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Interview Craig Barton on improving your teaching, lethal mutations and problem solving

How he wishes he'd taught maths

Over the last few years, Craig Barton has made quite a name for himself in the field of mathematics education. In his first book, *How I Wish I'd Taught Maths: Lessons learned from research, conversations with experts, and 12 years of mistakes* (2017), he describes in much detail exactly what its title suggests. Three years ago, the book was translated into Dutch by René Kneyber and given the appropriate name *Volgens Barton*. It's a play of words on the Dutch saying "volgens Bartjens", which means "from basic reasoning it follows that" – that saying in turn refers to the seventeenth century schoolmaster Willem Bartjens and is also the title of a Dutch magazine on mathematics education. A workshop based on the book, which was supposed to take place in March 2020, was postponed several times due to the Covid pandemic and finally took place in June 2022. In the meantime a second book was published: *Reflect, Expect, Check, Explain: Sequences and behaviour to enable mathematical thinking in the classroom* (2020). Mark Timmer and Wim Caspers were present to attend the workshop and interview Craig afterwards, and learnt that a third book may be on its way.

Craig studied Economics at the University of Cambridge, but mathematics was his favourite subject at school and he always had the wish to become a teacher. So, after finishing his studies and doing some travelling, he got his Postgraduate Certificate in Education and started teaching in secondary school at the age of 22. Already after a couple of years, he started the website www.mrbartonmaths.com to spread ideas about teaching mathematics. He was also maths advisor for *The Times*, and started his own podcast to broadcast interviews about teaching mathematics. While talking to many mathematics teachers and educational scientists, he started realising: "Pretty much everything I had done up

until that point in my career was wrong." Instead of giving up, Craig started reading research papers and books about education, cognitive psychology and mathematics teaching, culminating in his first book that describes what he did before, what he learned from research and what his current views are. In this article we discuss several themes from the first book, as well as new ideas Craig developed afterwards.

In the foreword of your book, Dylan William writes that teaching can never be fully research-based, in the sense that it's impossible to precisely dictate what works and what doesn't. But, he also states that it would be incorrect to call it an art that

can never be improved by science. So, it must be somewhere in between. With that idea in mind, which aspects of the teaching methodology that you prefer and describe in the book do you regard as objective truths about the best way to teach?

"I've started now doing this 'Tips for Teachers' thing, where teachers come on and speak to me about different advice from different subjects. I spoke to a lady recently, Sarah Cottingham, who is an English teacher and now a researcher and a teacher trainer. One of her tips was that teachers should treat research as a *compass*, not a prescription. I think that's probably the best way I've heard it articulated. Research can always guide us in a certain direction, but there are so many different variables at play within different classrooms, different contexts. Thirty students are never going to be the same across different schools, even different times of the day. So research can always guide us to certain things, but it can never tell us exactly what to do. But I think there are certain 'best bets': certain things that you're pretty sure are a good idea to do with anyone.

So the number one of course would be *retrieval practice*. It is a really good idea to keep asking students to try and recall things they've done in the past. And yet it's

still something that I don't think you would see in enough classrooms, certainly not done well. So retrieval practice; I think you will do well to find anyone who would argue that's a bad idea. I think, though, with retrieval practice, some people would argue that testing students is bad, but then as we discussed today, you can do that in a very low-stakes way, that gets all the benefits of retrieval without any of the anxiety-inducing effects. So retrieval would be one.

I think another best bet would be from cognitive load theory. It's probably a good idea to *identify extraneous load and reduce it*. So, it's probably a bad idea to have text and diagrams separated on a slide, it's probably a bad idea to put some text up and then read it out straight away. There are certain things like that, that I think are pretty non-controversial. I think they're pretty good bets.

Then there are lots of things that I passionately agree about, but other teachers wouldn't. For example, I think it's a very good idea to only challenge students to solve problem with material that they're secure with. Whereas you would get a whole load of literature on productive failure, which would suggest it's a really good idea to give students problems to solve with material they're not secure with. So, there are lots of things where research would suggest two different things. But I think there are certain areas of research where there are perhaps not objective truths, but certainly best bets."

Are there also certain aspects of your book, or in your way of teaching, that are just your style? And for which you think 'well, people could do that, but another way would be just as fine'?

"Yes, lots of things. Worked examples would be the big one. We talked today about how I use *silent teacher* (where I teach and model something in silence). I think it's a good idea and I think there are certain parts of research that suggest it's a good idea. But I certainly don't think it's the only way to do worked examples. I've seen plenty of poor examples of silent teacher, including given by myself. And I've seen plenty of examples of excellent modelling that doesn't use silent teacher. So I think that would be something that is very much up to interpretation.

