Scale-Free Collaborative Protocol Design for Output Synchronization of Heterogeneous Multi-Agent Systems With Nonuniform Communication Delays

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***Abstract*—In this paper, we study regulated output synchronization for continuous- or discrete-time heterogeneous multi-agent systems with linear right-invertible agents subject to unknown, nonuniform and arbitrarily large communication delays. It is assumed that all the agents are introspective, meaning that they have access to their own local measurements. A scale-free design framework utilizing localized information exchange has been adopted. The scale-free protocol design is solely based on the knowledge of the agent models such that we do not require any information about the communication networks and the number of agents.**

***Index Terms*—Heterogeneous multi-agent systems, output synchronization, scale-free collaborative protocols, communication delays.**

I. Introduction

Synchronization problem of multi-agent systems (MAS) has become a hot topic among researchers in recent years. Cooperative control of MAS is used in practical application such as robot networks, autonomous vehicles, distributed sensor networks, and others. The objective of synchronization is to secure an asymptotic agreement on a common state or output trajectory by local interaction among agents (see [1]–[4] and references therein).

We identify two classes of MAS: homogeneous and heterogeneous. The agents dynamics are nonidentical in heterogeneous networks. For such networks it is more reasonable to consider output synchronization since the dimensions of states and their physical interpretation may be different. Meanwhile a common assumption, especially for heterogeneous MAS, is that agents are introspective; that is, agents possess some knowledge about their own states. There exist many results about this type of agents. Examples include homogenization-based synchronization via local feedback [5], [6], an internal model principle based design [7], distributed high-gain observer-based design [8], low-and-high-gain-based, purely distributed, linear time invariant protocol design [9], H_∞ design [10], and HJB based optimal synchronization [11]. Other designs can also be found such as feed forward design for nonlinear agents [12], output synchronization under attacks [13], protocol design for agents with input constraints [14], and the work of heterogeneous synchronizations with a novel event triggered protocol [15].

In practical applications, the network dynamics are not-ideal and may be subject to delays. Time delays may afflict system performance or even lead to instability. As discussed in [16], two types of delays have been considered in the literature: input delays and communication delays. The former encapsulates the processing time to execute an input for each agent, whereas the latter can be considered as the time it takes to transmit information from an origin agent to its destination.

Some research work has been done for both constant and time-varying input delay, specifically with the objective of deriving an upper bound on the input delays such that agents can still achieve synchronization; see, for example [17]–[24]. In the case of communication delay, some research work has been done; see [20], [25]–[32]. Time-varying communication delays for a general multi-agent system have been considered in [33]. As it is well-known that in order to withstand large communication delays one needs to preserve diffusiveness (namely to ensure the invariance of the synchronization manifold). This can be achieved via two methods:

1) The first method is the standard state/output synchronization by regulating the states/outputs to a constant trajectory. This method is intensively utilized in the literature [16]. A notable phenomenon in this case is that the final consensus is constant where in many practical problems this would be the case; see for example [34].
2) The second method is to consider delayed state/output synchronization which is introduced in [25], [35]–[38] to allow non-constant or dynamic desired output/state trajectory.

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In this paper, we have considered output synchronization and as such we have utilized the first method for preserving the diffusiveness of the network by regulating the outputs to a constant synchronized trajectory.

It is worth noting that all of the existing literature as reviewed above require some knowledge of the communication network, commonly a bound on the spectrum of the Laplacian matrix and the number of agents where this data is explicitly utilized in the design of protocols. In particular, most of the existing protocols utilize the bound on the real part of the second smallest eigenvalue of the associated Laplacian matrix of the communication network which means the protocols are not *scale-free*, see the results on heterogeneous MAS [5]–[15] and homogeneous MAS with communication delays [31]–[33], [36]. There is a current body of research that shows for a certain class of non-exhaustive graphs, the algebraic connectivity converges to zero as the size of the network increase, see [39]. Therefore, for the networks with protocols that are not scale-free if the size of the network increases the algebraic connectivity of the network tends to zero which leads to loss of synchronization (i.e., instability of the disagreement dynamic).

Recently, we have introduced a new generation of scale-free collaborative protocols for synchronization of homogeneous and heterogeneous MAS where the agents are subject to input saturation, disturbances, and input delays, see [40]–[43]. The *scale-free collaborative* protocol means the design is independent of the information about the associated communication graph or the size of the network, i.e., the number of agents.

In this paper, we deal with scale-free regulated output synchronization problem for heterogeneous MAS with continuous- or discrete-time introspective right-invertible agents in the presence of unknown, non-uniform, and arbitrarily large communication delays. The main contribution of this paper is designing protocols for MAS subject to unknown, nonuniform, and arbitrarily large communication delays in *scale-free* framework such that:

- The linear dynamic protocols are designed solely based on the knowledge of agent models and do not depend on information about the communication network such as the spectrum of the associated Laplacian matrix.
- The design is scale-free and does not require knowledge of the number of agents. That is to say, the universal dynamical protocols work for any communication network as long as it is connected.
- The proposed protocols achieve regulated output synchronization for heterogeneous continuous- or discrete-time MAS with any unknown, nonuniform, and arbitrarily large communication delays.

Notations and preliminaries

We denote the set of real numbers by $\mathbb{R}$, the set of integer numbers by $\mathbb{Z}$, the set of non-negative real numbers by $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geq 0\}$, the set of non-negative integer numbers by $\mathbb{Z}_{\geq 0}$, and the entire complex plane by $\mathbb{C}$. Given a matrix $A \in \mathbb{R}^{n\times m}$, A^{T} denotes the transpose of A. Let $\mathbf{j}$ indicate $\sqrt{-1}$. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane, and A is said to be Schur stable if all its eigenvalues are in the closed unit disk. We denote by $\mathrm{diag}\{A_1, \ldots, A_N\}$, a block-diagonal matrix with $A_1, \ldots, A_N$ as its diagonal elements. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context.

For $\bar{A} \in \mathbb{C}^{n\times m}$ and $\bar{B} \in \mathbb{C}^{p\times q}$, the Kronecker product of $\bar{A}$ and $\bar{B}$ is defined as

$$\bar{A} \otimes \bar{B} = \begin{pmatrix} \bar{a}_{11}\bar{B} & \ldots & \bar{a}_{1m}\bar{B} \\ \vdots & \ddots & \vdots \\ \bar{a}_{n1}\bar{B} & \ldots & \bar{a}_{nm}\bar{B} \end{pmatrix},$$

where $[\bar{A}]_{ij} = \bar{a}_{ij}$. The following properties of the Kronecker product will be particularly useful:

$$(\bar{A} \otimes \bar{B})(\bar{C} \otimes \bar{D}) = (\bar{A}\bar{C}) \otimes (\bar{B}\bar{D}),$$
$$\bar{A} \otimes (\bar{B} + \bar{E}) = \bar{A} \otimes \bar{B} + \bar{A} \otimes \bar{E},$$

where $\bar{C} \in \mathbb{C}^{m\times r}$, $\bar{D} \in \mathbb{C}^{q\times s}$, and $\bar{E} \in \mathbb{C}^{p\times q}$.

To describe the information flow among the agents we associate a *weighted graph* $\mathcal{G}$ to the communication network. The weighted graph $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix with non-negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e., we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. A *directed tree* is a subgraph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. The *root set* is the set of root nodes. A *directed spanning tree* is a subgraph which is a directed tree containing all the nodes of the original graph. If a directed spanning tree exists, the root has a directed path to every other node in the tree.

