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Dynamic behavior in a Cournot duopoly with social responsibility

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ABSTRACT

In an oligopoly with isoelastic demand, the paper analyzes the quantity competition between N_{PM} profit-maximizing firms and N_{RS} socially responsible firms whose objective function is a linear combination of profit and consumer surplus.

From the static analysis it follows that greater social responsibility has a competitive effect, since reduces the equilibrium price and increases the market share of socially responsible firms. In addition, it increases both the consumer surplus and total surplus.

For the duopoly case, the dynamic study leads to the conclusion that, if at least one of the firms follows the gradient rule as an adjustment mechanism, an increase in the speed of adjustment is a source of instability. An increase in the value of the elasticity of demand as well as a reduction in the marginal cost has a stabilizing effect on the Cournot equilibrium. A higher level of social responsibility exerts a stabilizing role on the dynamics as long as demand is sufficiently elastic.

1. Introduction

In recent years, the term Corporate Social Responsibility (CSR) has gained special relevance, not only in academic literature, but also in articles for public diffusion. This term refers to a form of corporate self-regulation that takes into account the interests of different stakeholders in the firm and includes ethical, social or environmental objectives. For an extensive review of the topic see [1].

From an empirical point of view, there are several contributions that investigate the benefits of including social responsibility objectives, although most of them focus on isolated initiatives, and therefore may overestimate the initiatives of individual agents [2], and have mainly focused on financial outcomes, showing that there is no clear relationship between social responsibility outcomes and financial results [3,4].

From a theoretical perspective, CSR has been studied under different approaches. Some contributions identify CSR with the creation of public goods [5–7], deducing a parallelism between CSR and results obtained in models of private provision of public goods. Other papers analyze the desirability of social responsibility [8], its role in the selection of motivated agents [9] and study competition in the presence of "green" consumers [10,11]. Other authors present CSR as a strategic weapon to gain competitive advantage and increase profits [12], thus corroborating the original idea of [13], who points out that CSR is simply to increase profits. Along these lines, Kopel and Brand [14] analyze a duopoly à la Cournot in which a profit-maximizing firm and a socially responsible firm compete, which, in addition to incorporating profit into its objective, considers consumer surplus. The authors show that the socially responsible firm achieves a higher market share and a higher profit than its competitor. The strategic effects between a set of profit-maximizing firms and a firm that takes into account consumer surplus and the amount of pollutant emissions are analyzed in [15]. The authors show that, if the market is large enough, the socially responsible firm earns higher profits than the other firms and leads to a higher level of social welfare.

The strategic nature of CSR under imperfect competition has been analyzed by other authors. For example, in [16], in a differentiated duopoly, it is shown that under Cournot competition, the adoption of a CSR strategy by at least one firm can be the result of a subgame perfect Nash equilibrium, in both symmetric and asymmetric equilibria, depending on the degree of differentiation and the level of social responsibility.

In a duopoly with perfectly substitute products, and assuming a linear demand, Fanti and Bucella [17] analyze CSR in a context of strategic delegation. The authors show that, in the subgame perfect Nash

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equilibrium, both firms adopt a strategy based on CSR, leading to higher profits and lower total surplus.

More recently, Planer-Friedrich and Sahm [18] analyze the strategic use of CSR in an oligopoly with linear demand, and considering both homogeneous and differentiated product. The authors show that under Cournot competition, firms assign a positive level of CSR to their objective function, regardless of the number of competitors, even if this leads to lower profits. Furthermore, it is shown that CSR can constitute a barrier to entry, whereby social responsibility can increase market concentration. Furthermore, the results show that, under quantity competition, the levels of social responsibility decrease as the degree of differentiation increases, and are zero under competition à la Bertrand.

Theoretical contributions in the analysis of CSR are carried out in a static context, with few works that develop a dynamic analysis in the presence of social responsibility. As exceptions we can mention the work of [2], who analyze in a dynamic context how a firm should achieve its social responsibility objectives over time. In a dynamic optimization model, Becchetti et al. [19] analyze the competition between a profit-maximizing incumbent and a socially responsible entrant. The authors show that the incumbent firm reacts both through price and social commitment to the entry of the socially responsible firm. Moreover, if consumers' social interest grows sufficiently, the incumbent firm decides to invest in CSR activities.

From an evolutionary perspective, Kopel et al. [20] analyze quantity-based competition in an oligopoly. The authors consider two groups of firms: those whose objective is profit maximization and those that maximize a linear combination of profit and consumer surplus. In the evolutionary model, the objective of the firms is endogenously determined, deducing that it can be profitable to implement a socially responsible strategy if consumers are willing to pay a higher price for the product of the firm that pursues a social objective. From the dynamic point of view, it follows that, if the propensity of firms to switch from one strategy to another is sufficiently large, steady states are unstable and complex dynamics may emerge.

More recently, in the context of a differential game, Lambertini et al. [21] study competition in a Cournot duopoly with accumulation of output and a negative environmental externality (pollution), assuming that one of the firms has the CSR objective and the other is a profit maximizer. The authors conclude that, if the market is large enough, the socially responsible firm sells more, accumulates more capital and earns higher profits than the rival firm.

This paper provides results both in the analysis of competition in the presence of firms with CSR objectives, and in the study of non-linear dynamics in oligopoly models [22–25]. On the one hand, in a static context, we analyze quantity competition in an oligopoly with isoelastic demand, assuming that a set of profit-maximizing firms compete with other socially responsible firms which incorporate in their objective function the consumer surplus. On the other hand, in an asymmetric duopoly model, the influence on the asymptotic stability of the Cournot equilibrium of the elasticity of demand, as well as the degree of social responsibility, is analyzed. To that end, it is assumed that firms adjust quantities by adopting different expectations schemes, in the context of discrete time.

The rest of the paper is organized as follows. Section 2 introduces the oligopoly model and develops the static analysis. Section 3 analyzes the dynamic competition in quantities and the local stability of the Cournot equilibrium, considering a duopoly model under different expectations schemes. Section 4 concludes the paper with the main conclusions.

2. The model. Static analysis

We assume an oligopoly where firms produce a homogeneous product and compete in quantities. Market demand is isoelastic, so that the inverse demand curve is given by: $p(Q) = Q^{\frac{-1}{\eta}}$ where p is the market price, $Q = \sum_{i=1}^{N} q_i$ is the total output, with $q_i \geq 0$ being the quantity supplied by firm i, and $\eta > 1$ is a parameter representing the elasticity of demand. We consider a market with N firms being N_{PM} “profit maximizing firms”, and N_{RS} “socially responsible firms”, with $N = N_{PM} + N_{RS}$. Following [14], the subscripts PM and RS refer to a profit maximizing firm and a socially responsible firm, respectively. We assume that all firms have identical unit production costs, i.e., $c_i = c > 0,\ i = 1,\dots,N$, and the firms belonging to the same group are symmetrical. Therefore, the objective of a PM firm is the profit maximization, being the objective function:

$$\pi_{iPM}(q_{iPM}, q_{-iPM}) = (p-c)q_{iPM} = \left(Q^{\frac{-1}{\eta}} - c\right)q_{iPM}, \quad i = 1,\dots,N_{PM} \tag{1}$$

By contrast, the objective of a RS firm is to maximize the sum of its profit and a share of consumer surplus (see [14]). Formally, the objective function of a RS firm is given by:

$$V_{jRS}(q_{jRS}, q_{-jRS}) = \pi_{jRS}(q_{jRS}, q_{-jRS}) + \theta\, CS(q_{jRS}, q_{-jRS}) = \left(Q^{\frac{-1}{\eta}} - c\right)q_{jRS} + \theta \frac{Q^{\frac{-1}{\eta}+1}}{\eta - 1}, \quad j = 1,\dots,N_{RS} \tag{2}$$

where $\theta \in [0,1)$ is a parameter representing the weight any RS firm puts on consumer surplus given by $CS(q_{PM}, q_{RS}) = \frac{Q^{\frac{-1}{\eta}+1}}{\eta-1}$ for all $\eta > 1$.[1]

All firms simultaneously set the quantities that maximize their respective objective functions, given by (1) and (2).

