A Stochastic Fluid Model Approach to the Stationary Distribution of the Maximal Priority Process

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ABSTRACT

In traditional priority queues, we assume that every customer upon arrival has a fixed, class-dependent priority, and that a customer may not commence service if a customer with a higher priority is present in the queue. However, in situations where a performance target in terms of the tails of the class-dependent waiting time distributions has to be met, such models of priority queueing may not be satisfactory. In fact, there could be situations where high priority classes easily meet their performance target for the maximum waiting time, while lower classes do not.

Kleinrock introduced a time-dependent priority queue in [5], and derived results for a delay dependent priority system in which a customer's priority is increasing, from zero, linearly with time in proportion to a rate assigned to the customer's priority class. The advantage of such priority structure is that it provides a number of degrees of freedom with which to manipulate the relative waiting times for each customer class. Upon a departure, the customer with highest priority in queue (if any) commences service.

Stanford, Taylor and Ziedins [9] pointed out that the performance of many queues, particularly in the healthcare and human services sectors, is specified in terms of tails of waiting time distributions for customers of different classes. They used this time-dependent priority queue, which they referred to as the accumulating priority queue, and derived its waiting time distributions, rather than just the mean waiting times. They did this via an associated stochastic process, the so-called *maximum priority process*.

Here, we are interested in the stationary distribution at the times of commencement of service of this maximum priority process. Until now, there has been no explicit expression for this distribution. We construct a mapping of the maximum priority process in [9] to a *tandem fluid queue* analysed by O'Reilly and Scheinhardt in [6, 7], which enables us to find expressions for this stationary distribution using techniques derived in [6, 7] and also in [1–3, 8].

Previous work on this topic includes that of Dams [4]. We build on this work, and for the first time derive the results for the stationary distribution of the maximum priority process at the times of the commencement of service.

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