For a weighted graph $\mathcal{G}$, the *weighted in-degree* of node i is given by $d_{\mathrm{in}}(i) = \sum_{j=1}^{N} a_{ij}$. The matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$ [44]. Moreover, if the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane [3].

A linear time-invariant system Σ described by

$$\Sigma : \begin{cases} \dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t), \\ y(t) = \tilde{C}x(t), \end{cases}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$. The system Σ is right-invertible if, given a smooth reference output y_r, there exists an initial condition $x(0)$ and an input $u(t)$ that ensures $y(t) = y_r$ for all $t \geq 0$.

Remark 1: The system Σ is right-invertible if and only if the rank of $\begin{pmatrix} sI - \tilde{A} & -\tilde{B} \\ \tilde{C} & 0 \end{pmatrix}$ is $n + p$ for all but finitely many $s \in \mathbb{C}$.

Remark 2: The definition of right-invertibility for discrete-time systems is same as the definition for the continuous-time systems stated above.

II. Problem Formulation

Consider a MAS consisting of N non-identical linear agents:

$$\begin{cases} x_i^+(t) = A_i x_i(t) + B_i u_i(t), \\ y_i(t) = C_i x_i(t), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^p$ are the state, input, and output of agent i for $i = 1, \ldots, N$. In the aforementioned presentation, for continuous-time systems, s denotes the time derivative, i.e., $x_i^+(t) = \dot{x}_i(t)$ for $t \in \mathbb{R}$; while for discrete-time systems, s denotes the time shift, i.e., $x_i^+(t) = x_i(t+1)$ for $t \in \mathbb{Z}$.

The agents are introspective, meaning that each agent collects a local measurement $z_i \in \mathbb{R}^{q_i}$ of its internal dynamics. In other words, each agent has access to the quantity

$$z_i(t) = C_i^m x_i(t). \tag{2}$$

First, we focus on *continuous-time* networks. The communication network provides agent i with the following information which is a linear combination of its own output relative to that of other agents:

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})), \tag{3}$$

where $\tau_{ij} \in \mathbb{R}_{\geq 0}$ represents an unknown communication delay from agent j to agent i and $\tau_{ii} = 0$. $a_{ij} \geq 0$, and $a_{ii} = 0$. This communication topology of the network, presented in (3), can be associated with a weighted graph $\mathcal{G}$ in with each node indicating an agent in the network and the weight of an edge is given by the coefficient a_{ij}. The communication delay implies that it takes τ seconds for agent j to transfer its state information to agent i.

In terms of the coefficient of the associated Laplacian matrix L, $\zeta_i(t)$ can be represented as

$$\zeta_i(t) = \sum_{j=1}^{N} \ell_{ij} y_j(t - \tau_{ij}). \tag{4}$$

Our goal is to achieve regulated output synchronization among all agents while the synchronized dynamics equals to a constant trajectory. Clearly, we need some level of communication between the reference trajectory and the agents. We assume that a nonempty subset $\mathscr{C}$ of the agents have access to their own output relative to the reference trajectory $y_r \in \mathbb{R}^p$. In other words, each agent has access to the quantity

$$\psi_i(t) = \iota_i(y_i(t) - y_r), \qquad \iota_i = \begin{cases} 1, & i \in \mathscr{C}, \\ 0, & i \notin \mathscr{C}. \end{cases} \tag{5}$$

Therefore, the information available for agent $i \in \{1, \ldots, N\}$, is given by

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})) + \iota_i(y_i(t) - y_r). \tag{6}$$

Hereinafter, we refer to the node set $\mathscr{C}$ as root set. For any graph with the Laplacian matrix L, we define the expanded Laplacian matrix as

$$\bar{L} = L + \operatorname{diag}\{\iota_i\} = [\bar{\ell}_{ij}]_{N \times N}, \tag{7}$$

which is not a regular Laplacian matrix associated to the graph, since the sum of its rows does not need to be zero. Meanwhile, it should be emphasized that $\bar{\ell}_{ij} = \ell_{ij}$ for $i \neq j$ in $\bar{L}$. Then, (6) can be rewritten as

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} \bar{\ell}_{ij}(y_j(t - \tau_{ij}) - y_r). \tag{8}$$

To guarantee that each agent can achieve the required regulation, we need to make sure that there exists a pass to each node starting with node from the set $\mathscr{C}$. Therefore, we denote the following set of graphs.

Definition 1: Given a node set $\mathscr{C}$, we denote by $\mathbb{G}_{\mathscr{C}}^N$ the set of all directed graphs with N nodes containing the node set $\mathscr{C}$, such that every node of the network graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^N$ is a member of a directed tree which has its root contained in the node set $\mathscr{C}$. Note that this definition does not require necessarily the existence of directed spanning tree.

Remark 3: From [8, Lemma 7] it follows for any $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^N$ defined in Definition 1, the associated expanded Laplacian matrix $\bar{L}$ as defined by (7) is invertible and all the eigenvalues of $\bar{L}$ have positive real parts.

We also introduce a localized information exchange among agents. In particular, each agent $i = 1, N$ has access to the following information denoted by $\hat{\zeta}_i(t)$, of the form

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\xi_i(t) - \xi_j(t - \tau_{ij})), \tag{9}$$

where $\xi_j \in \mathbb{R}^n$ is a variable produced internally by agent j and will be defined in next sections. Given that agents communicate y_i and ξ_i over the same communication networks, the communication delays τ_{ij} between agent j and agent i are the same in (4) and (9).

In the case of networks with *discrete-time* agents, each agent has access to the following information

$$\bar{\zeta}_i(t) = \frac{1}{2+D_{\text{in}}(i)}\sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t-\tau_{ij})) + \iota_i(y_i(t) - y_r), \tag{10}$$

where $D_{\text{in}}(i)$ is the upper bound of $d_{in}(i) = \sum_{j=1}^{N} a_{ij}$ for $i = 1,\dots,N$. Here, $\tau_{ij} \in \mathbb{Z}_{\geq 0}$ and $\tau_{ii} = 0$.

For any graph $\mathcal{G} \in \mathbb{G}_{\mathscr{C}}^N$, with the associated Expanded Laplacian matrix $\bar{L}$, we define

$$\bar{D} = I_N - (2I_N + D_{\text{in}})^{-1}\bar{L}, \tag{11}$$

where $D_{\text{in}} = \text{diag}\{D_{\text{in}}(1), D_{\text{in}}(2),\dots,D_{\text{in}}(N)\}$. It is easily verified that the matrix $\bar{D}$ is a matrix with all elements being non-negative and the sum of each row is less than or equal to 1. The matrix $\bar{D}$ has all eigenvalues in the open unit disk if and only if every node of the network graph $\mathcal{G}$ is a member of a directed tree which has its root contained in the set $\mathscr{C}$ [32, Lemma 1].

