2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC)
October 8-12, 2022, Macau, China

A dynamic stop-skipping model for preventing public transport overcrowding beyond the pandemic-imposed capacity levels

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2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC) | 978-1-6654-6880-0/22/$31.00 ©2022 IEEE | DOI: 10.1109/ITSC55140.2022.9922302

***Abstract*— We consider the dynamic scheduling of skip-stop patterns in public transportation. Operators of fixed-line public transportation services seek to reduce their vehicle running times by skipping stops that will not impact significantly the waiting times of passengers. This can result in an improved trade-off between vehicle running times and passenger waiting times allowing to slightly increase the travel times of passengers in order to reduce the operational costs. Although there exist several decision support models for dynamic stop-skipping, these models do not consider the impact of skipped stops to in-vehicle crowding. That is, apart from the increased passenger waiting times, a skipped stop might result in an increased amount of passengers boarding the next trip of the line resulting in overcrowding. To rectify this, we propose a mixed-integer nonlinear model that incorporates the objective of avoiding in-vehicle overcrowding when making stop-skipping decisions. This is particularly relevant in situations where passengers are no longer able to find a seat or they have to maintain social distancing inside the vehicle because of a pandemic. Because the stop-skipping problem is NP-Hard, we introduce a number of valid inequalities that tighten its solution space and we demonstrate the performance of our model in benchmark problem instances.**

Keywords: Expressing; stop-skipping; COVID-19; social distancing; public transport.

I. Introduction

As public transport operators resume their services, they have to operate under reduced capacities due to COVID-19 [1]. Because demand can exceed capacity at different areas and across different times of the day, drivers have to refuse passenger boardings at specific stops to avoid overcrowding [2]. In practice, the coronavirus pandemic has resulted in the consideration of two vehicle capacities: the nominal (actual) capacity of the vehicle, hereafter referred to as "hard capacity", and the pandemic-imposed capacity of the vehicle that satisfies the recommended social distancing, hereafter referred to as "soft capacity". The soft capacity is lower than the hard one and it can differ among public transport service providers based on the local pandemic regulations. Contrary to the hard capacity, the value of the soft capacity can vary over time subject to the status of the pandemic in a study area.

Given the urgent need to develop decision support tools that can meet the soft capacity of public transport vehicles, this study introduces a dynamic integer nonlinear program to derive the optimal service patterns of skipped/served stops for individual vehicle trips. In addition to the objective of meeting the soft capacity due to COVID-19, the proposed stop-skipping model accounts for the waiting time of passengers at stops. The model decides the skipped and the served stops of a public transport vehicle and it is tested in a bus line connecting the University of Twente with its surrounding cities demonstrating the trade-off between the reduced in-vehicle crowding levels, the trip travel times, and the waiting times of passengers.

The developed mixed-integer nonlinear model (MINLP) has an exponential computational cost and it cannot be solved to global optimality in near real-time for bus lines with regular sizes. To rectify this, the model is solved with the implementation of a branch-and-bound algorithm that terminates its branching and bounding operations within a pre-determined time limit returning an improved feasible solution with a guaranteed optimality gap.

The developed model is implemented in a bus line connecting the University of Twente with its two neighboring cities. Its performance is compared against the "as-is" setting that does not skip any stops. The evaluation results demonstrate that the proposed stop-skipping model can reduce the total in-vehicle passenger load that exceeds the pandemic capacity by 31% and the total trip travel times by 2%. This, however, results in skipping (i.e., not serving) a number of stops (approximately 1 stop per two trips).

II. Literature Review

Public transport is one of the most affected sectors of the pandemic. Many public transport service providers have reported ridership losses of up to 90% compared to the pre-pandemic levels. In addition, social distancing requirements have frequently resulted in the adaptation of public transport schedules, including stop closures at crowded areas, cancelation of services at night times, changes of service frequencies, and timetable modifications [3]–[5]. These modifications to the (pre-pandemic) public transport schedules are predominantly made at the tactical planning stage. For instance, Transport for London announced the closure of specific metro stations at the early stages of the pandemic. This study will also address the problem of not serving specific stops to reduce the in-vehicle crowding. Instead of semi-permanent stop closures, however, this study proposes a dynamic stop-skipping model that decides which stops to skip and which stops to serve for all vehicles of a service line operating within a time horizon.

