit the definition of its binary operations is demonstrated by an excerpt from the work. The subject M-15 cannot represent any element of the set that applies to the closed property while forming a conclusion. The subject M-15 is unable to properly connect its representation, structural connections, procedural connections, implications, generalization connections, and hierarchy connections due to

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its associative property, the presence of identity elements, and inverse elements.

It was validated further during an interview with the subject M-15, which is seen in the following excerpt of the interview.

Excerpts of interviews with subject M-15 on item 2:

R : Alright, what about the elements chosen in the proof of item 2?
M-15 : The elements I chose were "x, y", ma'am?
R : Does the operation match?
M-15 : Yes.
R : How is the result with the three elements operated?
M-15 : They match, ma'am.
R : Could you write a conclusion that can represent all elements of the set?
M-15 : I don't understand, ma'am.
R : What about proving the existence of identity elements?
M-15 : I think the identity element is 5.
R : How could you get 5?
M-15 : I solved in the form of an equation, ma'am.
R : What about the proof for inverse elements?
M-15 : The inverse of $is - x$, ma'am.
R : How could you get it?
M-15 : From the form of the equation "x $\oplus x^{-1} = i$", ma'am.
R : What's the next?
M-15 : I don't really understand, ma'am

Based on the interview excerpt with subject M-21, it is compatible with the excerpt of subject M-15's work. Subject M-15 can relate set elements to their binary operations, but cannot provide explanations for associative proofs, the existence of identical elements, or inverses. Subject M-15 indicates a propensity for failing to build representational, structural, procedural, implication, generalization, and hierarchical connections.

The subjects of M-8 and M-15 who utilized the type-2 approach to item 2 questions are unable to identify errors in their work, then attempt to respond without understanding the procedure for finding answers. Due to the subject's inadequacies in describing the limitations of the components of the set that can satisfy the specification of its binary operations, the two subjects are incapable of constructing six forms of mathematical connections.

c. Exploration Results of Mathematical Connections on Item 3

The parts that follow describe the outcomes of the item 3 test work and the results of interviews with the two research subjects M-8 and M-15. Based on their work showing closed and associative properties, subjects M-8 and M-15 were unable to draw implicative, general, and hierarchical implications. In addition, the two subjects were unable to represent identity elements and inverses representationally, structurally, procedurally, implicatively, generically, and hierarchically.

In addition, confirmation is conducted through interviews with M-8, which are described in the next section.

Excerpts of interviews with subject M-8 on item 3:

R : What about your work on item 3, do you understand?
M-8 : Yes, ma'am.
R : What about the closed property conclusion, has it been represented?
M-8 : Yes, ma'am.
R : What about the symbol "$aob = c, (\forall a, b \in S)(\exists c \in S)aob = c$", which element does the symbol represent?
M-8 : Ir represents the results of the operation, ma'am.
R : Can the symbol "$\exists$" represent all elements?
M-8 : Yes, ma'am.
R : So, what is the correct symbol?
M-8 : I'm sorry, my ma'am, I don't know.
R : Okay, now, what about the conclusions on the proof of associative properties?
M-8 : This one *(pointing to the written answer).*
R : Do you think the conclusions are already representative?
M-8 : Yes, I do.
R : Do you know whether the conclusion is complete or not?
M-8 : No, I don't.

Based on the aforementioned excerpts from the M-8 subject's interview, it appears that the M-8 subject's capacity to demonstrate the closed, associative, identity and inverse elements tends to be unable to build on all forms of connections.

In addition, a part of the interview is included in the subsequent section.

Excerpts of interviews with subject M-15 on item 3:

R : How about the proof of closed properties in item 3?
M-15 : It's because the results are contained in S, ma'am.
R : Which one indicates the fulfillment of a closed property?
M-15 : This one, ma'am. *(Pointing to the written answer)*
R : Could you explain the proof of closed property?
M-15 : I can only show like this, ma'am. *(Pointing to the written answer)*
R : Good

Based on excerpts of interviews with M-15 participants, it is less able to fully symbolize the operation's effects, and the conclusion is neither implicative, generic, or hierarchical. In addition, M-15 cannot establish the capabilities of all forms of connections based on evidence of associative qualities, the presence of identical elements, and the subject's inverse.