Therefore, for discrete-time networks we can rewrite the information exchange (10) as

$$\begin{aligned}\bar{\zeta}_i(t) &= \frac{1}{2+D_{\text{in}}(i)}\sum_{j=1}^{N} \bar{\ell}_{ij}(y_j(t-\tau_{ij}) - y_r)\\ &= y_i(t) - y_r - \sum_{j=1}^{N} \bar{d}_{ij}(y_j(t-\tau_{ij}) - y_r)\end{aligned} \tag{12}$$

with $\bar{D} = [\bar{d}_{ij}]_{N\times N}$, and we can rewrite $\hat{\zeta}_i(t)$ as

$$\hat{\zeta}_i(t) = \frac{1}{2+D_{\text{in}}(i)}\sum_{j=1}^{N} a_{ij}(\xi_i(t) - \xi_j(t-\tau_{ij})), \tag{13}$$

where $\tau_{ij} \in \mathbb{Z}_{\geq 0}$ with $\tau_{ii} = 0$.

We formulate the problem of regulated output synchronization for heterogeneous networks subject to unknown, nonuniform and arbitrarily large communication delays utilizing linear scale-free collaborative protocols as follows.

Problem 1: Consider a MAS described by (1) with local information (2) and a given constant reference trajectory $y_r \in \mathbb{R}^p$. Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}_{\mathscr{C}}^N$.

Then, the *scalable regulated output synchronization problem based on localized information exchange utilizing collaborative protocols* for heterogeneous networks subject to unknown, nonuniform and arbitrarily large communication delays is to find, if possible, a linear dynamic protocol for each agent $i \in \{1,,N\}$, using only knowledge of agent model, i,e. (A_i, B_i, C_i), of the form

$$\begin{cases} x_{i,c}^+ = A_{i,c}x_{i,c} + B_{i,c}\bar{\zeta}_i + C_{i,c}\hat{\zeta}_i + D_{i,c}z_i,\\ u_i = E_{i,c}x_{i,c} + F_{i,c}\bar{\zeta}_i + G_{i,c}\hat{\zeta}_i + H_{i,c}z_i,\end{cases} \tag{14}$$

where $\bar{\zeta}_i$ and $\hat{\zeta}_i$ are defined by (8) and (9) for continuous-time MAS and are defined by (12) and (13) for discrete-time MAS with $\xi_i = H_{i,c}x_{i,c}$, and $x_{c,i} \in \mathbb{R}^{n_{c_i}}$, such that regulated output

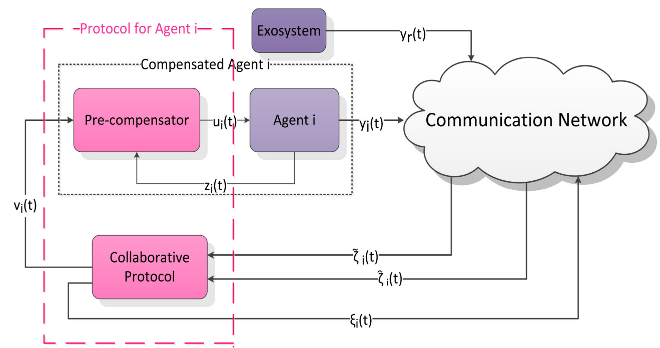

Fig. 1. Architecture of the protocol for regulated output synchronization.

synchronization

$$\lim_{t\to\infty}(y_i(t) - y_r) = 0, \text{ for } i \in \{1,\dots,N\} \tag{15}$$

is achieved for any N and any graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^N$ and any communication delay $\tau_{ij} \in \mathbb{R}_{\geq 0}$ and $\tau_{ij} \in \mathbb{Z}_{\geq 0}$ for continuous- and discrete-time MAS with $i \neq j$ respectively.

We make the following assumptions for the agents:

Assumption 1: For agents $i \in \{1,\dots,N\}$,

1) (C_i, A_i, B_i) is stabilizable, detectable and right-invertible.
2) (C_i^m, A_i) is detectable.

Remark 4: Assumption 1 is the sufficient condition on designing pre-compensator. Right-invertibility means that there exist the initial conditions $x_i(0)$ and $u_i(0)$ such that output $y_i(t) = y_r$ for all $t \geq 0$. This condition is usually required in the literature on heterogeneous MAS since synchronization requires output regulation and right-invertibilty is a well-known condition for output regulation. Meanwhile, stabilizability and detectability of (C_i, A_i, B_i) and detectability of (C_i^m, A_i) guarantee the compensated system is stabilizable and detectable. The detailed design process on how to apply Assumption 1 can be found in [6, Appendix B] for continuous-time and [45, Appendix A.1] for discrete-time systems.

III. Protocol Design

In this section, we design scale-free protocols to solve regulated output synchronization problem for heterogeneous networks of continuous- or discrete-time agents in the presence of communication delays.

Architecture of the protocol

The protocol for regulated output synchronization of heterogeneous MAS in the presence of communication delays has two main modules as shown in Fig. 1.

1) The first module designs precompensators given the chosen target model to homogenize the heterogeneous agents following our previous results stated in [6], [45].
2) The second module designs collaborate protocols for almost homogenized agents to achieve regulated output synchronization in the presence of communication delays.

For solving scalable regulated output synchronization, our design procedure consists of three steps.

A. Step 1: Choosing the Target Model

First, let $\bar{n}_d$ denote the maximal order of infinite zeros of (C_i, A_i, B_i), $i = 1, \ldots, N$. We choose target model (C, A, B) is invertible, of uniform rank $n_q \geq \bar{n}_d$, and has no invariant zeros. Without loss of generality, we choose the target model, i.e., (C, A, B) as

$$A = \begin{pmatrix} 0 & I_{p(n_q-1)} \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ I_p \end{pmatrix}, \quad C = \begin{pmatrix} I_p & 0 \end{pmatrix}$$

for continuous-time case and

$$A = \begin{pmatrix} 0 & I_{p(n_q-1)} \\ 0 & 0 \end{pmatrix} + I_{pn_q}, \quad B = \begin{pmatrix} 0 \\ I_p \end{pmatrix}, \quad C = \begin{pmatrix} I_p & 0 \end{pmatrix}$$

for discrete-time case.

B. Step 2: Designing Pre-Compensators

In this step, we design pre-compensators to reshape agent models to almost identical agents. Given chosen target model (C, A, B), by utilizing the design methodology from [6, Appendix B] for continuous-time and [45, Appendix A.1] for discrete-time systems, we design a pre-compensator for each agent $i \in \{1, \ldots, N\}$, of the form

$$\begin{cases} \eta_i^+ = A_{i,h}\eta_i + B_{i,h}z_i + E_{i,h}v_i, \\ u_i = C_{i,h}\eta_i + D_{i,h}v_i, \end{cases} \tag{16}$$

which gives the compensated agents as

$$\begin{cases} \bar{x}_i^+ = A\bar{x}_i + B(v_i + \rho_i), \\ y_i = C\bar{x}_i, \end{cases} \tag{17}$$

where ρ_i is given by

$$\begin{cases} w_i^+ = A_{i,s}w_i, \\ \rho_i = C_{i,s}w_i, \end{cases} \tag{18}$$

and $A_{i,s}$ is Hurwitz stable (Schur stable for discrete-time systems). Note that the compensated agents are homogenized and have the target model (C, A, B).

C. Step 3: Designing Collaborative Protocols for the Compensated Agents

In this section, to achieve regulated output synchronization, we design collaborative protocols for almost homogenized continuous- or discrete-time agents in the presence of unknown, non-uniform and arbitrarily large communication delays. Since we use stochastic matrices with proper scaling of data in the case of discrete-time MAS (see (10)), it requires completely different methodology for analyzing the stability. Then, we split this step to two parts to show the completed design clearly.