In past literature, there are mathematical models for deciding about the skipped stops of a service line at the tactical and the operational (dynamic) level. Studies at the tactical

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level typically decide about the skipped stops of a service line before the start of the daily operations and they do not proceed to real-time adjustments of these decisions [6]–[11]. In contrast, dynamic stop-skipping models applied at the operational level can use real-time information regarding the passenger demand and the status of the operations [12].

A comprehensive survey of dynamic stop-skipping models is provided in [13] and [14]. There are two strands of research in dynamic stop-skipping: works that focus on the stop-skipping decisions of a single vehicle that is about to be dispatched, and works that focus on the stop-skipping decisions of a number of future vehicle-trips that operate within a specific time horizon. Studies that decide the skipped stops of a single vehicle result in simpler models that can be solved with brute force for medium-sized service lines with up to 20 stops [15], [16]. This notwithstanding, their decisions are myopic because the effect of a vehicle's skipped stops on its subsequent trips is not considered. Studies that decide about the skipped stops of a set of future trips provide more comprehensive decisions, but, at the same time, result in more computationally complex mathematical models with an increased number of decision variables. This study belongs to the latter category and proposes a comprehensive MINLP model that is solved with branch-and-bound.

The study of [17] was one of the first to propose the development of dynamic stop-skipping strategies for all trips operating in a time horizon (also known as "rolling horizon"). Eberlein et al. [17] modeled the stop-skipping problem as an integer nonlinear program considering the passenger waiting times as an objective function. Given the problem's complexity, [17] simplified the model formulation and proposed an analytic solution that can be applied to the simplified problem. [13] introduced also a dynamic stop-skipping model that decides about the skipped stops of vehicle-trips of a service line in rolling horizons. The model was an integer nonlinear program and it was solved to global optimality for small-scale instances.

Stop-skipping can also be applied in conjunction with other control measures [18], [19]. Li et al. [20] proposed a combined stop-skipping and short-turning model that was formulated as a 0–1 stochastic programming problem. Their formulation was stochastic because they considered operational disruptions. Given the complexity of the problem, [20] used heuristics and tested the solution performance with sample data from the Shanghai Transit Company. Stop-skipping is also combined with vehicle holding at public transport stops [21]–[23] and vehicle scheduling [24].

In the aforementioned studies, the main focus of stop-skipping models is to reduce the trip travel times without increasing significantly the waiting times of passengers that are waiting at skipped stops. This trade-off is the essence of stop-skipping models, which are also known as "expressing models" because they strive to reduce the trips' travel times by serving fewer stops. During the current pandemic, however, stop-skipping can be used for different purposes. Stop-skipping can be applied to reduce the in-vehicle crowding levels instead of only seeking to reduce the in-vehicle travel times. [25] proposed a model in this direction, but this model did not consider the trip travel times in the optimization process and returned feasible solutions only if the number of vehicle-trips was enough to satisfy the soft capacities of vehicles. This study provides a more comprehensive formulation that considers the trade-off between: (i) the passenger waiting times at stops; (ii) the trip travel times; and (iii) the exceedance of the soft capacity in overcrowded line segments. Specific contributions are:

- the expansion of dynamic stop-skipping to incorporate the soft capacities of vehicles as an additional objective.
- the reformulation of the expanded model to a MINLP that can be solved with branch-and-bound.
- the investigation of the potential benefits in a Dutch case study.

