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Research Article

Mathematical Modeling of Coronavirus Dynamics with Conformable Derivative in Liouville–Caputo Sense

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Coronavirus has become a serious global phenomenon in recent times and has negative effects on the entire world economy. In this study, a fractional mathematical model formulated in fractional conformable derivative is studied. The model hinges on the concept of mammal hosts and humans. The basic properties of the coronavirus model are investigated. The stability analysis is carried out as well as sensitivity analysis based on the reproduction number. Numerical simulation is undertaken to give impetus to the analytical results which indicate that both fractional conformable order derivative and fractional-order derivative have serious consequences in numerical result outcomes.

1. Introduction

The world has not been free of diseases since creation, and mankind has not also done well in the immediate environment. Technology and science have brought fast changes, making lives easy and more comfortable [1, 2]. The natural environment is undergoing serious degradation in almost all parts of the world. Humans' advancement through science and technology has brought a dramatic shift in many natural certain, including marital principles, modernized agriculture, transportation, education, culture and many more. Currently, through technology, man has been able to develop some genetically modified foods [3–5]. Some of these advancements have serious consequences in human endeavor. Many plants and animals are going into extinction because of man's overexploitation of the natural environment. Why does society, therefore, blame the occurrence of epidemics and pandemic in the world? Are we not the creation of our own problems? There have been several pandemics in the past including the recent Ebola menace which killed many people and totally destroyed many countries' economy [6, 7].

The coronavirus is not an exception and would not probably the last one man would ever encounter. The outbreak of the current pandemic begun at Wuhan in the province of Hubei in the Republic of China. The association of the pandemic has to do with a seafood market centre which dealt with live animals. It is believed that the virus was associated with a host animal that humans infected [8]. Subsequently, human-to-human infection began. Due to migration, as people are now very mobile, the disease has spread to almost all parts of the continents. Notably, countries that have suffered severely are Italy, United Kingdom, United States of America, France, South Korea, and so on [1, 5]. Sub-Saharan Africa is not sped with this menace. South Africa is the leading country, and most countries have recorded the coronavirus infection. They have fragile economies with poor health infrastructure and would have serious effect on patients if care is not taken [1, 4, 5].

The symptoms of the disease include the following: severe headache, respiratory infection, and high temperature. The latent period of the disease is fourteen days, and during this period, the person can infect others. Currently, there is no cure for the disease; however, the WHO suggests preventive mechanisms such as social distancing, frequent washing of hands with running water, and application of hand sanitizer [5]. Several research studies are currently underway in the hope of obtaining drugs and vaccines to prevent the spread and mortality of the disease. It is important that quantitative and qualitative information on etiology be studied. It has been found that mathematical modeling is capable of providing qualitative information on many important parameters that are important for decision making by health professionals. There have been number of mathematical models on many epidemics both current and past [9, 10].

In recent times, non-integer models have gained tremendous advancement due to their ability to predict complex models or phenomena in light of engineering, technology, economics, etc. Fractional derivatives and integrals possess the past memory and the present state of phenomenon which helps in the accurate predictions of models. There are many fractional operators including Liouville–Caputo, Grunwald–Letnikov, Atangana–Gomez, Caputo–Fabrizio, Atangana–Baleanu, and others which are commonly used by researchers [11–14].

The recently introduced operator by Khalil et al. [15] has attracted several researchers because of its applicability and wide usefulness in many scientific problems. The operator boasts of some interesting properties such as conformable vectors, conformable partial derivatives, Taylor series expansion, Laplace transform, and others [16]. Thus, conformable fractional derivative is just local derivative in Riemann–Liouville and Caputo sense whose purpose is to give rise to non-local fractional derivative. Qureshi [17] investigated the effects of vaccination on measles dynamics under fractional conformable derivative with Liouville–Caputo operator and obtained a threshold in conformable derivative form that reduces infection. Khan and Aguilar [18] explored the dynamics of tuberculosis (TB) model and presented results that prove superiority of the conformable operator. This study is motivated by the effective and efficient results obtained by the previous authors with the fractional conformable order derivative. Harir et al. [19] employed conformable fractional-order derivative to examine SIR epidemic model and obtained a result that provided a qualitative information in 2021. In the same year, Hosseini et al. [20] utilized conformable derivative to demonstrate the effectiveness of numerical scheme result by comparing it with the analytical solutions in a heat transfer problem. Then, Allahamou et al. [21] also used conformable approach to study co-infection model of Hantavirus and validated the model using European moles in 2021.

The analytical and numerical results based on conformable derivative meet all the standard derivative criteria and are easy to compute which makes the results more efficient and reliable for predicting the model. The aim of this paper is to employ the fractional conformable derivative in Liouville–Caputo sense to examine the dynamics of the coronavirus model and also to present some qualitative information on coronavirus menace.

The rest of the paper is arranged as follows. In Section 2, mathematical preliminaries in both analysis and numerical simulations of our model are presented. Section 3 is solely devoted to the formulation of mathematical modeling. In the next section, the existence of bounded solutions in a biologically feasible region is presented. Section 5 contains the disease-free equilibrium and its stability, and Section 6 deals with the sensitivity analysis for the threshold quantity R0. Sections 7 and 8 present numerical algorithms and simulation results, respectively. MATLAB 2016a has been used to obtain numerical solutions. Finally, the study ends up with a conclusion.

2. Preliminarcies

Some of the basic results needed in the qualitative analysis and numerical simulations of the proposed coronavirus model (7) are presented.