1) Continuous-Time MAS: Collaborative protocols based on localized information exchanges are designed for the continuous-time compensated agents (17) and (18) with $i \in \{1, \ldots, N\}$ as follows:

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i - BK\hat{\zeta}_i + H(\bar{\zeta}_i - C\hat{x}_i) + \iota_i Bv_i, \\ \dot{\chi}_i = A\chi_i + Bv_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ v_i = -K\chi_i, \end{cases} \tag{19}$$

where H and K are design matrices such that $A - HC$ and $A - BK$ are Hurwitz stable. The exchanging information $\hat{\zeta}_i$ is defined as (9) and $\bar{\zeta}_i$ is defined as (8) with the agents communicate $\xi_i = \chi_i$.

Finally, we combine the designed protocol for homogenized network with pre-compensators and present our protocols as:

$$\begin{cases} \dot{\eta}_i = A_{i,h}\eta_i + B_{i,h}z_i - E_{i,h}K\chi_i, \\ \dot{\hat{x}}_i = A\hat{x}_i - BK\hat{\zeta}_i + H(\bar{\zeta}_i - C\hat{x}_i) - \iota_i BK\chi_i, \\ \dot{\chi}_i = A\chi_i - BK\chi_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ u_i = C_{i,h}\eta_i - D_{i,h}K\chi_i. \end{cases} \tag{20}$$

Then, we have the following theorem for scalable regulated output synchronization of continuous-time heterogeneous MAS in the presence of communication delays.

Theorem 1: Consider a heterogeneous network of N agents (1) and (3) satisfying Assumption 1 with local information (2). Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}_{\mathscr{C}}^N$.

Then, the scalable regulated output synchronization problem utilizing localized information exchange via linear dynamic protocol as stated in Problem 1 is solvable for any $y_r \in \mathbb{R}^p$. More specifically, for any given constant reference trajectory $y_r \in \mathbb{R}^p$, protocol (20) achieves scalable regulated output synchronization for any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$ and any graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^N$ with any size of the network N.

To obtain the result of Theorem 1, we need the following lemmas where Lemma 1 is a classical result for the stability of linear time-delayed system (see [31], [46]) and Lemma 2 has been used in the literature for consensus of MAS in the presence of delay.

Lemma 1: ([31, Lemma 3]) Consider a linear time-delay system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{21}$$

where $x(t) \in \mathbb{R}^n$ and $\tau_i \in [0, \bar{\tau}]$ with $\bar{\tau} > 0$. Assume that $A + \sum_{i=1}^{m} A_i$ is Hurwitz stable. Then, (21) is asymptotically stable for $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$ if

$$\det\left[\mathbf{j}\omega I - A - \sum_{i=1}^{m} e^{-\mathbf{j}\omega\tau_i} A_i\right] \neq 0 \tag{22}$$

for all $\omega \in \mathbb{R}$, and for all $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$.

Lemma 2: ([31, Lemma 1]) Let α be a lower bound for the eigenvalues of $\bar{L}$. Then, for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$, $(i, j = 1, \ldots, N)$ and all $\omega \in \mathbb{R}$, the real part of all eigenvalues of $\bar{L}_s(\tau)$ will be larger than or equal to α, where

$$\bar{L}_s(\tau) = \begin{pmatrix} \bar{\ell}_{11} & \cdots & \bar{\ell}_{1k}e^{-s\tau_{1k}} & \cdots & \bar{\ell}_{1N}e^{-s\tau_{1N}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\ell}_{k1}e^{-s\tau_{k1}} & \cdots & \bar{\ell}_{kk} & \cdots & \bar{\ell}_{kN}e^{-s\tau_{kN}} \\ \vdots & \cdots & \vdots & \ddots & \vdots \\ \bar{\ell}_{N1}e^{-s\tau_{N1}} & \cdots & \bar{\ell}_{Nk}e^{-s\tau_{Nk}} & \cdots & \bar{\ell}_{NN} \end{pmatrix} \tag{23}$$

is the expanded Laplacian matrix in the frequency domain and τ denotes a vector consisting of all $\tau_{ij}(i \neq j)$ with $i \in \{1, \ldots, N\}$.

Proof of Theorem 1: For continuous-time MAS, the compensated MAS can be written as

$$\begin{cases} \dot{\bar{x}}_i = A\bar{x}_i + B(v_i + C_{i,s}w_i), \\ \dot{w}_i = A_{i,s}w_i, \\ y_i = C\bar{x}_i, \end{cases} \tag{24}$$

where $A_{i,s}$ is Hurwitz stable.

Let $\Pi = \begin{pmatrix} I_p \\ 0 \end{pmatrix}$ such that we have

$$A\Pi = 0 \text{ and } C\Pi = I_p.$$

Then, by defining $\tilde{y}_i = y_i - y_r$ and $\tilde{x}_i = \bar{x}_i - \Pi y_r$, we have

$$\begin{cases} \dot{\tilde{x}}_i = A\bar{x}_i + B(v_i + C_{i,s}w_i), \\ \dot{w}_i = A_{i,s}w_i, \\ y_i = C\bar{x}_i. \end{cases} \tag{25}$$

Next, we obtain the closed-loop system as

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i - BK\chi_i + BC_{i,s}w_i, \\ \dot{\hat{x}}_i &= A\hat{x}_i - BK\hat{\zeta}_i + H(\bar{\zeta}_i - C\hat{x}_i) - \iota_i BK\chi_i, \\ \dot{\chi}_i &= A\chi_i - BK\chi_i + \hat{x}_i - \hat{\zeta}_i - \iota_i\chi_i, \\ \dot{w}_i &= A_{i,s}w_i, \\ \tilde{y}_i &= C\tilde{x}_i. \end{aligned}$$

By defining

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_N \end{pmatrix}, \hat{x} = \begin{pmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{pmatrix}, \chi = \begin{pmatrix} \chi_1 \\ \vdots \\ \chi_N \end{pmatrix}, w = \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix}, \tilde{y} = \begin{pmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_N \end{pmatrix},$$

we have the following closed-loop system in frequency domain as

$$\begin{aligned} s\tilde{x} &= (I_N \otimes A)\tilde{x} - (I_N \otimes (BK))\chi + (I_N \otimes B)C_s w, \\ s\hat{x} &= (I_N \otimes (A - HC))\hat{x} - (\bar{L}_s(\tau) \otimes (BK))\chi \\ &\quad + (\bar{L}_s(\tau) \otimes (HC))\tilde{x}, \\ s\chi &= (I_N \otimes (A - BK) - \bar{L}_s(\tau) \otimes I)\chi + \hat{x}, \\ sw &= A_s w, \\ \tilde{y} &= (I_N \otimes C)\tilde{x}, \end{aligned}$$

where $A_s = \text{diag}(A_{i,s})$ and $C_s = \text{diag}(C_{i,s})$ for $i = \{1, \ldots, N\}$. Let $\delta = \tilde{x} - \chi$, and $\bar{\delta} = (\bar{L}_s(\tau) \otimes I)\tilde{x} - \hat{x}$, then we have

$$s\tilde{x} = (I_N \otimes (A - BK))\tilde{x} + (I_N \otimes (BK))\delta + (I_N \otimes B)C_s w, \tag{26}$$

$$s\bar{\delta} = (I_N \otimes (A - HC))\bar{\delta} + (\bar{L}_s(\tau) \otimes B)C_s w, \tag{27}$$

$$s\delta = (I_N \otimes A - \bar{L}_s(\tau) \otimes I)\delta + \bar{\delta} + (I_N \otimes B)C_s w, \tag{28}$$

$$sw = A_s w. \tag{29}$$

We need to show the asymptotic stability of (26)–(29) for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$. Since A_s and $A - HC$ are stable, we have w and $\bar{\delta}$ is asymptotically stable. As such, asymptotic stability of (27) and (29) are implied by asymptotic stability of the following reduced system:

$$\begin{pmatrix} s\tilde{x} \\ s\delta \end{pmatrix} = \begin{pmatrix} I_N \otimes (A - BK) & I_N \otimes (BK) \\ 0 & I_N \otimes A - \bar{L}_s(\tau) \otimes I \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \delta \end{pmatrix}. \tag{30}$$