III. Problem Formulation

A. Assumptions and Nomenclature

The modeling part of this work extends the dynamic stop-skipping model of [13] to incorporate the soft capacity restrictions due to the COVID-19 pandemic. The formulation of the problem relies on the following assumptions:

- Buses that serve the same line do not (typically) overtake each other [26], [27].
- The passenger arrivals at stops are random at high-frequency services [28].
- An origin-destination pair cannot be skipped by two consecutive bus trips of the same line [15], [16], [29].
- Passengers use the same door channels for boardings and alightings.
- Passenges who are skipped by a vehicle will be accommodated by another service line or they will exit the public transport system.

Our formulation decides about the skipped and served stops of a set of vehicle-trips of a bus line that are planned to operate in the near future (i.e., within the time window of the next hour). Let $N = \{1, ..., n..., |N|\}$ be the set of trips and $S = \{1, .., |S|\}$ be the ordered set of stops of the bus line. In addition to the sets of trips and line stops, there are also the following parameters that provide the necessary input to the dynamic stop-skipping model.

Parameters:

- $t_{n,s}$ is the estimated inter-station travel time of trip n from stop $s-1$ to stop s
- The soft capacity of each vehicle-trip, g_n, $\forall n \in N$
- The hard capacity of each vehicle-trip, $\tilde{g}_n$, $\forall n \in N$
- p_1 and p_2 are the average boarding and alighting times per passenger, respectively
- δ is the average bus acceleration plus deceleration time for serving a bus stop
- λ_{sy} is the average arrival rate at stop s of passengers who travel to destination stop y
- c_1 is the unit time value associated with the passenger waiting time increase

- c_2 is the unit time value associated with the vehicle operation time
- c_3 is the unit time value associated with the exceedance of the soft capacity of a vehicle-trip
- $\tilde{d}_{n,1}$ is the planned departure time of every trip $n \in N$ from the first stop of the line
- $\tilde{w}_{1,sy}$ is the number of passengers waiting for the first trip and are traveling from stop s to $y > s$.

The decision variables of the skip-stop decision problem are the binary variables $x_{n,s}$, where $x_{n,s} = 1$ if stop s is served by vehicle-trip n and 0 otherwise. This results in a combinatorial problem with $2^{|N||S|}$ potential combinations of stop-skipping strategies. The variables of the optimization problem are presented below.

Variables:

- the departure time $d_{n,s}$ of any trip n from stop s
- the arrival times $a_{n,s}$
- the dwell times $k_{n,s}$
- the headway $h_{n,s}$ between trips $n-1$ and n at stop s
- the number of passengers waiting for vehicle-trip n and traveling from stop s to y, $w_{n,sy}$
- the number of passengers boarding trip n at stop s, $u_{n,s}$
- the number of passengers boarding trip n at stop s whose destination is stop y, $b_{n,sy}$
- the number of passengers alighting trip n at stop s, $v_{n,s}$
- the in-vehicle passenger load of vehicle-trip n when departing from stop s, $\gamma_{n,s}$

B. Mathematical Formulation

The number of passengers waiting for vehicle-trip n and traveling from stop s to y, $w_{n,sy}$, is:

$$w_{n,sy} = \lambda_{sy} h_{n,s} \quad \forall n \in N \setminus \{1\}, s \in S \setminus \{|S|\}, y \in S : y > s \tag{1}$$

where $w_{n,sy} = \tilde{w}_{sy}$ when $n = 1$.

The objective function of the stop-skipping decision problem includes three terms that are multiplied by the respective weight factor c_1, c_2 and c_3:

$$f(x) \doteq c_1 \sum_{n=2}^{|N|} \sum_{s=1}^{|S|-1} u_{n,s} \frac{h_{n,s}}{2} + c_2 \sum_{n=2}^{|N|} \sum_{s=2}^{|S|} (t_{n,s} + (k_{n,s} + \delta) x_{n,s}) + c_3 \sum_{n=1}^{|N|} \sum_{s=2}^{|S|} \max(0, \gamma_{n,s} - g_n) \tag{2}$$

The first term computes the total waiting time of passengers. In more detail, $u_{n,s} \frac{h_{n,s}}{2}$ is the waiting time of passengers who arrive after the departure (or passing) of bus $n-1$ at stop s, assuming uniformly distributed passenger arrivals. The second term in the objective function computes the total travel time of all bus trips in set N. The third term penalizes in-vehicle crowding $\gamma_{n,s}$ when it is beyond the soft capacity limit g_n.

