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Research article

Forced waves and their asymptotic behaviors in a Lotka-Volterra competition model with spatio-temporal nonlocal effect under climate change

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Abstract: In this paper, we propose a modified Lotka-Volterra competition model under climate change, which incorporates both spatial and temporal nonlocal effect. First, the theoretical analyses for forced waves of the model are performed, and the existence of the forced waves is proved by using the cross-iteration scheme combining with appropriate upper and lower solutions. Second, the asymptotic behaviors of the forced waves are derived by using the linearization and limiting method, and we find that the asymptotic behaviors of forced waves are mainly determined by the leading equations. In addition, some typical numerical examples are provided to illustrate the analytical results. By choosing three kinds of different kernel functions, it is found that the forced waves can be both monotonic and non-monotonic.

Keywords: nonlocal effect; forced wave; competitive system; climate change

1. Introduction

Global climate change has caused the greater loss of sea ice, more intense drought, heat waves and hurricanes and accelerated sea level rise. Scientists predicted that Arctic seemed to be a high probability of becoming ice-free in summer before mid-century [1]. Climate change has already resulted in region shifting and then observable environmental heterogeneity, which is very unfavourable for the world's species of plants and animals to survive and spread. The topic studied here has provided some insights into the effect of shifting heterogeneity on ecological species.

Reaction-diffusion models often incorporate both dispersal and local rates of change. From the view of mathematical biology, reaction-diffusion models can help us to describe the population change and derive the coexistence of interacting species [2–4]. For scalar reaction-diffusion equations modelling the single species in a shifting environment, one prominent work comes from Berestycki et al. [5]. Therein, the reaction-diffusion equation can be described as

$$\partial_t u(t,x) = d\partial_{xx}u(t,x) + g(x-ct, u(t,x)), \qquad t>0, x\in\mathbb{R}, \tag{1.1}$$

where $u(t,x)$ represents the density of individuals at time t and position x, g denotes the effective rate of reproduction and mortality, and $d > 0$ denotes the diffusion coefficient. In this model (1.1), it is assumed that the favorable environment is moved forward at the speed c (per time step), and the minimum favorable patch size for the persistence of species can be derived. Traveling wave solutions can describe a species that distributed over some certain range would gradually expand its range to the whole environment. If we seek for the solution of (1.1) in the form of $u(t,x) = U(x-ct)$ with the moving coordinate $x-ct$ as the independent variable $\xi := x-ct$, i.e., the solution is traveling with given forced speed c, then such special kind of solution is called the forced (traveling) waves. In brief, the solution of (1.1) travels with the same speed as the environment changes. In order to gain more insights into the genetic diversity under the shifting environment, Garnier and Lewis [6] investigated the existence of the forced waves for equation (1.1). Later, Berestycki and Rossic [7,8] further extended the results established in [5] by taking higher $d-$dimension space and general function g into account. Vo [9] considered the analogous situations without the condition, as presented in [7], that the favorable zone was compact. Biologically, it means that the environment is unfavorable outside a compact set $[0,L]$ and favorable inside. In favorable zone $[0,L]$, species can disperse, grow or other activities, while outside of which the species die at a given rate, with no reproduction. Very recently, Berestycki and Fang [10] reconsidered the equation (1.1), where $g(\xi,\cdot)$ was chosen to be asymptotically KPP type as $\xi\to-\infty$, and they derived its existence, multiplicity and attractive property of forced waves. One special form of (1.1) is written in the following equation by considering the Logistic growth

$$\partial_t u(t,x) = d\partial_{xx}u(t,x) + u(t,x)(r(x-ct) - u(t,x)), \qquad t>0, x\in\mathbb{R}. \tag{1.2}$$

By assuming that $r(x)\in C(\mathbb{R},\mathbb{R})$ is a nondecreasing function with $-\infty < r(-\infty) < 0 < r(\infty) < \infty$, Hu and Zou [11] proved that the forced waves for (1.2) exist. As a further step, Fang, Lou and Wu [12] derived the conditions of the existence and non-existence of the pulse forced waves for the limiting equation (1.2), which originates from the typical susceptible-infective-susceptible epidemic model.

Notice that the species do not live isolated, interspecies interaction always exists. Cooperation, competition and predater-prey relations are three common interspecies interactions [13]. For the two species reaction-diffusion models under climate change, several works have explored the interplay between species and dynamical characteristics. For instance, Yang et al. [14] derived the existence and asymptotic behaviors of forced waves of a cooperative Lotka-Volterra model. Dong et al. [15] investigated several kinds of forced waves, which connect different steady states to a competitive Lotka-Volterra model. Especially, Wang et al. [16] discussed the uniqueness and the stability of forced waves in a competitive Lotka-Volterra model. Choi et al. [17] studied the existence of forced waves in a predator-prey model. For a general cooperative model, Wu and Xu [18] established the existence of forced waves, and then derived its uniqueness and stability of the forced waves.

Spatio-temporal nonlocal effect first introduced by Britton [19], which is beyond the limit of time delay and the scope of spatial location, is attracting more and more attentions [20–22]. As an example, Banerjee and Volpert [23] showed that the nonlocal consumption for the prey species has often led to stationary inhomogeneous space solutions while the classical and local version was unable to produce them. Hutchinson and Williams [24] pointed out that the nonlocal structured assemblage processes seem to be more important when compared to the local processes on Hong Kong shores. Song and Yang [25] demonstrated that the joint interaction of the nonlocal spatial averaging and delay caused the appearance of spatio-temporal patterns. For early works, readers can refer to the references [26–28].

Motivated by these works, we discuss the following Lotka-Volterra competition model under the climate change and spatio-temporal nonlocal effect

$$
\begin{aligned}
\partial_t u(t,x) &= d_1\partial_{xx}u(t,x) + u(t,x)[r_1(x-ct) - u(t,x)] \\
&\quad - a_1 u(t,x)\int_0^{\infty}\int_{\mathbb{R}} J_1(s,y)v(t-s,x-y)\mathrm{d}y\mathrm{d}s, \\
\partial_t v(t,x) &= d_2\partial_{xx}v(t,x) + v(t,x)[r_2(x-ct) - v(t,x)] \\
&\quad - a_2 v(t,x)\int_0^{\infty}\int_{\mathbb{R}} J_2(s,y)u(t-s,x-y)\mathrm{d}y\mathrm{d}s,
\end{aligned}
\tag{1.3}
$$

where $d_1, d_2 > 0$ are the diffusion rates, $a_1, a_2 > 0$ are interspecies effect and $c > 0$ is the rightward shifting speed. The model (1.3) is a generized two-species competition model. Every species follows the inhomogeneous Logistic growth, and sustains interspecific competition. What is noteworthy is that the interspecific competition term is a spatio-temporal average weighted toward the current time and position, which can cover a series of models by choosing some special kernels. We assume for $i = 1, 2$,

(A1) $J_i(\cdot,\cdot) \in C(\mathbb{R}_+ \times \mathbb{R}, \mathbb{R}_+)$, $J_i(s,-y) = J_i(s,y)$, $\int_0^{\infty}\int_{\mathbb{R}} J_i(s,y)\mathrm{d}y\mathrm{d}s = 1$, and there exists some $\alpha^* > 0$ such that $\int_0^{\infty}\int_{\mathbb{R}} \mathrm{e}^{\alpha(y-cs)}J_i(s,y)\mathrm{d}y\mathrm{d}s < \infty$ for any $\alpha \in (0,\alpha^*]$ and $c > 0$;
(A2) $r_i(\cdot) \in C(\mathbb{R},\mathbb{R})$ is nondecreasing with $-\infty < r_i(-\infty) < 0 < r_i(\infty) < \infty$;
(A3) $a_1 < \frac{r_1(\infty)}{r_2(\infty)} < \frac{1}{a_2}$;
(A4) there exist positive numbers k_i, ν_i such that

$$\lim_{x\to\infty}\frac{r_i(\infty) - r_i(x)}{\mathrm{e}^{-\nu_i x}} = k_i.$$

It is noted that there is a positive equilibria $E_*(u_*, v_*)$ in the reaction system corresponding to (1.3) when the growth function $r_i(x-ct)$ is replaced by $r_i(\infty)$, where

$$u_* = \frac{r_1(\infty) - a_1 r_2(\infty)}{1 - a_1 a_2}, \quad v_* = \frac{r_2(\infty) - a_2 r_1(\infty)}{1 - a_1 a_2}.$$

The assumptions (A1)-(A4) have some ecological meanings. In (A1), the even assumption ensures the spatio-temporal weights depend on distances from the original position. The normalisation assumption guarantees that the kernel will not affect the spatially uniform steady state solutions of (1.3) when the growth function $r_i(x-ct)$ replaced by $r_i(\infty)$, i.e., $(0,0)$ and (u_*, v_*). (A2) tells us each species is befitting far to the right (corresponding to the North). As time goes, then the inappropriate environment $\{x \in \mathbb{R} : r_i(x-ct) < 0\}$ for each species is moving along the real axis from left to right. Therewith the appropriate environment $\{x \in \mathbb{R} : r_i(x-ct) > 0\}$ is rightward shrinking. The edge of the proper environment is shifting at a speed c. (A3) is equivalent to $r_1(\infty) - a_1 r_2(\infty) > 0$ and $r_2(\infty) - a_2 r_1(\infty) > 0$. It shows that two competing species are able to coexist near ∞. In (A4), the exponential decay of $r_i(x)$ for x sufficiently large is used in the construction of the lower solution. Biologically, the species cannot spread to infinity, so we assume the growth rate of species near the infinity will decline rapidly as time goes by.

The remaining sections of this paper are organized as follows. Motivated by the recent work of [15], in Section 2, we first construct the appropriate upper and lower bounds of solutions to (1.3), and then obtain the existence of forced waves through the cross-iteration method on the basis of the derived solutions. In Section 3, the asymptotic properties of forced waves along two tails are studied by the linearization techniques. Finally, in Section 4, three different numerical examples are provided to confirm the obtained analytical results.