What I tried to do in the first book, but I really try to do it now: never give



Craig Barton

advice. I will never say 'do this, do that'. I'll always say: 'Here is an idea. Here's what I do, here's why I do it. If you like it, how can you make it work for you?' And I think that's all we can ever do. As soon as you start saying 'this will definitely work', you're in a load of trouble. As soon as you start saying 'do this', you're in a load of trouble as well. It goes back to that idea of using research, but in this case advice, as a compass, not a prescription. I suggest ways, and then put it up to teachers to modify.

The idea of *lethal mutation* is interesting to mention. It's the idea that a teacher reads some research, and it comes out differently in the classroom. A really good example is *knowledge organizers*, where at the start of a unit you present the stu-

dents often with a one page summary of all the key facts and things that are really useful to remember for that unit. Seems fine, seems a smart thing to do. And, there is research to suggest that knowledge organizers indeed are a really good thing to do. The problem is, if it turns into a lethal mutation.

For a knowledge organizer to be good, it has got to be quizzable. Students need to be able to cover up one bit and then test themselves. So you cover up the definition of a word, can you remember the definition? Cover up a word, see the definition; can you remember the word? That's useful. But what happens is: people latch on to the idea that knowledge organizers are good and then all of a sudden every topic in every subject in every school has

a knowledge organizer. But they're useless. They're just a page of notes, and they're just given out to kids for no reason. So that's an example of a lethal mutation.

But sometimes these mutations are good things. Sometimes research will suggest something, and you take an aspect of that research, but you mutate it in a way that works for you. And actually it turns out to be something that the researchers hadn't suggested, but you found a way to make it work.

So I think teachers have got to be really careful. Firstly in actually understanding exactly what the research is suggesting. But then not being afraid to take the core idea and adding their own modifications to make it work. I think that's a smart way to do things."

Today, and also in the book, you talked about many 'effects' that teachers can use to improve their teaching: the self-explanation effect, the hypercorrection effect, the modality effect, and so on. And you also said: don't do everything at the same time. What would you say is the most important one if people want to just try one, or if they have to prioritise some?

"I've been thinking about this a lot. So, I mentioned Sarah Cottingham on my Tips for Teachers podcast, and she also suggested another thing which I've really started thinking about recently. She said: 'There is so much advice out there for teachers. You go on Twitter and there's a hundred ideas for you. You read a book, there's another hundred ideas. You go on a website, another hundred ideas.' And the classic thing is: the fear of missing out. You think: 'If you're not doing this idea, you're a terrible teacher. So I best do that and I best do this and I best do that.' And all of a sudden things are out of control. You tried to change too much, you don't know what you're doing, your kids don't know what's going on, and crucially: if something works, you don't know what has worked, and if something doesn't work you don't know what hasn't worked. So it's really terrible. You've got to prioritise.

And what Sarah suggests is: a good way to decide what to prioritise is to either *look for something you already do and see if there is a way to make it better*. And she would call this a 'tip'. An example of this would be: let's imagine that you use mini-whiteboards in your lessons, and you're

really happy using mini-whiteboards. But you just wonder: 'Is there a way I could use them better?' So, what you do is: you go looking for tips that can improve your mini-whiteboard use. A good example of this for me is: when you say '3...2...1... show me your boards', the key is saying '3...2...1... at the back, show me your boards. Okay, put your boards down. Middle, show me your boards. Okay, front, show me your boards'. That way you don't get this visual overwhelm. So that is an example of a tip. And I went hunting that because it's something I already do, and it's not going to lead to a major change in my practice. So I think looking for tips that build on something that you already do is a good idea.

Your other option is *look for something in your practice that you know isn't working*. An example here would be: you know that your checking for understanding isn't great. So here you think: 'How can I improve my checking for understanding?' And there you might come across mini-whiteboards as a solution, but now that's going to require a bigger change. Now you need to learn all the techniques of mini-whiteboards.

So, I think looking for tips that build on something you already do, or looking for techniques that will fill in a deficiency of something you've identified as a problem with your teaching, that's quite a good strategy to help you prioritise. Having a way to direct what you want to improve on either by identifying a gap in your teaching or identifying something you already do that you can improve, I think is far better than just looking for any old tip and saying 'try that, try that, try that'."

Are there some topics in maths education that you think you could still delve into a bit more?

"When I first started doing the research, I read two things. First, I read research papers and books specifically about maths education. Second, I would read generic teaching books and generic research papers. The Bjorks' *desirable difficulties* would be a classic example, or Doug Lemov's *Teach Like A Champion*, which isn't aimed at any particular subject.