Remark: If the public transport service provider places more importance on meeting the soft capacity requirement that is imposed due to COVID-19, then c_3 should take an adequately higher value than c_1, c_2. In this case, c_3 can be seen as a Big-M term where $c_3 >> c_1$ and c_2.

The objective function is nonlinear because of the first two terms that include multiplications between variables and the (implicit) conditional expression $\max(0, \gamma_{n,s} - g_{n,s})$ of the third term. Later in this study, the third term will be reformulated to a linear expression by adding continuous and binary variables.

The expected number of passengers who will board bus trip n at stop s (assuming bus n stops at stop s) depends on the number of passengers traveling between stops s and y ($y > s$) and whether the bus will stop at stop y:

$$u_{n,s} = x_{n,s} \sum_{y=s+1}^{|S|} w_{n,sy} x_{n,y}, \quad \forall n \in N, s \in S \setminus \{|S|\} \tag{3}$$

At the last stop of the service line we have no boardings. Thus, $u_{n,|S|} = 0, \forall n \in N$. From the total amount of passengers boarding bus trip n at stop s ($u_{n,s}$), the number of passengers boarding bus trip n at stop $s \in S \setminus \{|S|\}$ whose destination is stop y is:

$$b_{n,sy} = x_{n,s} w_{n,sy} x_{n,y}, \quad \forall n \in N, s \in S \setminus \{|S|\}, y > s \tag{4}$$

Clearly, $b_{n,sy} = 0$ for $y \leq s$. The expected number of alighting passengers for bus trip n at stop s is:

$$v_{n,s} = x_{n,s} \sum_{y=1}^{s-1} w_{n,ys} x_{n,y}, \forall n \in N, s \in S \setminus \{1\} \tag{5}$$

A special case is the first stop of a bus trip where we do not have passenger alightings. This introduces the boundary condition $v_{n,1} = 0, \forall n \in N$. The dwell time of each bus trip n at each stop s depends on the number of passengers that will board and alight at the stop, denoted by $u_{n,s}$ and $v_{n,s}$, respectively:

$$k_{n,s} = p_1 u_{n,s} + p_2 v_{n,s}, \quad \forall n \in N, s \in S \setminus \{1\} \tag{6}$$

The in-vehicle passenger load of any vehicle-trip $n \in N$ traveling from stop s to stop $s+1$ is also derived by:

$$\gamma_{n,s} = \gamma_{n,s-1} + u_{n,s} - v_{n,s} \quad, \forall n \in N, s \in S \setminus \{1, |S|\} \tag{7}$$

where $\gamma_{n,s} = u_{n,s}$ for $s = 1$. In addition, the in-vehicle passenger load should not exceed the hard capacity limit of its vehicle:

$$\gamma_{n,s} \leq \tilde{g}_n, \quad \forall n \in N, \forall s \in S \setminus \{|S|\} \tag{8}$$

The arrival time of bus trip n at stop s is equal to its departure time at stop $s-1$ ($d_{n,s-1}$), plus the travel time between the two stops, plus the time lost in acceleration and deceleration:

$$a_{n,s} = d_{n,s-1} + t_{n,s} + \frac{\delta}{2}(x_{n,s-1} + x_{n,s}), \quad \forall n \in N, s \in S \setminus \{1,2\} \tag{9}$$

where the arrival time at the second stop is derived from the boundary condition:

$$a_{n,2} = \tilde{d}_{n,1} + t_{n,2} + \frac{\delta}{2}(x_{n,1} + x_{n,2}), \quad \forall n \in N \tag{10}$$

In addition, the departure time of vehicle-trip n from stop $s \in S \setminus \{1\}$ is:

$$d_{n,s} = a_{n,s} + k_{n,s}, \quad \forall n \in N, s \in S \setminus \{1\} \tag{11}$$

