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# Physics of the universe transparency in a deformed kinematics

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We present a first study of the possible effects of a relativistic deformation of special relativity in the recent observations of very high-energy gamma rays by the LHAASO experiment, which has opened a new phenomenological window to study deformations in the kinematics of special relativity. Our analysis of the interaction of high-energy photons with the CMB background complements theoretical studies based on Lorentz invariance violation scenarios, while making predictions that would allow one to distinguish between a violation and a deformation of the symmetries of special relativity.

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## 1. Introduction: LHAASO, LIV and the COST Action CA18108

The detection of $\gamma$ rays reaching the PeV scale by the LHAASO collaboration [1] has been one of the scientific highlights in 2021 and a revolutionary discovery in the field of $\gamma$-ray astronomy. Even though the 12 ultrahigh-energy $\gamma$-ray sources of photons of energies above 100 TeV reported by LHAASO have not yet been firmly localized and identified (except for the Crab Nebula) [1], the detection of photons with energies far beyond 100 TeV demonstrate the existence of galactic *PeVatrons*. The possibility that some of these sources were extragalactic, although it cannot be ruled out by present data, would necessarily mean new physics beyond special relativity. The reason is that photons of PeV energies lose their energy very effectively, so that they could not arrive to us from outside our galaxy. The LHAASO detection has opened then a new window to the universe, allowing us in particular the exploration of possible effects in the propagation of ultrahigh-energy $\gamma$ rays.

High-energy photons get absorbed in their interaction with background photons with the emission of an electron-positron pair through a process which is allowed in special relativity (SR), $\gamma + \gamma_b \to e^+ + e^-$. The kinematics of this process is modified in a Lorentz invariance violation (LIV) scenario, where the dispersion relation between energy and momentum of the photon gets modified,

$$E_\gamma^2 - p_\gamma^2 = \pm \frac{E^{n+2}}{E_{\text{LIV}}^n}, \tag{1}$$

where $n > 0$ is the order of the correction, which is parameterized by the high-energy scale $E_{\text{LIV}}$. Pair creation can only happen beyond a certain energy threshold. With the positive sign above, the energy of the high-energy photon is larger than the one it should have in SR and the threshold is smaller; as a consequence, the reaction is favoured (*superluminal* case, $v_\gamma = dE/dp > 1$); with the negative sign (*subluminal* case, $v_\gamma < 1$), the threshold is higher than in SR, so that the process is harder. The minimum energy needed by the background photon, $\varepsilon_{\text{th}}$, is then modified with respect to its value in SR as [2]: [1]

$$\varepsilon_{\text{th}} = \frac{m_e^2}{E_\gamma} \mp \frac{E_\gamma^{n+1}}{4E_{\text{LIV}}^n}. \tag{2}$$

A modification in the transparency of the universe to very energetic photons is then predicted in LIV scenarios, which are then constrained (at the level of the Planck scale for $n = 1$ in the subluminal case [2], and even beyond that scale in the superluminal case [3–7]) by the detection of such high-energy photons.

The implications of the LHAASO results are particularly relevant for the objectives of the COST Action CA18108 "Quantum gravity phenomenology in the multi-messenger approach" [8], whose aim is to look for signatures of quantum gravity models in the observation of different cosmic messengers. These non-conventional effects could certainly include a modification of the kinematics of special relativity induced by a "quantum spacetime" affecting the propagation and/or interactions of the high-energy photons observed by LHAASO. Such modification may entail a violation of the relativity principle (the above mentioned scenario of LIV), although a much less explored scenario is also possible. A relativistic deformed kinematics (RDK) is a deformation of

[1] If one modifies the dispersion relation not only for the photon, but also for the electron-positron pair, the coefficient 1/4 in the expression of $\varepsilon_{\text{th}}$ gets converted to 1/8; compare with Eq. (12).

special relativity which is compatible with a relativity principle, that is, with deformed Lorentz transformations that connect a set of inertial reference frames. The purpose of this talk[2] is to consider the implications of such possibility in the physics that may be explored in the near future by LHAASO and similar experiments.