Following Lemma 1, we prove the stability of (30) in two steps. In the first step, we prove the stability in the absence of communication delays and in the second step we prove the stability of (30) by checking condition (22).

1) When there is no communication delay in the network, the stability of system (30) is equivalent to asymptotic stability of the matrix

$$\begin{pmatrix} I_N \otimes (A - BK) & I_N \otimes (BK) \\ 0 & I_N \otimes A - \bar{L} \otimes I \end{pmatrix}. \tag{31}$$

According to Remark 3, since eigenvalues $\lambda_1, \cdots, \lambda_N$ of $\bar{L}$ have positive real part, we have

$$\begin{aligned} &(T \otimes I)(I_N \otimes A - \bar{L} \otimes I)(T^{-1} \otimes I) \\ &\quad = I_N \otimes A - \bar{J} \otimes I \end{aligned} \tag{32}$$

for a non-singular transformation matrix T satisfying $T^{-1}\bar{J}T = \bar{L}$, where $\bar{J}$ denotes the upper triangular Jordan form of $\bar{L}$. And (32) is upper triangular Jordan form with $A - \lambda_i I$ for $i = 1, \ldots, N$ on the diagonal. Since A has all eigenvalues at the origin, $A - \lambda_i I$ is stable. Therefore, all eigenvalues of $I_N \otimes A - \bar{L} \otimes I$ have negative real parts. Then, since we have $I_N \otimes A - \bar{L} \otimes I$ is asymptotically stable, we just need to prove the stability of

$$\dot{\tilde{x}} = I_N \otimes (A - BK)\tilde{x} \tag{33}$$

with $A - BK$ being Hurwitz stable. Therefore, we can obtain the asymptotic stability of (30), i.e.,

$$\lim_{t\to\infty} \tilde{x}_i \to 0,$$

which implies that $\tilde{y}_i \to 0$, i.e., $y_i \to y_r$.

2) Next, in the light of Lemma 1, the closed-loop system (30) is asymptotically stable for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$, if

$$\det\left[\ \mathbf{j}\omega I - \begin{pmatrix} I_N \otimes (A - BK) & I_N \otimes (BK) \\ 0 & I_N \otimes A - \bar{L}_{\mathbf{j}\omega}(\tau) \otimes I \end{pmatrix}\right] \neq 0 \tag{34}$$

for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$. Inequality (34) is satisfied if the matrix

$$\begin{pmatrix} I_N \otimes (A - BK) & I_N \otimes (BK) \\ 0 & I_N \otimes A - \bar{L}_{\mathbf{j}\omega}(\tau) \otimes I \end{pmatrix} \tag{35}$$

does not have any eigenvalue on the imaginary axis for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$. Utilizing Lemma 2, we have that all eigenvalues of $\bar{L}_{\mathbf{j}\omega}(\tau)$ have positive real part for any τ_{ij}. Therefore,

$$I_N \otimes \bar{A} - \bar{L}_{\mathbf{j}\omega}(\tau) \otimes I$$

has all negative real part eigenvalues. It implies that all eigenvalues of matrix (35) have negative real parts, i.e., matrix (35) does not have any eigenvalue on the imaginary axis for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$. Thus we have

$$\tilde{x}_i \to 0 \text{ i.e., } y_i \to y_r$$

which means the synchronization $y_i \to y_j$ is achieved. ■

2) Discrete-Time MAS: In this subsection, we design collaborative protocols with localized information exchange for the discrete-time compensated agent models (17) and (18) with $i \in \{1, \ldots, N\}$ as follows:

$$\begin{cases} \chi_i(t+1) = A\chi_i(t) + Bv_i(t) + A\hat{x}_i(t) \\ \qquad -A\hat{\zeta}_i(t) - \frac{\iota_i}{2+D_{\text{in}}(i)} A\chi_i(t), \\ \hat{x}_i(t+1) = A\hat{x}_i(t) + H(\bar{\zeta}_i(t) - C\hat{x}_i(t)) \\ \qquad -BK\hat{\zeta}_i(t) + \frac{\iota_i}{2+D_{\text{in}}(i)} Bv_i(t), \\ v_i(t) = -K\chi_i(t), \end{cases} \tag{36}$$

where H and K are matrices such that $A - HC$ and $A - BK$ are Schur stable.

The network information $\bar{\zeta}_i$ is defined by (12) and $\hat{\zeta}_i$ is defined by (13) with the agents communicate $\xi_i = \chi_i$. Similar to the design procedure in the previous section, we combine the collaborative protocols and pre-compensators to get the final protocol as:

$$\begin{cases} \eta_i(t+1) = A_{i,h}\eta_i(t) + B_{i,h}z_i(t) - E_{i,h}K\chi_i(t), \\ \hat{x}_i(t+1) = A\hat{x}_i(t) + H(\bar{\zeta}_i(t) - C\hat{x}_i(t)) \\ \qquad -BK\hat{\zeta}_i(t) - \frac{\iota_i}{2+D_{\text{in}}(i)} BK\chi_i(t), \\ \chi_i(t+1) = (A - BK)\chi_i(t) + A\hat{x}_i(t) \\ \qquad -A\hat{\zeta}_i(t) - \frac{\iota_i}{2+D_{\text{in}}(i)} A\chi_i(t), \\ u_i(t) = C_{i,h}\eta_i(t) - D_{i,h}K\chi_i(t). \end{cases} \tag{37}$$

Then, we have the following theorem for scalable regulated output synchronization of discrete-time heterogeneous MAS in the presence of communication delays.

Theorem 2: Consider a heterogeneous network of N agents (1) and (12) satisfying Assumption 1 with local information (2). Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}_{\mathscr{C}}^N$.

Then, the scalable regulated output synchronization problem utilizing localized information exchange via linear dynamic protocol as stated in Problem 1 is solvable for any $y_r \in \mathbb{R}^p$. More specifically, for any given constant reference trajectory $y_r \in \mathbb{R}^p$, protocol (37) achieves scalable regulated output synchronization for any communication delays $\tau_{ij} \in \mathbb{Z}_{\geq 0}$ $(i \neq j)$ and any graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^N$ with any size of the network N.

We also recall the following lemma from [23] for stability of discrete-time systems.