Assuming that overtaking between buses of the same line is not allowed, the time headway between the arrival of bus trip n at stop s and the departure of its preceding one from stop s is:

$$h_{n,s} = a_{n,s} - d_{n-1,s}, \quad \forall n \in N \setminus \{1\}, s \in S \setminus \{1\} \tag{12}$$

Finally, the time headway at the first stop is calculated based on the boundary condition:

$$h_{n,1} = \tilde{d}_{n,1} - \tilde{d}_{n-1,1}, \quad \forall n \in N \setminus \{1\} \tag{13}$$

The aforementioned constraints and the objective function in (2) represent the formulation of the stop-skipping problem that considers the soft capacity of vehicles due to COVID-19. To this problem, we add the following constraints:

$$x_{n,1} = x_{n,|S|} = 1, \quad \forall n \in N \tag{14}$$

$$(x_{n-1,s}x_{n-1,y}) + (x_{n,s}x_{n,y}) \geq 1 \qquad \forall n \in N \setminus \{1\}, s \in S, y \in S : y \geq s \tag{15}$$

Constraint (14) enforces that we cannot skip the first and the last stop of each vehicle-trip. In addition, constraint (15) ensures that if an origin-destination pair is skipped by one trip, it will be served by the next one.

The mathematical program expressed in Eqs.(1)-(15) is a mixed-integer nonlinear optimization problem (MINLP) with linear and nonlinear constraints and a nonlinear objective function. This results in a hard-to-solve problem because its feasible region is not a convex set. To reduce the complexity, the third term of the objective function, $c_3 \sum_{n=1}^{|N|} \sum_{s=2}^{|S|} \max(0, \gamma_{n,s} - g_n)$, can be linearized by replacing $\max(0, \gamma_{n,s} - g_n)$ with a continuous variable $r_{n,s}$ and adding the following constraints:

$$r_{n,s} \geq \gamma_{n,s} - g_n, \quad \forall n \in N, s \in S \setminus \{1\} \tag{16}$$

$$r_{n,s} \geq 0, \quad \forall n \in N, s \in S \setminus \{1\} \tag{17}$$

$$r_{n,s} \leq \gamma_{n,s} - g_n + M z_{n,s}, \quad \forall n \in N, s \in S \setminus \{1\} \tag{18}$$

$$r_{n,s} \leq M(1 - z_{n,s}), \forall n \in N, s \in S \setminus \{1\} \tag{19}$$

where M is a very large positive number (Big-M), and $z_{n,s} \in \{0,1\}$ a binary variable that can be seen as a binary $|N||S|$-dimensional matrix. The MINLP expressed in Eqs.(1)-(19) can be solved with branch-and-bound by solving the continuous relaxation of the problem and constructing a rooted decision tree with branching and bounding operations.

IV. EXPERIMENTS

A. Case study description

The case study is bus line 9 in Twente operated by Keolis. This bus line connects the University of Twente with its two neighboring cities: Hengelo with 80 thousand inhabitants and Enschede with 160 thousand inhabitants. The bus line consists of 13 stops per direction. The topology of the line is presented in Figure 1.

Fig. 1. Topology of bus line 9 in Twente.

The line operates from 6:29 until 23:29 during weekdays and its average trip travel time per direction is 16 minutes.

B. Application

The application focuses on the peak hour of the weekdays, starting at 8 am and ending at 9 am. The mean passenger demand in this period is presented in the origin-destination demand matrix of Table 1.