2. Solutions of modified Lotka-Volterra model and appearance of forced waves

In this section, we will obtain the existence of forced waves to (1.3). The method is to construct a pair of proper upper and lower solutions and employing the cross-iteration to reach a fixed point in corresponding integral equations. Remembering the moving coordinate $x-ct$ and setting $u(t,x) = U(\xi)$ and $v(t,x) = V(\xi)$ with $\xi = x - ct$, then (1.3) becomes

$$\begin{aligned} &d_1U''(\xi) + cU'(\xi) + U(\xi)[r_1(\xi) - U(\xi)] \\ &\quad - a_1U(\xi)\int_0^\infty\int_{\mathbb{R}} J_1(s,y)V(\xi - y + cs)\mathrm{d}y\mathrm{d}s = 0, \\ &d_2V''(\xi) + cV'(\xi) + V(\xi)[r_2(\xi) - V(\xi)] \\ &\quad - a_2V(\xi)\int_0^\infty\int_{\mathbb{R}} J_2(s,y)U(\xi - y + cs)\mathrm{d}y\mathrm{d}s = 0. \end{aligned} \tag{2.1}$$

Next, we will consider the solution to (2.1) satisfying the following asymptotic boundary conditions:

$$\lim_{\xi\to-\infty}(U(\xi),V(\xi)) = (0,0), \quad \lim_{\xi\to\infty}(U(\xi),V(\xi)) = (u_*,v_*). \tag{2.2}$$

For $\xi \in \mathbb{R}$, denote

$$\begin{aligned} &\overline{U}(\xi) = \min\{r_1(\infty), u_* + \beta_1u_*\mathrm{e}^{-\alpha\xi}\}, \quad \underline{U}(\xi) = \max\{0, u_* - \beta_3u_*\mathrm{e}^{-\alpha\xi}\}, \\ &\overline{V}(\xi) = \min\{r_2(\infty), v_* + \beta_2v_*\mathrm{e}^{-\alpha\xi}\}, \quad \underline{V}(\xi) = \max\{0, v_* - \beta_4v_*\mathrm{e}^{-\alpha\xi}\}, \end{aligned}$$

where $\alpha > 0$ to be determined later, $\beta_1,\beta_2 > 0,\beta_3,\beta_4 > 1$ satisfy

$$a_1\max\left\{\frac{\beta_4}{\beta_1},\frac{\beta_2}{\beta_3}\right\} < \frac{r_1(\infty) - a_1r_2(\infty)}{r_2(\infty) - a_2r_1(\infty)} < \frac{1}{a_2}\min\left\{\frac{\beta_4}{\beta_1},\frac{\beta_2}{\beta_3}\right\}. \tag{2.3}$$

Lemma 2.1. *Assume that the inequality* (2.3) *holds. Then there exist sufficiently small parameters* $\alpha_1 > 0$ *and* $\alpha_2 > 0$ *such that*

$$\begin{aligned} &\Delta_1(\alpha) := a_1\beta_4v_*\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s - \beta_1u_* \le 0, \\ &\Delta_2(\alpha) := a_2\beta_3u_*\int_0^\infty\int_{\mathbb{R}} J_2(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s - \beta_2v_* \le 0, \end{aligned}$$

for $\alpha \in (0,\alpha_1]$ *and*

$$\begin{aligned} &\Delta_3(\alpha) := -2k_1\beta_3^{\frac{\alpha-\nu_1}{\alpha}} + \beta_3u_* - a_1\beta_2v_*\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s \ge 0, \\ &\Delta_4(\alpha) := -2k_2\beta_4^{\frac{\alpha-\nu_2}{\alpha}} + \beta_4v_* - a_2\beta_1u_*\int_0^\infty\int_{\mathbb{R}} J_2(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s \ge 0, \end{aligned}$$

for $\alpha \in (0,\alpha_2]$.

Proof. By (2.3), we follow that

$$\Delta_1(0) = a_1\beta_4 v_* - \beta_1 u_* = \beta_1 v_* \left[a_1 \frac{\beta_4}{\beta_1} - \frac{r_1(\infty) - a_1 r_2(\infty)}{r_2(\infty) - a_2 r_1(\infty)} \right] < 0,$$
$$\Delta_2(0) = a_2\beta_3 u_* - \beta_2 v_* = a_2\beta_3 v_* \left[\frac{r_1(\infty) - a_1 r_2(\infty)}{r_2(\infty) - a_2 r_1(\infty)} - \frac{1}{a_2}\frac{\beta_2}{\beta_3} \right] < 0,$$

and $\Delta_1(\alpha), \Delta_2(\alpha)$ are continuous functions in α, then there exists some small $\alpha_1 > 0$ such that $\Delta_1(\alpha) \leq 0$ and $\Delta_2(\alpha) \leq 0$ for $\alpha \in (0, \alpha_1]$.

Starting from (2.3), we also have

$$\lim_{\alpha \to 0+} \Delta_3(\alpha) = \beta_3 u_* - a_1\beta_2 v_* = \beta_3 v_* \left[\frac{r_1(\infty) - a_1 r_2(\infty)}{r_2(\infty) - a_2 r_1(\infty)} - a_1 \frac{\beta_2}{\beta_3} \right] > 0,$$
$$\lim_{\alpha \to 0+} \Delta_4(\alpha) = \beta_4 v_* - a_2\beta_1 u_* = a_2\beta_1 v_* \left[\frac{1}{a_2}\frac{\beta_4}{\beta_1} - \frac{r_1(\infty) - a_1 r_2(\infty)}{r_2(\infty) - a_2 r_1(\infty)} \right] > 0,$$

and $\Delta_3(\alpha), \Delta_4(\alpha)$ are continuous functions in α, then there exists some small $\alpha_2 > 0$ such that $\Delta_3(\alpha) \geq 0$ and $\Delta_4(\alpha) \geq 0$ for $\alpha \in (0, \alpha_2]$.

□

Lemma 2.2. *Let $c > 0$ and $0 < \alpha < \min\{\frac{c}{d_1}, \frac{c}{d_2}, \alpha_1, \alpha_2, \nu_1, \nu_2\}$ be sufficiently small. Then $(\overline{U}(\xi), \overline{V}(\xi))$ and $(\underline{U}(\xi), \underline{V}(\xi))$ are the upper and lower solutions of (2.1), respectively.*

Proof. **The proof of upper solution.** For $\xi > \frac{1}{\alpha} \ln \frac{\beta_1 u_*}{r_1(\infty) - u_*}$, then $\overline{U}(\xi) = u_* + \beta_1 u_* e^{-\alpha\xi}$. Note that $\underline{V}(\xi) \geq v_* - \beta_4 v_* e^{-\alpha\xi}$ for any $\xi \in \mathbb{R}$. As $r_1(\xi) \leq r_1(\infty)$, we have

$$\begin{aligned}
& d_1\overline{U}''(\xi) + c\overline{U}'(\xi) + \overline{U}(\xi)[r_1(\xi) - \overline{U}(\xi)] \\
& \quad - a_1\overline{U}(\xi) \int_0^\infty \int_{\mathbb{R}} J_1(s,y)\underline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\
\leq & \beta_1 u_* e^{-\alpha\xi} \alpha(d_1\alpha - c) + u_* e^{-\alpha\xi}(1 + \beta_1 e^{-\alpha\xi}) \\
& \quad \times \left[a_1\beta_4 v_* \int_0^\infty \int_{\mathbb{R}} J_1(s,y) e^{\alpha y - \alpha cs} \mathrm{d}y\mathrm{d}s - \beta_1 u_* \right] \\
\leq & u_* e^{-\alpha\xi}(1 + \beta_1 e^{-\alpha\xi}) \left[a_1\beta_4 v_* \int_0^\infty \int_{\mathbb{R}} J_1(s,y) e^{\alpha(y - cs)} \mathrm{d}y\mathrm{d}s - \beta_1 u_* \right].
\end{aligned} \tag{2.4}$$

By (2.4) and Lemma 2.1, we have

$$\begin{aligned}
& d_1\overline{U}''(\xi) + c\overline{U}'(\xi) + \overline{U}(\xi)[r_1(\xi) - \overline{U}(\xi)] \\
& \quad - a_1\overline{U}(\xi) \int_0^\infty \int_{\mathbb{R}} J_1(s,y)\underline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\
\leq & u_* e^{-\alpha\xi}(1 + \beta_1 e^{-\alpha\xi})\Delta_1(\alpha) \leq 0.
\end{aligned} \tag{2.5}$$

For $\xi < \frac{1}{\alpha} \ln \frac{\beta_1 u_*}{r_1(\infty) - u_*}$, then $\overline{U}(\xi) = r_1(\infty)$. It needs to be noted that $\underline{V}(\xi) \geq 0$ for any $\xi \in \mathbb{R}$. According to $r_1(\xi) \leq r_1(\infty)$, we have

$$\begin{aligned}
& d_1\overline{U}''(\xi) + c\overline{U}'(\xi) + \overline{U}(\xi)[r_1(\xi) - \overline{U}(\xi)] \\
& \quad - a_1\overline{U}(\xi) \int_0^\infty \int_{\mathbb{R}} J_1(s,y)\underline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\
\leq & r_1(\infty)[r_1(\xi) - r_1(\infty)] \leq 0.
\end{aligned} \tag{2.6}$$