What I'm reading now, though, and I'm finding fascinating, is books or blogs written by non-maths teachers but who are practicing in the classroom. So for me the best by a long way at the moment is Adam Boxer, who is a science teacher, and I'm

currently reading his book *Teaching Secondary Science: A Complete Guide*. And just like I did in my *How I Wish I'd Taught Maths*, he makes clear in the introduction that this is just for science teachers. Every example is either about physics or chemistry or biology; there is nothing else in there. He never says: 'If you are a geography teacher, this will work for teaching volcanos. If you're a French teacher, this will work for whatever.' So, all about science. I think that's fascinating, because you get a real good picture as to what he does with content that he is familiar with. And then the challenge for the reader is: what does this look like for me as a maths teacher. Interestingly, maybe paradoxically, I find that much easier to apply to my subject than a generic teaching book that tries to come up with this broader principle that will work in history, will work in French, will work in maths. I don't find that as powerful as something so focused on a particular subject that actually I can read something and either immediately dismiss it or immediately see 'you know what, I think this could work in maths'.

So I'm reading Adam Boxer's book at the moment and I'm making pages of notes. I can just see how clearly this applies to maths, because you see how it gets implemented in practice. In Adam's book, he writes a whole chapter on how he explains how the heart works. And you think: 'What on earth is that going to teach me about maths?' But I read that and I think: 'You know what, I can use those exact same principles for how I would teach how a quadratic equation works, or quadratic graphs.' So I think seeing how something works specifically in a classroom, that's what I'm interested in at the moment as well. I've not been so brave as to read a book from an English teacher, because I think English and maths are possibly so far removed that it would not be as transferable. But then I don't know, I've not tried that. That's on my list."

Is there a topic that you think requires more research?

"There's lots, actually. Let me mention three quickly. I think *problem solving* is interesting. The problem with problem solving is: lots of the research defines problem solving in a way that we as mathematicians wouldn't. When John Sweller talks about solving problems in maths with cognitive

load theory, his problems are just what we would consider to be kind of fluency problems; it's just examples like 'solve this linear equation', whereas we would consider problems to be non-routine things with different contexts and so on. So, I'd like to see more research into genuine maths problems and either ways to solve them or effective strategies. Because as I say, it seems to be dominated by research into productive failure. And my problem with that research is: I think, from reading the studies, it seems to work best with motivated successful students — students who have experienced success in mathematics, so failure is seen as a short-term state. And if they keep working, they can be successful. Whereas you try that with a student who experienced a lot of failing in mathematics, and you ask them to struggle again, I'm not convinced that works. So I think more research into problem solving would be one thing.

More research into *modelling and explanations* I think is interesting. What makes a good explanation in mathematics, is something that I think a lot of teachers just try to make up on their own. Is there a better way to explain how to solve an equation? Because teachers have all their own favourite methods. Joanne Morgan, a teacher in the UK, has written a brilliant book called *Mathematical Methods*. With lots of different maths topics, there are seven or eight different ways you can go ahead and solve the problem. And often as teachers we either choose our favourite or we expose students to three or four methods, but that feels to me like a bit of a risky policy. It feels to me that there should be the best method to do something. There should be a way to test teaching students different methods and see which one is most effective. But it's difficult, because you've got to control for teacher quality and all that kind of thing. Just feels to me that there has not been enough research into methods, and a lot of teachers tend to just use their favourites. I'm not convinced that's the best way. Imagine you've got a novice teacher, so a teacher who has only been teaching for two or three years. Are they in the best position to determine the best way to teach solving linear equations to a high-attaining 11-year-old versus a low-attaining 14-year-old? They're just guessing. It just feels to me that that's an area where we

could be a bit more informed. But teachers feel very passionate about methods; you get teachers who are very attached to the way they were taught something. And the problem with that is: a lot of maths teachers are good at maths, and the way they were taught it, is perhaps not going to be the best way for someone who is lower-achieving and who struggles with mathematics. So I think, methods would be somewhere I would like to see a bit more research.

And the final one I would just mention is *retrieval practice*. I think this is another example of a lethal mutation; not all practice is retrieval practice and not all retrieval practice is created equal. I think there are good examples of retrieval practice and bad examples of retrieval practice, and I think a bit more support and guidance in that would be useful for teachers."

In your book you say that everything you did before was really bad. Everything was wrong: you didn't take working memory into account, you didn't encourage students to self-explain, and so on. But still, it seems that things were going quite well. Your manager was happy, you were happy, your students seemed to be happy. And the results were also quite okay. So, how bad could it have been, if everyone was happy and the results were good? What was so bad about it?