Definition 1. The definition of fractional derivative presented by Riemann and Liouville of order α (RL^{α}) in terms of the power law type kernel $(x-\xi)^{p-\alpha-1}$ with convolution of a function $z(\xi)$ [16] is as follows:

$$_{RL^{\alpha}}\mathfrak{D}^{\alpha}_{a_1,x}z(x)=\frac{1}{\Gamma(p-\alpha)}\frac{d^p}{dx^p}\int_{a_1}^{x}(x-\xi)^{p-\alpha-1}z(\xi)d\xi\ ,\ \alpha\in(p-1,p]. \quad (1)$$

Definition 2. The definition of fractional derivative presented by Liouville and Caputo of order β (LC^{β}) in terms of the power law type kernel $(x-\xi)^{p-\beta-1}$ with convolution of the local derivative of a function $z(\xi)$ is as follows:

$$_{LC^{\beta}}\mathfrak{D}^{\beta}_{a_1,x}z(x)=\frac{1}{\Gamma(p-\beta)}\int_{a_1}^{x}(x-\xi)^{p-\beta-1}\frac{d^{\beta}}{d\xi^{\beta}}z(\xi)d\xi\ ,\ \beta\in(p-1,p]. \quad (2)$$

Definition 3. The conformable fractional derivative of order α (CFD^{α}) is defined as

$$ {}_{CFD^{\alpha}}\mathfrak{D}^{\alpha}_{a_1,x} z(x) = \lim_{\zeta\to 0} \frac{z\left(x+\zeta x^{1-\alpha}\right)-z(x)}{\zeta}, \quad t,\alpha>0. \tag{3} $$

Remark 1. The relation between CFD^{α} and local ordinary derivative is

$$ {}_{CFD^{\alpha}}\mathfrak{D}^{\alpha}_{a_1,x} z(x) = \left(x-a_1\right)^{1-\alpha}\frac{d}{dx}z(x). \tag{4} $$

Definition 4. The CFD^{α} in the sense of LC^{β} is defined as

$$ \begin{aligned} {}^{\beta}_{C}\mathfrak{D}^{\alpha}_{a_1,x} z(x) &= \frac{1}{\Gamma(p-\beta)}\int_{a_1}^{x}\left[\frac{\left(x-a_1\right)^{\alpha}-\left(\xi-a_1\right)^{\alpha}}{\alpha}\right]^{p-\beta-1} \\ &\quad\cdot \frac{{}^{p}_{CFD^{\alpha}}\mathfrak{D}^{\alpha}_{a_1,\xi} z(\xi)}{\left(\xi-a_1\right)^{1-\alpha}}d\xi, \\ {}^{\beta}_{C}\mathfrak{D}^{\alpha}_{a_1,x} zx &= {}^{p-\beta}_{CF\,D}\mathbb{I}^{\alpha}_{a_1,x}\left({}^{p}_{CFD^{\alpha}}\mathfrak{D}^{\alpha}_{a_1,x} z(x)\right), \end{aligned} \tag{5} $$

where $z(x)\in C^{p}_{\alpha,a_1}([a_1,a_2])$, $\operatorname{Re}(\beta)\ge 0$, and $p=\lceil\operatorname{Re}(\beta)\rceil+1$.

3. Mathematical Model Formulation

This section presents a coronavirus model of fractional conformable derivative version by Bonyah et al. [22] in which the total host mammal population N_a is apportioned into susceptible mammal class S_a, latent mammal class L_a, infected mammal class I_a, and recovered mammal class R_a. Hence, total host mammal population is denoted by $N_a = S_a + L_a + I_a + R_a$. Human total population is also subdivided into susceptible human class S_b, latent human class L_b, infected human class I_b, and recovered human class R_b. Mammal and human recruitment rates are Λ_a and Λ_b, respectively. Natural mortality rate for humans and mammals is μ_a and μ_b in that order. Effective contact rate between infected mammals and susceptible mammals is given by β_1. The effective contact rate between infected mammals and susceptible human is denoted by β_2, β_3. The waning rate of recovered human losses immunity to be part of the susceptible class is γ. The recovery rate of human and mammal is τ_b and τ_a, respectively. The rate human and mammal move into infected classes is denoted by θ_b and θ_a while human disease induced mortality rate is ω. With initial conditions $S^0_m = S_a(0), L^0_a = L_a(0), I^0_a = I_a(0), R^0_a = R_a(0)$, $S^0_b = S_b(0), L^0_b = L_b(0), I^0_b = I_b(0)$, $R^0_b = R_b(0)$, the following non-linear differential equations represent the interactions among the various compartments:

$$ \begin{aligned} \frac{dS_a}{dt} &= \Lambda_a - \beta_1 S_a I_a - \mu_a S_a, \\ \frac{dL_a}{dt} &= \beta_1 S_a I_a - (\mu_a+\theta_a)L_a, \\ \frac{dI_a}{dt} &= \theta_a L_a - (\tau_a+\mu_a)I_a, \\ \frac{dR_a}{dt} &= \tau_a I_a - \mu_a R_a, \\ \frac{dS_b}{dt} &= \Lambda_b - \beta_2 S_b I_a - \beta_3 S_b I_b + \gamma R_b - \mu_b S_b, \\ \frac{dL_b}{dt} &= \beta_2 S_b I_a + \beta_3 S_b I_b - (\mu_b+\theta_b)L_b, \\ \frac{dI_b}{dt} &= \theta_h L_b - (\tau_b+\mu_b+\omega)I_b, \\ \frac{dR_b}{dt} &= \tau_b I_b - (\mu_b+\gamma)R_b. \end{aligned} \tag{6} $$

Now, replacing the integer-order derivatives in coronavirus system (6) with CFD^{α} in the sense of LC^{β}, we obtain the following system:

$$ \begin{aligned} {}^{\beta}_{C}D^{\alpha}_{0,t}S_a &= \Lambda_a - \beta_1 S_a I_a - \mu_a S_a, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}L_a &= \beta_1 S_a I_a - (\mu_a+\theta_a)L_a, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}I_a &= \theta_a L_a - (\tau_a+\mu_a)I_a, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}R_a &= \tau_a I_a - \mu_a R_a, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}S_b &= \Lambda_b - \beta_2 S_b I_a - \beta_3 S_b I_b + \gamma R_b - \mu_h S_b, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}L_b &= \beta_2 S_b I_a + \beta_3 S_b I_b - (\mu_b+\theta_b)L_b, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}I_b &= \theta_b L_b - (\tau_b+\mu_b+\omega)I_b, \\ {}^{\beta}_{C}D^{\alpha}_{0,t}R_b &= \tau_b I_b - (\mu_b+\gamma)R_b. \end{aligned} \tag{7} $$

4. Existence of Bounded Solutions in a Biologically Feasible Region

Here, we will study the boundedness of the solution of model (7) in a positively invariant region.