By (2.5) and (2.6), we have shown that

$$\begin{aligned} &d_1\overline{U}''(\xi) + c\overline{U}'(\xi) + \overline{U}(\xi)[r_1(\xi) - \overline{U}(\xi)] \\ &\quad - a_1\overline{U}(\xi)\int_0^\infty \int_{\mathbb{R}} J_1(s,y)\underline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \le 0, \end{aligned} \tag{2.7}$$

for $\xi \neq \frac{1}{\alpha}\ln\frac{\beta_1 u_*}{r_1(\infty)-u_*}$. Similarly, for $\xi > \frac{1}{\alpha}\ln\frac{\beta_2 v_*}{r_2(\infty)-v_*}$, by Lemma 2.1, then

$$\begin{aligned} &d_2\overline{V}''(\xi) + c\overline{V}'(\xi) + \overline{V}(\xi)[r_2(\xi) - \overline{V}(\xi)] \\ &\quad - a_2\overline{V}(\xi)\int_0^\infty \int_{\mathbb{R}} J_2(s,y)\underline{U}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\ &\le v_*\mathrm{e}^{-\alpha\xi}(1+\beta_2\mathrm{e}^{-\alpha\xi})\Delta_2(\alpha) \le 0. \end{aligned} \tag{2.8}$$

For $\xi < \frac{1}{\alpha}\ln\frac{\beta_2 v_*}{r_2(\infty)-v_*}$, then we have

$$\begin{aligned} &d_2\overline{V}''(\xi) + c\overline{V}'(\xi) + \overline{V}(\xi)[r_2(\xi) - \overline{V}(\xi)] \\ &\quad - a_2\overline{V}(\xi)\int_0^\infty \int_{\mathbb{R}} J_2(s,y)\underline{U}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\ &\le r_2(\infty)[r_2(\xi) - r_2(\infty)] \le 0. \end{aligned} \tag{2.9}$$

Based on (2.8) and (2.9), we have shown that

$$\begin{aligned} &d_2\overline{V}''(\xi) + c\overline{V}'(\xi) + \overline{V}(\xi)[r_2(\xi) - \overline{V}(\xi)] \\ &\quad - a_2\overline{V}(\xi)\int_0^\infty \int_{\mathbb{R}} J_2(s,y)\underline{U}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \le 0, \end{aligned}$$

for $\xi \neq \frac{1}{\alpha}\ln\frac{\beta_2 v_*}{r_2(\infty)-v_*}$.

The proof of lower solution. For $\xi > \frac{1}{\alpha}\ln\beta_3 > 0$, then $\underline{U}(\xi) = u_* - \beta_3 u_*\mathrm{e}^{-\alpha\xi}$. Note that $\overline{V}(\xi) \le v_* + \beta_2 v_*\mathrm{e}^{-\alpha\xi}$ for any $\xi \in \mathbb{R}$. Let $\alpha > 0$ be sufficiently small such that $\frac{1}{\alpha}\ln\beta_3, \frac{1}{\alpha}\ln\beta_4$ are sufficiently large. By (A4), we get

$$r_i(\infty) - r_i(\xi) \le 2k_i\mathrm{e}^{-\nu_i\xi}, \qquad \xi \ge \frac{1}{\alpha}\ln\beta_{i+2},\ i = 1,2. \tag{2.10}$$

Now recalling $r_1(\infty) = u_* + a_1 v_*$ and in view of (2.10) and Lemma 2.1, we obtain

$$\begin{aligned} &d_1\underline{U}''(\xi) + c\underline{U}'(\xi) + \underline{U}(\xi)[r_1(\xi) - \underline{U}(\xi)] \\ &\quad - a_1\underline{U}(\xi)\int_0^\infty \int_{\mathbb{R}} J_1(s,y)\overline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \\ &\ge \beta_3 u_*\mathrm{e}^{-\alpha\xi}\alpha(c - d_1\alpha) + u_*\mathrm{e}^{-\alpha\xi}(1-\beta_3\mathrm{e}^{-\alpha\xi}) \\ &\quad \times\left[(r_1(\xi) - r_1(\infty))\mathrm{e}^{\alpha\xi} + \beta_3 u_* - a_1\beta_2 v_*\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s\right] \\ &\ge u_*\mathrm{e}^{-\alpha\xi}(1-\beta_3\mathrm{e}^{-\alpha\xi}) \\ &\quad \times\left[-2k_1\mathrm{e}^{\frac{\alpha-\nu_1}{\alpha}\ln\beta_3} + \beta_3 u_* - a_1\beta_2 v_*\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\mathrm{e}^{\alpha(y-cs)}\mathrm{d}y\mathrm{d}s\right] \\ &= u_*\mathrm{e}^{-\alpha\xi}(1-\beta_3\mathrm{e}^{-\alpha\xi})\Delta_3(\alpha) \ge 0. \end{aligned} \tag{2.11}$$

For $\xi < \frac{1}{\alpha}\ln\beta_3$, then $\underline{U}(\xi) = 0$. Hence,

$$\begin{aligned} &d_1\underline{U}''(\xi) + c\underline{U}'(\xi) + \underline{U}(\xi)[r_1(\xi) - \underline{U}(\xi)] \\ &\quad - a_1\underline{U}(\xi)\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\overline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s = 0. \end{aligned} \tag{2.12}$$

By (2.11) and (2.12), we have shown that

$$\begin{aligned} &d_1\underline{U}''(\xi) + c\underline{U}'(\xi) + \underline{U}(\xi)[r_1(\xi) - \underline{U}(\xi)] \\ &\quad - a_1\underline{U}(\xi)\int_0^\infty\int_{\mathbb{R}} J_1(s,y)\overline{V}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \geq 0. \end{aligned} \tag{2.13}$$

for $\xi \neq \frac{1}{\alpha}\ln\beta_3$. Similarly, we can show that

$$\begin{aligned} &d_2\underline{V}''(\xi) + c\underline{V}'(\xi) + \underline{V}(\xi)[r_2(\xi) - \underline{V}(\xi)] \\ &\quad - a_2\underline{V}(\xi)\int_0^\infty\int_{\mathbb{R}} J_2(s,y)\overline{U}(\xi - y + cs)\mathrm{d}y\mathrm{d}s \geq 0. \end{aligned}$$

for $\xi \neq \frac{1}{\alpha}\ln\beta_4$. □

Assume $\mathcal{BC}$ to be $\mathcal{L}^\infty(\mathbb{R}) \cap C(\mathbb{R})$, where $\mathcal{L}^\infty(\mathbb{R})$ means the collection of all bounded functions and $C(\mathbb{R})$ denotes the set of all continuous and real functions, which are mapped from $\mathbb{R}$ to $\mathbb{R}$. Let $\mathcal{Y} = \mathcal{BC} \times \mathcal{BC}$. For $u = (u_1, u_2) \in \mathcal{Y}$, based on the definition of norm $\|u\|_{\mathcal{Y}}=\|u_1\|_{\mathcal{BC}}+\|u_2\|_{\mathcal{BC}}$ with $\|u_i\|_{\mathcal{BC}}= \sup_{x\in\mathbb{R}}|u_i(x)|$, and we can introduce the following functional space

$$\Gamma := \{(U, V) \in \mathcal{Y} : \underline{U} \leq U \leq \overline{U}, \underline{V} \leq V \leq \overline{V} \text{ on } \mathbb{R}\}.$$

For $(U, V) \in \Gamma$, picking $\rho_1 \geq -r_1(-\infty) + 2r_1(\infty) + a_1r_2(\infty)$ and $\rho_2 \geq -r_2(-\infty) + 2r_2(\infty) + a_2r_1(\infty)$, denote

$$\begin{aligned} F_1(U,V)(\xi) &= \rho_1U(\xi) + U(\xi)[r_1(\xi) - U(\xi)] \\ &\quad - a_1U(\xi)\int_0^\infty\int_{\mathbb{R}} J_1(s,y)V(\xi - y + cs)\mathrm{d}y\mathrm{d}s, \\ F_2(U,V)(\xi) &= \rho_2V(\xi) + V(\xi)[r_2(\xi) - V(\xi)] \\ &\quad - a_2V(\xi)\int_0^\infty\int_{\mathbb{R}} J_2(s,y)U(\xi - y + cs)\mathrm{d}y\mathrm{d}s. \end{aligned} \tag{2.14}$$

Then, F_1 is nondecreasing in $U \in [0, r_1(\infty)]$ and F_2 is nondecreasing in $V \in [0, r_2(\infty)]$. Rewrite the system (2.1) as

$$\begin{aligned} d_1U''(\xi) + cU'(\xi) - \rho_1U(\xi) + F_1(U,V)(\xi) = 0, \\ d_2V''(\xi) + cV'(\xi) - \rho_2V(\xi) + F_2(U,V)(\xi) = 0. \end{aligned} \tag{2.15}$$

It is clearly shown that the entire solution (U, V) of (2.15) is bounded if and only if $(U, V) = Q(U, V)$ is continuous , and $Q = (Q_1, Q_2)$ can be computed as follows,

$$Q_i(U,V)(\xi) = \frac{1}{d_i(\lambda_{i2} - \lambda_{i1})}\int_{-\infty}^{\infty} \chi_i(\xi - s)F_i(U,V)(s)\mathrm{d}s,$$

where

$$\chi_i(\xi) = \begin{cases} \mathrm{e}^{\lambda_{i2}\xi}, & \xi \leq 0, \\ \mathrm{e}^{\lambda_{i1}\xi}, & \xi > 0, \end{cases} \qquad \lambda_{ij} = \frac{-c + (-1)^j\sqrt{c^2 + 4d_i\rho_i}}{2d_i},\ i, j = 1, 2. \tag{2.16}$$

Lemma 2.3. *The operators Q_1 and Q_2 are non-decreasing and non-increasing in U, respectively. Meanwhile, Q_1 and Q_2 are non-increasing and non-decreasing in V. Moreover, Q maps Γ into Γ.*