"Yeah, so it's the classic thing: the results were good, but how much better could they have been? That's the big question. The problem was: a lot of teaching in the UK as recent as five years ago, and maybe even through to today, was driven by what senior leaders in schools perceive that Ofsted (our national inspectorate of schools who come in and make judgements on teaching in schools) wanted. So if they got this idea that Ofsted wanted to see group work in lessons, then everybody better be doing group work. If they believed that Ofsted really liked it when students were up at the board writing, you could be sure that everyone was doing it. So my teaching was very whizz-bang, it had all the gimmicks in there. Any time I was observed, whether it was by teachers of my own school or by Ofsted, it went down a storm; it was brilliant, because it was ticking all the boxes.

And I was just thinking about this the other day: my first head of department, Debbie, was a fantastic teacher. When

I first started teaching when I was 22, Debbie had been teaching by then 20–25 years. In my first year — I cannot believe this — I was giving Debbie some training on how to use interactive whiteboards, how to facilitate group discussions, how to do all these things that I didn't have a clue what I was doing. Debbie, meanwhile, and this is 16 years ago, in her lessons, was using mini whiteboards, was doing mass participation, was doing really good self-explanation questions. And I watched Debbie, and I wouldn't even pay any attention to that, because it wasn't the fancy stuff of teaching that I wanted to do.

I look back and think: you know what, all right, my lessons were fine, I could get the kids on board, so the kids probably did a lot of work outside of lessons. Probably kids did well despite me, not because of me. But if I had known what I know now, back then, the kids would have done so much better, so much better. And crucially, the teachers I worked with back then, they would have been so much better as well."

Would that then have been better regarding motivation of the students, or the results on tests, or maybe it's not even visible on tests because it's more regarding how much they really understand the material?

"Yeah, I think the test scores would have been better. I think our tests are pretty good. Like any test they're not a perfect test of understanding, but our high-stakes tests are pretty good. So I think their tests would have been better. I would have been quizzing my kids more frequently, so they would have remembered more. And because they would have remembered more, we would have been able to do more complex things with that basic understanding, so their problem solving would have been better.

Knowing what I know about motivation, I think the kids probably would also have been more motivated, because they would have been more successful. I probably would have been happier as well. I love teaching, but I would have been happier because I think I had probably been doing less work. Because I used to spend hours planning lessons, because of all these fancy activities.

I think everything would have been better. The only thing that wouldn't have been better is: I would not have been able to

write a book about it, as I wouldn't have been able to go through the crisis."

You wrote that your ideas may transfer to primary school teaching, but how do you think they would transfer to university teaching?

"Interesting. So, I'm always careful to say two things. One: even if you're a secondary maths teacher who teaches the same age group as me, there's no guarantee anything I say will work. It's just ideas and suggestions, and as soon as you go outside of my domain, I'm always sceptical about whether it works. I've been told it works in primary school, but the challenge is always on teachers to see if they can apply the strategies. I think they work better for the older years of primary school.

University, again, I can't see any reason why some of the core things wouldn't work. I mean, you'd be mad not to do retrieval practice, whatever you want your kids to learn. But then we get something like silent teacher and you're doing some more complex multi-step procedure. I think that's when you need to do what we talked about today: you're doing perhaps a little bit in silence, and then get them to reflect on that, then a little bit more in silence. I think you need to adapt. But I think it's more likely to transfer to university maths than it is to, say, secondary school geography or secondary school history and so on. I think probably most of my ideas will transfer okay."

In the Netherlands, the idea of 'guided reinvention' by Hans Freudenthal is quite popular. Guided reinvention looks a bit like guided discovery, a concept that you mention in the book and that you do not seem to be a big fan of. The idea of guided reinvention is that students reinvent the materials, but not just by trying; the teacher ask questions and guides them in the right direction. Is this different from what you call guided discovery?

"This was one of the biggest changes I had to make as a teacher, as a result of reading research. I had it in my head that students should 'discover' something, and by that I certainly don't mean: 'Here is a blank piece of paper, prove Pythagoras.' I mean: 'Here are some prompts for things you should try, what do you notice?' I was convinced that if students did it that way, they were bound to either understand or remember more than if they were simply told that. It just made perfect sense. I can't say hand on heart that I'm convinced anymore.

I think it works better the more structured it is, and the more the teacher is in control. I think that's the key. The 'angle at the centre' theorem is a really good example. It doesn't work when students are given a load of blank circles, and you tell them: 'Put three points on the circumference, connect two of the points to the centre, connect the same two points to the third point on the circumference, measure the angles.' The reason that doesn't work

is: it takes ages and it relies on students being able to fluently measure angles to 'discover' the theorem. I had one child that said: 'The angle at the centre is three times the angle at the circumference.' Whereas, if you use something like GeoGebra, you can control those examples in a certain way, to make it clear to students what the relationship is. They still get that power of being convinced it is true, but without the potential room for misunderstanding that lesser guided reinvention or guided discovery brings into play. So I would always, where possible, try and show students something as well as telling it them, but in the past I was very much of: see if you can figure it out yourself. And I'm not convinced anymore that this works."