Lemma 3: ([23, Lemma 6]) Consider a linear time-delay system

$$x(t+1) = Ax(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{38}$$

where $x(t) \in \mathbb{R}^n$ and $\tau_i \in \mathbb{N}^+$. Suppose $A + \sum_{i=1}^m A_i$ is Schur stable. Then, (38) is asymptotically stable if

$$\det[e^{j\omega} I - A - \sum_{i=1}^{m} e^{-j\omega\tau_i^r} A_i] \neq 0 \tag{39}$$

for all $\omega \in [-\pi, \pi]$ and for all $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$.

Proof of Theorem 2: For discrete-time MAS, the compensated MAS model can be written as

$$\begin{cases} \bar{x}_i(t+1) = A\bar{x}_i(t) + B(v_i(t) + C_{i,s}w_i(t)), \\ w_i(t+1) = A_{i,s}w_i(t), \\ y_i(t) = C\bar{x}_i(t), \end{cases} \tag{40}$$

where $A_{i,s}$ is Schur stable. Similar to the case of continuous-time in Theorem 1, we let $\Pi = \begin{pmatrix} I_p \\ 0 \end{pmatrix}$, then we have

$$A\Pi = \Pi \text{ and } C\Pi = I_p.$$

Let $\tilde{y}_i(t) = y_i(t) - y_r$ and $\tilde{x}_i(t) = \bar{x}_i(t) - \Pi y_r$ such that

$$\begin{cases} \tilde{x}_i(t+1) = A\tilde{x}_i(t) + B(v_i(t) + C_{i,s}w_i(t)), \\ w_i(t+1) = A_{i,s}w_i(t), \\ \tilde{y}_i(t) = C\tilde{x}_i(t). \end{cases} \tag{41}$$

We obtain the closed-loop system as

$$\begin{aligned}
\tilde{x}_i(t+1) &= A\tilde{x}_i(t) - BK\chi_i(t) + BC_{i,s}w_i(t),\\
\hat{x}_i(t+1) &= A\hat{x}_i(t) + H(\bar{\zeta}_i(t) - C\hat{x}_i(t))\\
&\quad - BK\hat{\zeta}_i(t) - \frac{\iota_i}{2+D_{\text{in}}(i)}BK\chi_i(t),\\
\chi_i(t+1) &= A\chi_i(t) - BK\chi_i(t) + A\hat{x}_i(t)\\
&\quad - A\hat{\zeta}_i(t) - \frac{\iota_i}{2+D_{\text{in}}(i)}A\chi_i(t),\\
w_i(t+1) &= A_{i,s}w_i(t),\\
\tilde{y}_i(t) &= C\tilde{x}_i(t).
\end{aligned}$$

By defining

$$\tilde{x}(t) = \begin{pmatrix}\tilde{x}_1(t)\\ \vdots \\ \tilde{x}_N(t)\end{pmatrix}, \hat{x}(t) = \begin{pmatrix}\hat{x}_1(t)\\ \vdots \\ \hat{x}_N(t)\end{pmatrix}, \chi(t) = \begin{pmatrix}\chi_1(t)\\ \vdots \\ \chi_N(t)\end{pmatrix},$$
$$w(t) = \begin{pmatrix}w_1(t)\\ \vdots \\ w_N(t)\end{pmatrix}, \tilde{y}(t) = \begin{pmatrix}\tilde{y}_1(t)\\ \vdots \\ \tilde{y}_N(t)\end{pmatrix},$$

we have the following closed-loop system in z domain as:

$$\begin{aligned}
z\tilde{x} &= (I_N\otimes A)\tilde{x} - (I_N\otimes (BK))\chi + (I_N\otimes B)C_s w,\\
z\hat{x} &= (I_N\otimes (A-HC))\hat{x} - ((I_N - \bar{D}_z(\tau))\otimes (BK))\chi\\
&\quad + ((I_N - \bar{D}_z(\tau))\otimes (HC))\tilde{x},\\
z\chi &= (I_N\otimes (A-BK) - (I_N - \bar{D}_z(\tau))\otimes A)\chi + (I_N\otimes A)\hat{x},\\
zw &= A_s w,\\
\tilde{y} &= (I_N\otimes C)\tilde{x},
\end{aligned}$$

where $A_s = \text{diag}(A_{i,s})$ and $C_s = \text{diag}(C_{i,s})$ for $i = \{1,\dots,N\}$, and

$$\bar{D}_z(\tau) = \begin{pmatrix}
\bar{d}_{11} & \cdots & \bar{d}_{1k}e^{-z\tau_{1k}} & \cdots & \bar{d}_{1N}e^{-z\tau_{1N}}\\
\vdots & \ddots & \vdots & \ddots & \vdots\\
\bar{d}_{k1}e^{-z\tau_{k1}} & \cdots & \bar{d}_{kk} & \cdots & \bar{d}_{kN}e^{-z\tau_{kN}}\\
\vdots & \cdots & \vdots & \ddots & \vdots\\
\bar{d}_{N1}e^{-z\tau_{N1}} & \cdots & \bar{d}_{Nk}e^{-z\tau_{Nk}} & \cdots & \bar{d}_{NN}
\end{pmatrix}.$$

Let $\delta = \tilde{x} - \chi$, and $\bar{\delta} = ((I_N - \bar{D}_z(\tau))\otimes I)\tilde{x} - \hat{x}$, then we have

$$z\tilde{x} = (I_N\otimes (A-BK))\tilde{x} + (I_N\otimes (BK))\delta + (I_N\otimes B)C_s w, \tag{42}$$
$$z\bar{\delta} = (I_N\otimes (A-HC))\bar{\delta} + ((I_N - \bar{D}_z(\tau))\otimes B)C_s w, \tag{43}$$
$$z\delta = (\bar{D}_z(\tau)\otimes A)\delta + (I_N\otimes A)\bar{\delta} + (I_N\otimes B)C_s w, \tag{44}$$
$$zw = A_s w. \tag{45}$$

We need to show the asymptotic stability of (42)–(45) for all communication delays $\tau_{ij}\in\mathbb{Z}_{\geq 0}$. Since A_s and $A-HC$ are stable, then we have w and $\bar{\delta}$ is asymptotically stable. As such, asymptotic stability of (43) and (45) are implied by asymptotic stability of the following reduced system:

$$\begin{pmatrix} z\tilde{x}\\ z\delta\end{pmatrix} = \begin{pmatrix} I_N\otimes (A-BK) & I_N\otimes (BK)\\ 0 & \bar{D}_z(\tau)\otimes A\end{pmatrix}\begin{pmatrix}\tilde{x}\\ \delta\end{pmatrix}. \tag{46}$$

Following Lemma 3, we prove the stability of (46) in two steps. In the first step, we prove the stability in the absence of communication delays and in the second step we prove the stability of (46) by checking condition (39).

1) When there is no communication delay in the network, the stability of system (46) is equivalent to asymptotic stability of the matrix

$$\begin{pmatrix} I_N\otimes (A-BK) & I_N\otimes (BK)\\ 0 & \bar{D}\otimes A\end{pmatrix}, \tag{47}$$

where $\bar{D} = [\bar{d}_{ij}]\in\mathbb{R}^{N\times N}$ and we have that the eigenvalues of $\bar{D}$ are in open unit disk. The eigenvalues of $\bar{D}\otimes A$ are of the form $\lambda_i\mu_j$, with λ_i and μ_j eigenvalues of $\bar{D}$ and A, respectively [47, Theorem 4.2.12]. Since $|\lambda_i| < 1$ and $|\mu_j| = 1$, we find $\bar{D}\otimes A$ is Schur stable. Thus, we just need to prove the stability of

$$\tilde{x}(t+1) = (I_N\otimes (A-BK))\tilde{x}(t), \tag{48}$$

with $A-BK$ being Schur stable. Therefore, we can obtain the asymptotic stability of (46), i.e.,

$$\lim_{t\to\infty}\tilde{x}_i(t)\to 0.$$

It implies that $\tilde{y}_i(t)\to 0$, i.e., $y_i(t)\to y_r$.