Proof. For any $(U_1, V_1), (U_2, V_2) \in \Gamma$ with $U_1 \geq U_2$ and $V_1 \leq V_2$, recalling $0 \leq U_i \leq r_1(\infty), 0 \leq V_i \leq r_2(\infty)$, we obtain

$$\begin{aligned}
&F_1(U_1, V_1)(\xi) - F_1(U_2, V_2)(\xi) \\
=&[\rho_1 + r_1(\xi) - U_1(\xi) - U_2(\xi)][U_1(\xi) - U_2(\xi)] \\
&- a_1 \int_0^{\infty} \int_{\mathbb{R}} J_1(s, y)[U_1(\xi)V_1(\xi - y + cs) - U_2(\xi)V_2(\xi - y + cs)]\mathrm{d}y\mathrm{d}s \\
=&\left[\rho_1 + r_1(\xi) - U_1(\xi) - U_2(\xi) - a_1 \int_0^{\infty} \int_{\mathbb{R}} J_1(s, y)V_1(\xi - y + cs)\mathrm{d}y\mathrm{d}s\right] \\
&\times [U_1(\xi) - U_2(\xi)] \\
&- a_1 \int_0^{\infty} \int_{\mathbb{R}} J_1(s, y)[V_1(\xi - y + cs) - V_2(\xi - y + cs)]U_2(\xi)\mathrm{d}y\mathrm{d}s \geq 0.
\end{aligned}$$

In a similar way, we can show that $F_2(U, V)$ is nondecreasing in V and nonincreasing in U. If $U_1 \geq U_2$ and $V_1 \leq V_2$, then we have

$$\begin{aligned}
&Q_1(U_1, V_1)(\xi) - Q_1(U_2, V_2)(\xi) \\
=&\frac{1}{d_1(\lambda_{12} - \lambda_{11})} \int_{-\infty}^{\infty} \chi_1(\xi - s)[F_1(U_1, V_1)(s) - F_1(U_2, V_2)(s)]\mathrm{d}s \geq 0,
\end{aligned}$$

and

$$\begin{aligned}
&Q_2(U_1, V_1)(\xi) - Q_2(U_2, V_2)(\xi) \\
=&\frac{1}{d_2(\lambda_{22} - \lambda_{21})} \int_{-\infty}^{\infty} \chi_2(\xi - s)[F_2(U_1, V_1)(s) - F_2(U_2, V_2)(s)]\mathrm{d}s \leq 0,
\end{aligned}$$

since $\chi_i \geq 0$ in (2.16). The above inequality implies that

$$\begin{aligned}
Q_1(\underline{U}, \overline{V})(\xi) \leq Q_1(U, V)(\xi) \leq Q_1(\overline{U}, \underline{V})(\xi), \\
Q_2(\overline{U}, \underline{V})(\xi) \leq Q_2(U, V)(\xi) \leq Q_2(\underline{U}, \overline{V})(\xi),
\end{aligned} \tag{2.17}$$

which means the existence of the upper and lower bounds for Q_1 and Q_2 in the entire space of $(U, V) \in \Gamma$ and $\xi \in \mathbb{R}$.

Next, it will be proved that Q is mapped from Γ to Γ. Let $\xi_0 = \frac{1}{\alpha} \ln \beta_3$. For $\xi \neq \xi_0$, we can assume that $\xi < \xi_0$ without lacking the generality. After that, in accordance with (2.13) and (2.14), it can be followed that

$$
\begin{aligned}
&Q_1(\underline{U},\overline{V})(\xi)\\
=&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left[\int_{-\infty}^{\xi}\mathrm{e}^{\lambda_{11}(\xi-s)}F_1(\underline{U},\overline{V})(s)\mathrm{d}s+\int_{\xi}^{\infty}\mathrm{e}^{\lambda_{12}(\xi-s)}F_1(\underline{U},\overline{V})(s)\mathrm{d}s\right]\\
\geq&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left(\int_{-\infty}^{\xi}\mathrm{e}^{\lambda_{11}(\xi-s)}+\int_{\xi}^{\xi_0}\mathrm{e}^{\lambda_{12}(\xi-s)}+\int_{\xi_0}^{\infty}\mathrm{e}^{\lambda_{12}(\xi-s)}\right)\\
&\times\left[-d_1\underline{U}''(s)-c\underline{U}'(s)+\rho_1\underline{U}(s)\right]\mathrm{d}s\\
=&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\Big\{d_1(\lambda_{12}-\lambda_{11})\underline{U}(\xi)+d_1\mathrm{e}^{\lambda_{12}(\xi-\xi_0)}[\underline{U}'(\xi_0+0)-\underline{U}'(\xi_0-0)]\\
&\quad+(d_1\lambda_{12}+c)\mathrm{e}^{\lambda_{12}(\xi-\xi_0)}[\underline{U}(\xi_0+0)-\underline{U}(\xi_0-0)]\Big\}.
\end{aligned}
$$

For $\xi>\xi_0$, we get

$$
\begin{aligned}
&Q_1(\underline{U},\overline{V})(\xi)\\
\geq&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left(\int_{-\infty}^{\xi_0}\mathrm{e}^{\lambda_{11}(\xi-s)}+\int_{\xi_0}^{\xi}\mathrm{e}^{\lambda_{11}(\xi-s)}+\int_{\xi}^{\infty}\mathrm{e}^{\lambda_{12}(\xi-s)}\right)\\
&\times\left[-d_1\underline{U}''(s)-c\underline{U}'(s)+\rho_1\underline{U}(s)\right]\mathrm{d}s\\
=&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\Big\{d_1(\lambda_{12}-\lambda_{11})\underline{U}(\xi)+d_1\mathrm{e}^{\lambda_{11}(\xi-\xi_0)}[\underline{U}'(\xi_0+0)-\underline{U}'(\xi_0-0)]\\
&\quad+(d_1\lambda_{11}+c)\mathrm{e}^{\lambda_{11}(\xi-\xi_0)}[\underline{U}(\xi_0+0)-\underline{U}(\xi_0-0)]\Big\}.
\end{aligned}
$$

Note that $\underline{U}(\xi_0+0)=\underline{U}(\xi_0-0)$ and $\underline{U}'(\xi_0+0)\geq\underline{U}'(\xi_0-0)$. Hence, according to the aforementioned inequalities in (2.17), we have $Q_1(\underline{U},\overline{V})(\xi)\geq\underline{U}(\xi)$ for $\xi\neq\xi_0$. By the continuity, $Q_1(\underline{U},\overline{V})(\xi)\geq\underline{U}(\xi)$ for all ξ.

Similarly, let $\xi_1=\frac{1}{\alpha}\ln\frac{\beta_1 u_*}{r_1(\infty)-u_*}$. If $\xi<\xi_1$, by using (2.7), we also obtain

$$
\begin{aligned}
&Q_1(\overline{U},\underline{V})(\xi)\\
=&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left[\int_{-\infty}^{\xi}\mathrm{e}^{\lambda_{11}(\xi-s)}F_1(\overline{U},\underline{V})(s)\mathrm{d}s+\int_{\xi}^{\infty}\mathrm{e}^{\lambda_{12}(\xi-s)}F_1(\overline{U},\underline{V})(s)\mathrm{d}s\right]\\
\leq&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left(\int_{-\infty}^{\xi}\mathrm{e}^{\lambda_{11}(\xi-s)}+\int_{\xi}^{\xi_1}\mathrm{e}^{\lambda_{12}(\xi-s)}+\int_{\xi_1}^{\infty}\mathrm{e}^{\lambda_{12}(\xi-s)}\right)\\
&\times\left[-d_1\overline{U}''(s)-c\overline{U}'(s)+\rho_1\overline{U}(s)\right]\mathrm{d}s\\
=&\frac{1}{d_1(\lambda_{12}-\lambda_{11})}\Big\{d_1(\lambda_{12}-\lambda_{11})\overline{U}(\xi)+d_1\mathrm{e}^{\lambda_{12}(\xi-\xi_1)}[\overline{U}'(\xi_1+0)-\overline{U}'(\xi_1-0)]\\
&\quad+(d_1\lambda_{12}+c)\mathrm{e}^{\lambda_{12}(\xi-\xi_1)}[\underline{U}(\xi_1+0)-\underline{U}(\xi_1-0)]\Big\}.
\end{aligned}
$$

For $\xi > \xi_1$, we get

$$
\begin{aligned}
& Q_1(\overline{U}, \underline{V})(\xi) \\
\leq & \frac{1}{d_1(\lambda_{12}-\lambda_{11})}\left(\int_{-\infty}^{\xi_1} e^{\lambda_{11}(\xi-s)} + \int_{\xi_1}^{\xi} e^{\lambda_{11}(\xi-s)} + \int_{\xi}^{\infty} e^{\lambda_{12}(\xi-s)}\right) \\
& \times\left[-d_1\overline{U}''(s) - c\overline{U}'(s) + \rho_1\overline{U}(s)\right] \mathrm{d}s \\
= & \frac{1}{d_1(\lambda_{12}-\lambda_{11})}\Big\{d_1(\lambda_{12}-\lambda_{11})\overline{U}(\xi) + d_1 e^{\lambda_{11}(\xi-\xi_1)}[\overline{U}'(\xi_1+0) - \overline{U}'(\xi_1-0)] \\
& \quad + (d_1\lambda_{11}+c)e^{\lambda_{11}(\xi-\xi_1)}[\overline{U}(\xi_1+0) - \overline{U}(\xi_1-0)]\Big\}.
\end{aligned}
$$

Note that $\overline{U}(\xi_1+0) = \overline{U}(\xi_1-0)$ and $\overline{U}'(\xi_1+0) \leq \overline{U}'(\xi_1-0)$. Hence, based on (2.17), it can be derived that $Q_1(\overline{U}, \underline{V})(\xi) \leq \overline{U}(\xi)$ for $\xi \neq \xi_1$. Thanks to the continuity, $Q_1(\overline{U}, \underline{V})(\xi) \leq \overline{U}(\xi)$ for all ξ.