You say that there is a big difference between novices and experts in students, and in people in general. So, that also holds for teachers? And, if so, what would that imply for initial teacher training and for professional development? Do you think we should do different things there?

"I think novice teachers struggle a lot with the planning phases of sequences of lessons, because they often find it hard to pitch things. They often find it hard particularly to, what Doug Lemov would say, plan for error, or predict where students are going to go wrong. I think that's quite difficult.

When I look back at my teacher training, I almost wish that I could have had another year of training after I had been teaching for five years. Now I can be taught some of those techniques that I was introduced to before I had the opportunity to try them out. We do have things like 'Master's in Teaching' courses, but it's voluntary. Your school would need to support you in a lot of occasions to do it. I think it's one of those things that should be mandatory, in the same way that teachers should be, obviously, trained before they teach. I think there is a good argument to be made that it should be mandatory to upscale teachers after a certain amount of years.

I think what's really interesting to mention about teacher training: often novice teachers are advised to watch the so-called best teachers in the school, to learn their techniques. The problem with that is: novices can't really learn that much from experts, particularly regarding behaviour. Behaviour is a really interesting concept that

The workshop

It was not entirely clear what we could expect from the workshop we were to be attending on 7 June 2022. The books of course contain enough ideas and examples to elaborate upon. And the website www.mrbartonmaths.com developed into a gateway to a whole universe of websites, not only containing podcasts and links to research papers, but also a nicely accessible collection of teaching resources ordered by topic. Additionally, there are specific collections for teachers concerning diagnostic questions, variation theory and SSDD Problems (Same Surface, Different Depth).

All material can be applied instantly when preparing next day's lessons. And that turned out to be the approach of the workshop: prepare next day's lesson, choosing from an array of ideas and possibilities that were presented. So, we started with breaking down the subject of our lesson to atoms of knowledge, then prioritising the necessary prior knowledge, finding diagnostic questions and deciding how to use them. Then, we covered worked examples. Maybe we could use the 'Self Explanation Effect' or the 'Silent Teacher' approach. The final part of the morning session questioned the benefits and problems regarding students copying worked examples.

The afternoon session was devoted to retrieval (asking questions about material that has already been covered) with an emphasis on 'Low-Stakes Quizzes'. Also, the latest addition to the website family was introduced: 'Tips For Teachers' — an attempt to categorise and make accessible the ideas and lessons learnt over the years.

novice teachers often struggle with. I did myself. So what's the advice given? Well, 'go and watch a certain teacher because he is an expert at behaviour'. So you go watch him or her and the kids are behaving well, but you have no idea why. Because the things they do, they're invisible to novices. So you're much better placed to go and watch somebody who has been teaching for one year or two years, because there isn't as much of a gap between where you're at and where they're at. And you can probably start learning some of their techniques. The problem with that is, not many schools have the culture where trainee teachers observe other relatively novice teachers. And the novice teachers who've been teaching for one or two years, they've got to be quite confident themselves to have other teachers coming in to watch them.

So, I think you need almost a culture in the school, or some programme within universities, where novice teachers work with teachers who are only slightly less novice than them, so you can have those discussions. And then, later on, teachers who have been teaching for five years, they're the ones who can work with the teachers who have been working for ten years and see what they can learn from them. If you've been teaching for six weeks and you go watch someone who has been teaching for twenty years, I'm not convinced that is all that useful an experience."

You make use of diagnostic questions a lot. It can be difficult, though, to get all the students to participate truthfully. When you use the 'fingers in the air' approach that you mention in your book, some of the students quickly look around before answering, to cheat, and this is hard to prevent. How do you do this?

"This is a classic problem. My diagnostic questions are multiple-choice questions: one right answer, three wrong answers. And the idea is that you want mass participation, that's the whole point. The whole reason you're asking these is so you can get the understanding of every child in the class. But what inevitably happens: you say to students '3...2...1... show me your answers' and you'll always get students who do exactly what you describe; they wait. What they end up doing then, is they end up switching their answers to the answer that matches whom they perceive

to be the cleverest student in the class. And all of a sudden, all your advantages of mass participation have gone, because you might as well have just asked the cleverest kid in the class.