Lemma 1. *The region* $\Omega = \{(S_a(t), L_a(t), I_a(t), R_a(t), S_b(t), L_b(t), I_b(t), R_b(t)) \in R^8_+ \colon N_a(t) \le \Lambda_a/\mu_a, N_b(t) \le \Lambda_b/\mu_b\}$ *is positively invariant for coronavirus model (7) with* CFD^{α} *in the sense of* LC^{β} *and initial conditions in* R^8_+.

Proof 1. Adding all the equations of the host mammal population, we get

$$ {}^{\beta}_{C}D^{\alpha}_{0,t}N_a(t) = \Lambda_a - \mu_a N_a(t). \tag{8} $$

By separating variables and integrating, we get

$$ N_a(t) = \frac{1}{\mu_a}\left(\Lambda_a - \exp\left(-\frac{\mu_a}{\alpha}t^{\alpha}\right)\right). \tag{9} $$

Thus, it is deduced that

$$\lim_{t\longrightarrow\infty} \sup N_a(t) \leq \frac{\Lambda_a}{\mu_a}. \tag{10}$$

Now, similarly adding all the equations of the human population, we obtain

$$\lim_{t\longrightarrow\infty} \sup N_b(t) \leq \frac{\Lambda_b}{\mu_b}. \tag{11}$$

These results show that the solutions are bounded for time and model (7) possesses the positively invariant region Ω. □

5. Disease-Free Equilibrium (DFE) and Its Stability

Solving model (7) under no infection condition, we obtain the following disease-free steady state (DFSS) D_0:

$$\begin{aligned} D_0 &= \left(S_a^0, L_a^0, I_a^0, R_a^0, S_b^0, L_b^0, I_b^0, R_b^0\right),\\ D_0 &= \left(\frac{\Lambda_a}{\mu_a}, 0, 0, 0, \frac{\Lambda_b}{\mu_b}, 0, 0, 0\right). \end{aligned} \tag{12}$$

For the next generation method [23], we have

$$F = \begin{pmatrix} 0 & \frac{\beta_1\Lambda_a}{\mu} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & \frac{\beta_2\Lambda_b}{\mu_b} & 0 & \frac{\beta_3\Lambda_b}{\mu_b}\\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{13}$$

$$V = \begin{pmatrix} \mu_a + \theta_a & 0 & 0 & 0\\ -\theta_a & \tau_a + \mu_a & 0 & 0\\ 0 & 0 & \mu_b + \theta_b & 0\\ 0 & 0 & -\theta_b & \tau_b + \mu_b + \omega \end{pmatrix}. \tag{14}$$

Therefore, the reproduction number is $\mathscr{R}_0 = \mathscr{R}_1 + \mathscr{R}_2$, where $\mathscr{R}_1 = \beta_1\theta_a\Lambda_a/\mu_a(\theta_a + \mu_a)(\mu_a + \tau_a)$ and $\mathscr{R}_2 = \beta_3\theta_b\Lambda_b/\mu_b(\theta_b + \mu_b)(\omega + \mu_b + \tau_b)$.

Theorem 1. *If $R_0 < 1$, then D_0 of coronavirus model (7) satisfies $Re(\lambda_j) < 0$, for $j = 1(1)8$, and D_0 is locally asymptotically stable (LAS), where $Re(\lambda)$ represents the real part of an eigenvalue of the corresponding Jacobian matrix of coronavirus model (7) at D_0.*

Proof 2. For the required result, the corresponding Jacobian matrix calculated at D_0 is

$$J_{D_0} = \begin{pmatrix} -\mu_a & 0 & -\frac{\beta_1\Lambda_a}{\mu_a} & 0 & 0 & 0 & 0 & 0\\ 0 & -\theta_a - \mu_a & \frac{\beta_1\Lambda_a}{\mu_a} & 0 & 0 & 0 & 0 & 0\\ 0 & \theta_a & -\mu_a - \tau_a & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \tau_a & -\mu_a & 0 & 0 & 0 & 0\\ 0 & 0 & -\frac{\beta_2\Lambda_h}{\mu_b} & 0 & -\mu_b & 0 & -\frac{\beta_3\Lambda_b}{\mu_b} & \gamma\\ 0 & 0 & \frac{\beta_2\Lambda_b}{\mu_b} & 0 & 0 & -\theta_b - \mu_b & \frac{\beta_3\Lambda_b}{\mu_b} & 0\\ 0 & 0 & 0 & 0 & 0 & \theta_b & -\omega - \mu_b - \tau_b & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_b & -\gamma - \mu_b \end{pmatrix}. \tag{15}$$