## 2. Relativistic deformed kinematics

Within SR, the analysis of the kinematics of a process such as $A_1(p^{(1)})+A_2(p^{(2)}) \to A_3(p^{(3)})+A_4(p^{(4)})$, takes into account as essential ingredients the conservation law and the dispersion relation

$$p_\mu^{(1)} + p_\mu^{(2)} = p_\mu^{(3)} + p_\mu^{(4)}\,, \qquad p^{(i)2} = m_i^2\,, \tag{3}$$

which are invariant under linear Lorentz transformations $p_\mu^{(i)\prime} = L_\mu^{\ \nu} p_\nu^{(i)}$ connecting inertial systems of reference. We would like now to *deform* the kinematics by introducing an energy scale $\Lambda$ which modifies the addition of momenta in the conservation law to a non-linear $\Lambda$-dependent composition law of momenta and the dispersion relation to a new one which includes a function $C(p^{(i)}, \Lambda)$,

$$\left[p^{(1)} \oplus p^{(2)}\right]_\mu = \left[p^{(3)} \oplus p^{(4)}\right]_\mu\,, \qquad C(p^{(i)}, \Lambda) = m_i^2\,, \tag{4}$$

in such a way that the modified composition law (MCL) and modified dispersion relation (MDR) reduce to their expressions in SR in the $\Lambda \to \infty$ limit.

The deformed kinematics can be relativistic or not. In the latter case, there is a privileged system of reference and we have a LIV scenario. In the first case, which corresponds to doubly special relativity (DSR) models [11], there is a relativistic principle. It implies the introduction of $\Lambda$-deformed Lorentz transformations, which need to be defined in the two-particle system, $(k, l) \to (k', l')$, to guarantee the invariance of the previously defined MCL and MDR:

$$\left[p^{(1)\prime} \oplus p^{(2)\prime}\right]_\mu = \left[p^{(3)\prime} \oplus p^{(4)\prime}\right]_\mu\,, \qquad C(p^{(i)\prime}, \Lambda) = m_i^2\,. \tag{5}$$

As an example, let us consider a first order deformation, which includes only corrections to SR kinematics proportional to $1/\Lambda$. Imposing rotational invariance, a generic MDR will depend on two coefficients $\alpha_1$ and $\alpha_2$,

$$C(p) \;=\; p_0^2 - \vec{p}^{\,2} + \frac{\alpha_1}{\Lambda} p_0^3 + \frac{\alpha_2}{\Lambda} p_0 \vec{p}^{\,2} = m^2, \tag{6}$$

and we can also write a generic MCL in the form

$$[p \oplus q]_0 \;=\; p_0 + q_0 + \frac{\beta_1}{\Lambda} p_0 q_0 + \frac{\beta_2}{\Lambda} \vec{p} \cdot \vec{q}\,, \tag{7}$$

$$[p \oplus q]_i \;=\; p_i + q_i + \frac{\gamma_1}{\Lambda} p_0 q_i + \frac{\gamma_2}{\Lambda} p_i q_0, \tag{8}$$

depending on coefficients $\beta_1$, $\beta_2$, $\gamma_1$ and $\gamma_2$. In the case of a RDK, the relativity principle relates both ingredients and establishes the following relations between the previous coefficients [12]:

$$\alpha_1 \;=\; -\beta_1\,, \qquad \alpha_2 \;=\; \gamma_1 + \gamma_2 - \beta_2\,, \tag{9}$$

[2] Presentation based on Refs. [9, 10].

which generalize the "golden rule" introduced in Ref. [13].

Because of the "golden rules" (9), the kinematics of the relativistic invariance (RDK) and non-invariance cases (LIV) is very different, as the following example will show. Let us consider the pair production process by a high-energy photon of energy $E$ in its interaction with a background (from the cosmic microwave background (CMB) or the extragalactic background light (EBL)) photon of energy $\varepsilon \ll E$,

$$\gamma(E) + \gamma_{\text{EBL/CMB}}(\varepsilon) \to e^- + e^+. \tag{10}$$

If all particles have a first-order deformed kinematics as defined in Eqs. (6)-(8), one obtains the following threshold equation:

$$\frac{\gamma_1 + \gamma_2 - \beta_1 - \beta_2 - \alpha_1 - \alpha_2}{8\Lambda} E_{\text{th}}^3 + O\left(\frac{E_{\text{th}}^2 \varepsilon}{\Lambda}, \frac{E_{\text{th}} m_e^2}{\Lambda}\right) + E_{\text{th}}\varepsilon - m_e^2 = 0, \tag{11}$$

which of course contains the SR threshold solution, $E_{\text{th}}^{\text{SR}} = m_e^2/\varepsilon$, in the $\Lambda \to \infty$ limit.