So, a couple of strategies for this. In the long term, you want a culture where students realise that there is no point doing that. And the best way to do that is for you to say to kids: 'I'm not collecting in your marks or your responses. The only reason I am asking you this question is so I know what you know and what you don't know, so I can help you.' Kids never believe you the first time you say that, but if you've done diagnostic questions for six weeks and at no point have you ever collected in marks for this, then all of a sudden kids start to realise 'you know what, maybe he is telling the truth here'. It takes time.

But in the short term, there are things you can do as well. Mini-whiteboards are good for this. Fingers have the problem that they're visible; if you're sat behind somebody and they hold up two fingers, you know they voted B. You voted C, so all of a sudden you drop one of your fingers down, and so on. Mini-whiteboards are better, because you can't see from the back of the board what the student has written on the front of the board. So mini-whiteboards work quite well for that.

ABCD-cards are my preferred way for students to vote at diagnostic questions, because if they're different colours, they're a lot easier to distinguish than one finger, two fingers, three fingers, four fingers. The problem with ABCD-cards is: they're also easier to distinguish for the kids. So you can see very quickly that the child in front of you is going for something different than you have and you can switch your answer. Not as quickly as you can with fingers, but you can still switch. The back of the cards can be white, that helps, but then you can have a quick glance down your row, which is still problematic.

I think you can do one thing that can solve a lot of these problems, and it sounds stupid. But just have the students close their eyes. You don't want them closing them before they have seen the question. But if they've seen the question and you say: 'Okay, decide on your answer. Okay, close your eyes. 3...2...1... hold up your answers.' That works quite well. Some teachers will have the kids put their heads on the desk, or cover their eyes, or whatever

works for you. If it's tied in with this idea of 'low stakes' — the fact that no one is going to be judged on this — it works well.

Imagine a scenario where you're one child in the class and you think the correct answer is A, and everyone else thinks it's B, and you hold up A, and everyone else holds up B. Then the teacher says to you: 'Why do you think the answer is A?' You've got to be pretty confident to say 'I think it's this because...'. What you'll probably do, is either say 'Well there's no point of me saying, since I'm wrong', or you'll say 'Everyone else thinks it's B, actually I meant to say B'. Whereas in fact A could be right. Or, more importantly, I want to know why you think it's A so I can help you. But if you close your eyes and you hold up A, and then the teacher says 'everyone put your cards down', then you don't know how many there were. It tends to lend itself better to student discussions when you don't know how many have said it. So that would be a short term fix that works quite well."

In the Netherlands, like in the UK, students have been at home a lot due to Covid. Many of them now have a hard time paying attention in class. They're distracted and they're not used to working hard anymore. Sometimes they don't do their homework anymore. It's difficult for teachers, we hear that quite a lot. The students have a hard time to be motivated, and many of them have also developed a lot of gaps in their knowledge. And, as you cite one of your former students in the first book: 'It's kind of hard to have a growth mindset when I keep doing shit on tests, sir.' This problem seems to apply to many students at the moment. What do you think would be the best course of action over the next months, or even years maybe, to get them back on track?

"It's the same problem in the UK. It's been a nightmare for teachers and students over these last couple of years. The first thing to say is that, like we talked about today in the workshop: assessment of relevant prior knowledge is more important than ever. Because we simply cannot assume students know something. When things were normal, we already couldn't make assumptions. But certainly now, considering that kids have had a disruptive two years, we can't assume anything at all. So for anything we teach students, there has

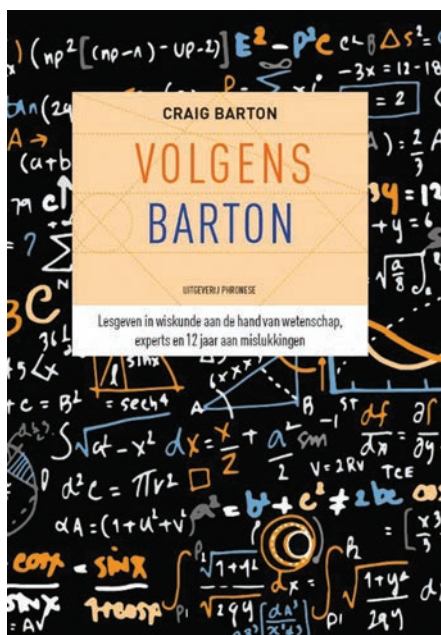
to be a really robust assessment of prior knowledge first. And then based on that assessment, just like we talked about today, we need to respond to it. So if there are gaps in students' knowledge, we need to either reteach it, offer an explanation and retest, or whatever it is — we need to respond. Because, the danger is, our curriculums haven't adapted to what has happened in the last two years. There is still the same amount of content to cover, but if that prior knowledge isn't there and we just try to race through it, we can't cover it.