TABLE I
PASSENGER DEMAND FROM 8AM TO 9AM OF BUS LINE 9

	Destinations												
Origins	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	8	16	16	24	24	16	16	24	16	8	32	44
2	0	0	4	8	8	16	16	8	16	24	40	24	52
3	0	0	0	4	4	4	32	24	16	16	32	40	32
4	0	0	0	0	8	8	16	24	32	40	8	24	56
5	0	0	0	0	0	4	8	8	20	20	8	28	28
6	0	0	0	0	0	0	8	4	8	24	12	20	32
7	0	0	0	0	0	0	0	4	4	4	12	24	48
8	0	0	0	0	0	0	0	0	8	8	4	8	36
9	0	0	0	0	0	0	0	0	0	4	8	16	44
10	0	0	0	0	0	0	0	0	0	0	4	12	48
11	0	0	0	0	0	0	0	0	0	0	0	8	12
12	0	0	0	0	0	0	0	0	0	0	0	0	4
13	0	0	0	0	0	0	0	0	0	0	0	0	0

Bus line 9 is a high-frequency line with a headway of 5 min from 8am to 9am. That is, the frequency is 12 vehicles per hour. Due to this high frequency, passenger arrivals at stops are considered to be random because passengers do not coordinate their arrivals with the arrival times of the buses [28]. The considered soft capacity due to the pandemic regulations is $g_n = 59$ passengers (38 seated, 21 standees) and the hard (actual) capacity is $\tilde{g}_n = 81$ passengers (38 seated, 43 standees). The boarding and alighting time per passenger is $p_1 = 2$ s and $p_2 = 1$ s, respectively. In accordance to Fu et al. (2003), the values of c_1 and c_2 are

\$20/h and \$50/h, respectively. If a bus stops at a particular stop, there is an additional time $\delta = 20$ s for acceleration and deceleration purposes. The values of the parameters are summarized in Table 2.

TABLE II
PARAMETER VALUES

Name	Unit	Value
δ	s	20
p_1	s	2
p_2	s	1
c_1	\$/h	20
c_2	\$/h	50
c_3	\$/h	3.6· 108 (a very large number)
M	-	104 (a very large number)
g_n	passengers	59
$\tilde{g}_n$	passengers	81

The mathematical model presented in Eqs.(1)-(19) is programmed in Python 3.8 and it is solved with branch-and-bound using the optimization solver Gurobi Optimizer, version 9.1.2. The optimizer is run on the cloud (on Microsoft Azure - F2s v2) using a virtual machine with 2 CPUs and 4096 MB RAM. The continuous variables of the problem are 5075 and the binary ones 1027. The model has also 3451 quadratic constraints.

Initially, the continuous relaxation of the model is solved to provide the lower bound of the optimization problem. The solution of the continuous relaxation is computed in 3.08 s and results in an objective function score of 6.987078e+06. However, this solution is not feasible because the continuous relaxation violates the integrality constraints of the MINLP. Using this initial solution as a starting point, the branch-and-bound algorithm is implemented in Gurobi. Because of the dynamic nature of the problem, we provide a computation time budget of 10 minutes (600 s). After the end of the 10-min period, the solver returned a feasible solution with an objective function score of 5.08285e+07. The best known lower bound at the time of the termination of the branch-and-bound algorithm was 3.4822e+07, indicating that the optimality gap of the derived solution is, at most, 31.5%. We note that this optimality gap is between the derived feasible solution after running the optimization for 10 min and the best known lower bound solution at the time of the termination of the optimization. That is, the actual optimality gap between the derived solution and the globally optimal solution might be much lower than 31.5% because the lower bound is not a feasible solution and it is allowed to take a lower value than the globally optimal solution.

The branch-and-bound algorithm explored 9885 nodes of the rooted decision tree. Figure 2 shows the performance change of the lower bound at each exploration step and the improved score of the best known feasible solution that takes the value of 5.08285e+07 after running the algorithm for 600 s. Note that, as mentioned above, the gap between the lower bound and the best known feasible solution at the termination of the algorithm is 31.5%.