In the LIV case, where usually one does not consider a modification in the composition law ($\gamma_i = \beta_i = 0$), one gets

$$E_{\text{th}}^{\text{LIV}} \approx \frac{m_e^2}{\varepsilon}\left[1 + \frac{(m_e^2)^2}{\varepsilon^3}\frac{\alpha_1 + \alpha_2}{8\Lambda}\right] = \frac{m_e^2}{\varepsilon}\left[1 + \frac{m_e^2}{\varepsilon^2}\frac{m_e^2}{\varepsilon\Lambda_{\text{eff}}}\right], \tag{12}$$

where we have defined and effective scale $\Lambda_{\text{eff}} = 8\Lambda/(\alpha_1 + \alpha_2)$.

In the relativistic case, however, from the golden rules (9), we have that $\gamma_1+\gamma_2-\beta_1-\beta_2-\alpha_1-\alpha_2 = 0$, and then the leading term in Eq. (11) vanishes. Including the subdominant terms then gives

$$E_{\text{th}}^{\text{RDK}} \approx \frac{m_e^2}{\varepsilon}\left[1 + \frac{\beta_1 + \beta_2 + 3\gamma_2 - \gamma_1}{4\Lambda}\frac{m_e^2}{\varepsilon}\right] = \frac{m_e^2}{\varepsilon}\left[1 + \frac{m_e^2}{\varepsilon\Lambda_{\text{eff}}}\right], \tag{13}$$

which is suppressed by a factor $(\varepsilon^2/m_e^2)$ with respect to the LIV case, Eq. (12). This is a generic situation: corrections in RDK are softer and then, less visible, than in the LIV scenario.

For typical CMB energies, and allowing for a 10% correction of the threshold in SR, we can get bounds on $\Lambda_{\text{eff}}$ from Eqs (12) and (13), which are beyond the Planck scale in the first case, and of the order of the PeV in the second case. This means that, in order to study the effects of a RDK in the physics of photons at the PeV scale, we will need to go beyond the first-order approximation used above and consider an all-order relativistic deformed kinematics, as we will do in the next sections.

## 3. Mean free path of high-energy photons in SR

Let us remind the computation of the mean free path of high-energy photons in SR for the process (10). One can obtain the threshold from $s = 4m_e^2$, where $s$ is the the squared center of mass energy, corresponding to the creation of the electron-positron pair at rest in the center of mass system, getting

$$\varepsilon_{\text{th}} = \frac{2\,m_e^2}{E\,(1-\cos\theta)}, \tag{14}$$

where $\theta$ is the angle formed by the directions of both photons. The cross section of this process is the well-known Breit-Wheeler cross section

$$\sigma_{\gamma\gamma}(E,\varepsilon,\theta) \;=\; \frac{2\pi\alpha^2}{3m_e^2}\,W(\beta), \tag{15}$$

which depends on the square of the fine structure constant $\alpha$, since it is a two-vertex process at tree level, with

$$W(\beta) \;=\; (1-\beta^2)\left[2\beta(\beta^2-2)+(3-\beta^4)\,\ln\frac{1+\beta}{1-\beta}\right], \tag{16}$$

written in terms of $\beta$, the speed of the electron and positron in the center of mass system,

$$\beta(\varepsilon,E,\theta) \;=\; \sqrt{1-\frac{2\,m_e^2}{\varepsilon\,E\,(1-\cos\theta)}}. \tag{17}$$

For an isotropic background of photons, the cross-section is maximized for

$$\varepsilon(E) \approx \left(\frac{900\,\text{GeV}}{E}\right)\,\text{eV}, \tag{18}$$

so that, depending on the energy of the high-energy photon, the dominant background is different: EBL, for $10^9 \leq E \leq 10^{14}$ eV; CMB, for $10^{14} \leq E \leq 10^{19}$ eV, and radio-background (RB), for $E \geq 10^{19}$ eV. For PeV photons, then, the relevant background will be that of the CMB.