I'll just use the 'leaky pipe analogy' that I shared today; a primary head teacher in the UK shared this with me. If you want to build a pipe to get from point A to point B, one option is that you start building it, and loads of holes appear, and you think: 'Forget it, it doesn't matter, I've got a deadline here, I'm just going to keep building.' And sure you'll get to point B, but the pipe is useless, because water is leaking out left, right and centre. Whereas option two is: you build the pipe, you notice a hole, so you stop everything and you plug that hole. And then you build a bit more. And then another hole, and you plug that hole. By the end of the time you're probably nowhere near point B, but at least you've got something that is useful. You can carry some water, you've made progress. So I think more than ever we've got to be realistic. If prior knowledge is not there, we have to respond to it.

Talking about confidence, talking about motivation: students are anxious now, more anxious than ever. And if we keep trying to teach the material on shaky grounds, so they keep failing that new material, it's not going to help anybody."

But how do you deal with students that did pay attention last year? You do want them to get to point B, and we don't want to hold them back by doing everything again this year for the ones that didn't do anything last year.

"So this is the eternal problem that teachers always face: differentiation. It's even worse now, and there is definitely no easy solution. There are things that we can do that make things easier. When we're assessing prior knowledge, and we find out that prior knowledge is present in some students, but absent in others, what we certainly shouldn't do, in my opinion, is



The Dutch translation: Craig Barton, *Volgens Barton – Lesgeven in wiskunde aan de hand van wetenschap, experts, en 12 jaar aan mislukkingen*, Uitgeverij Phronese, 2020, 2e druk, 540 p., ISBN 9789490120368, prijs € 34,99.

move one group of students onto the next thing, whilst we work with the other group. I don't think that works. Because then what happens if this group of students has problems with the new thing, then it's a disaster. What we can do is: we can come up with tasks that help students who are secure with that prior knowledge think deeper about that prior knowledge. I outlined one today, where if you assess prior knowledge with a diagnostic question, and it reveals that half your class know and half your class don't know, the half of the class who don't know you can reteach, re-explain and so on; the half of the class that do know can study that diagnostic question. There are two things I like to ask students to do with a diagnostic question.

One is: let's say the correct answer is A, explain why a student might think the correct answer is B, and then write an explanation how you would help them. Now that is a good challenge. And even if a student knows the correct answer, now they're thinking deeper about that subject matter. That's a useful thing to do that helps with differentiation.

The second thing that I spoke about today is: 'The correct answer to this question is A, you know that, fantastic. Change one thing in the question that makes B the right answer. Change one thing in the question that makes C the right answer.' So, now all of a sudden, students aren't just thinking

about that one area. They're now thinking about the other areas that the three wrong answers flag up. But you're doing it in a way that doesn't mean the students are off doing something completely different. And if you've got two or three students that are doing this, or five or ten students, they can swap questions with each other and check each other's work and so on and so on.

So I think you have to prioritise for your time to teach the kids whose prior knowledge isn't there, but I still think you can do really useful things with students whose prior knowledge is secure and get them thinking deeply, but I think that's as best you can do."

Still, the students are then working on the old material. Getting to point B will remain a problem then, probably.

"Yeah, it will still be a problem, but if you look at results of PISA, there is always this classic thing that comes out: students still do well on material they haven't been taught, if they have been taught the other material well. They do far better than students who have raced through everything and don't have that depth of knowledge. So I'd still always go for slower and depth, as opposed to trying to get people through it. But it's not an easy solution."

In your second book you wrote that problem solving was the most problematic topic from the first book, in the sense that it was received with quite some criticism. Can you elaborate a bit on that?

"Yes, that's what a lot of people get mad at me about. Problem solving is a really interesting area. I wouldn't say I've changed my mind on problem solving, but over the last couple of years I've read a lot more and thought a lot more about it. So, I still don't think generic problem solving skills such as 'draw a picture', 'be systematic' or 'solve a simpler problem' are useful. I think they're useful as a label, but I don't think they're useful as problem solving strategies. So, you would get somebody who is an expert in a certain domain, and you say to them 'here's a challenging problem', and they say 'I don't know how to do it' and you say 'can you solve a simpler problem?'. I think maybe they find that useful, maybe they can reduce that problem to something that they can solve and then scale back up again. If they're not secure in the knowledge for that domain, though,

that's not going to help, because... what is a simpler problem? I mean, $2+2$ is a simpler problem, but it's not going to help you solve a graphical challenge or a geometrical challenge. So I don't think those generic problem solving strategies are useful.

But I do think that there are two things that are useful. One is the obvious one: I think *domain-specific strategies* are useful. So, for example, if students have got a problem involving a triangle, it's probably a good idea to make that triangle right-angled. Drawing a radius in a circle is another good example. There are things like that I think you can teach students.