Therefore, subjects M-8 and M-15 tend to be unable of establishing a variety of connections. Consequently, the subjects tend to be able to establish the ability of

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representational connections on questions of items 1, 2, and 3 in relating two or three proofs of closed property and associative property. Nevertheless, the two subjects are unable to form a representational, structural, procedural, implicative, generalizing, or hierarchical connection. Concerning item 2, M-8 and M-15 students have a propensity to be incapable of establishing all types of mathematical connections. The inadequacy of these two subjects stems from their inability to tie the constituents of the set of numbers to the binary operating structure of non-standard forms in terms of representation, structure, and procedure.

4) *Exploration Results of Two Subjects Using Abstraction-Construction Stages (Type-1):* The following is a summary of the interpretation of the work of M-4 and M-13 study participants based on the results of the mathematical connection exam in questions 1, 2, and 3 as shown in Table VI below.

TABLE VI
ASUMMARY OF MATHEMATICAL CONNECTION TEST WORK INTERPRETATION BASED ON INDICATOR PREDICTION RUBRICS FOR SUBJECTS M-4 AND M-13

Initials of the Research Subject	**Interpretation of Mathematical Connection Type Ability**
M-4	The subject can only represent when it takes the elements of the set that meet in the proof, but it cannot operate procedurally on the evidence of the closed, associative, identity, and inverse properties. Therefore, M-4 subjects cannot build representation connections, structural connections, procedural connections, implication connections, generalization connections, or hierarchical connections.
	In question 2, the incapacity to construct representational connections, structural connections, procedural connections, implication connections, generalization connections, and hierarchical connections are demonstrated. This inability is due to the subject's inability to connect the structure of the existence of the set element with the non-standard structure of binary operation definition.
	In the case of item 3, the subject cannot build representational, structural, implication, or hierarchical connections based on the proof of closed property, associative qualities, the presence of elements of identity, or inverse. The subject just inputs the set's elements into the Cayley table.
M-13	At the time of determining identity and inverse elements, the subject is unable to construct implication, generalization, and hierarchical connections.
	The subject is unable to provide evidence of the closed property and associative property. The incapacity of the M-13 subject to recognize the structure of set elements and the structure of non-standard binary operations. Therefore, it has been unable to adequately build a variety of connections.
	In the case of item 3, the subject was unable to construct representational connections, structural connections, implication connections, hierarchical connections based on proof of closed property, associative properties, the presence of elements identity, and inverse. But just writing down the set's elements in the form of a Cayley table without presenting anything else.

Based on the prediction rubric of the connection type indication, the two subjects M-4 and M-13 demonstrated an inability to build representational connection skills, but not the other 5 types of connections. i.e., structural and procedural linkages to the proof conclusions of a closed property. The two subjects were unable to make procedural, structural, implication, generalization, and hierarchical connections to the evidence that identity and inverse elements exist. The ineptitude of these two subjects conforms to the type-1 (abstraction-construction) method, according to which, if the subject is to comprehend the definition, simple (familiar) instances should precede it, for instance in the form of its already familiar set. In contrast, when it comes to developing binary set structures and operations that differ from the subject's knowledge, they typically cannot yet relate to them.

Based on the results of the work of the two subjects, writing the replies to the stage revealed a closed property to the operational item, but no conclusion has been reached. The two subjects are typically unable to answer or explain the cause when interviewed. Both interviewees demonstrate the same behavior. M-4 and M-13 students continue to rely on the examples and responses of their peers. This incapacity implies consistency with the abstraction-construction processes they employ.

Abstractions-constructions, analogies-abstractions, and constructions-analogies have different tendencies to build the ability of six types of mathematical connections, according to the results of a study of the mathematical connection ability of eight research subjects categorized using construction approaches. Subjects who employ the construction stage are typically capable of establishing six types of mathematical connection links in a set and standard and non-standard binary operation. Research subjects who utilize the construction-analogical stage likely to be able to establish three forms of representation connections, structural connections, and procedural connections in a set and standard binary operation. Subjects that employ analogy-abstraction stages tend to describe the symbol of a set element and the binary operation of the default form in a closed property using representational links. In describing the symbol of a set element and the binary operation of the standard form in the closed property of the standard form and the non-standard form (in binary operations defined by Caley's table), subjects who use abstraction-construction stages tend to establish representational connections. The authors' previous publications present research data that has been published

B. Discussion

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Based on the findings of the study conducted on eight research subjects grouped using four types of stages (abstraction-construction, analogy-abstraction, construction, and construction-analogy), it is possible to develop numerous variants of six types of connections in abstract algebra. Subjects who employ the construction stage are typically capable of establishing six types of mathematical connection links in a set and standard and non-standard binary operation. Suominen (2018) and Junarti (2020c) describe mathematics as the types of mathematical linkages. In proving a group, these six sorts of mathematical connections were used, in addition to other types of connections that were not predicted by this study. While the steps needed to construct these six sorts of connections are the same steps that students should master in abstract algebra or analytic classes, these six forms of connections are unique.