Its corresponding characteristic equation is given by

$$(\lambda + \mu_b)(\lambda + \mu_b + \gamma)(\lambda + \mu_a)^2\left(\lambda^4 + D_1\lambda^3 + D_2\lambda^2 + D_3\lambda + D_4\right) = 0, \tag{16}$$

where the coefficients D_j for $j = 1, 2, 3, 4$ are given by

$$\begin{aligned}
D_1 &= \omega + \theta_b + \tau_b + 2\mu_b + \theta_a + \tau_a + 2\mu_a,\\
D_2 &= (\omega + \theta_b + \tau_b + 2\mu_b)(\theta_a + \tau_a + 2\mu_a) + (\theta_a + \mu_a)(\mu_a + \tau_a)(1 - \mathscr{R}_1)\\
&\quad + (\theta_b + \mu_b)(\omega + \tau_b + \mu_b)(1 - \mathscr{R}_2),\\
D_3 &= (\omega + \theta_b + \tau_b + 2\mu_b)(\theta_a + \mu_a)(\mu_a + \tau_a)(1 - \mathscr{R}_1)\\
&\quad + (\theta_a + \tau_a + 2\mu_a)(\theta_b + \mu_b)(\omega + \tau_b + \mu_b)(1 - \mathscr{R}_2),\\
D_4 &= (\theta_b + \mu_b)(\omega + \tau_b + \mu_b)(\theta_a + \mu_a)(\mu_a + \tau_a)(1 - \mathscr{R}_1)(1 - \mathscr{R}_2).
\end{aligned} \tag{17}$$

Since the eigenvalues $-\mu_b$, $-\mu_b$, $-\mu_a$, and $-(\mu_a + \gamma)$ are negative, all the other coefficients D_j for $j = 1(1)4$ of the characteristic polynomial are positive if $R_0 < 1$. The Routh–Hurwitz [24] criteria $D_j > 0$ for $j = 1(1)4$ and $D_1 D_2 D_3 > D_1^2 D_4 + D_3^2$ can be satisfied easily. So, the DFSS D_0 of coronavirus model (7) is LAS if $R_0 < 1$. □

Theorem 2. *The DFSS D_0 of coronavirus model (7) is globally asymptotically stable (GAS) for $R_0 < 1$ and unstable for $R_0 > 1$.*

Proof 3. For the proof, let us construct a Lyapunov function at DFSS D_0:

$$\mathscr{L}(t) = \mathscr{C}_1 L_a(t) + \mathscr{C}_2 I_a(t) + \mathscr{C}_3 L_b(t) + \mathscr{C}_4 I_b(t), \tag{18}$$

where the constants$C_j > 0$, for $j = 1, 2, 3, 4$and they are chosen later. Calculating the CFD^{α} in the sense of LC^{β}, we get

$$\begin{aligned}
{}^{\beta}_{C}\mathfrak{D}^{\alpha}_{a_1,x}\mathscr{L}(t) &= \mathscr{C}_1\, {}^{\beta}_{C}D^{\alpha}_{0,t} L_a(t) + \mathscr{C}_2\, {}^{\beta}_{C}D^{\alpha}_{0,t} I_a(t)\\
&\quad + \mathscr{C}_3\, {}^{\beta}_{C}D^{\alpha}_{0,t} L_b(t) + \mathscr{C}_4\, {}^{\beta}_{C}D^{\alpha}_{0,t} I_b(t).
\end{aligned} \tag{19}$$

Using the proposed coronavirus model (7), we obtain

$$\begin{aligned}
{}^{\beta}_{C}\mathfrak{D}^{\alpha}_{a_1,t}\mathscr{L}(t) &= \mathscr{C}_1\left\{\beta_1 S_m^0 I_a - (\mu_a + \theta_a) L_a\right\} + \mathscr{C}_2\{\theta_a L_a - (\tau_a + \mu_a) I_a\}\\
&\quad + \mathscr{C}_3\left\{\beta_2 S_b^0 I_a + \beta_3 S_b^0 I_b - (\mu_b + \theta_b) L_b\right\} + \mathscr{C}_4\{\theta_h L_b - (\tau_b + \mu_b + \omega) I_b\},\\
&= \left\{\beta_1 S_a^0 \mathscr{C}_1 - (\tau_a + \mu)_a \mathscr{C}_2 + \beta_2 S_b^0 \mathscr{C}_3\right\} I_a + \{-(\theta_a + \mu_a)\mathscr{C}_1 + \theta_a \mathscr{C}_2\} L_a\\
&\quad + \{-(\theta_b + \mu_b)\mathscr{C}_3 + \theta_b \mathscr{C}_4\} L_b + \left\{\beta_3 S_b^0 \mathscr{C}_3 - (\tau_b + \mu_b + \omega)\mathscr{C}_4\right\} I_b.
\end{aligned} \tag{20}$$

Now, choose $C_1 = \theta_a$, $C_2 = \mu_a + \theta_a$, $C_3 = \{(\mu_a + \tau_a)(\mu_a + \theta_a) - \beta_1 \Lambda_a \theta_b / \mu_a\}\mu_b / \beta_2 \Lambda_b$, and $C_4 = (\theta_b + \mu_b)\{(\mu_a + \tau_a)(\mu_a + \theta_a) - \beta_1 \Lambda_a \theta_b / \mu_a\}\mu_b / \beta_2 \Lambda_b \theta_b$. After simplification, we obtain

$${}^{\beta}_{C}D^{\alpha}_{0,t}\mathscr{L}(t) = \frac{\mu_b}{\beta_2 \Lambda_b \theta_b}(\mu_a + \tau_a)(\mu_a + \theta_a)(\theta_b + \mu_b)(\mu_b + \tau_b + \omega)(1 - \mathscr{R}_1)(\mathscr{R}_2 - 1). \tag{21}$$

Clearly, if $R_0 < 1$, then the derivative presented in equation (21) is negative. □