Thus, given q_{-iPM}, the first order condition of the problem of profit maximization by a PM firm is:

$$\frac{\partial \pi_{iPM}}{\partial q_{iPM}} = 0 \Leftrightarrow Q^{-1/\eta} - \frac{q_{iPM}}{\eta} Q^{-(1+\eta)/\eta} - c = 0, \quad i = 1,\dots,N_{PM} \tag{3}$$

The Eq. (3) allows us to define implicitly the best response function of any PM firm, $q_{iPM} = R_{iPM}(q_{-iPM})$, since the second order condition of the problem of profit maximization is verified:

$$\frac{\partial^2 \pi_{iPM}}{\partial q_{iPM}^2} = -\frac{Q^{-(1+2\eta)/\eta}}{\eta}\left[2\left(\sum_{k=1,k\neq i}^{N_{PM}} q_{kPM} + \sum_{j=1}^{N_{RS}} q_{jRS}\right) + \left(1 - \frac{1}{\eta}\right)q_{iPM}\right] < 0, \quad i = 1,\dots,N_{PM}$$

The first order condition of the maximization problem for RS firms is given by:

$$\frac{\partial V_{jRS}}{\partial q_{jRS}} = 0 \Leftrightarrow Q^{-1/\eta} - \frac{q_{jRS}}{\eta} Q^{-(1+\eta)/\eta} + \theta \frac{Q^{-1/\eta}}{\eta} - c = 0, \quad j = 1,\dots,N_{RS} \tag{4}$$

This equation implicitly defines the best response function of RS firms, $q_{jRS} = R_{jRS}(q_{-jRS})$, given that the second order condition of the maximization problem is satisfied:

$$\frac{\partial^2 V_{jRS}}{\partial q_{jRS}^2} = -\frac{Q^{-(1+2\eta)/\eta}}{\eta}\left[\left(2 + \frac{\theta}{\eta}\right)\left(\sum_{i=1}^{N_{PM}} q_{iPM} + \sum_{k=1,k\neq i}^{N_{RS}} q_{kRS}\right) + \left(1 - \frac{1-\theta}{\eta}\right)q_{jRS}\right] < 0, \quad j = 1,\dots,N_{RS}$$

[1] For $0 \leq \eta \leq 1$ consumer surplus is undefined, given that the integral $\int_0^{\overline{Q}} Q^{\frac{-1}{\eta}} dQ$ does not converge.

Adding (3) and (4), the total quantity produced and demanded in equilibrium is deduced:

$$Q^* = \left(\frac{N\eta + \theta N_{RS} - 1}{N\eta c}\right)^{\eta} \tag{5}$$

Substituting this expression in (3) and (4), and under the assumption of symmetry $\left(q_{iPM} = q_{PM}, q_{jRS} = q_{RS}\right)$ we can deduce the quantities produced by each firm *PM* and *RS*, respectively, in the Cournot-Nash equilibrium $E^* = (q^*_{PM}, q^*_{RS})$ being each component of $q^*_{\mathbf{PM}}$ and $q^*_{\mathbf{RS}}$:

$$\left.\begin{aligned} q^*_{PM} &= \frac{1 - N_{RS}\theta}{N^{1+\eta}}\left(\frac{N\eta + \theta N_{RS} - 1}{c\eta}\right)^{\eta} = \frac{1 - N_{RS}\theta}{N}Q^* \\ q^*_{RS} &= \frac{1 + N_{PM}\theta}{N^{1+\eta}}\left(\frac{N\eta + \theta N_{RS} - 1}{c\eta}\right)^{\eta} = \frac{1 + N_{PM}\theta}{N}Q^* \end{aligned}\right\} \tag{6}$$

Note that the condition $\theta N_{RS} < 1$, ensures that profit-maximizing firms capture market share. It follows, therefore, that a very high aggregate social intensity, given by θN_{RS}, can be a barrier to entry for firms that are not socially responsible. This result is in line with the conclusions of [18] where it is concluded that social responsibility can increase market concentration.

The price equilibrium and the equilibrium values of the objective functions (1) and (2) are respectively given by:

$$p^* = \frac{N\eta c}{N\eta + \theta N_{RS} - 1}$$

$$\pi^*_{PM} = \left(\frac{N\eta + \theta N_{RS} - 1}{\eta c}\right)^{\eta-1}\frac{(1 - \theta N_{RS})^2}{N^{\eta+1}\eta} \leq$$

$$\leq \pi^*_{RS} = \left(\frac{N\eta + \theta N_{RS} - 1}{\eta c}\right)^{\eta-1}\frac{(1 - \theta N_{RS})(1 + \theta N_{PM})}{N^{\eta+1}\eta}$$

$$V^*_{RS} = \left(\frac{2\eta + \theta - 1}{\eta c}\right)^{\eta-1}\frac{(1 - \theta N_{RS})(1 + \theta N_{PM})(\eta - 1) + N^2\eta\theta}{N^{\eta+1}\eta(\eta - 1)}$$

Through a comparative statics analysis, we can deduce that, from an overall market point of view, greater corporate responsibility on the part of the *RS* firms has a competitive effect, as it leads to a reduction in price $\left(\frac{\partial p^*}{\partial \theta} < 0\right)$ and an increase in the total quantity offered $\left(\frac{\partial Q^*}{\partial \theta} > 0\right)$. Similarly, an increase in the number of socially responsible firms leads to a reduction in price and an increase in the total quantity demanded. $\left(\frac{\partial p^*}{\partial N_{RS}} < 0 \frac{\partial Q^*}{\partial N_{RS}} > 0\right)$.

It is deduced that $\underset{N_{PM}\to\infty}{Lim}\, p^* = c$, and $\underset{N_{RS}\to\infty}{Lim}\, p^* = \frac{\eta c}{\eta + \theta} < c$. This corroborates the competitive effect of social responsibility. Indeed, it is easy to deduce that $p^* > c$ if the condition $\theta N_{RS} < 1$ holds. Thus, a sufficiently high overall level of social responsibility can eliminate the market power.

Individually, in the presence of higher social responsibility, the profit-maximizing firms will have incentives to reduce their quantity offered $\left(\frac{\partial q^*_{PM}}{\partial \theta} < 0\right)$, while the socially responsible firm will increase it $\left(\frac{\partial q^*_{RS}}{\partial \theta} > 0\right)$.

From the point of view of the *PM* firms, it is easy to deduce that the higher the social responsibility, the lower the *PM* firm's profitability, since $\frac{\partial \pi^*_{PM}}{\partial \theta} < 0$ and $\frac{\partial \pi^*_{PM}}{\partial N_{RS}} < 0$.

These results corroborate the conclusions obtained by [14] under the assumption of linear demand.

The effect exerted by the value of the elasticity of demand is the well-known one. Indeed, a higher elasticity reduces the market power of both firms, since $\frac{\partial p^*}{\partial \eta} < 0$. As a consequence, all firms increase their quantity offered $\left(\frac{\partial q^*_{PM}}{\partial \eta} > 0, \frac{\partial q^*_{RS}}{\partial \eta} > 0\right)$.

By contrast, a higher value of marginal cost implies a lower quantity produced, since $\frac{\partial q^*_{PM}}{\partial c} < 0$ and $\frac{\partial q^*_{RS}}{\partial c} < 0$, which means a higher price $\left(\frac{\partial p^*}{\partial c} > 0\right)$ and increased market power for all firms. Moreover, the value of the objective functions is reduced in the presence of an increase in marginal cost $\left(\frac{\partial V^*_{RS}}{\partial c} < 0 \frac{\partial \pi^*_{PM}}{\partial c} < 0\right)$.

The influence of social responsibility can also be analyzed from a welfare point of view. The values of consumer and producer surplus in equilibrium are given as, respectively:

$$CS^* = \left(\frac{N\eta + \theta N_{RS} - 1}{\eta c}\right)^{\eta-1}\frac{1}{(\eta - 1)N^{\eta-1}}$$

$$PS^* = N_{PM}\pi^*_{PM} + N_{RS}\pi^*_{RS} = \left(\frac{N\eta + \theta N_{RS} - 1}{\eta c}\right)^{\eta-1}\frac{1 - \theta N_{RS}}{N^{\eta}\eta}$$

From these expressions, it can be deduced that both a greater weighting of social responsibility (θ increases), as a higher number of socially responsible firms (N_{RS} increases), increases the value of consumer surplus and reduces the value of producer surplus: $\frac{\partial CS^*}{\partial \theta} > 0, \frac{\partial CS^*}{\partial N_{RS}} > 0, \frac{\partial PS^*}{\partial \theta} < 0, \frac{\partial PS^*}{\partial N_{RS}} < 0$.

Globally, the total surplus is given by:

$$W^* = CS^* + PS^* = \left(\frac{N\eta + \theta N_{RS} - 1}{\eta c}\right)^{\eta-1}\frac{N\eta + (\eta - 1)(1 - \theta N_{RS})}{\eta(\eta - 1)N^{\eta}}$$

It is deduced that: $\frac{\partial W^*}{\partial \theta} > 0, \frac{\partial W^*}{\partial N_{RS}} > 0$, under the assumption $\theta N_{RS} < 1$. Consequently, these results are in line with the conclusions obtained in [16], in a Cournot duopoly model with differentiated products.