Then, combined with (2.17), it can be obtained that

$$
\underline{U}(\xi) \leq Q_1(U, V)(\xi) \leq \overline{U}(\xi), \tag{2.18}
$$

Similarly, we can obtain the following inequality

$$
\underline{V}(\xi) \leq Q_2(U, V)(\xi) \leq \overline{V}(\xi). \tag{2.19}
$$

Therefore, it can be proved that Q is mapped from Γ into Γ. □

Theorem 2.1. *Assume (A1)-(A4) hold. Then,* (1.3) *will generate a forced wave* $(u(t,x), v(t,x)) = (U(\xi), V(\xi))$ *connecting* $(0,0)$ *and* (u_*, v_*).

Proof. Define the following cross-iteration:

$$
\begin{aligned}
U_1 &= Q_1(\underline{U}, \overline{V}), & V_1 &= Q_2(\underline{U}, \overline{V}), \\
U_{k+1} &= Q_1(U_k, V_k), & V_{k+1} &= Q_2(U_k, V_k), \quad k \geq 1.
\end{aligned}
$$

In terms of (2.17), (2.18) and (2.19), we see that

$$
\underline{U} \leq U_1 = Q_1(\underline{U}, \overline{V}) \leq \overline{U}, \quad \underline{V} \leq V_1 = Q_2(\underline{U}, \overline{V}) \leq \overline{V}.
$$

Then $(U_1, V_1) \in \Gamma$ and by Lemma 2.3,

$$
\begin{aligned}
\overline{U} \geq U_2 = Q_1(U_1, V_1) \geq Q_1(\underline{U}, \overline{V}) = U_1 \geq \underline{U}, \\
\underline{V} \leq V_2 = Q_2(U_1, V_1) \leq Q_2(\underline{U}, \overline{V}) = V_1 \leq \overline{V}.
\end{aligned}
$$

Assume $(U_k, V_k) \in \Gamma$ with $U_k \geq U_{k-1}$ and $V_k \leq V_{k-1}$. Then it follows that

$$
\begin{aligned}
\overline{U} \geq Q_1(U_k, V_k) \geq Q_1(U_{k-1}, V_{k-1}) = U_k \geq \underline{U}, \\
\underline{V} \leq Q_2(U_k, V_k) \leq Q_2(U_{k-1}, V_{k-1}) = V_k \leq \overline{V}.
\end{aligned}
$$

Hence, the induction implies $(U_{k+1} = Q_1(U_k, V_k), V_{k+1} = Q_2(U_k, V_k)) \in \Gamma$ and U_k is increasing in k while V_k is decreasing in k. Then there exist U and V such that $U_k \to U$ and $V_k \to V$ pointwisely as $k \to \infty$.

Obviously, it can be shown that (U, V) is a fixed-point of $Q = (Q_1, Q_2)$. It is easily known that $F_i(U_k, V_k)$ converges to $F_i(U, V)$ pointwise when F_i is continuous. Since $F_i(U_k, V_k)$ is uniform and bounded, by the Lebesgue's dominated convergence theorem it follows,

$$\begin{aligned}
U(\xi) &= \lim_{k\to\infty} U_{k+1}(\xi) \\
&= \lim_{k\to\infty} Q_1(U_k, V_k)(\xi) \\
&= \lim_{k\to\infty} \frac{1}{d_1(\lambda_{12}-\lambda_{11})} \int_{-\infty}^{\infty} \chi_1(\xi - s) F_1(U_k, V_k)(s) \mathrm{d}s \\
&= \frac{1}{d_1(\lambda_{12}-\lambda_{11})} \int_{-\infty}^{\infty} \chi_1(\xi - s) F_1(U, V)(s) \mathrm{d}s = Q_1(U, V)(\xi).
\end{aligned}$$

and

$$\begin{aligned}
V(\xi) &= \lim_{k\to\infty} V_{k+1}(\xi) \\
&= \lim_{k\to\infty} Q_2(U_k, V_k)(\xi) \\
&= \lim_{k\to\infty} \frac{1}{d_2(\lambda_{22}-\lambda_{21})} \int_{-\infty}^{\infty} \chi_2(\xi - s) F_2(U_k, V_k)(s) \mathrm{d}s \\
&= \frac{1}{d_2(\lambda_{22}-\lambda_{21})} \int_{-\infty}^{\infty} \chi_2(\xi - s) F_2(U, V)(s) \mathrm{d}s = Q_2(U, V)(\xi).
\end{aligned}$$

Henceforth, $(U, V) \in \Gamma$ satisfies (2.1). Next, it will be easily proved that (U, V) follows the boundary conditions (2.2). Since $(\underline{U}(\xi), \underline{V}(\xi)) \le (U(\xi), V(\xi)) \le (\overline{U}(\xi), \overline{V}(\xi))$ and $\lim_{\xi\to\infty}(\underline{U}(\xi), \underline{V}(\xi)) = (u_*, v_*) = \lim_{\xi\to\infty}(\overline{U}(\xi), \overline{V}(\xi))$, therefore, $\lim_{\xi\to\infty}(U(\xi), V(\xi)) = (u_*, v_*)$.

Let $W(\xi)$ be a forced wave of the following equation

$$w_t(t, x) = d_i w_{xx}(t, x) + w(t, x)[r_i(x - ct) - w(t, x)].$$

Then, by Theorem 1.1 in [11], we have the limit $\lim_{\xi\to-\infty} W(\xi) = 0$. Since $W(\xi)$ is an upper solution of the following two equations

$$\begin{aligned}
&d_1 U''(\xi) + cU'(\xi) + U(\xi)[r_1(\xi) - U(\xi)] \\
&\quad - a_1 U(\xi) \int_0^{\infty} \int_{\mathbb{R}} J_1(s, y) V(\xi - y + cs) \mathrm{d}y \mathrm{d}s = 0,
\end{aligned}$$

and

$$\begin{aligned}
&d_2 V''(\xi) + cV'(\xi) + V(\xi)[r_2(\xi) - V(\xi)] \\
&\quad - a_2 V(\xi) \int_0^{\infty} \int_{\mathbb{R}} J_2(s, y) U(\xi - y + cs) \mathrm{d}y \mathrm{d}s = 0,
\end{aligned}$$

the comparison principle implies that

$$\begin{aligned}
&\limsup_{\xi\to-\infty} U(\xi) \le \lim_{\xi\to-\infty} W(\xi) = 0, \\
&\limsup_{\xi\to-\infty} V(\xi) \le \lim_{\xi\to-\infty} W(\xi) = 0.
\end{aligned} \tag{2.20}$$

It can be also noted that

$$\liminf_{\xi\to-\infty} U(\xi) \geq \lim_{\xi\to-\infty} \underline{U}(\xi) = 0,$$
$$\liminf_{\xi\to-\infty} V(\xi) \geq \lim_{\xi\to-\infty} \underline{V}(\xi) = 0. \tag{2.21}$$

Then, (2.20) and (2.21) result in $\lim_{\xi\to-\infty} U(\xi) = 0$ and $\lim_{\xi\to-\infty} V(\xi) = 0$. Therefore, (U, V) admits the asymptotic boundary conditions. □

3. Asymptotic behaviors of forced waves

To understand the rate of convergence in two tails, we now study the asymptotic behaviors of forced waves. Applying some delicate analyses, we could obtain the exactly exponential asymptotic decay of the forced wave with nonzero forced speed.

Theorem 3.1. *Let (A1)-(A4) be true. Then, there exist four positive constants A_1, A_2, B_1 and B_2 such that the forced wave $(U(\xi), V(\xi))$ generated by* (1.3) *has the asymptotic properties as follows*

$$\begin{pmatrix} U(\xi) \\ V(\xi) \end{pmatrix} = \begin{pmatrix} (A_1 + o(1))\mathrm{e}^{\frac{-c+\sqrt{c^2-4d_1r_1(-\infty)}}{2d_1}\xi} \\ (A_2 + o(1))\mathrm{e}^{\frac{-c+\sqrt{c^2-4d_2r_2(-\infty)}}{2d_2}\xi} \end{pmatrix}$$

as $\xi \to -\infty$; and

$$\begin{pmatrix} U(\xi) \\ V(\xi) \end{pmatrix} = \begin{pmatrix} u_* - (B_1 + o(1))\mathrm{e}^{-\sigma_1\xi} \\ v^* - (B_2 + o(1))\mathrm{e}^{-\sigma_1\xi} \end{pmatrix},$$

as $\xi \to \infty$. Here $\sigma_1 \in (0, \min\{\sigma_1^+, \sigma_2^+\})$, $\sigma_1^+ = \frac{c+\sqrt{c^2+4d_1u_*}}{2d_1}$, $\sigma_2^+ = \frac{c+\sqrt{c^2+4d_2v_*}}{2d_2}$.

Proof. Linearizing the system (1.3) around the equilibrium $(0, 0)$, we then get

$$d_1\phi''(\xi) + c\phi'(\xi) + r_1(\xi)\phi(\xi) = 0,$$
$$d_2\psi''(\xi) + c\psi'(\xi) + r_2(\xi)\psi(\xi) = 0. \tag{3.1}$$

The limit form of (3.1) for $\xi \to -\infty$ is written as

$$d_1\phi''(\xi) + c\phi'(\xi) + r_1(-\infty)\phi(\xi) = 0,$$
$$d_2\psi''(\xi) + c\psi'(\xi) + r_2(-\infty)\psi(\xi) = 0. \tag{3.2}$$

Note that the positive solution of (3.2) requires to satisfy $\lim_{\xi\to-\infty}(\phi(\xi), \psi(\xi)) = (0, 0)$. Henceforth, we can obtain one positive solution of (3.2) in the following form

$$\phi(\xi) = A_1\mathrm{e}^{\frac{-c+\sqrt{c^2-4d_1r_1(-\infty)}}{2d_1}\xi}, \quad \psi(\xi) = A_2\mathrm{e}^{\frac{-c+\sqrt{c^2-4d_2r_2(-\infty)}}{2d_1}\xi}, \tag{3.3}$$

for some positive constants A_1, A_2. Then, we can prove that (3.3) describes the dominating behaviors of (2.1). Relating (3.1) and (2.1), we have another equivalent form of (2.1)

$$d_1U''(\xi) + cU'(\xi) + r_1(-\infty)U(\xi) + G_1(U, V)(\xi) = 0,$$
$$d_2V''(\xi) + cV'(\xi) + r_2(-\infty)V(\xi) + G_2(U, V)(\xi) = 0, \tag{3.4}$$

where

$$\begin{aligned} G_1(U,V)(\xi) &= U(\xi)[r_1(\xi) - r_1(-\infty) - U(\xi)] \\ &\quad - a_1 U(\xi)\int_0^\infty \int_{\mathbb{R}} J_1(s,y)V(\xi - y + cs)\mathrm{d}y\mathrm{d}s, \\ G_2(U,V)(\xi) &= V(\xi)[r_2(\xi) - r_2(-\infty) - V(\xi)] \\ &\quad - a_2 V(\xi)\int_0^\infty \int_{\mathbb{R}} J_2(s,y)U(\xi - y + cs)\mathrm{d}y\mathrm{d}s. \end{aligned}$$

Among them, $G(U,V) = (G_1(U,V), G_2(U,V))$ becomes the smaller term, which is higher order than the linear term in (3.4). Similar to Theorem 2 in [16] and Theorem 3.1 in [29], it can be proved that the system of equations (3.4) gives the asymptotic behaviors of (2.1) as $\xi \to -\infty$.