6. Sensitivity Analysis for the Threshold Quantity $\mathscr{R}_0$

In mathematical models of infectious diseases, $\mathscr{R}_0$ has a very vital role in the prediction of an infectious disease that either the infection will die out or remain in the population. In this regard, it is good to know which parameter has more influence on the value of threshold quantity $\mathscr{R}_0$, for which we use sensitivity indices for $\mathscr{R}_0$ known as forward sensitivity indices [16] with the help of

$$\prod_{\rho}^{\mathscr{R}_0} = \frac{\partial \mathscr{R}_0}{\partial \rho} \times \frac{\rho}{\mathscr{R}_0}, \tag{22}$$

where ρ represents the biological parameters used in the proposed coronavirus model (7). Using definition (22), we obtain

$$\begin{aligned}
&\prod_{\beta_1}^{\mathscr{R}_1} = 1, \prod_{\Lambda_a}^{\mathscr{R}_1} = 1, \prod_{\theta_a}^{\mathscr{R}_1} = \frac{\mu_a}{\theta_a + \mu_a}, \prod_{\mu_a}^{\mathscr{R}_1} = -\left\{1 + \frac{\mu_a}{\theta_a + \mu_a} + \frac{\mu_a}{\tau_a + \mu_a}\right\},\\
&\prod_{\tau_a}^{\mathscr{R}_1} = -\frac{\tau_a}{\tau_a + \mu_a}, \prod_{\beta_3}^{\mathscr{R}_2} = 1, \prod_{\Lambda_h}^{\mathscr{R}_2} = 1, \prod_{\theta_h}^{\mathscr{R}_2} = \frac{\mu_b}{\theta_b + \mu_b},\\
&\prod_{\mu_b}^{\mathscr{R}_2} = -1 - \frac{\mu_b}{\theta_b + \mu_b} - \frac{\mu_b}{\tau_b + \mu_b + \omega}, \prod_{\tau_b}^{\mathscr{R}_2} = -\frac{\tau_b}{\tau_b + \mu_b + \omega}, \prod_{\omega}^{\mathscr{R}_2} = -\frac{\omega}{\tau_b + \mu_b + \omega}.
\end{aligned} \tag{23}$$

7. Numerical Algorithms and Results

Here, we derive the numerical schemes for coronavirus model (7) with CFD^{α} in the LC^{β} sense. An Adams–Moulton iterative (AMI) technique [17] will be implemented for the numerical approximations of state variables $(S_a, L_a, I_a, R_a, S_b, L_b, I_b, R_b)$ used in the proposed coronavirus model (7).

Consider a Cauchy initial value problem with the operator LC^{β}

$$\begin{aligned}
&{}_{LC^{\beta}}^{\beta}\mathfrak{D}_{0,t} z(t) = h(t, z(t)), \quad \beta > 0,\ t \in [0, T],\\
&z^{(q)}(0) = z_0^{(q)},
\end{aligned} \tag{24}$$

where $q = 0, 1, \ldots, \lceil \beta \rceil - 1$. The Volterra integral equation of the second kind can be obtained from the above-mentioned Cauchy problem as follows:

$$z(t) = \sum_{q=0}^{p-1} z_0^{(q)} \frac{t^q}{q!} + \frac{1}{\Gamma(\beta)} \int_0^t (t - \tau)^{\beta-1} h(\tau, z(\tau)) d\tau, \quad \beta \in (p - 1, p]. \tag{25}$$

Discretize the interval $[0, T]$ such that $b = T - 0/P$, $t_k = kb$, $k = 0, 1, \ldots, P$ with the CFD^{α}. We obtain the following AMI scheme for the CFD^{α} in the sense of LC^{β}:

$$z\left(t_{p+1}\right) = z(0) + \frac{b^{\beta}}{\beta\Gamma(\beta)} \sum_{k=0}^{p} \left[(p + 1 - k)^{\beta} - (p - k)^{\beta}\right]_{CFD^{\alpha}} \mathfrak{D}_{0,t}^{\alpha} h\left(t_k, z\left(t_k\right)\right), \quad k \in [0, p], \tag{26}$$

where

$${}_{CFD^{\alpha}}\mathfrak{D}_{0,t}^{\alpha} h\left(t_k, z\left(t_k\right)\right) = \frac{1}{t_k^{\alpha-1}} \frac{d}{dt} h\left(t_k, z\left(t_k\right)\right), \quad \alpha > 0. \tag{27}$$

Now, take $X(t_k) = (S_a(t_k), L_a(t_k), I_a(t_k), R_a(t_k), S_b(t_k), L_b(t_k), I_b(t_k), R_b(t_k))$ and using (25) and (26), we obtain the following iterative schemes for the proposed coronavirus model (7) with CFD^{α} in the LC^{β} sense:

$$\begin{aligned}
S_a(t_{p+1}) &= S_a(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_1(X(t_k)),\\
L_a(t_{p+1}) &= L_a(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_2(X(t_k)),\\
I_a(t_{p+1}) &= I_a(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_3(X(t_k)),\\
R_a(t_{p+1}) &= R_a(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_4(X(t_k)),\\
S_b(t_{p+1}) &= S_b(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_5(X(t_k)),\\
L_b(t_{p+1}) &= L_b(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_6(X(t_k)),\\
I_b(t_{p+1}) &= I_b(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_7(X(t_k)),\\
R_b(t_{p+1}) &= R_b(0) + \frac{b^\beta}{\beta\Gamma(\beta)}\sum_{k=0}^{p}\left[(p+1-k)^\beta - (p-k)^\beta\right]h_8(X(t_k)),
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
h_1(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\Lambda_a - \beta_1 S_a(t_k)I_a(t_k) - \mu_a S_a(t_k)\right),\\
h_2(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\beta_1 S_a(t_k)I_a(t_k) - (\mu_a + \theta_a)L_a(t_k)\right),\\
h_3(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\theta_a L_a(t_k) - (\tau_a + \mu_a)I_a(t_k)\right),\\
h_4(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\tau_a I_a(t_k) - \mu_a R_a(t_k)\right),\\
h_5(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\Lambda_b - \beta_2 S_b(t_k)I_a(t_k) - \beta_3 S_b(t_k)I_b(t_k) + \gamma R_h(t_k) - \mu_b S_b(t_k)\right),\\
h_6(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\beta_2 S_b(t_k)I_a(t_k) + \beta_3 S_b(t_k)I_b(t_k) - (\mu_b + \theta_b)L_b(t_k)\right),\\
h_7(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\theta_b L_b(t_k) - (\tau_b + \mu_b + \omega)I_b(t_k)\right),\\
h_8(X(t_k)) &= \frac{1}{t_k^{1-\alpha}}\left(\tau_b I_b(t_k) - (\mu_b + \gamma)R_b(t_k)\right).
\end{aligned} \tag{29}$$