3. Dynamic analysis

In this section, we introduce a dynamic adjustment process in the quantity competition. In this setting, the dynamic stability of the equilibria will be analyzed, as well as, the influence of the social responsibility, demand elasticity, and marginal cost on the stability of equilibria and the dynamic behavior of quantity trajectories. For the purpose of simplification and to facilitate the presentation and understanding of the results, we will focus on the case of a duopoly with one profit-maximizing firm and one socially responsible firm $(N_{PM} = N_{RS} = 1)$.

The dynamic process depends on the assumed time scale and on the way the firms adjusts quantities, which in turn, is conditional on their expectation formation.

We assume the discrete time case and three expectations rules: adaptive expectations, gradient rule based on marginal utility, and the Local Monopolistic Approximation (LMA). Each scheme of expectations implies a certain degree of bounded rationality.

In addition, we will consider two scenarios: one in which both firms adopt the same type of expectations (homogeneous expectations) and the other, in which each firm follows a different type of expectation (heterogeneous expectations).

3.1. Homogeneous expectations

In this section we analyze the dynamic of the model assuming that both firms choose the quantity produced according to the same adjustment rule.

3.1.1. Adaptive expectations

We assume that firm i uses an adjustment mechanism based on its best response function at any time t to determine production at time $t+1$. Each firm changes its output quantity proportionally to the difference between the value given by the reaction function and the quantity for the last period (see [23]). Formally:

$$q_{i,t+1} - q_{i,t} = \beta_i\left(R_i\left(q_{j,t}\right) - q_{i,t}\right),\ i,j = PM, RS,\ i \neq j \tag{7}$$

with $0 < \beta \leq 1$.

From the previous expression we obtain the dynamic system:

$$T_A : \begin{cases} q_{PM,t+1} = (1-\beta_{PM})q_{PM,t} + \beta_{PM} R_{PM}(q_{RS,t}) \\ q_{RS,t+1} = (1-\beta_{RS})q_{RS,t} + \beta_{RS} R_{RS}(q_{PM,t}) \end{cases} \tag{8}$$

By setting the fixed point conditions $q_{i,t+1} = q_{i,t} = q_i$ in the system (8), we obtain a unique steady state which is the Cournot-Nash equilibrium E^* given in (6).

In a discrete-time dynamic system, the condition for local asymptotic stability of an equilibrium is, as is well known, that the eigenvalues of the Jacobian matrix of system calculated at the equilibrium point should be inside the unit circle. In the two-dimensional case, the condition for local stability of the equilibrium can be given in terms of trace (*Tr*) and determinant (*Det*) of the associated Jacobian matrix (Schur's conditions, see [26]):

$$\left.\begin{array}{l} (i)\ 1 - Tr + Det > 0 \\ (ii)\ 1 + Tr + Det > 0 \\ (iii)\ 1 - Det > 0 \end{array}\right\} \tag{9}$$

If any single inequality in (9) becomes an equality, with the other two being simultaneously fulfilled, the equilibrium loses stability through either, a transcritical bifurcation, when $1 - Tr + Det = 0$, or a Flip bifurcation, when $1 + Tr + Det = 0$, or a Neimark-Sacker bifurcation, when $1 - Det = 0$.

The Jacobian matrix of system (8) evaluated at E^* is:

$$JT_A(E^*) = \begin{pmatrix} 1-\beta_{PM} & \beta_{PM} R'_{PM}(q^*_{RS}) \\ \beta_{RS} R'_{RS}(q^*_{PM}) & 1-\beta_{RS} \end{pmatrix} = \begin{pmatrix} 1-\beta_{PM} & -\beta_{PM}\dfrac{\eta - 1 + \theta(1+\eta)}{2\eta(1+\theta) + (\eta-1)(1-\theta)} \\ -\beta_{RS}\dfrac{(\eta-1)(1-\theta)}{2\eta + (\eta-1)(1-\theta)} & 1-\beta_{RS} \end{pmatrix}$$

We can deduce the following result for the local stability of the Nash equilibrium E^*.

Proposition 1. Under an adaptive expectations scheme, for all $\eta > 1$, $0 \le \theta < 1$ and $c > 0$, the Cournot-Nash equilibrium is locally asymptotically stable.

Proof. The trace and determinant of the Jacobian matrix $JT_A(E^*)$ are:

$$Tr = 2 - (\beta_{PM} + \beta_{RS}) \ge 0$$

$$Det = 1 - (\beta_{PM} + \beta_{RS}) + \beta_{PM}\beta_{RS}\left[1 - \frac{(1-\theta)(\eta-1)[\eta - 1 + \theta(1+\eta)]}{[2\eta(1+\theta) + (\eta-1)(1-\theta)][2\eta + (\eta-1)(1-\theta)]}\right]$$

It is verified that $Det = Tr - 1 + M$, with

$$M = \beta_{PM}\beta_{RS}\frac{4\eta(2\eta + \theta - 1)}{[2\eta(1+\theta) + (\eta-1)(1-\theta)][2\eta + (\eta-1)(1-\theta)]} > 0$$

Substituting these expressions into (9), we can deduce that Schur's conditions are verified:

$(i) 1 - Tr + Det = M > 0$

$(ii) 1 + Tr + Det = 2Tr + M > 0$

$(iii) 1 - Det = 2 - (Tr + M) =$

$$\beta_{PM} + \beta_{RS}\left[1 - \beta_{PM}\frac{4\eta(2\eta+\theta-1)}{[2\eta(1+\theta)+(\eta-1)(1-\theta)][2\eta+(\eta-1)(1-\theta)]}\right] > 0$$

The third condition is fulfilled since

$$0 < \frac{4\eta(2\eta+\theta-1)}{[2\eta(1+\theta)+(\eta-1)(1-\theta)][2\eta+(\eta-1)(1-\theta)]} < 1$$

$$0 < \beta_{PM} \le 1$$

then $1 - \beta_{PM}\frac{4\eta(2\eta+\theta-1)}{[2\eta(1+\theta)+(\eta-1)(1-\theta)][2\eta+(\eta-1)(1-\theta)]} > 0$ □.

3.1.2. Gradient rule expectations

Now, we assume that both firms follow the gradient rule. Under this rule, each firm decides to increase (decrease) its output quantity for period $t+1$, according to its marginal utility is positive (negative) at time t (see [23]). Formally:

$$q_{i,t+1} - q_{i,t} = \alpha_i(q_{i,t})\frac{\partial U_i(q_{i,t}q_{j,t})}{\partial q_{i,t}}, \quad i,j = PM, RS,\ i \ne j \tag{10}$$

where $\alpha_i(q_{i,t})$ is a positive function, which gives the extent of quantity variation of firm i following a given utility signal. A linear function is usually assumed, $\alpha_i(q_{i,t}) = \alpha_i q_{i,t}$, with $\alpha_i > 0$. $U_i(q_{i,t}, q_{j,t})$ denotes de objective function for each firm.

Thus, we obtain the following dynamic system:

$$\begin{cases} q_{PM,t+1} = q_{PM,t} + \alpha_{PM}\, q_{PM,t}\dfrac{\partial \pi_{PM,t}}{\partial q_{PM,t}} \\ q_{RS,t+1} = q_{RS,t} + \alpha_{RS}\, q_{RS,t}\dfrac{\partial V_{RS,t}}{\partial q_{RS,t}} \end{cases} \tag{11}$$

Substituting $\frac{\partial \pi_{PM,t}}{\partial q_{PM,t}}$ and $\frac{\partial V_{RS,t}}{\partial q_{RS,t}}$ into (11), the two-dimensional system that describes the dynamics of the game is given by the following nonlinear map:

$$T_G : \begin{cases} q_{PM,t+1} = q_{PM,t} + \alpha_{PM}\, q_{PM,t}\left(Q_t^{-1/\eta} - \dfrac{q_{PM,t}}{\eta}Q_t^{-(1+\eta)/\eta} - c\right) \\ q_{RS,t+1} = q_{RS,t} + \alpha_{RS}\, q_{RS,t}\left(Q_t^{-1/\eta}\left(1 + \dfrac{\theta}{\eta}\right) - \dfrac{q_{RS,t}}{\eta}Q_t^{-(1+\eta)/\eta} - c\right) \end{cases} \tag{12}$$

This system has three steady states: $E_1 = \left(\left(\frac{\eta-1}{c\eta}\right)^\eta, 0\right)$, $E_2 = \left(0, \left(\frac{\eta-1+\theta}{c\eta}\right)^\eta\right)$ and the Cournot-Nash equilibrium E^* given in (6).