Linearizing the system (1.3) at the equilibrium (u_*, v_*) and setting $\xi \to \infty$, then

$$\begin{aligned} &d_1\bar{\phi}''(\xi) + c\bar{\phi}'(\xi) - u_*\bar{\phi}(\xi) - a_1 u_* \int_0^\infty \int_{\mathbb{R}} J_1(s,y)\bar{\psi}(\xi - y + cs)\mathrm{d}y\mathrm{d}s = 0, \\ &d_2\bar{\psi}''(\xi) + c\bar{\psi}'(\xi) - v_*\bar{\psi}(\xi) - a_2 v_* \int_0^\infty \int_{\mathbb{R}} J_1(s,y)\bar{\phi}(\xi - y + cs)\mathrm{d}y\mathrm{d}s = 0. \end{aligned} \tag{3.5}$$

The characteristic equation of (3.5) is

$$\begin{aligned} &P(\sigma) := (d_1\sigma^2 - c\sigma - u_*)(d_2\sigma^2 - c\sigma - v_*) - N_1(\sigma)N_2(\sigma), \\ &N_i(\sigma) := \int_0^\infty \int_{\mathbb{R}} J_i(s,y)\mathrm{e}^{\sigma(y-cs)}\mathrm{d}y\mathrm{d}s, \quad i = 1,2. \end{aligned} \tag{3.6}$$

We next find the positive roots of (3.6). Note that $d_1\sigma^2 - c\sigma - u_* = 0$ has one positive root

$$\sigma_1^+ = \frac{c + \sqrt{c^2 + 4d_1 u_*}}{2d_1}.$$

Also, $d_2\sigma^2 - c\sigma - v_* = 0$ has one positive root

$$\sigma_2^+ = \frac{c + \sqrt{c^2 + 4d_2 v_*}}{2d_2}.$$

Then, $P(\sigma)$ admits at least one positive root. In fact,

$$P(0) = (1 - a_1 a_2)u_* v_* > 0, \;\; P(\sigma_1^+) = P(\sigma_2^+) < 0,$$

so $P(\sigma)$ has at least one real root σ_1 in the interval $(0, \sigma^+)$, where $\sigma^+ = \min\{\sigma_1^+, \sigma_2^+\}$. Thus, we can get one positive solution of (3.5) in the following form

$$\bar{\phi}(\xi) = B_1\mathrm{e}^{-\sigma_1\xi}, \quad \bar{\psi}(\xi) = B_2\mathrm{e}^{-\sigma_1\xi}.$$

Furthermore, the linear equations (3.5) give the asymptotic behaviors of (2.1) as $\xi \to \infty$.

$\square$

4. Numerical validations

In order to further understand the above-mentioned analytical results, we will provide three numerical examples by choosing different kernel functions J_i in this section. To ensure the theoretical results can be realized numerically, we will pick up some special functions, such as Delta function, Gaussion function, exponential function and Sine-Cosine function. Delta function $\delta(\cdot)$ means it can take ∞ at some certain point, while other values near this point are 0. Therefore, it can be seen as a very narrow pulse signal and used in describing the competitive effect depending solely on the density of individuals at the current position or time. Mathematically, it can reduce the double integrals into single integral. Gaussion function $\mathrm{e}^{-\frac{y^2}{4\rho}}/\sqrt{4\pi\rho}$ is the density function of normal distribution, which is a weight function and used to measure the competition at location y of speices. Exponential function $\mathrm{e}^{-\frac{s}{\tau}}/\tau$ decays monotonically as time goes by, and Sine-Cosine function $\sin(\frac{s}{\tau})+\cos(\frac{s}{\tau})$ is vibrating periodically over time, which are helpful to understand certain interspecific competition of species at previous time $s>0$.

Example 1. Let $J_i(s,y)=\delta(s)\frac{1}{\sqrt{4\pi\rho}}\mathrm{e}^{-\frac{y^2}{4\rho}}$, where $\rho>0$ and $\delta(s)$ is the delta function. From the point of biological meanings, we change the nonlocal structure of time variable into local situation by choose the Delta function $\delta(s)$, while the space variable is still globally dependent. The spatial structure is represented by the Gaussion type kernel function, which means the nearby reaction is more important than distant reaction toward the original position. In this case, the competitive term which originally has double-nonlocal property with respect to time and space reduces to one-nonlocal property with respect to space variable.

In this example, the system (2.1) can be reduced t

$$\begin{aligned} &d_1U''(\xi)+cU'(\xi)+U(\xi)\left[r_1(\xi)-U(\xi)-a_1\int_{\mathbb{R}}\frac{1}{\sqrt{4\pi\rho}}\mathrm{e}^{-\frac{y^2}{4\rho}}V(\xi-y)\mathrm{d}y\right]=0,\\ &d_2V''(\xi)+cV'(\xi)+V(\xi)\left[r_2(\xi)-V(\xi)-a_2\int_{\mathbb{R}}\frac{1}{\sqrt{4\pi\rho}}\mathrm{e}^{-\frac{y^2}{4\rho}}U(\xi-y)\mathrm{d}y\right]=0. \end{aligned} \tag{4.1}$$

Truncate $\mathbb{R}=(-\infty,\infty)$ by $[-\kappa,\kappa]$ for some large κ and adopt the uniform partition of $[-\kappa,\kappa]$ as

$$-\kappa=\xi_1<\xi_2<\cdots<\xi_{2n-1}<\xi_{2n}<\xi_{2n+1}=\kappa,$$

where $\xi_i=\xi_1+(i-1)h$, $h=\frac{\kappa}{n}$, $i=1,2,\cdots,2n+1$. Corresponding to the truncation, the asymptotic boundary conditions then become

$$(U(\xi_1),V(\xi_1))=(0,0),\ (U(\xi_{2n+1}),V(\xi_{2n+1}))=(u_*,v_*).$$

Let $[\boldsymbol{W}_1^T,\boldsymbol{W}_2^T]=[[W(1),\cdots,W(2n+1)],[W(2n+2),\cdots,W(4n+2)]]\in\mathbb{R}^{4n+2}$ be defined by

$$W(i)=\begin{cases}U(\xi_i), & \text{for } 1\le i\le 2n+1,\\ V(\xi_{i-2n-1}), & \text{for } 2n+2\le i\le 4n+2.\end{cases}$$

Then we have the following algebraic system in the form of matrix by discretizing the system (4.1):

$$\begin{bmatrix}\boldsymbol{M}_1 & 0\\ 0 & \boldsymbol{M}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\end{bmatrix}+\left(\begin{bmatrix}\boldsymbol{N}_1 & 0\\ 0 & \boldsymbol{N}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{W}_2\\ \boldsymbol{W}_1\end{bmatrix}\right)\circ\begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\end{bmatrix}+\begin{bmatrix}\boldsymbol{L}_1 & 0\\ 0 & \boldsymbol{L}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{W}_1\circ\boldsymbol{W}_1\\ \boldsymbol{W}_2\circ\boldsymbol{W}_2\end{bmatrix}+\begin{bmatrix}\boldsymbol{C}_1\\ \boldsymbol{C}_2\end{bmatrix}=\boldsymbol{0}, \tag{4.2}$$

where $A \circ B$ represents the Hadamard products of matrix A and B. Here

$$\boldsymbol{M}_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -ch + d_1 & ch - 2d_1 + h^2(r_1(\xi_2) - \frac{a_1 v_*}{2}\tilde{J}(\xi_2)) & d_1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -ch + d_1 & ch - 2d_1 + h^2(r_1(\xi_{2n}) - \frac{a_1 v_*}{2}\tilde{J}(\xi_{2n})) & d_1 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

with $\tilde{J}(\xi_i) = 1 - \int_{\xi_i - \xi_{2n+1}}^{\xi_{2n+1}-\xi_i} \tilde{G}(y)\mathrm{d}y$ and $\tilde{G}(y) = \frac{1}{\sqrt{4\pi\rho}}\mathrm{e}^{-\frac{y^2}{4\rho}}$,

$$\boldsymbol{M}_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -ch + d_2 & ch - 2d_2 + h^2(r_2(\xi_2) - \frac{a_2 u_*}{2}\tilde{J}(\xi_2)) & d_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -ch + d_2 & ch - 2d_2 + h^2(r_2(\xi_{2n}) - \frac{a_2 u_*}{2}\tilde{J}(\xi_{2n})) & d_2 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{N}_1 = -\frac{h^3 a_1}{3}\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ \tilde{G}(\xi_2 - \xi_1) & 4\tilde{G}(\xi_2 - \xi_2) & 2\tilde{G}(\xi_2 - \xi_3) & \cdots & 4\tilde{G}(\xi_2 - \xi_{2n}) & \tilde{G}(\xi_2 - \xi_{2n+1}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{G}(\xi_{2n} - \xi_1) & 4\tilde{G}(\xi_{2n} - \xi_2) & 2\tilde{G}(\xi_{2n} - \xi_3) & \cdots & 4\tilde{G}(\xi_{2n} - \xi_{2n}) & \tilde{G}(\xi_{2n} - \xi_{2n+1}) \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{N}_2 = \frac{a_2}{a_1}\boldsymbol{N}_1, \quad \boldsymbol{L}_1 = \boldsymbol{L}_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & -h^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -h^2 & 0 \\ 0 & \cdots & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{C}_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -u_* \end{bmatrix} \quad \text{and} \quad \boldsymbol{C}_2 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -v_* \end{bmatrix}.$$