2) Next, in the light of Lemma 3, the closed-loop system (46) is asymptotically stable for all communication delays $\tau_{ij}\in\mathbb{Z}_{\geq 0}$, if

$$\det\left[e^{\mathbf{j}\omega}I - \begin{pmatrix} I_N\otimes (A-BK) & I_N\otimes (BK)\\ 0 & \bar{D}_{\mathbf{j}\omega}(\tau)\otimes A\end{pmatrix}\right]\neq 0 \tag{49}$$

for all $\omega\in\mathbb{R}$ and any communication delays $\tau_{ij}\in\mathbb{Z}_{\geq 0}$. Inequality (49) is satisfied if the matrix

$$\begin{pmatrix} I\otimes (A-BK) & I\otimes (BK)\\ 0 & \bar{D}_{\mathbf{j}\omega}(\tau)\otimes A\end{pmatrix} \tag{50}$$

does not have any eigenvalue on the unit disk for all $\omega\in\mathbb{R}$ and any communication delays $\tau_{ij}\in\mathbb{Z}_{\geq 0}$. In the light of [32, Lemma 2], we have that all eigenvalues of $\bar{D}_{\mathbf{j}\omega}(\tau)$ are less than or equal to all eigenvalues of $\bar{D}$ for any $\tau_{ij}\in\mathbb{Z}_{\geq 0}$. It means that the eigenvalues of $\bar{D}_{\mathbf{j}\omega}(\tau)$ are in open unit disk. Therefore,

$$\bar{D}_{\mathbf{j}\omega}(\tau)\otimes A$$

is Schur stable. It implies that matrix (50) does not have any eigenvalue on the unit disk for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{Z}_{\geq 0}$. Thus, we have

$$\tilde{x}_i \to 0 \text{ i.e., } y_i(t) \to y_r$$

which means the synchronization $y_i(t) \to y_j(t)$ is achieved. ■

Remark 5: From the above results, the agents must track a constant trajectory generated by the exosystem in order to tolerate arbitrarily large constant communication delays and keeping the diffusiveness of the network. In most of the application such as power network it is desirable that the agents follow the constant trajectory. Moreover, for MAS subject to communication delays, in the case that the desired trajectory is not constant, one has to consider the delayed synchronization, such as literature [36].

IV. Numerical Examples

In this section, we illustrate the effectiveness of our protocols with numerical examples for continuous-and discrete-time heterogeneous MAS in achieving scale-free regulated output synchronization in the presence of unknown, nonuniform, and arbitrarily large communication delays.

A. Continuous-Time MAS

Consider agents models (1) satisfying Assumption 1 where

$$A_i = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, C_i^m = I,$$

for $i = 1, 6$, and

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, C_i^m = I,$$

for $i = 2, 7$, and

$$A_i = \begin{pmatrix} -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 & 1 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$C_i^{\mathrm{T}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, C_i^m = I,$$

for $i = 3, 4, 8, 9$, and finally

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, C_i^m = I,$$

for $i = 5, 10$. Let $y_r = 5$.

We consider the following examples for MASs with $N = 5$ and $N = 10$.

Example C.1: Consider a heterogeneous MAS consisting of 5 agents with (C_i, A_i, B_i) for $i \in \{1, \cdots, 5\}$. The associated adjacency matrix to the communication network is assumed to be $\mathcal{A}_1$ where $a_{21} = a_{32} = a_{43} = a_{54} = a_{25} = a_{35} = a_{13} = 1$ where communication delays equal to $\tau_{13} = 0.2\,\mathrm{sec}$, $\tau_{32} = 1\,\mathrm{sec}$, and $\tau_{35} = 0.5\,\mathrm{sec}$.

Example C.2: Next, consider a MAS consisting of 10 agents with (C_i, A_i, B_i) for $i \in \{1, \cdots, 10\}$, and communication network with associated adjacency matrix $\mathcal{A}_2$, where $a_{21} = a_{5,10} = a_{32} = a_{43} = a_{54} = a_{65} = a_{76} = a_{87} = a_{98} = a_{10,9} = a_{15} = 1$. In this example, in order to show that our protocol can withstand any arbitrarily large communication delays, we consider two cases as following.

- Firstly, we consider communication delays are $\tau_{54} = 2.5\,\mathrm{sec}$, $\tau_{65} = 1\,\mathrm{sec}$, $\tau_{98} = 3\,\mathrm{sec}$, and the rest equal to zero.
- In the second case, we consider the same MAS with 10 agents where the delays equal to $\tau_{54} = 4\,\mathrm{sec}$, $\tau_{65} = 6\,\mathrm{sec}$, $\tau_{98} = 8\,\mathrm{sec}$, and the delays between the rest of the links are zero.

Note that in both examples, $p = 1$ and $\bar{n}_d = 3$, which is the degree of infinite zeros of (C_2, A_2, B_2). It is obtained that $n_q = 3$. We choose the target model as

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}.$$

We design precompensators, stated in Step II, as follows.

$$u_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} v_i + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} z_i$$

for $i = 1, 6$, and

$$u_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_i + \begin{pmatrix} 1 & -1 & 1 & -2 & -2 \\ 0 & -1 & 1 & 0 & -1 \end{pmatrix} z_i$$

for $i = 3, 4, 8, 9$, and finally

$$u_i = v_i + \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} z_i$$

for $i = 5, 10$.

By choosing matrices $K = \begin{pmatrix} 210 & 107 & 18 \end{pmatrix}$ and $H^{\mathrm{T}} = \begin{pmatrix} 18 & 107 & 210 \end{pmatrix}$, we obtain the following collaborative protocols for the compensated agents $i = 1, \ldots, N$,

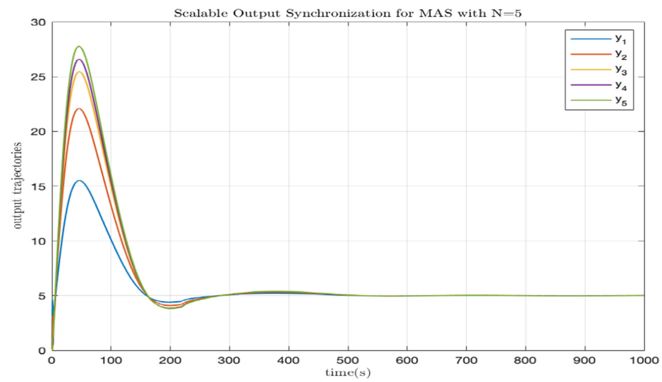

Fig. 2. Scalable output synchronization for MAS of *Example C.1* with $N = 5$.