The Jacobian matrix of (12) is as follows:

$$JT_G(q_{PM}q_{RS}) = \begin{pmatrix} 1 + \alpha_{PM}\dfrac{\partial \pi_{PM}}{\partial q_{PM}} + \alpha_{PM}\, q_{PM}\dfrac{\partial^2 \pi_{PM}}{\partial q^2_{PM}} & \alpha_{PM}\, q_{PM}\dfrac{\partial^2 \pi_{PM}}{\partial q_{PM}\partial q_{RS}} \\ \alpha_{RS}\, q_{RS}\dfrac{\partial^2 V_{RS}}{\partial q_{RS}\partial q_{PM}} & 1 + \alpha_{RS}\dfrac{\partial V_{RS}}{\partial q_{RS}} + \alpha_{RS}\, q_{RS}\dfrac{\partial^2 V_{RS}}{\partial q^2_{RS}} \end{pmatrix}$$

This matrix evaluated in the steady state E_1 leads to:

$$JT_G(E_1) = \begin{pmatrix} 1 - \alpha_{PM}\dfrac{c}{\eta} & \alpha_{PM}\dfrac{c}{\eta(\eta-1)} \\ 0 & 1 + \alpha_{RS}\dfrac{(1+\theta)c}{\eta - 1} \end{pmatrix}$$

which eigenvalues are:

$$\lambda_1 = 1 - \alpha_{PM}\frac{c}{\eta} < 1$$

$$\lambda_2 = 1 + \alpha_{RS}\frac{(1+\theta)c}{\eta - 1} > 1$$

and we deduce E_1 is an unstable boundary equilibrium of (12). Moreover, it is verified:

$$\alpha_{PM} < \frac{2\eta}{c} \Rightarrow -1 < \lambda_1 < 1 \Rightarrow E_1 \text{ is a saddle point}$$

$$\alpha_{PM} > \frac{2\eta}{c} \Rightarrow \lambda_1 < -1 \Rightarrow E_1 \text{ is a source}$$

The matrix $JT_G(q_{PM}, q_{RS})$ evaluated in the steady state E_2 leads to:

$$JT_G(E_2) = \begin{pmatrix} 1 + \alpha_{PM}\dfrac{c(1-\theta)}{\eta - 1 + \theta} & 0 \\ \alpha_{RS}\dfrac{c(1-\theta)}{\eta(\eta - 1 + \theta)} & 1 - \alpha_{RS}\dfrac{c}{\eta} \end{pmatrix}$$

which eigenvalues are:

$$\lambda_1 = 1 + \alpha_{PM}\frac{c(1-\theta)}{\eta - 1 + \theta} > 1$$

$$\lambda_2 = 1 - \alpha_{RS}\frac{c}{\eta} < 1$$

and we deduce E_2 is an unstable boundary equilibrium of (12). Moreover, it is verified:

$$\alpha_{RS} < \frac{2\eta}{c} \Rightarrow -1 < \lambda_2 < 1 \Rightarrow E_2 \text{ is a saddle point}$$

$$\alpha_{RS} > \frac{2\eta}{c} \Rightarrow \lambda_2 < -1 \Rightarrow E_2 \text{ is a source}$$

The matrix $JT_G(q_{PM}, q_{RS})$ evaluated in the Nash equilibrium E^* leads to:

$$JT_G(E^*) = \begin{pmatrix} 1 - \alpha_{PM}\dfrac{3\eta + \theta - 1 + \eta\theta}{4\eta^2}(1-\theta)Q^{*-1/\eta} & -\alpha_{PM}\dfrac{\eta + \theta - 1 + \eta\theta}{4\eta^2}(1-\theta)Q^{*-1/\eta} \\ -\alpha_{RS}\dfrac{\eta + \theta - 1 - \eta\theta}{4\eta^2}(1+\theta)Q^{*-1/\eta} & 1 - \alpha_{RS}\dfrac{3\eta + \theta - 1 - \eta\theta}{4\eta^2}(1+\theta)Q^{*-1/\eta} \end{pmatrix} \tag{13}$$

For the sake of simplicity we will assume from now on that $\alpha_{PM} = \alpha_{RS} = \alpha > 0$.[2]

The trace and the determinant of matrix given in (13) are, respectively:

$$Tr = 2 - \alpha\frac{(1-\theta)(3\eta + \theta - 1 + \eta\theta) + (1+\theta)(3\eta + \theta - 1 - \eta\theta)}{4\eta^2}Q^{*-1/\eta}$$

$$Det = Tr - 1 + \alpha^2\frac{(1-\theta^2)c}{2\eta^2}Q^{*-1/\eta} = Tr - 1 + \alpha^2\frac{(1-\theta^2)c}{\eta(2\eta + \theta - 1)} \tag{14}$$

Proposition 2. Assuming that both firms follow a gradient rule, for all $\eta > 1, 0 \le \theta < 1$ and $c > 0$, the Nash equilibrium is locally asymptotically stable provided that $\alpha < \alpha_G(\eta, c) = \frac{2\eta}{c}$.

Proof. Introducing (14) into the stability conditions, given in (9), we deduce that condition (*i*) is always satisfied.

Condition (*iii*) is satisfied, provided that:

$\alpha < \alpha_1$, with $\alpha_1 = \frac{2\eta + \theta - 1 + \eta(1-\theta^2)}{c(1-\theta^2)} = \frac{2\eta + \theta - 1}{c(1-\theta^2)} + \frac{\eta}{c}$.

Condition (*ii*) is fulfilled when the value of the speed of adjustment belongs to the set $(0, \alpha_2) \cup (\alpha_3, +\infty)$, being $\alpha_2 = \frac{2\eta}{c}$ and $\alpha_3 = \frac{2(2\eta + \theta - 1)}{c(1-\theta^2)}$.

By comparing of α_1, α_2 and α_3, we deduce that $0 < \alpha_2 < \alpha_1 < \alpha_3$. In consequence, the Nash equilibrium is asymptotically stable for all $0 < \alpha < \alpha_2$ □.

From the above proposition, it is deduced that the Nash equilibrium loses its dynamic stability through a flip bifurcation if $\alpha \ge \alpha_G(\eta, c)$, being:

$$\alpha_G(\eta, c) = \frac{2\eta}{c} \tag{15}$$

Moreover, when α exceeds this threshold, the unstable boundary equilibria E_1 and E_2 change from being saddle points to being sources.

It follows immediately from the threshold expression given in (15), that the stability of the Nash equilibrium increases as η increases and as c decreases.

This result is in line with the conclusions obtained by [28], for the n firms case, and without social responsibility. As we have seen in a static context, a higher elasticity of demand implies less market power on the part of the firms, therefore, a more competitive behavior favors the stability of the Nash equilibrium. The opposite is true from the marginal cost point of view.

Figs. 1, 2, 3 and 4 show how the Nash equilibrium loses its attractor character when the adjustment speed of the firms exceeds the stability threshold, and how more complex attractors (from a 2-cycle to a strange attractor) appear as the adjustment speed increases.

The evolution in the dynamic behavior of the system given in (12) shown in the figures above is corroborated by the bifurcation diagram of q_i ($i = PM, RS$) and the maximum exponent of Lyapunov with respect to the adjustment speed (see Fig. 5).

Thus, when firms adopt a relatively slow adjustment, market stability improves. More precisely, if the speed of adjustment is below the threshold $\alpha_G(\eta, c)$, any disturbances that move the market away from the Nash equilibrium disappear in the long run. If this speed is higher than the threshold, the long-run behavior of quantities around the Nash equilibrium becomes more complex the faster the adjustment, giving the market greater instability.

From (15) we observe that the adjustment speed threshold does not depend on the parameter θ. The intuition behind this result is the following: as previously shown, the value of the objective functions of the firms in the Nash equilibrium varies in an opposite way with a change in θ, $\left(sign\frac{\partial V_{RS}^*}{\partial\theta} \ne sign\frac{\partial \pi_{PM}^*}{\partial\theta}\right)$, and, since both have the same speed of adjustment, the effect of this parameter on the equilibrium dynamics is neutralized.

However, once the speed of adjustment exceeds the threshold, the long-run evolution of the trajectories around the Nash Equilibrium depends on the value of the social responsibility parameter as shown in Fig. 6 (bifurcation diagram of q_i ($i = PM, RS$) and the maximum exponent of Lyapunov with respect to the parameter θ).

[2] This assumption is usually made in the literature in order to simplify the dynamic analysis and to obtain formal results. See, among others, [27].