Let $r_1(x)$ and $r_2(x)$ be $\frac{2}{1+\mathrm{e}^{-x}} - 1.2$ and $\frac{1.2}{1+\mathrm{e}^{-x}} - 0.2$, respectively. Meanwhile, other parameters are set to be $a_1 = 0.4, a_2 = 0.5, d_1 = d_2 = \rho = 1, c = 2$. Then, let $(u_*, v_*) = (0.5, 0.75)$ and (A1)-(A4) hold. By solving the solutions of (4.2) with Matlab, we can obtain the numerical solution of (4.1). In Figure 1, we see that the monotonic forced wave front connecting $(0, 0)$ and (u_*, v_*) in Example 1 propagates to the right at the rate $c = 2$. So as to be more readable, in Figure 2, we present all possible evolutions of the forced wave front of u-spices and v-spices, respectively.

Example 2. Let $J_i(s, y) = \frac{1}{\tau}\mathrm{e}^{-\frac{s}{\tau}}\delta(y)$, where $\delta(y)$ is the delta function with y and $\tau > 0$. From the point of biological meanings, we change the nonlocal structure of space variable into local situation by choose the Delta function $\delta(y)$, while the time variable is still globally dependent. The time structure is represented by the exponential decay function, which means the effect of short time is more important than the effect of longer time. In this case, the competitive term which originally has double-nonlocal property with respect to time and space reduces to one-nonlocal property with respect to time variable.

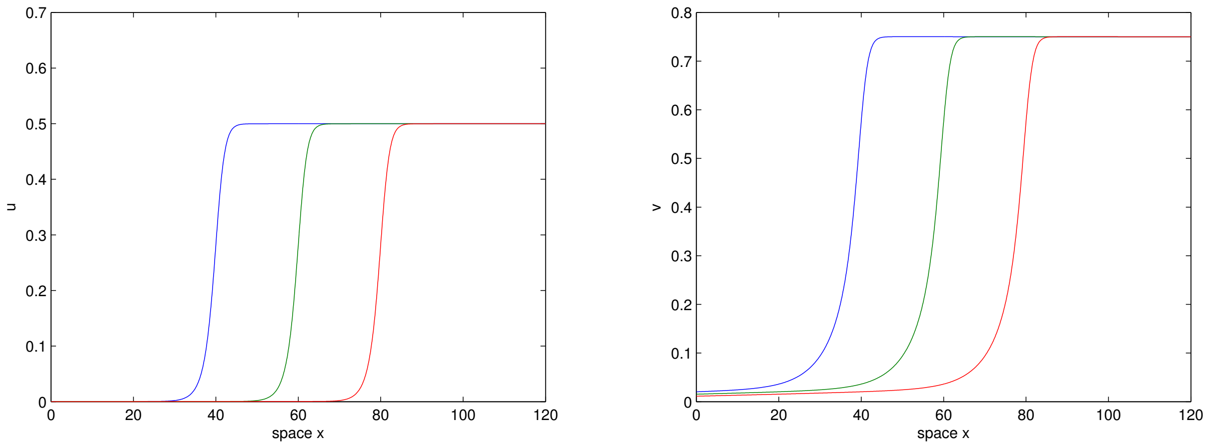

Figure 1. For the system in Example 1, the forced wave front is plotted for $t = 20, 30, 40$ (corresponding to blue line, green line and red line). The left panel shows the wave front of u-species every 10 time steps, and the right panel presents the wave front of v-species.

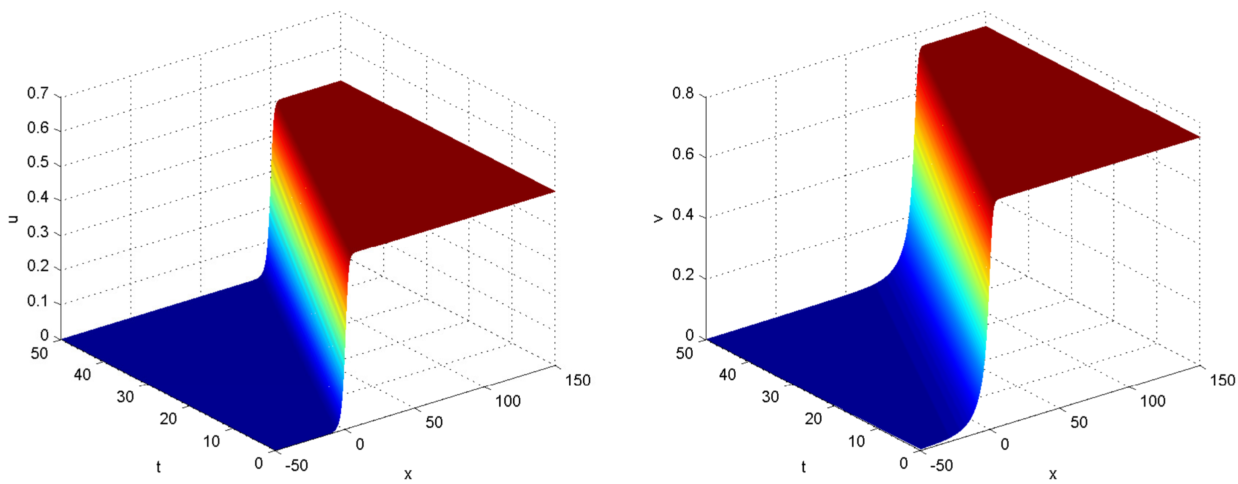

Figure 2. For the system in Example 1, the evolutions of the forced wave front are fully plotted with regard to the possible time and position in 3-D form. The left panel shows the wave front of u-species, and the right panel shows the wave front of v-species.

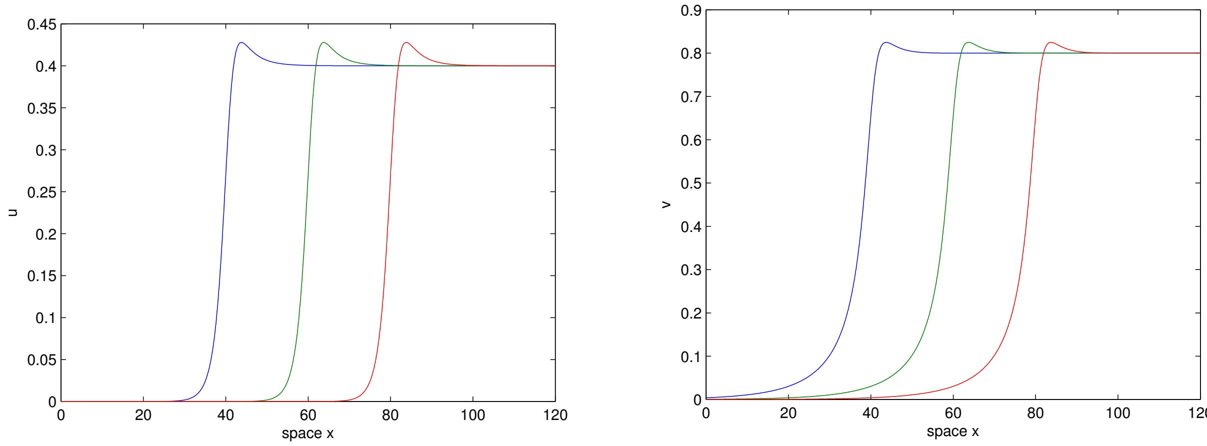

Figure 3. For the system in Example 2, the forced wave front is plotted for $t = 20, 30, 40$ (corresponding to blue line, green line and red line). The left panel shows the wave front of u-species every 10 time steps, and the right panel presents the wave front of v-species.

In this example, the system (2.1) reduces to

$$\begin{aligned} &d_1 U''(\xi) + cU'(\xi) + U(\xi)\left[r_1(\xi) - U(\xi) - a_1 \int_0^{\infty} \frac{1}{\tau} \mathrm{e}^{-\frac{s}{\tau}} V(\xi - cs) \mathrm{d}s\right] = 0, \\ &d_2 V''(\xi) + cV'(\xi) + V(\xi)\left[r_2(\xi) - V(\xi) - a_2 \int_0^{\infty} \frac{1}{\tau} \mathrm{e}^{-\frac{s}{\tau}} U(\xi - cs) \mathrm{d}s\right] = 0. \end{aligned}$$

The procedure of numerical simulation in this example is similar as that in Example 1, so we omit the details. Let $r_1(x), r_2(x)$ be $\frac{2}{1+\mathrm{e}^{-x}} - 1.2$, $\frac{1.2}{1+\mathrm{e}^{-x}} - 0.2$, and other parameters are set to be $a_1 = a_2 = 0.5, d_1 = d_2 = 1, \tau = 2, c = 2$. Then, $(u_*, v_*) = (0.4, 0.8)$ and (A1)-(A4) hold. In Figure 3, we can observe that the non-monotonic forced wave connecting $(0, 0)$ and (u_*, v_*) in Example 2 propagates to the right at the rate $c = 2$. Likewise, Figure 4 presents all the possible evolutions of the forced wave front of u-spices and v-spices, respectively.