The probability of survival of a high-energy photon can be written as

$$P_{\gamma\to\gamma}(E,z_s) \;=\; \exp\left(-\tau_\gamma(E,z_s)\right), \tag{19}$$

where the optical depth $\tau$ can be computed from

$$\tau_\gamma(E,z_s) \;=\; \int_0^{z_s} dz\,\frac{dl(z)}{dz}\int_{-1}^{1} d(\cos\theta)\,\frac{1-\cos\theta}{2}\int_{\varepsilon_{\text{th}}}^{\infty} d\varepsilon\,n_\gamma(\varepsilon,z)\,\sigma_{\gamma\gamma}(E(z),\varepsilon,\theta), \tag{20}$$

where $z_s$ is the redshift of the source, $dl(z)/z$ is the distance traveled by a photon per unit redshift at redshift $z$, which depends on the cosmological parameters, and $n_\gamma(\varepsilon,z)$ is the spectral density of background photons, which in general depends on $z$. When considering near sources, one can safely disregard the expansion of the Universe. We can define in this case the mean free path $\lambda_\gamma$ for the high-energy photon as

$$\frac{1}{\lambda_\gamma(E)} \;=\; \int_{-1}^{1} d(\cos\theta)\frac{1-\cos\theta}{2}\int_{\varepsilon_{\text{th}}}^{\infty} d\varepsilon\,n_\gamma(\varepsilon)\,\sigma_{\gamma\gamma}(E,\varepsilon,\theta)\,. \tag{21}$$

Using $s = 2E\varepsilon(1-\cos\theta)$, we can write the previous integrals as

$$\frac{1}{\lambda_\gamma(E)} = \frac{1}{8E^2}\int_{4m_e^2}^{\infty} ds\;s\,\sigma_{\gamma\gamma}(s)\int_{s/(4E)}^{\infty} d\varepsilon\frac{1}{\varepsilon^2}n_\gamma(\varepsilon). \tag{22}$$

As said above, for photons between 0.1 PeV and $10^4$ PeV, the interaction with the CMB becomes dominant. Inserting the CMB density $n_\gamma(\varepsilon) \;=\; (\varepsilon/\pi)^2(e^{\varepsilon/kT_0}-1)^{-1}$, $T_0 = 2.73$ K, in the previous expression, we get (the integral in $\varepsilon$ is analytical and the integral in $s$ is solved numerically) the blue curve in Fig. 1, which is superimposed on a general curve from Ref. [14], which is obtained with different EBL and RB models.

We will now follow the previous procedure of computation for the case of a relativistic deformed kinematics.

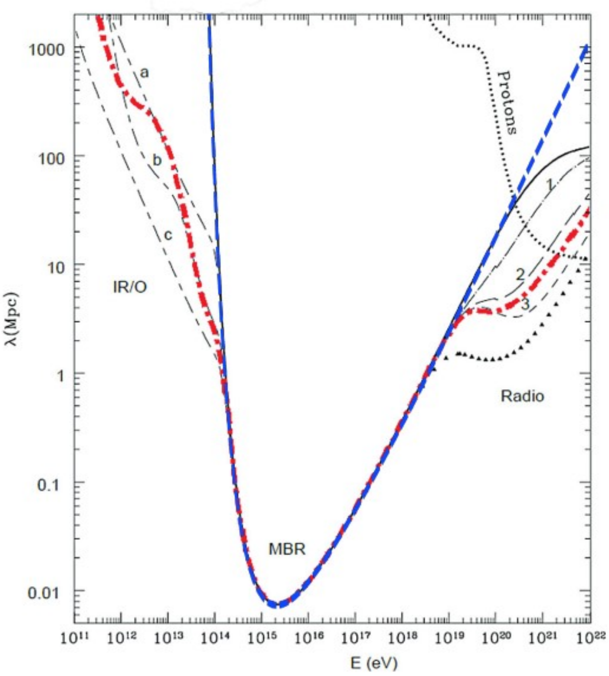


**Figure 1:** In blue, our results for the mean free path for the CMB energy range and in red the results showed in figure 2 of "Transparency of the Universe to gamma rays" [14].