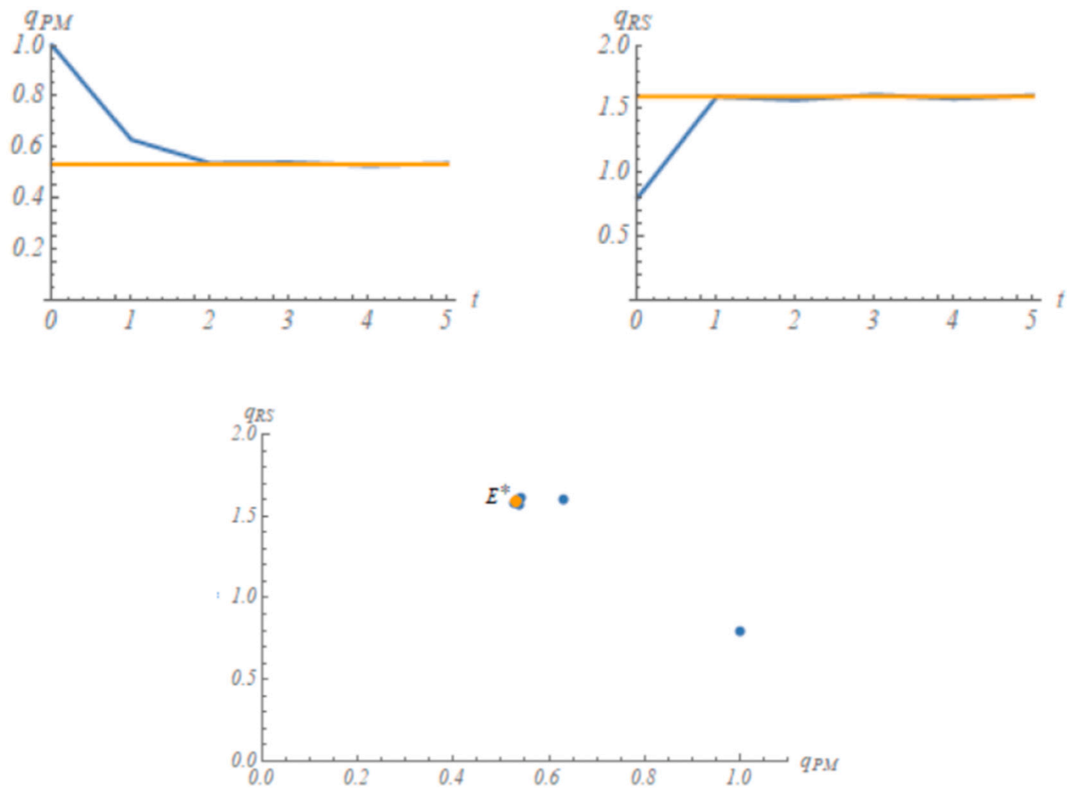

Fig. 1. Attractor (yellow): the Nash Equilibrium E^* (for $\eta = 2, \theta = 0.5, c = 0.6, \alpha = 6$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

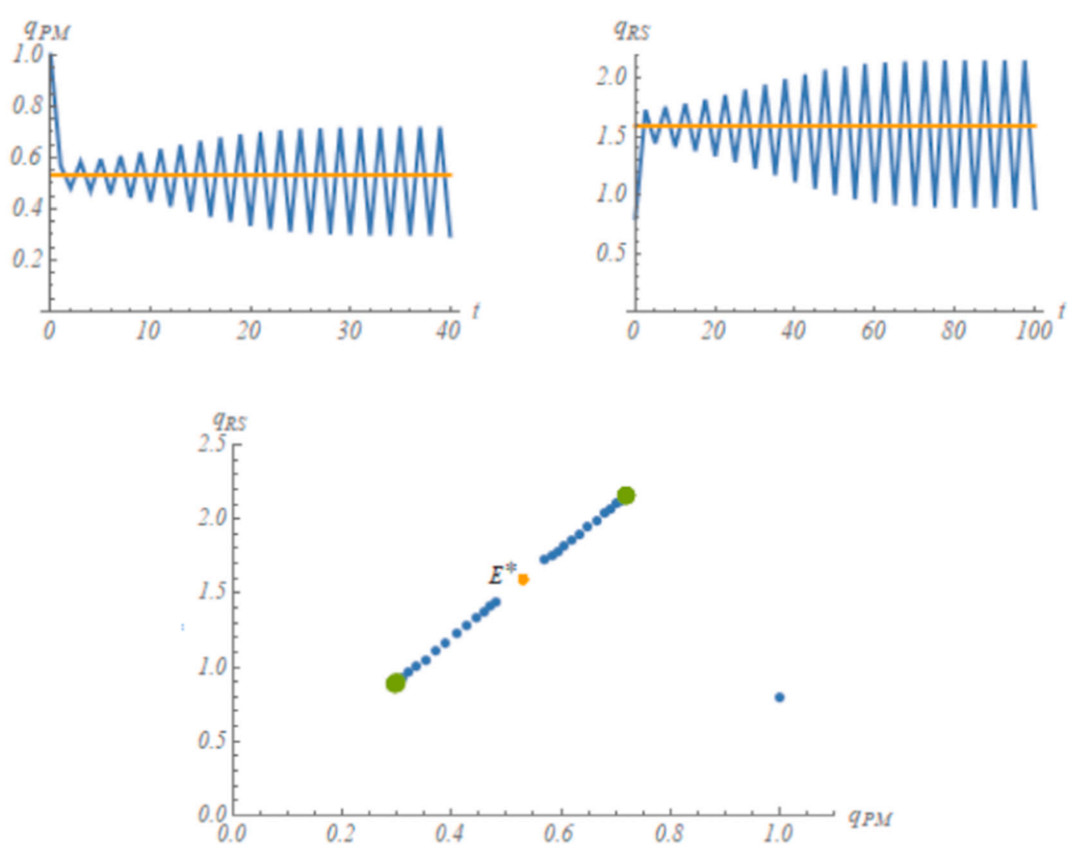

Fig. 2. Attractor (green): a 2-cycle (for $\eta = 2, \theta = 0.5, c = 0.6, \alpha = 7$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

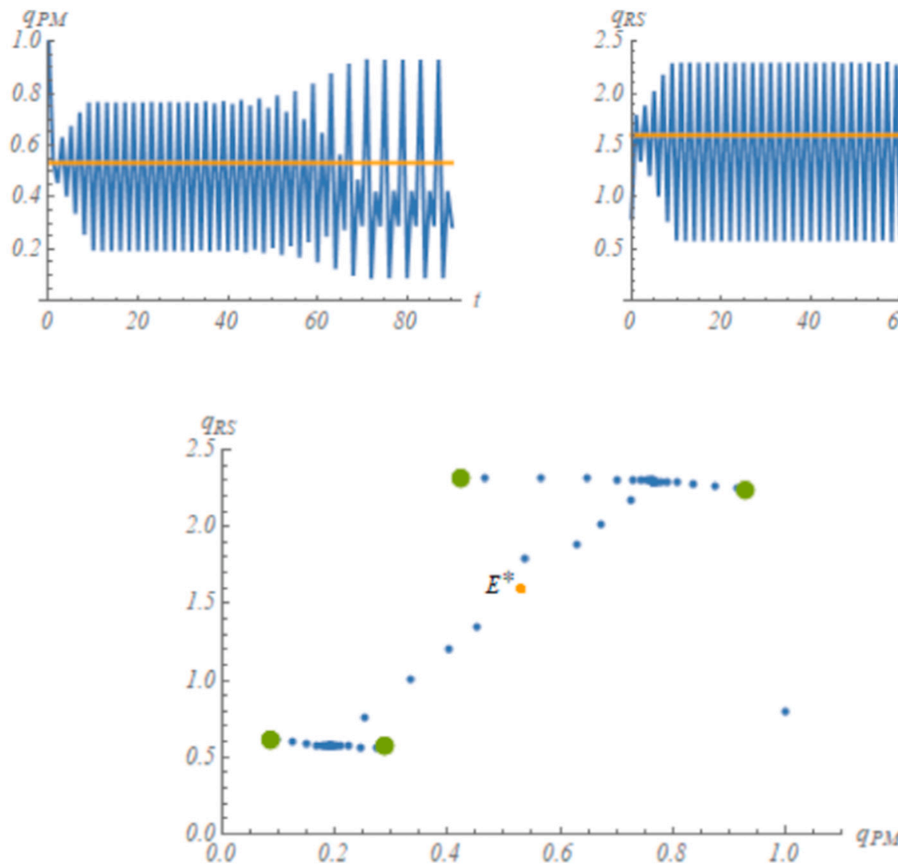

Fig. 3. Attractor (green): a 2^2-cycle (for $\eta = 2, \theta = 0.5, c = 0.6, \alpha = 7.5$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

We can deduce that an increase in the social responsibility parameter reduces the complexity of the attractor reached in the long run, giving more stability to the market.