The novelty of exploring these six types of mathematical connections reveals the existence of a correlation between students' inability in the topic of groups and their inability to connect the six types of mathematical connections that are crucial for proofs and problem-solving in abstract algebra. This aligns with Yoshioka and Higashibata's (2019) assertion that abstract algebra requires proficiency in these six types of mathematical connections.

The originality of uncovering these six types of mathematical connections specifically for the subject of abstract algebra has not been explored in previous studies or research. Based on the findings of this investigation, which maps the proficiency in these six types of mathematical connections, it greatly assists instructors in identifying the needs for these connections, particularly in the topic of groups, and simultaneously helps reduce the difficulty level for students in the abstract algebra course. This is consistent with Yoshioka and Higashibata's (2019) statement that mathematical connections are essential for linking ideas between concepts, and poor mathematical connection skills can lead to students' failure in solving mathematical problems (Pambudi et al., 2020).

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Research subjects who utilize the construction-analogical stage likely to be able to establish three forms of representation connections, structural connections, and procedural connections in a set and standard binary operation. Students utilize these stages of construction-analogy most frequently. This stage relates to the Junarti level (2020c) and is characterized by the ability to analogize the structure of new or unfamiliar mathematical characteristics or objects. At this point, students are able to abstract definitions without concrete instances. This conforms to the steps described by Gómez-Ferragua et al. (2013) and Junarti (2020c), in which participants are able to generate non-standard mathematical properties/objects based on their definition references. In addition, analogical reasoning, which (Richland & Begolli, 2016) encourages students' higher-level thinking, might be employed as a follow-up in the next stage of capacity development.

Subjects that employ analogy-abstraction stages tend to describe the symbol of a set element and the binary operation of the default form in a closed property using representational links. In accordance with Novotna (2016), Gómez-Ferragud et al (2013), Okotac (2016), Richland & Begolli (2017), and Oktac (2017), students can use analogies to extract known structures of mathematical properties or objects (2016). In addition, students are able to design unfamiliar or novel structures of mathematical characteristics or objects and then abstract their meanings. The stages begin with the analogy so that the definition can be more easily abstracted.

In describing the symbol of a set element and the binary operation of the standard form in the closed property of the standard form and the non-standard form (in binary operations defined by Caley's table), subjects who use abstraction-construction stages tend to establish representational connections. Beginning with abstracting the concept through specific existing examples, these two subjects then connect representations to closed, associative qualities, the presence of identity elements, and inverse elements. According to Tapahan Dubinsky et al. (1994), Novotna (2006), Oktac (2016), and Junarti (2020c), abstraction is achieved when the subject applies specifically known examples to the prerequisite material.

The type of mathematical connection utilized by students is contingent on the stages employed. There are just six sorts of mathematical connections required for this course's mathematical content connections. The linking of content in abstract algebra is a major focus of Wasserman's (2018) research, as well as Zbiek and Heid's (2018) work connecting high school subjects to the mathematical activity-based instruction of abstract algebra. The subjects' mastery of prerequisite material determines the type of concepts associated with abstract algebra (Junarti, 2020c), hence changing the type of mathematical connection it establishes.

Different conceptual understandings of mathematics across students necessitate distinct phases, methodologies, and strategies for connecting them. Individually practiced habits cause differences in comprehension (Junarti et al, 2019b). This is consistent with the objective of mathematics education in the twenty-first century, which blends different conceptual proximity (Golding, 2018) to accommodate the demands of subjects with different features and skills.

IV. Conclusions

The finding of this study is that the subjects of each stage used had a tendency to construct various forms of mathematical relationships. Subyek tends to be able to establish six types of mathematical connection links in a set, as well as standard and non-standard binary operations (as specified by Caley's table).

In the meantime, research studies that employ the construction-analogical stage tend to be able to establish three forms of representation links, structural connections, and procedural connections in a set of standard binary operations. These three forms of possible mathematical connections are influenced by the level of comprehension of prior knowledge (knowledge of prerequisite materials)

Research subjects that utilize both the analogy-abstraction and abstraction-construction stages have a similar propensity for establishing mathematical connections. The kind of connection established is the representation connection type when describing the symbol of a set element and the binary operation of the standard form in the closed property, particularly in the standard form as in binary operations specified in the form of Caley's table.

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