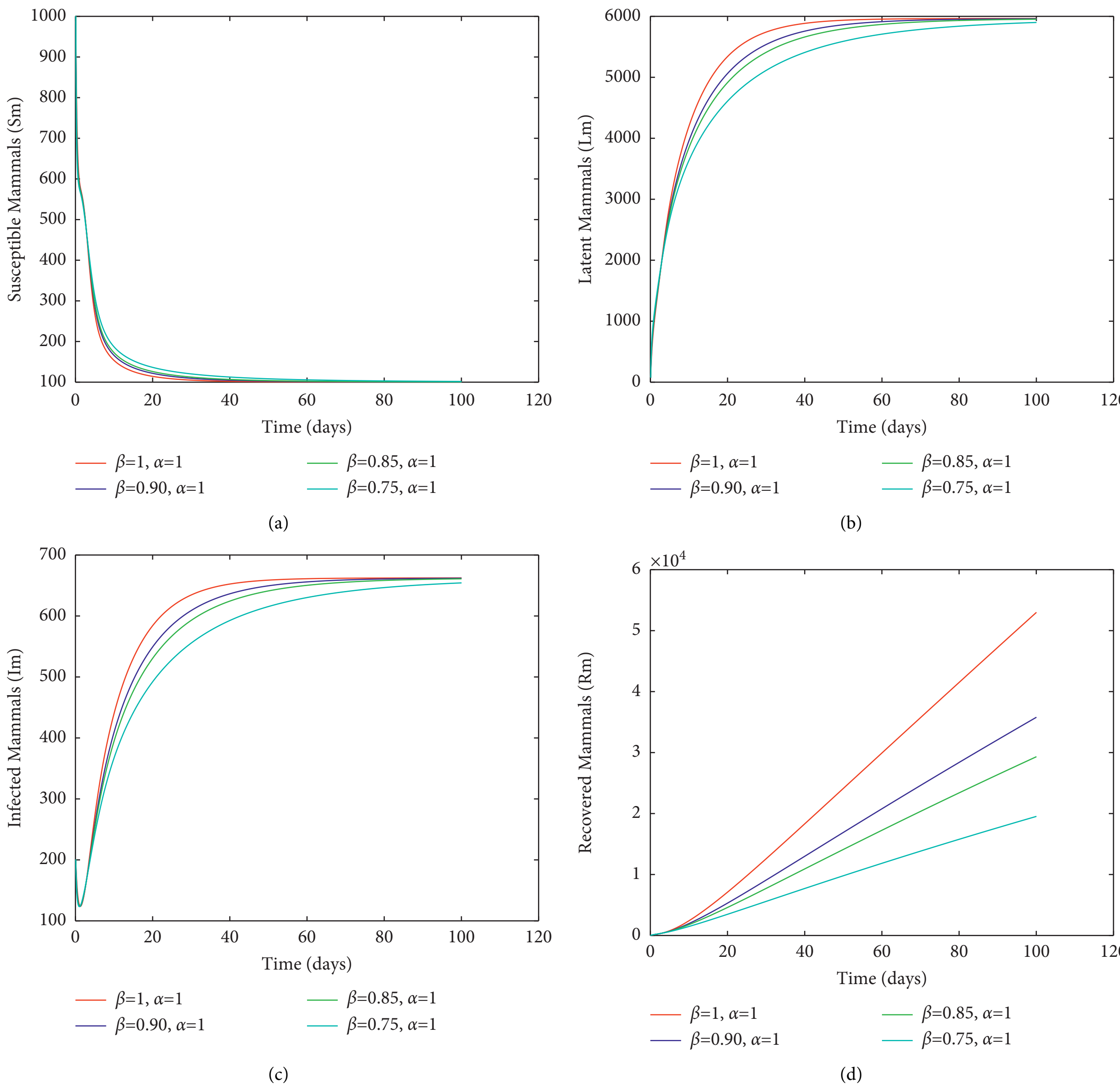

Figure 1: Continued.