3.2. Heterogeneous expectations

In this section we will consider the case of heterogeneous expectations. First, we will assume that the socially responsible firm adopts the gradient rule, while the profit maximizing firm adjusts the quantity according to the Local Monopolistic Approximation (LMA) proposed by [29].

For de *PM* firm, the adjustment mechanism is based on the estimation of the partial derivative of the demand function computed in the current state while ignoring the presence of competitors.

Following the same reasoning as in [28], we obtain that the *PM* firm adjusts its output quantity from one period to the next according to the equation:

$$q_{PM,t+1} = \frac{\eta Q_t\left(1 - c\,Q_t^{1/\eta}\right) + q_{PM,t}}{2} \tag{16}$$

It should be noted that the LMA adjustment rule assumes that only the partial derivative of the demand function at the current time is known. Therefore, the socially responsible firm cannot adopt this adjustment mechanism, since it needs to know the entire demand for the calculation of the consumer surplus, which is part of its objective function.

The *RS* firm follows an expectations scheme based on the gradient rule formalized in (10).

From (10) and (16), the following non-linear dynamic system is obtained:

$$T_{HT}: \begin{cases} q_{PM,t+1} = \dfrac{\eta Q_t\left(1 - c\,Q_t^{1/\eta}\right) + q_{PM,t}}{2} \\ q_{RS,t+1} = q_{RS,t} + \alpha_{RS}\, q_{RS,t}\left(Q_t^{-1/\eta}\left(1 + \dfrac{\theta}{\eta}\right) - \dfrac{q_{RS,t}}{\eta} Q_t^{-(1+\eta)/\eta} - c\right) \end{cases} \tag{17}$$

This system has two steady states: $E_1 = \left(\left(\frac{\eta-1}{c\eta}\right)^{\eta}, 0\right)$ and the Cournot-Nash equilibrium E^* given in (6).

The Jacobian matrix of (17) is as follows:

$$JT_{HT}(q_{PM},\ q_{RS}) = \begin{pmatrix} \dfrac{(1+\eta)\left(1 - cQ^{1/\eta}\right)}{2} & \dfrac{\eta - c(1+\eta)Q^{1/\eta}}{2} \\ \alpha\, q_{RS}\dfrac{\partial^2 V_{RS}}{\partial q_{RS}\partial q_{PM}} & 1 + \alpha\dfrac{\partial V_{RS}}{\partial q_{RS}} + \alpha\, q_{RS}\dfrac{\partial^2 V_{RS}}{\partial q_{RS}^2} \end{pmatrix}$$

This matrix evaluated in the steady state E_1 leads to:

$$JT_{HT}(E_1) = \begin{pmatrix} \dfrac{1+\eta}{2\eta} & \dfrac{1}{2\eta} \\ 0 & 1 + \alpha\dfrac{(1+\theta)c}{\eta - 1} \end{pmatrix}$$

which eigenvalues are:

$$\lambda_1 = \frac{1+\eta}{2\eta} \in (0, 1)$$

$$\lambda_2 = 1 + \alpha\frac{(1+\theta)c}{\eta - 1} > 1$$

and we deduce that E_1 is an unstable boundary equilibrium of (17), specifically it is a saddle point.

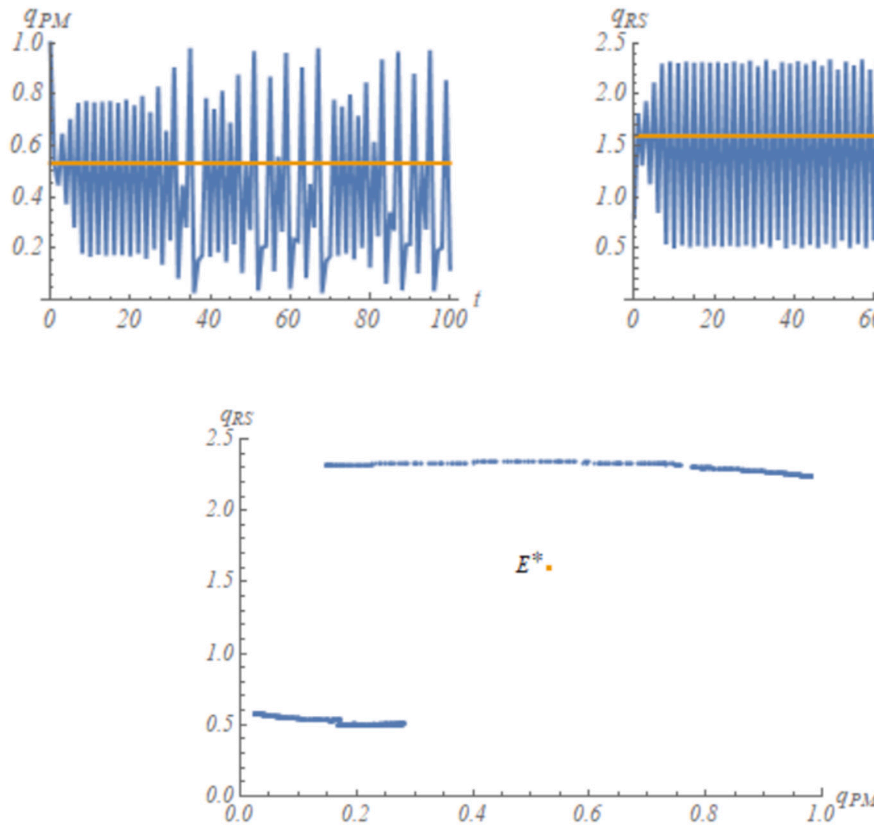

Fig. 4. Attractor (blue): a strange attractor (for $\eta = 2, \theta = 0.5, c = 0.6, \alpha = 7.65$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

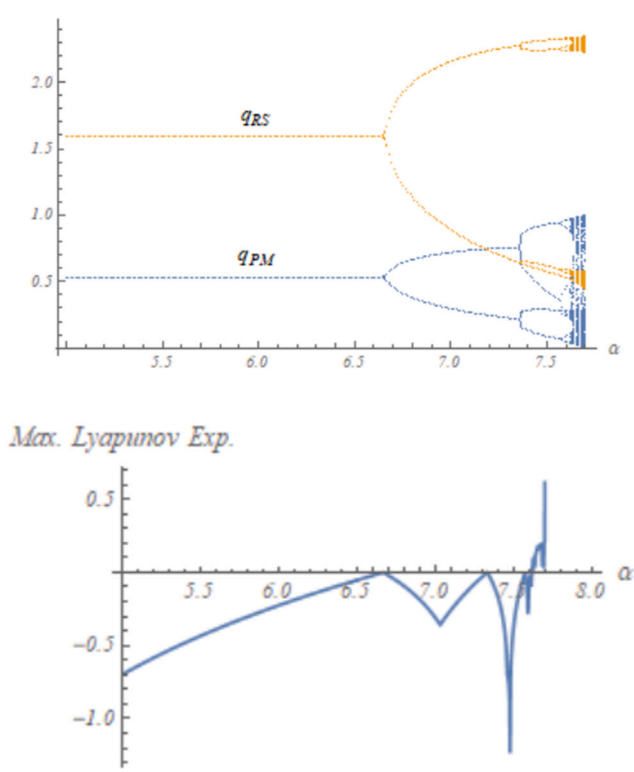

Fig. 5. Bifurcation diagrams q_i (i = *PM, RS*) and the maximum exponent of Lyapunov with respect to the parameter α (for $\eta = 2, \theta = 0.5, c = 0.6$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$).