The derived solution from the optimization process is presented in Figure 3. Figure 3 shows the skipped stops for

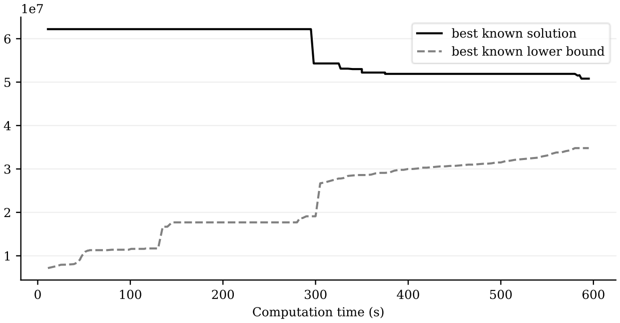

Fig. 2. Changes of the best known solution and the best known lower bound during the optimization process. The optimality gap after 600 s (10 min) is 31.5%.

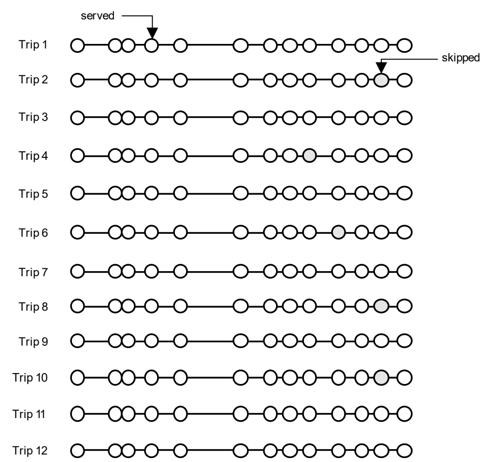

Fig. 3. Optimal skip-stop solution when considering the soft capacity constraints due to the pandemic. The skipped stops of a trip are presented in gray color.

each one of the 12 trips that operate from 8am until 9am. Notice that stops are skipped at every second trip in order to avoid refusing a passenger to board a bus two times in a row. This is a result of constraint (15) of our mathematical program.

The proposed stop-skipping strategy from 8am to 9am is compared against the 'as-is' situation that does not apply stop-skipping. The results of this comparison in terms of total passenger waiting times, total bus trip travel times, and in-vehicle crowding beyond the soft vehicle capacities are presented in Table 3.

TABLE III
PASSENGER DEMAND FROM 8AM TO 9AM OF BUS LINE 9.

	Stop-skipping	As-is
Total waiting times of passengers (in hours)	45.09	47.63
Total bus trip travel times (in hours)	4.07	4.15
In-vehicle crowding beyond the soft capacity limit (in passengers · line segments)	508	743

From Table 3 one can note that when applying stop-skipping, the in-vehicle crowding beyond the soft capacity level reduces by 31%. In addition, the total trip travel times are also reduced by 1.9% because skipping a stop results in a travel time decrease. The total waiting times are also not affected significantly when applying stop-skipping demonstrating the positive effect of implementing a skip-stop strategy.

V. Conclusion

This work studied the dynamic stop-skipping problem in public transport. The dynamic stop-skipping problem, which considered the soft vehicle capacities due to the pandemic, was modeled as a MINLP and was solved with branch-and-bound. The method was implemented in bus line 9 in Twente. The implementation showed that the MINLP formulation can return a solution with a proven optimality gap within a reasonable time for a regular-sized bus line. This is important given the dynamic nature of the model that might require solving it several times during the daily operations. Importantly, the skip-stop solution was able to reduce by 31% the in-vehicle crowding beyond the soft capacity limit compared to the as-is case where stop-skipping was not permitted. In addition, this improvement does not have a negative impact on the total trip travel times or the passenger waiting times. In fact, both the trip travel times and the passenger waiting times were slightly improved.

In future research, the dynamic stop-skipping model can be applied in public transport lines with more stops to further investigate the scalability of the model. In this direction, one can also explore the development of heuristic solution methods to further reduce the computational costs. Finally, the probability that some skipped passengers might wait for the next trip of the service line instead of using an alternative service can be further investigated in order to incorporate it into the mathematical model.

Acknowledgement

This work if funded by the The Netherlands Organisation for Health Research and Development (ZonMw) under the L4 project 'COVID 19 Wetenschap voor de Praktijk', project number: 10430042010018.

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