Example 3. Let $J_i(s, y) = \frac{1}{\tau}[\sin(\frac{s}{\tau}) + \cos(\frac{s}{\tau})]\mathrm{e}^{-\frac{s}{\tau}}\delta(y)$, where $\tau > 0$ and $\delta(y)$ is the delta function with the variable y. From the point of biological meanings, we change the nonlocal structure of space variable into local situation by choose the Delta function $\delta(y)$, while the time variable is still globally dependent. The time structure is represented by the Sine-Cosine function combined with exponential decay function, which means the effect decays fast as time goes on. But the Sine-Cosine function can induce the non-monotone phenomenon. In this case, the competitive term which originally has double-nonlocal property with respect to time and space reduces to one-nonlocal property with respect to time variable. There appear some complicated dynamics.

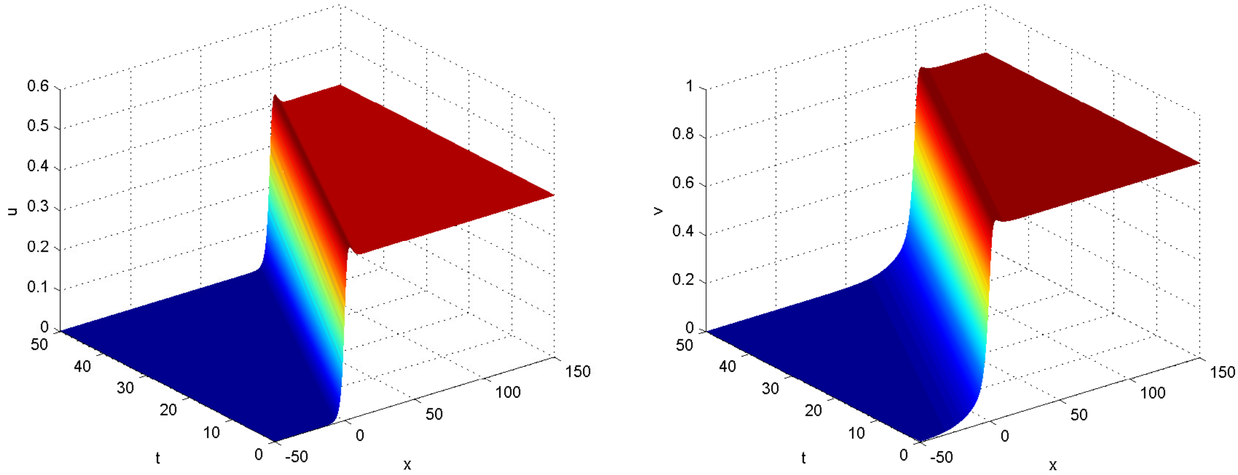

Figure 4. For the system in Example 2, the evolutions of the forced wave front are fully plotted with regard to the possible time and position in 3-D form. The left panel shows the wave front of u-species, and the right panel shows the wave front of v-species.

In this example, the system (2.1) is simplified as

$$\begin{aligned}
&d_1U''(\xi) + cU'(\xi) + U(\xi)\left[r_1(\xi) - U(\xi)\right] \\
&\quad - a_1U(\xi)\int_0^{\infty} \frac{1}{\tau}\left[\sin\left(\frac{s}{\tau}\right) + \cos\left(\frac{s}{\tau}\right)\right] e^{-\frac{s}{\tau}} V(\xi - cs)\mathrm{d}s = 0, \\
&d_2V''(\xi) + cV'(\xi) + V(\xi)\left[r_2(\xi) - V(\xi)\right] \\
&\quad - a_2V(\xi)\int_0^{\infty} \frac{1}{\tau}\left[\sin\left(\frac{s}{\tau}\right) + \cos\left(\frac{s}{\tau}\right)\right] e^{-\frac{s}{\tau}} U(\xi - cs)\mathrm{d}s = 0.
\end{aligned}$$

The procedure of numerical simulation in this example is similar as that in Example 1, so we omit the details. Similarly, let $r_1(x), r_2(x)$ be $\frac{2}{1+e^{-x}} - 1.2$, $\frac{1.2}{1+e^{-x}} - 0.2$, respectively. Meanwhile, we assume that $a_1 = a_2 = 0.5, d_1 = d_2 = 1, \tau = 3, c = 2$. Then, we set (u_*, v_*) to be $(0.4, 0.8)$ and (A1)-(A4) hold. In Figure 5, it can be observed that the non-monotonic forced wave connecting $(0, 0)$ and (u_*, v_*) in Example 3 moves to the right at the rate $c = 2$. As a further step, Figure 6 present all possible evolutions of the forced wave front of u-spices and v-spices, respectively.

5. Conclusions and outlooks

In summary, we propose an improved Lotka-Volterra competition model with the spatio-temporal nonlocal effect under climate change. By use of the cross-iteration techniques, we investigate the existence and asymptotical behaviors of the proposed model in detail, and then demonstrate the existence of forced waves generated by the model, where the solutions to the model need to fulfil the appropriate bounds. Meanwhile, it is found that the asymptotic behaviors of forced waves are dominated by the leading equations, and the numerical examples are also provided to validate the analytical predictions.

Although we do not prove the monotonicity of forced waves, by picking out three different kernels and interspecific competition coefficients a_1 and a_2, we find the forced waves could be monotonic or

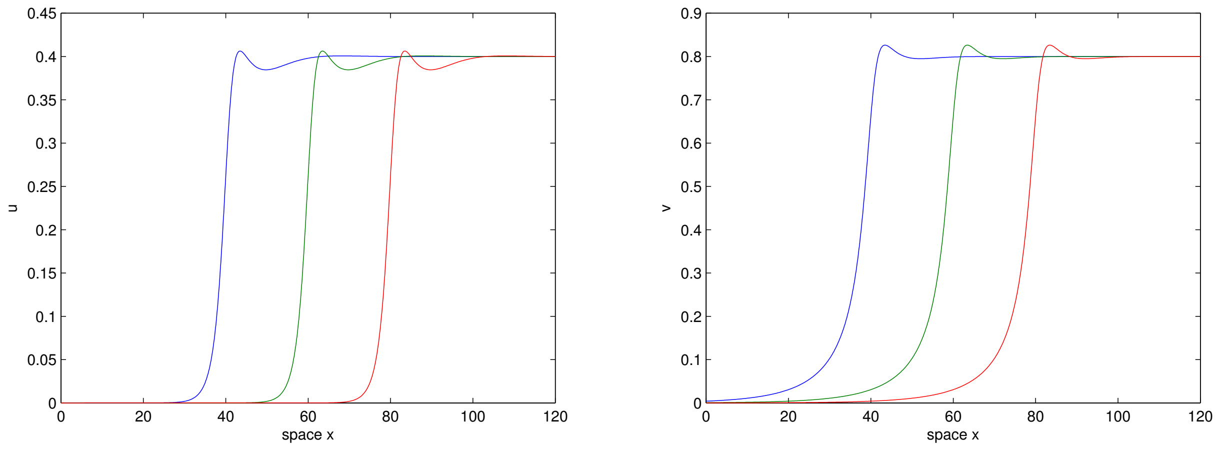

Figure 5. For the system in Example 3, the forced wave front is plotted for $t = 20, 30, 40$ (corresponding to blue line, green line and red line). The left panel shows the wave front of u-species every 10 time steps, and the right panel presents the wave front of v-species.

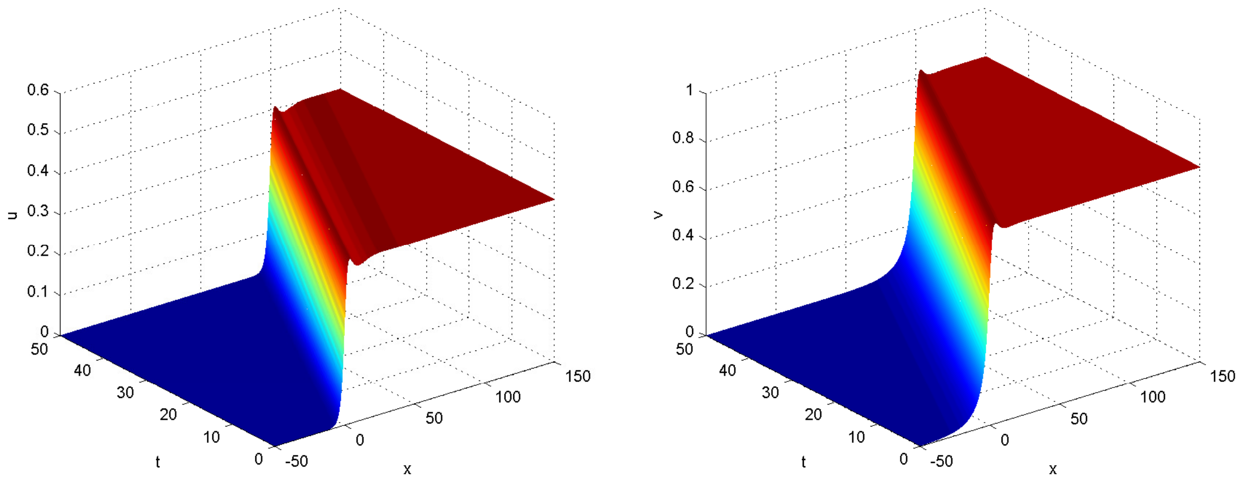

Figure 6. For the system in Example 3, the evolutions of the forced wave front are fully plotted with regard to the possible time and position in 3-D form. The left panel shows the wave front of u-species, and the right panel shows the wave front of v-species.

non-monotonic. Monotone propagation for species is particular and it can be estimated and controlled. Due to the spread of species potentially being affected by a series of factors, many species will appear to be in non-monotonic propagation mode. In this situation, human intervention would be difficult without knowing at what time the species will peak. The uniqueness and monotonicity of forced waves within some parameter ranges will be investigated in the future work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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