The matrix $JT_{HT}(q_{PM}, q_{RS})$ evaluated in the Nash equilibrium E^* leads to:

$$JT_{HT}(E^*) = \begin{pmatrix} \dfrac{(\eta+1)(1-\theta)}{4\eta} & -\dfrac{\eta-1+\theta(\eta+1)}{4\eta} \\ -\alpha\dfrac{c(1+\theta)(1-\theta)(\eta-1)}{2\eta(2\eta+\theta-1)} & 1-\alpha\dfrac{c(1+\theta)(2\eta+(\eta-1)(1-\theta))}{2\eta(2\eta+\theta-1)} \end{pmatrix} \quad (18)$$

which trace and determinant are, respectively:

$$\begin{aligned} Tr &= 1 - \alpha\frac{c(1+\theta)(2\eta+(\eta-1)(1-\theta))}{2\eta(2\eta+\theta-1)} + \frac{(\eta+1)(1-\theta)}{4\eta} \\ Det &= \frac{(\eta+1)(1-\theta)}{4\eta} - \alpha\frac{c(1+\theta)(1-\theta)}{2(2\eta+\theta-1)} = \\ &\quad Tr - 1 + \alpha\frac{c(1+\theta)}{2\eta} = Tr - 1 + M, \text{ with } M = \alpha\frac{c(1+\theta)}{2\eta} > 0 \end{aligned} \quad (19)$$

Proposition 3. Assuming that firm PM follows an expectations scheme based on Local Monopolistic Approximation and firm RS uses an expectations scheme based on Gradient rule, for all $\eta > 1, 0 \le \theta < 1$ and $c > 0$, the Nash equilibrium is locally asymptotically stable provided that $\alpha < \alpha_{HT}(\eta, \theta, c)$, where:

$$\alpha_{HT}(\eta, \theta, c) = \frac{(2\eta+\theta-1)(4\eta+(1+\eta)(1-\theta))}{c(1+\theta)(2\eta+(2\eta-1)(1-\theta))} \quad (20)$$

Proof. Introducing (19) into the stability conditions, given in (9), we obtain:

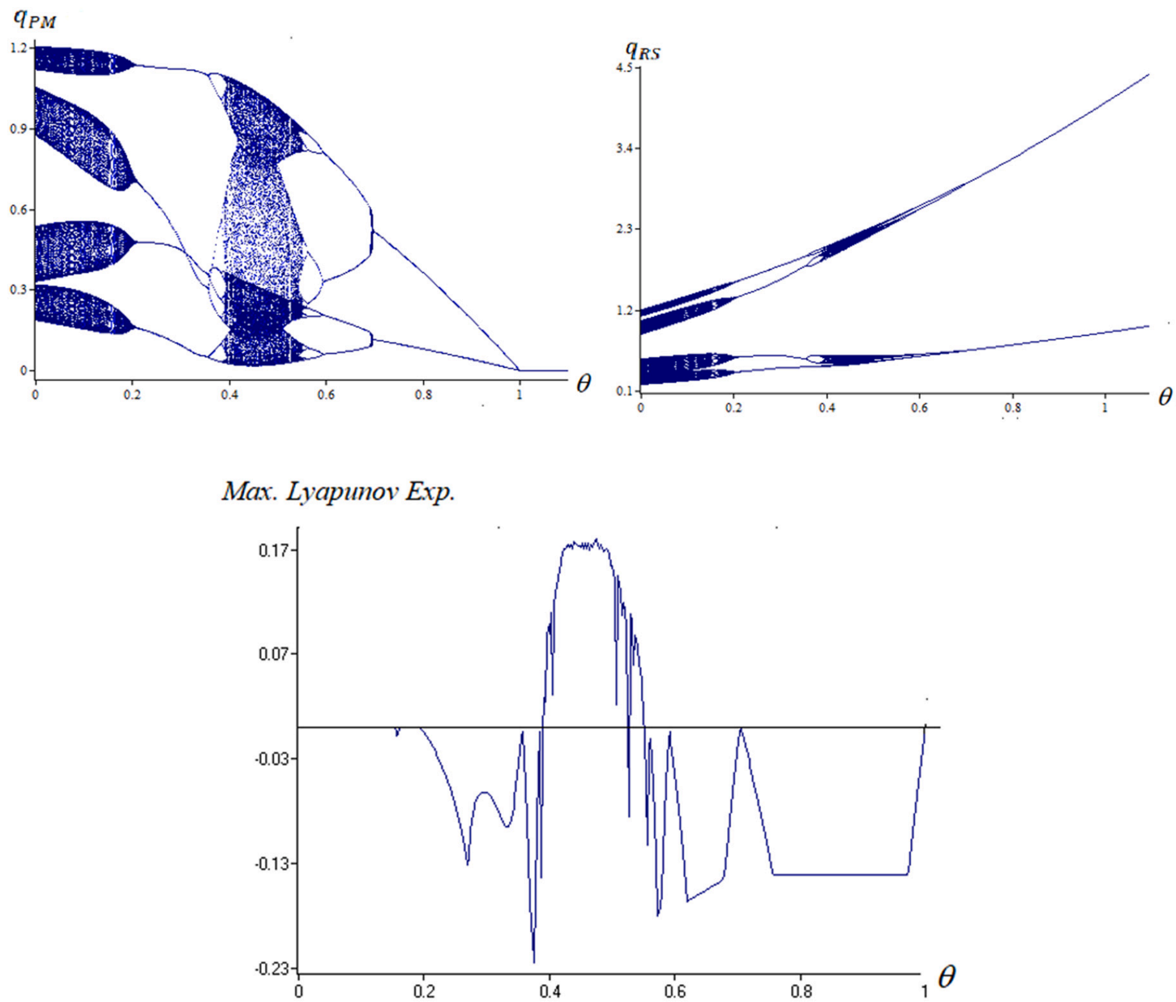

Fig. 6. Bifurcation diagram of q_i ($i = PM, RS$) and the maximum exponent of Lyapunov with respect to the parameter θ (for $\eta = 2, \alpha = 7.65, c = 0.6$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$).

$(i)\ 1 - Tr + Det = M > 0.$

$$(iii)\ 1 - Det = \frac{2\eta(1+\theta) + (\eta-1)(1-\theta)}{4\eta} + \alpha\frac{c(1+\theta)\left(1-\theta^2\right)}{2(2\eta+\theta-1)} > 0\ (ii)\ 1$$

$$+ Tr + Det = 2Tr + M = \frac{4\eta + (\eta+1)(1-\theta)}{2\eta} - \alpha\frac{c(1+\theta)(2\eta + (2\eta-1)(1-\theta))}{2\eta(2\eta+\theta-1)} > 0$$

. Condition (*iii*) is satisfied given that $\eta > 1$, $0 \le \theta < 1$ and $c > 0$. Condition (*ii*) is satisfied, provided that:

$$\alpha < \alpha_{HT}(\eta, \theta, c) = \frac{(2\eta+\theta-1)(4\eta + (1+\eta)(1-\theta))}{c(1+\theta)(2\eta + (2\eta-1)(1-\theta))}$$

In consequence, the Nash equilibrium is asymptotically stable for all $0 < \alpha < \alpha_{HT}(\eta, \theta, c)$ □.

From the above proposition, it is deduced that there is a threshold for the *RS* firm's adjustment speed below which the local dynamic stability of the Nash equilibrium is guaranteed. In Fig. 7 we can see the bifurcation diagram of q_i ($i = PM, RS$) and the maximum exponent of Lyapunov with respect to the adjustment speed, showing how the Nash equilibrium loses its stability through a flip bifurcation if $\alpha \ge \alpha_{HT}(\eta, \theta, c)$.

In this case, the threshold $\alpha_{HT}(\eta, \theta, c)$ depends on the rest of the parameters defining the model (17), including the parameter associated with social responsibility, which was not the case when both firms follow an adjustment given by the gradient rule. It follows immediately from the threshold expression given in (20) that the stability of the Nash equilibrium increases as c decreases.

Moreover, a higher demand elasticity leads to a higher stability, given that:

$$\frac{\partial\alpha_{HT}}{\partial\eta} = \frac{2[4\eta+(1+\eta)(1-\theta)](1-\theta)^2 + (2\eta+\theta-1)(5-\theta)[2\eta+(2\eta-1)(1-\theta)]}{c(1+\theta)[2\eta+(2\eta-1)(1-\theta)]^2} > 0$$

Deriving $\alpha_{HT}(\eta,\theta,c)$ with respect to θ, we obtain:

$$\frac{\partial\alpha_{HT}}{\partial\theta} = -\frac{2(\eta-1)\left[2\left(7-10\theta+\theta^2\right)\eta^2 + 2\left(1-\theta^2\right)\eta - (1-\theta)^2\right]}{c(1+\theta)^2[2\eta + (2\eta-1)(1-\theta)]^2}$$

whose sign depends on the relationship between the elasticity of demand, η, and the parameter associated with social responsibility, θ, as shown in Fig. 8.

From the inspection of Fig. 8 it can be deduced that, for any value of the elasticity of demand, there is a critical value of θ above which RSC exerts a stabilizing effect, since it increases the threshold value of the speed of adjustment $\alpha_{HT}(\eta,\theta,c)$.