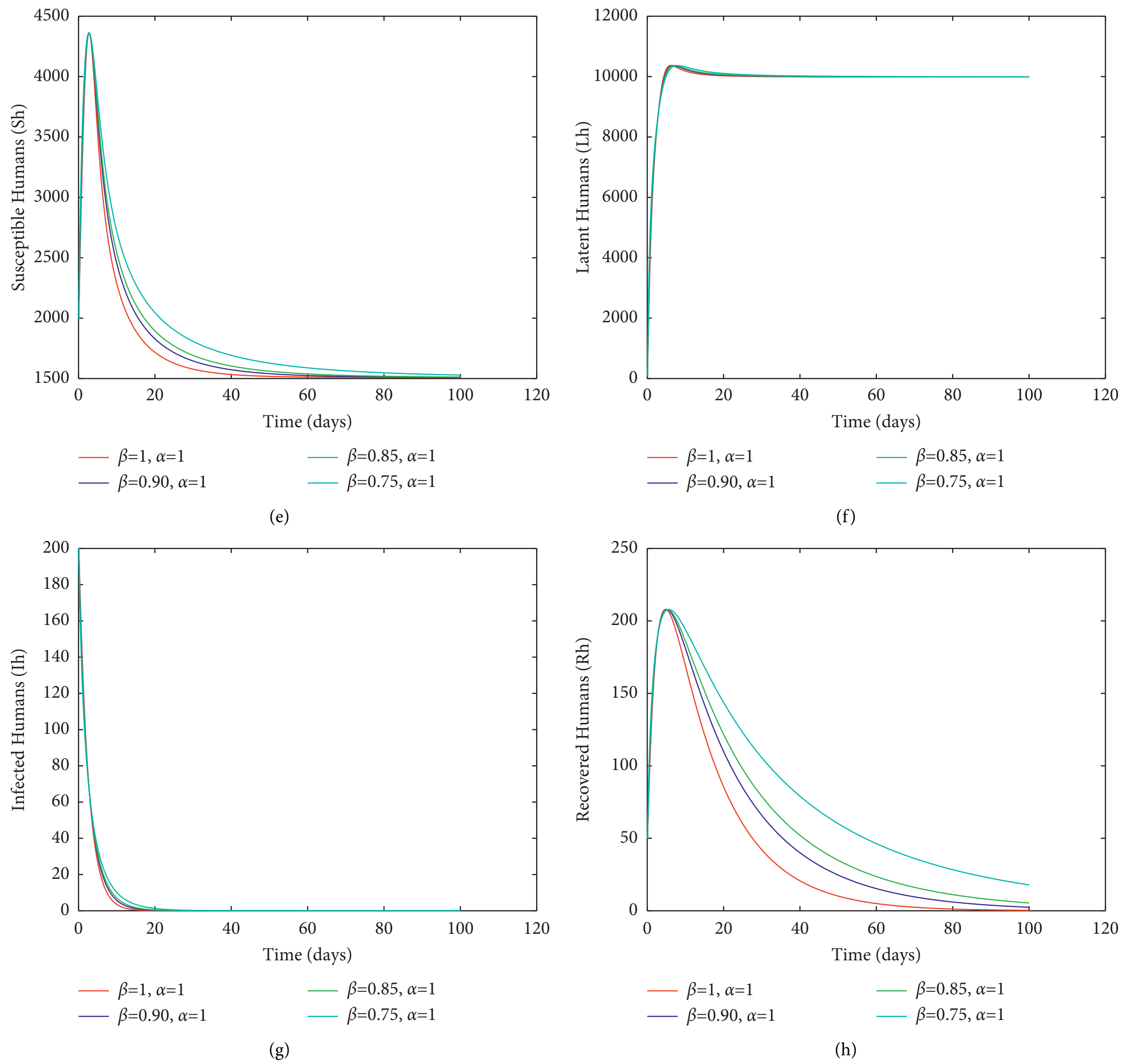

Figure 1: Simulation of fractional conformable model (7), when $\beta = 1, \beta = 0.90, \beta = 0.85, \beta = 0.75$, and $\alpha = 1$.

8. Numerical Simulation Results

The step size used for this work is 10^{-2} and the time interval considered is $[0, 30]$ with the following initial conditions: $(1000, 80, 200, 3, 2000, 80, 200, 50)$. The parameter values employed for the numerical simulation were obtained in [25] as follows: $\Lambda_a = 600, \Lambda_b = 9000, \beta_1 = 0.009$, $\mu_a = 0.000474, \theta_a = 0.1, \tau_a = 0.9, \beta_2 = 0.009, \beta_3 = 0.009, \gamma = 0.07, \mu_b = 0.0009, \theta_b = 0.9, \omega = 0.8, \tau_b = 0.5$ In this work, β represents the fractional conformable derivative order and αdepicts the Liouville–Caputo operator order in equation (7). In Figures 1(a)–1(h), the fractional conformable derivative order β is varied while the fractional order α derivative in Liouville–Caputo is kept constant. The number of mammals in Figure 1(a) decreases as the conformable fractional order β increases from 0.75 to 1 which implies that more mammals are getting infected with the virus. In Figures 1(b)–1(d), the number of mammals in these classes increases as the fractional conformable order β increases from 0.75 to 1. Figure 1(f) indicates that the number of latent humans increases as fractional conformable order β increases from 0.75 towards 1. For Figures 1(e), 1(g), and 1(h), the number of humans in these classes reduces as the fractional conformable order β approaches 1 as the number of individuals reduces. Figures 2(a)–2(h) represent the numerical simulation based on equation (7) with constant fractional conformable derivative and a varied fractional order α in sense of Liouville–Caputo. Figure 2(a) shows that as the fractional order increases towards 1, the number of susceptible mammals increases gradually. In Figures 2(b)–2(d), the number of mammals in the