## 4. Modification of the mean free path in a RDK

We will now see how the mean free path of SR is modified in the particularly simple model of relativistic deformed kinematics DCL1 [15], for which the all-order expression of the modified composition law contains only correction terms proportional to $\Lambda^{-1}$. The MCL of the two-particle system is defined by a total energy $E_T$ and a total momentum $\vec{p}_T$ from the individual energies $(E, \varepsilon)$ and momenta $(\vec{p}, \vec{\kappa})$,

$$E_T = E + \varepsilon + \frac{E\,\varepsilon}{\Lambda}, \qquad \vec{p}_T = \begin{cases} \vec{p} + \vec{\kappa} + E\vec{\kappa}/\Lambda \\ \vec{p} + \vec{\kappa} + \varepsilon\vec{p}\Lambda \end{cases}, \tag{23}$$

so that the composition law is associative (and the relativistic invariance of the kinematics can be studied at the two-particle level) and non-commutative (so that this kinematics is not the one of SR in weird momentum variables), and we have two possibilities for the total momentum $\vec{p}_T$.

The MDR which is (relativistically) compatible with the previous MCL is

$$\frac{E^2 - \vec{p}^2}{1 + E/\Lambda} = m^2 = 0. \tag{24}$$

This dispersion relation has the characteristics that for a massless particle, $E = |\vec{p}|$. One can see that it is possible to make a change of momentum variables so that the new MCL and MDR are those of the $\kappa$-Poincaré bicrossproduct basis [15].

The relativistic invariant $s$, which is simply $E_T^2 - \vec{p}_T^2$ in SR, gets then deformed to $(E_T^2 - \vec{p}_T^2)(1 + E_T/\Lambda)$, which gives the following two expressions, depending on which possibility one takes for

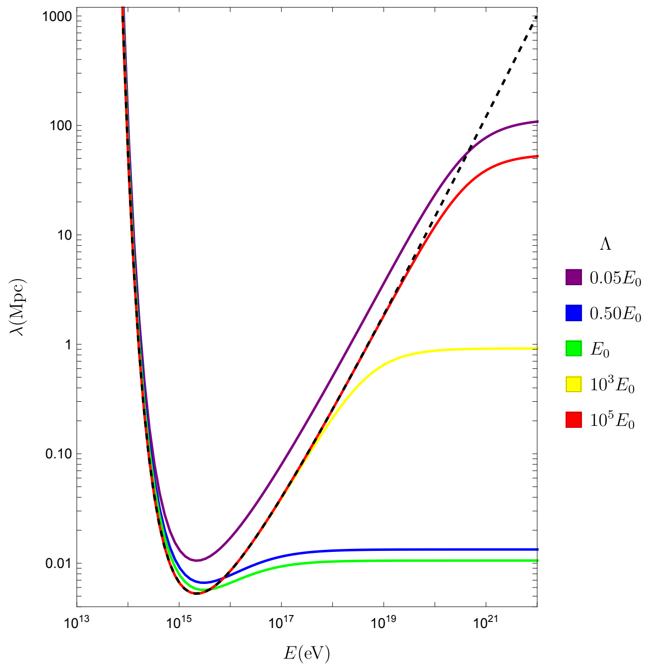


**Figure 2:** Mean free path for photons as a function of its energy. The dashed curve corresponds to the mean free path in SR, with a minumum at $E_0 \approx 2\,\text{PeV}$, and the solid curves represent the result in RDK for different values of the scale $\Lambda$ in units of $E_0$.