But when I wrote my first book, and even when I wrote my second book, what I didn't think were useful, are generic metacognitive strategies. And now I've read enough to know they are. So this is Alan Schoenfeld's work. He suggests that when students are stuck on a problem, you say things like 'what are you trying to solve?' or 'if you solved this part of the problem, how would it help you, what would you do with it?'. I think things like that do work. I've seen enough research to suggest they work. I've tried that out with enough students to suggest actually I think there is something in that. And those strategies I think can be applied to everything. But that would be an example of something that at the time, I grouped together with the Polya stuff and I thought 'no this doesn't work'. I think I've seen enough now to suggest that metacognitive strategies like that are probably worth building in with students."

In your latest book you describe a model of a learning episode using three components: the worked example, practice and problem solving. In the second component you invite the students to reflect, expect, check and explain. It seems possible to translate this to the four steps of problem solving Polya uses. So, aren't these generic skills after all that you can let students get acquainted with by using their labels and by pointing out 'this is what you're doing here, this is what you're doing there'?

"Yes, I think so. I think it's similar to what we talked about regarding Schoenfeld's metacognitive strategies. So Schoenfeld may ask a student 'why are you doing what you're doing?' and that would apply to any problem. I think the reflect-

expect-check-explain framework is really useful for different parts in the learning episode. So I use it during worked examples. Let's say you are doing silent teacher. You can imagine the teacher pausing for the student to *reflect* and ask himself: 'What has he just done there?' Then the *expect* part: what does the student think the teacher is going to do next? The teacher then does it, the student checks whether it matches what they expected, and they can then *explain*; if it is different, why is it different? If it is exactly what they expected to happen, how would they explain it to somebody who doesn't understand it? So, the framework certainly works in the modelling phase.

I designed the framework for the intelligent practice phase, so a student works through a sequence of related problems. They compare consecutive problems, reflect what's the same, what's different. 'What do I expect?', 'how is the answer going to change?', and so on. It can certainly work as a framework to work through isolated problems, but I think it works best when you have something to compare against. So whether it's something that has just happened and you're trying to predict what is going to happen next, or you've solved a previous problem and there is a relationship between the two problems — I think that's where it is at its most powerful. But it's a way of thinking; if students can exhibit it, they can do so at several points in the learning episode. I think it's a useful thing."

Problem solving seems to be the last stage in the learning episode. Are there other components, like applying mathematics or modelling?

"I would categorise problem solving as a broad label. A more useful label would be something like 'doing non-routine things'. That would, for me, include solving open-ended problems, doing application problems, maybe doing some inquiries. Basically, it's doing something non-routine with the material you have learned. I think Colin Foster has a useful way of describing this. He imagines two types of tunnel. One is where you go into the tunnel and you can see the exit. And the exit may be a long way away, but basically you know what you've got to do. You'll be in that tunnel for a long time, but there is only one way out of it; you just go through that

tunnel. For me that wouldn't be problem solving. That would be something routine. Maybe hard, but it would be routine.

Problem solving is where you walk into a tunnel and you have no idea how to get out. And you may walk around, you may go down some wrong paths and you need to come back on yourself. You may only be in that tunnel for a couple of minutes, but you also may be in there for a couple of days, trying to figure out your way out. I think for me that analogy encompasses what I mean by either 'non-routine' or 'problem solving'. Because, you can get an application problem and until you start messing around with that, you don't really know what that problem is about. You could get an inquiry problem, where you understand what the problem is about, but you have no idea how you are going to find the solution to it.

So, thinking in terms of routine versus non-routine, and the non-routine certainly coming after the routine, I think that is quite a useful way of doing it."

And this is what your next book will be about?

"No, haha. I *am* writing at the moment. I have not told anybody this, this is a world exclusive here. It will be surrounding my 'Tips for Teachers' thing. I really like the idea of a collection of 'tips'. As I said before: you can hunt through to find something that either helps you improve on something you already do or fixes a gap in something you've identified as a problem. And here are a list of techniques that you can try, like Doug Lemov does with *Teach Like A Champion*. But I think there is a slightly different way that I want to present this, that could be potentially useful. So at the moment, what I'm doing is: when I do talks, I'm experimenting with different tips, seeing which ones seem to work. Every time I go back to my phone if I feel like I've said something useful and I type it.

So, my next book will be based around 'Tips for Teachers', but it will be looking at all the things I've learned since I wrote *How I Wish I'd Taught Maths* and my second book. It will be taking core ideas from that and looking at what I think about them now. So, low-stakes quizzes for example. I talked today about fifteen ways to improve low-stakes quizzes. It will be things like that, I think that is what I'll be looking at next." ❦