The Teaching and Learning of Multiplication Bonds: A Position Statement

Pupils need to develop an understanding of the multiplicative composition of numbers, fluency in working flexibly with multiplication bonds and automaticity in knowing them.

This Position Statement arises from meetings of the Joint ATM/MA Primary Group which have frequently turned to discussing issues around the teaching of multiplication. In the light of these discussions, we hope this statement will help to inform the debate around how best to teach and learn multiplication bonds.

Addition and multiplication bonds: are they essentially different?

Children need to learn as many addition bonds as multiplication bonds:

> 100 addition bonds from 1 + 1 to 10 + 10
>
> 100 multiplication bonds from 1 x 1 to 10 x 10[1]

It is the case that multiplication bonds have been taught most often by learning 'tables', whereas automaticity in knowing the addition bonds has been attained without resorting to learning any addition 'tables'. Is this difference simply a result of curriculum history (it's always been done that way) or is there really a significant difference between how we learn addition bonds and how we learn multiplication bonds?

One of the difficulties in the debate around learning multiplication bonds arises from differing assumptions about what language to use and what is meant by using that language. We think that one way to move the debate forward is to achieve clarity and agreement on language.

Multiplication bonds

We use multiplication bonds in this paper rather than 'multiplication facts' because as Cockroft (1982) identified:

> *Facts are items of information which are essentially unconnected or arbitrary...The so-called 'number facts', for example 4 + 6 = 10, do not fit this category since they are not unconnected or arbitrary but follow logically from an understanding of the number system.* p. 71

For example, knowing the fact that 'four' is the name (in English) of the symbol '4' does not help determine the name of the symbol '6'. In contrast 4 x 6 = 24 is not a fact as, using Cockcroft's definition, it is not arbitrary. It represents a multiplicative relationship from which other relationships follow.

We also prefer the language of learning multiplication bonds, rather than learning 'times tables' as it indicates there are different ways to both learn and access these bonds, aside from chanting. We believe that the aim is for learners to develop an understanding of the multiplicative composition of numbers, in a similar way to developing an understanding of the additive composition of numbers. This means that when considering a number, for example 24, learners recognise that it can be thought of multiplicatively as: twenty-four ones, one twenty-four, twelve twos, two twelves, eight threes, three eights, six fours and four sixes (and eventually as 48 halves and so on). They know and connect multiplications and divisions to the number and also understand relationships between the numbers and bonds; fluently knowing 24 = 4 x 6 means a learner also knows that, for example, 6 is 24 divided by 4 and that 24 divided by 6 is 4.

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[1] We focus here on the knowledge of multiplication bonds to 10 x 10. Although currently up to 12 x 12 is a National Curriculum expectation, the case for knowing the additional 44 bonds is not well supported in evidence. See https://blog.wolfram.com/2013/06/26/is-there-any-point-to-the-12-times-table/

Fluency

Flexibility and decision making are key elements of fluency. The aims of the National Curriculum (2013) describe fluency as a combination of conceptual understanding and automaticity, developed side by side, and that all pupils:

> *become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately* p. 3

Automaticity

We prefer the language of automaticity, as this is not the same as recall; Fosnot and Dolk (2001) suggest automaticity relies on thinking about relationships rather than simply memorising facts. There is a choice to make: whether to focus on memorisation or automaticity.

There are dangers inherent in pursuing a 'learn the bonds in isolation' model (e.g. chanting tables alone):

- It promotes an image of mathematics as a subject focused on memorising rather than thinking. This may lead to an expectation of instrumental rather than relational understanding (Skemp, 1976).
- It can result in negative attitudes towards mathematics because of an untimely and inappropriate focus on speed.

  > *One of the challenges faced by the students in the first stages of elementary school is the task of memorizing the multiplication table; … students who are challenged by such a difficult memorization task in the early years of school life develop a negative attitude towards mathematics in later years.* Bahadıri, 2017, p. 128
- Verbal memory alone is not always reliable and it is subject to pattern interference.

  > *Despite many hours of practice, most people encounter great difficulty with the multiplication tables. Ordinary adults of average intelligence make mistakes roughly 10 percent of the time. Some multiplications, such as 8 x 7 or 9 x 7, can take up to 2 seconds, and the error rate goes up to 25 percent. (The answers are 8 x 7 = 54 and 9 x 7 = 64. Or are they? Oh dear! I'll leave it to you to sort out.)…The reason we have such trouble is we remember the table linguistically, and as a result many of the different entries interfere with one another.* Devlin, 2000, p. 60

Where multiplication bonds are learnt to automaticity through exploring the meaning of multiplication, connections between bonds and between tables become exposed (for example multiplying four and multiplying eight) (see Field 2020 and Field, Day and Vyas 2021). This supports learners in deriving multiplications at times when their memory proves unreliable or they experience pattern interference. It also benefits learners in helping them to use what they know to solve problems involving numbers beyond the multiplication tables.

> *Research suggests peer discussion, different representations and a broad selection of strategies are more effective than just repetition and practice alone (Brendefur et al, 2015).*
> Cambridge Espresso 1 p. 2

Focussing on understanding rather than memorisation does not mean automaticity will not happen, it **does** happen, as a result of compression:

> *Mathematics is amazingly compressible; you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to* ***see it as a whole****, there is often tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in other mental processes. The insight that goes with this compression is one of the real joys of mathematics.* Thurston, 1990, pp. 846-7

Returning to the question of whether or not learning and knowing multiplication bonds is intrinsically harder than learning addition bonds, the research of Davydov (1985) and followers of his work shows that a curriculum based on measuring can support children in thinking, talking and learning about multiplicative relations alongside additive ones. They also assert that learning multiplication bonds can be done by attending to mathematical structure, in a fashion similar to the way that working on structure underpins learning addition bonds. Their work also raises questions about whether initial teaching of multiplication is best done as repeated addition, as is often the case currently (see for example, Keith Devlin's thoughts on this: https://www.maa.org/external_archive/devlin/devlin_06_08.html).

In conclusion, we advocate teaching multiplication bonds to automaticity through approaches that are built on conceptual understanding and meaningful practice. This would enable pupils to understand multiplicative relationships, make connections and build fluency.

To share in the delight and the intellectual experience of mathematics – to fly where before we walked – that is the goal of a mathematical education. Thurston, 1990, p. 848

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