$\vec{p}_T$:

$$\tilde{s}_1 = \frac{2\,\varepsilon\,E\,(1-\cos\theta)}{1+\varepsilon/\Lambda} \approx 2\,\varepsilon\,E\,(1-\cos\theta) = s\,, \tag{25}$$

$$\tilde{s}_2 = \frac{2\,\varepsilon\,E\,(1-\cos\theta)}{1+E/\Lambda} = 2\,\varepsilon\,E'\,(1-\cos\theta)\,, \tag{26}$$

where we have taken into account that $\varepsilon \ll \Lambda$, while $E \sim \Lambda$, and we have introduced

$$E' = \frac{E}{1+E/\Lambda}\,. \tag{27}$$

In the absence of a full dynamical model for this RDK, we can simply assume that $\tilde{\sigma}_{\gamma\gamma}(\tilde{s}) = \sigma_{\gamma\gamma}(s \to \tilde{s})$, since we will have then a relativistically invariant expression for $\tilde{\sigma}_{\gamma\gamma}$ that reduces to the SR cross-section when $\tilde{s} \to s$. This allows to immediately obtain $\tilde{\lambda}^{-1}$ as the mean value of the inverses of mean free paths for each of the two possibilities for $\tilde{s}$ (and then, for the two possible values of the cross-section):

$$\frac{1}{\tilde{\lambda}(E)} = \frac{1}{2}\left(\frac{1}{\lambda(E)} + \frac{1}{\lambda(E')}\right), \tag{28}$$

where $\lambda(E)$ and $\lambda(E')$ are just the expressions of the mean free paths in SR, evaluated at energies $E$ and $E'$, respectively. The result is shown in Fig. 2 for different values of the high-energy scale $\Lambda$, which are given in units of $E_0 = 2\,\text{PeV}$, the energy where the SR mean free path reaches its minimum value.

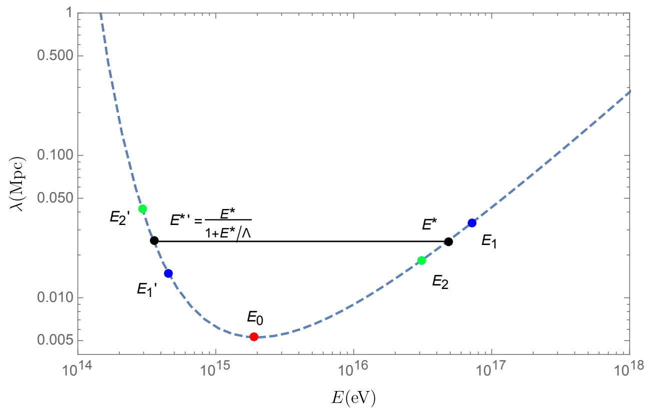


**Figure 3:** Graphical representation of the position of $E$ and $E^{*'}(\Lambda)$ as defined in the main text in the mean free path curve $\lambda(E)$. The red point is the minimum value of $\lambda(E)$; black points correspond to two different energies, $E^*(\Lambda)$ and $E^{*'}(\Lambda)$, which for a particular value of $\Lambda$ lead to the same $\lambda(E)$; blue and green points corresponds to greater and smaller energies than $E^*(\Lambda)$.

Since $\lim_{E\to\infty}(E/(1+E/\Lambda)) = \Lambda$,

$$\frac{1}{\tilde{\lambda}(\infty)} = \frac{1}{2}\left(\frac{1}{\lambda(\infty)} + \frac{1}{\lambda(\Lambda)}\right), \tag{29}$$

and then $\tilde{\lambda}(\infty) = 2\,\lambda(\Lambda)$, so that $\tilde{\lambda}$ goes to a constant for large energies, in contrast to what happens in SR (dashed curve in Fig. 2). If $\Lambda < E_0$, $\tilde{\lambda}(\infty)$ grows if $\Lambda$ decreases; if $\Lambda > E_0$, $\tilde{\lambda}(\infty)$ grows if $\Lambda$ grows; in fact, $\tilde{\lambda}(\infty)$ reaches a minimum for $\Lambda = E_0$.