More precisely, it can be deduced that this critical value is lower the higher the elasticity of demand. Consequently, in the presence of a

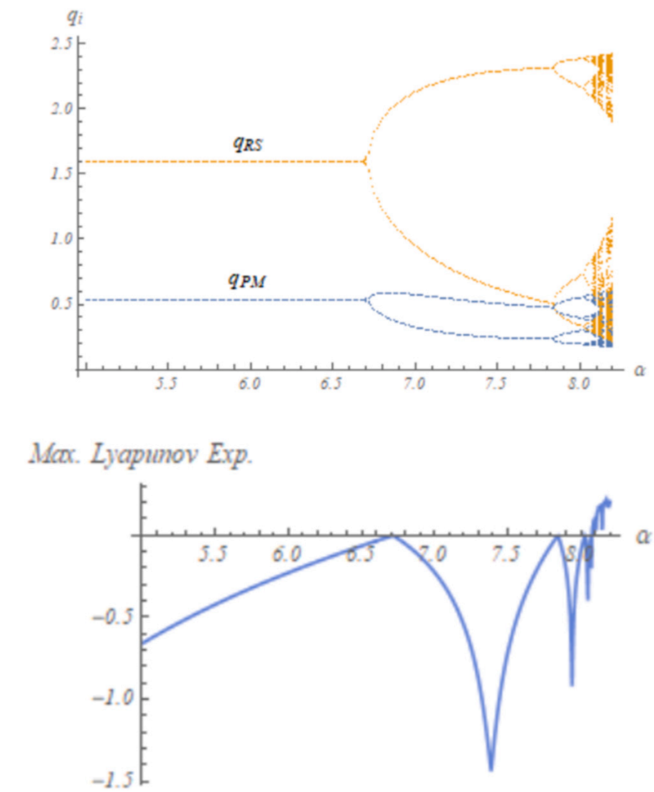

Fig. 7. Bifurcation diagram of q_i ($i = PM, RS$) and the maximum exponent of Lyapunov with respect to the parameter α (for $\eta = 2, \theta = 0.5, c = 0.6$ and initial conditions $q_{PM,0} = 1, q_{RS,0} = 0.8$).

sufficiently elastic demand, a higher specific weight of consumer surplus in the objective function of the socially responsible firm increases the set of values of the adjustment speed parameter that ensure Nash equilibrium stability.

A similar analysis would be carried out for the other combinations of expectation schemes, giving rise to other models that are not developed for reasons of length. Their study allows us to conclude that in the case where the *PM* firm follows *LMA* scheme and the *RS* firm adopts adaptive expectations, we obtain that the Nash equilibrium is asymptotically stable. In the cases where one firm adopts the scheme based on the gradient rule and the other adjusts the quantity according to adaptive expectations, again a threshold of the adjustment speed parameter is obtained which, once exceeded, the Nash equilibrium loses its attractor character. If the *PM* firm adopts the gradient rule as an expectation scheme, it is obtained as a stability threshold:

$$\alpha_{PM}(\beta_{RS},\eta,\theta,c) = \frac{2\eta(2-\beta_{RS})(2\eta+\theta-1)(3\eta+\theta-1-\eta\theta)}{c(1-\theta)\left[(3\eta+\theta-1)^2-\eta^2\theta^2-2\beta_{RS}\eta(2\eta+\theta-1)\right]} \tag{21}$$

If it is the *RS* firm that adopts the gradient rule as the adjustment mechanism, the threshold is given by:

$$\alpha_{RS}(\beta_{PM},\eta,\theta,c) = \frac{2\eta(2-\beta_{PM})(2\eta+\theta-1)(3\eta+\theta-1+\eta\theta)}{c(1+\theta)\left[(3\eta+\theta-1)^2-\eta^2\theta^2-2\beta_{PM}\eta(2\eta+\theta-1)\right]} \tag{22}$$

Assuming that $\beta_{PM} = \beta_{RS} = \beta$ we obtain:

$$\alpha_{PM}-\alpha_{RS} = \frac{4\theta\eta(2-\beta)(2\eta+\theta-1)^2}{c(1-\theta^2)\left[(3\eta+\theta-1)^2-\eta^2\theta^2-2\beta\eta(2\eta+\theta-1)\right]} > 0$$

Therefore, it follows that the market is more stable if the socially responsible firm is the one with the highest level of rationality.

4. Conclusions

In an oligopoly with isoelastic demand, the paper has analyzed the competition in quantities between a set of firms classified in two groups: profit maximizing firms and socially responsible firms, which consider consumer surplus in their objective function.

Static analysis has allowed us to deduce the efficient role of social responsibility, since it has a clear competitive effect.

In fact, developing the comparative statics analysis, a decreasing relationship between market price and the degree of social responsibility has been obtained, which reduces the market power of the firms. Moreover, it has been deduced that greater social responsibility increases consumer surplus and social welfare and reduces producer surplus.

The dynamic analysis of competition has been carried out for the duopoly case in a discrete time scenario, and assuming that firms adjust quantities according to the adaptive expectations, gradient rule based on the marginal profit and Local Monopolistic Approximation schemes.

From the study it is found that, under an adaptive expectations scheme, the Cournot equilibrium is locally asymptotically stable over the entire parameter space. The instability of the equilibrium arises in the presence of the gradient rule as an adjustment mechanism in one or both firms. If both firms adopt such an expectations scheme, the degree of social responsibility does not affect the stability of the equilibrium, being more stable the lower the marginal cost and the higher the elasticity of demand (the equilibrium is more stable when demand is more sensitive to changes in price). These effects are also reproduced in a context of heterogeneous expectations where the socially responsible firm acts according to the gradient rule and the profit-maximizing firm makes its decisions over time using the Local Monopolistic Approximation.

Regarding the parameter associated with social responsibility it has a

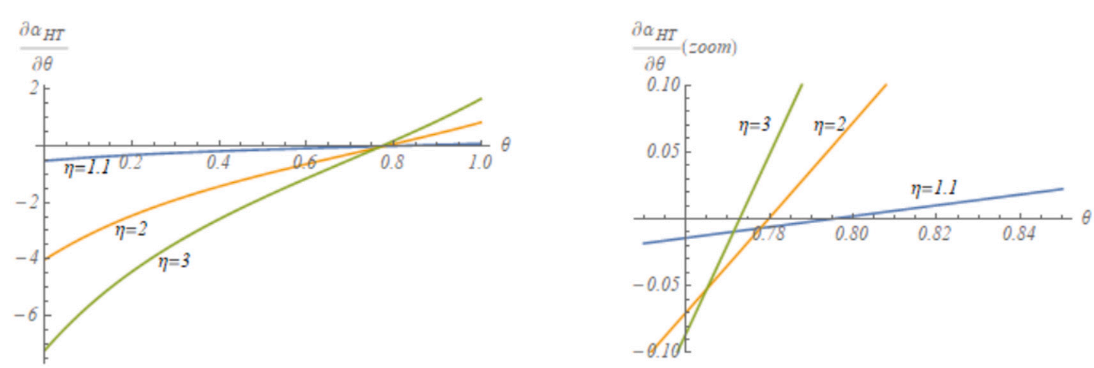

Fig. 8. Graphical representation and zoom of the derivative $\frac{\partial \alpha_{HT}}{\partial \theta}(\eta,\theta,0.6)$ for $\eta = 1.1$ (blue), $\eta = 2$ (yellow) and $\eta = 3$ (green).

direct effect on the asymptotic stability of the Nash equilibrium in the case of heterogeneous expectations. In this case, the degree of social responsibility plays a stabilizing role in the Nash equilibrium, its effect being greater the more elastic the demand function is.

In the case where both firms have expectations based on the gradient rule, the degree of social responsibility plays a stabilizing role in a different sense. If the speed of adjustment is sufficiently large so that the Nash equilibrium has lost its attractor character, the larger θ is, the less complex the evolution of the trajectories around the equilibrium (reducing the degree of complexity of the attractor evolving from a strange attractor to a 2-cycle).

In summary, the results show that the market is more stable the closer it is to the competitive scenario. This depends, on the one hand, on the value of the elasticity of demand (structure variable) and, on the other hand, on the level of corporate social responsibility (behavior variable).

Finally, we should note that the attractors appearing in the dynamic models (12) and (17) are not globally stable in the economically significant region, so to complete the results obtained in this paper we propose to carry out a study of the global dynamics of both models in a future research. For this purpose, we will follow the methodology used in [25,30,31] which is based on the study of the dynamic on invariant sets, critical curves, basins of attraction and global bifurcations.

It would be expected that we would get as a result the coexistence of attractors, multistability (path dependent situation or sensitivity of initial conditions), and qualitative changes in the structure of the basin of attraction when a global bifurcation occurs, among others.

As [31] points out, these results will provide information on parametric values and initial conditions that will allow firms to adjust their strategies over time, increasing market stability and avoiding unpredictability as much as possible.

CRediT authorship contribution statement

J. Andaluz: Conceptualization, Methodology, Writing – original draft. **A.A. Elsadany:** Software, Visualization, Writing – review & editing. **G. Jarne:** Methodology, Formal analysis, Software, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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