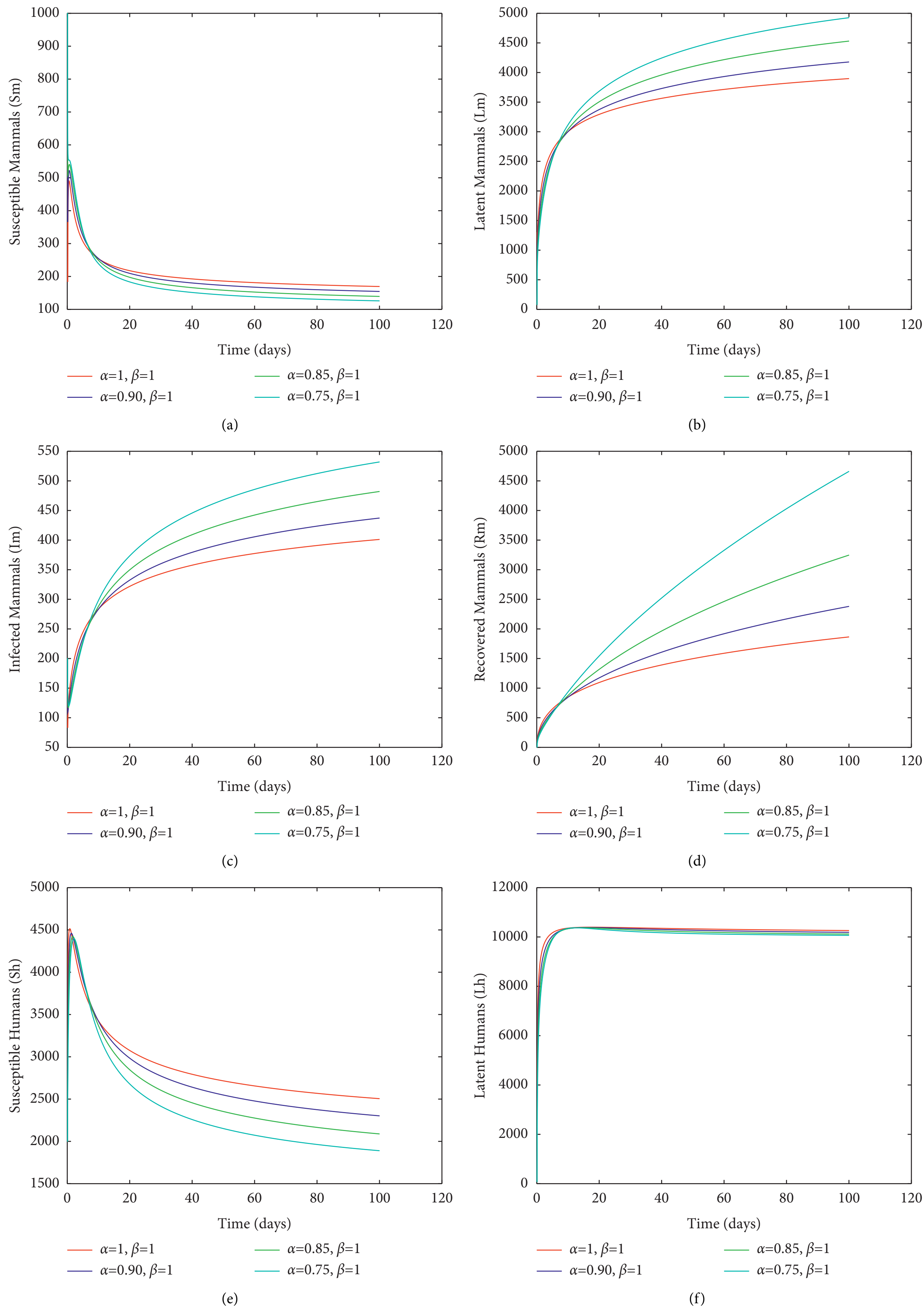

Figure 2: Continued.

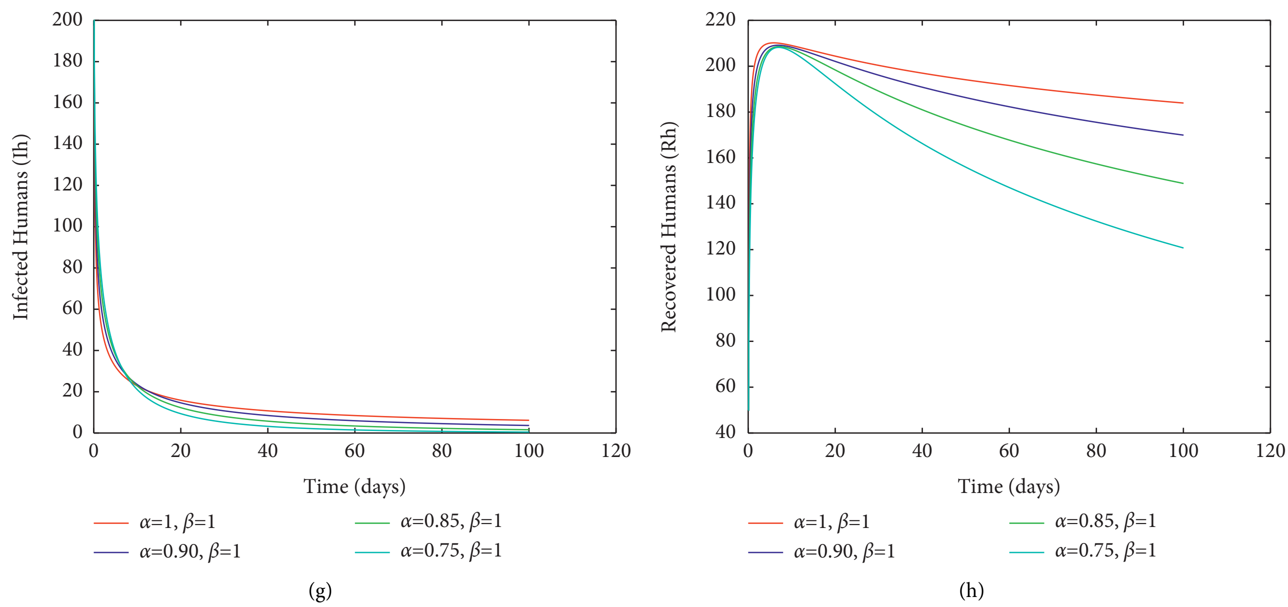

FIGURE 2: Simulation of fractional conformable model (7), when $\alpha = 1, \alpha = 0.90, \alpha = 0.85, \alpha = 0.75$, and $\beta = 1$.

respective classes eventually reduces as the fractional order α increases towards 1. Figure 2(e) indicates that the number of susceptible humans increase as the fractional order α decreases and a similar situation can be seen in Figure 2(g) for the infected humans class. The case is not different from Figures 2(f) and 2(h). As the fractional order α increases, the number of humans in these classes also increases, respectively.

9. Conclusions

In this work, a coronavirus model in the context of fractional conformable derivative in light of Liouville–Caputo sense was formulated. The basic property of model boundedness was investigated. The asymptotic stability of the steady states of the model has been studied. Sensitivity analysis was undertaken to have some basic idea about the parameter values involving the basic reproduction number. Numerical analysis based on the Adams–Moulton scheme was carried out, and the results indicated both fractional order conformable derivative order have an effect on the dynamics of coronavirus. It is, therefore, suggested that this derivative can be applied to other complex physical phenomena. In the future, similar models can be investigated using the fractional conformal approach because it conforms to the principles of derivatives and is easy to use. Other related models can be studied using the fractional conformal stochastic modeling approach. This could also be employed in financial and economic models since it is easy to utilize.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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