It is interesting to find the crossing point between the curves $\lambda(E)$ and $\tilde{\lambda}(E)$, i.e. one wants to find the energy $E^*$ such that

$$\frac{1}{\lambda(E^*)} = \frac{1}{\tilde{\lambda}(E^*)} = \frac{1}{2}\left(\frac{1}{\lambda(E^*)} + \frac{1}{\lambda(E^{*'})}\right). \tag{30}$$

The equality happens when $\lambda(E^*) = \lambda(E^{*'})$, where

$$E^{*'} = \frac{E^*}{1+E^*/\Lambda}. \tag{31}$$

The situation is sketched in Fig. 3, where the values $E^*$ and $E^{*'}$ are indicated. For $E_1 > E^*(\Lambda)$, we see from Fig. 3 that $\lambda(E_1') < \lambda(E_1)$, and then Eq. (28) gives $\tilde{\lambda}(E_1) < \lambda(E_1)$, and the Universe is less transparent than in SR; for $E_2 < E^*(\Lambda)$, $\tilde{\lambda}(E_2) > \lambda(E_2)$, and the Universe is more transparent than in SR. This conclusion can also be clearly seen in Fig. 4, which shows the mean free paths in SR and in RDK for values of $\Lambda$ around $E_0$: to the right of the crossing point between the curve of SR and each curve of RDK, the last one is below the one of SR, while the RDK curves are above the one of SR to the left of the corresponding crossing points.

## 5. Conclusions

The observation of high-energy photons at the PeV scale by LHAASO has opened a new phenomenology window to study deformations in the kinematics of SR.

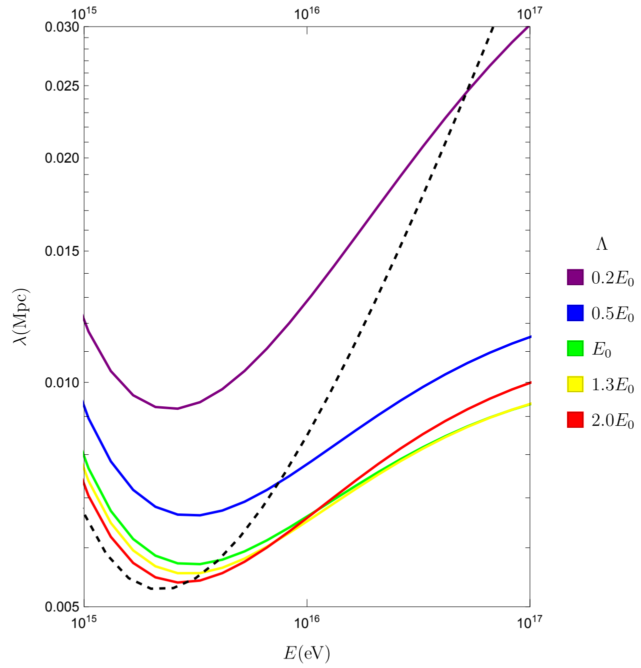


**Figure 4:** Photon mean free path as a function of the energy. The dashed black curve and the colored solid lines represent the mean free path of SR and RDK for different values of the high-energy scale (near the minimum $E_0$), respectively.

In the LIV scenario, the experimental data is sensitive to a high-energy scale of the order of the Planck mass in the subluminal case, predicting a higher transparency of the Universe to very high-energy photons with respect to SR (while the possibility of processes such as pair emission, $\gamma \to e^+e^-$, or photon splitting, $\gamma \to 3\gamma$, excludes the superluminal case up to several orders of magnitude above the Planck scale) .

In the case of a relativistic deformed kinematics, the LHAASO results could be sensitive to a much lower energy scale of deformation, thirteen orders of magnitude below the Planck mass. In contrast to the LIV case, our study shows a higher or lower transparency of the Universe depending on the energy range, so that the spectrum of very high-energy photons could distinguish between the LIV and the RDK cases.

In order to have a measurable effect, we need that the high-energy scale of the RDK be much lower than the Planck mass, which is the 'naïve' scale associated with quantum gravity phenomenology. There are however many scenarios (e.g., large extra dimensions) where this is a sound possibility, and, at the same time, this may be compatible with photon time delay and other kinematic constraints [16].

The computation of the mean free path we made here was particularly simple in the selected model of RDK, but other alternatives could be considered, that would correspond to different bases in DSR. The studied energy range could also be extended by considering other backgrounds of low-energy photons beyond the CMB. Nevertheless, this is an example of the interest of a "RDK phenomenology", complementary to a LIV phenomenology. In particular, this study could be generalized to other processes, opening the window to new astroparticle effects in the DSR framework.

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