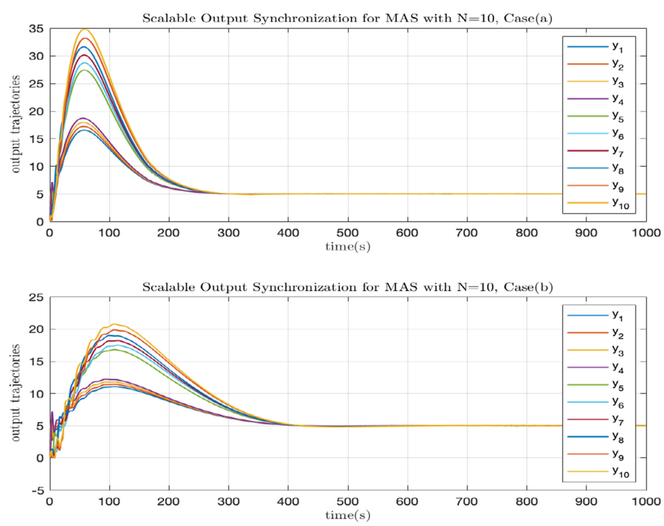

Fig. 3. Scalable output synchronization for MAS of *Example C.2* with $N = 10$.

$$
\begin{cases}
\dot{\hat{x}}_i = \begin{pmatrix} -18 & 1 & 0 \\ -107 & 0 & 1 \\ -210 & 0 & 0 \end{pmatrix} \hat{x}_i - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 210 & 107 & 18 \end{pmatrix} \hat{\zeta}_i \\
\qquad + \begin{pmatrix} 18 \\ 107 \\ 210 \end{pmatrix} \bar{\zeta}_i + \begin{pmatrix} 0 \\ 0 \\ \iota_i \end{pmatrix} v_i, \\
\dot{\chi}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \chi_i + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v_i + \hat{x}_i - \hat{\zeta}_i - \iota_i \chi_i, \\
v_i = -\begin{pmatrix} 210 & 107 & 18 \end{pmatrix} \chi_i.
\end{cases}
\tag{51}
$$

The simulation results are shown in Figs. 2 and 3 for MAS with $N = 5$ and $N = 10$, respectively. The outputs of all agents converge to $y_r = 5$, which means output synchronization can be achieved for any MAS with any connected communication network. Note that the protocol design only depends on the compensated agent model (A, B, C). Besides, the results in Fig. 3 illustrate that our protocols can withstand

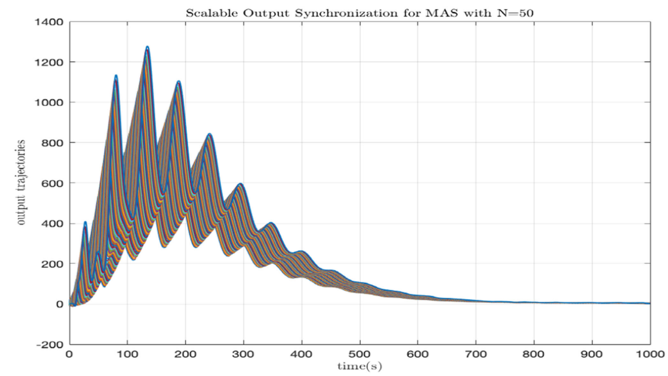

Fig. 4. Scalable output synchronization for MAS of *Example C.3* with $N = 50$.

arbitrarily large communication delays. As it is obvious in Fig. 3, large communication delays have an impact on the convergence rate.

To further illustrate the scalability nature of our protocols, in the next example, we will consider a MAS with $N = 50$. We will show that utilizing the same protocol in *Example C.1* and *Example C.2* we can achieve sacalable output synchronization for MAS with 50 agents.

Example C.3 In this example, we consider a heterogeneous MAS with 50 agents. The agent models (C_i, A_i, B_i) for $i \in \{1, \cdots, 50\}$ are randomly chosen from the agent models of *Example C.1*. The communication network of this MAS is considered to be *circular* and the communication delays are assumed to be random numbers between 0 and 1. The simulation result is shown in Fig. 4. We observe that our one-shot-designed protocol (51) works for any heterogeneous MAS with any communication networks and any number of agents N. In other words, it is *scale-free*.

B. Discrete-Time MAS

Consider discrete-time agents models (1), satisfying Assumption 1 where

$$
A_i = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix},
$$

$$
C_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, C_i^m = I,
$$

for $i = 1, 8$, and

$$
A_i = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},
$$

$$
(C_i^m)^{\mathrm{T}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},
$$

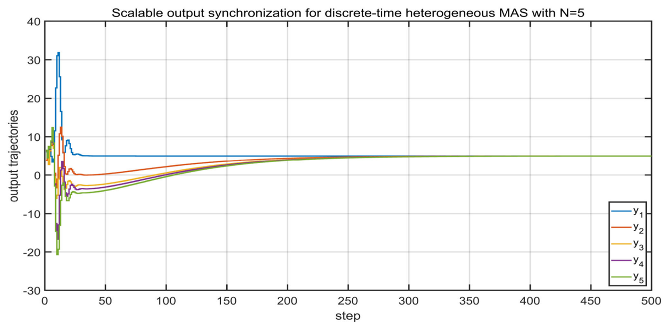

Fig. 5. Scalable output synchronization for MAS of *Example D*.1 with $N=5$.

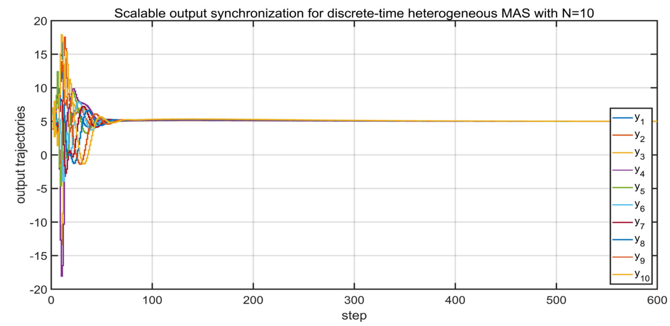

Fig. 6. Scalable output synchronization for MAS of *Example D*.2 with $N=10$.

for $i=2,4,7$, and

$$A_i=\begin{pmatrix}1&1\\0&1\end{pmatrix}, B_i=\begin{pmatrix}0\\1\end{pmatrix}, C_i^{\mathrm{T}}=\begin{pmatrix}1\\0\end{pmatrix}, C_i^m=I,$$

for $i=3,9$, and finally

$$A_i=\begin{pmatrix}1&1\\-5&-4\end{pmatrix}, B_i=\begin{pmatrix}0\\1\end{pmatrix}, C_i^{\mathrm{T}}=\begin{pmatrix}1\\0\end{pmatrix},$$
$$(C_i^m)^{\mathrm{T}}=\begin{pmatrix}1\\1\end{pmatrix},$$

for $i=5,6,10$. Let $y_r=5$.

We consider two different discrete-time heterogeneous MAS with $N=5$ and $N=10$ as following.

Example D.1: Consider a discrete-time heterogeneous MAS with 5 agents and agent models (C_i,A_i,B_i) for $i\in\{1,\cdots,5\}$. The associated adjacency matrix with the directed communication topology is $\mathcal{A}_1$ where $a_{21}=a_{32}=a_{43}=a_{54}=a_{25}=a_{35}=a_{13}=1$. The communication delays are assumed to be equal to $\tau_{13}=2$, $\tau_{32}=1$, and $\tau_{35}=3$.

Example D.2: Next, consider a discrete-time heterogeneous MAS with 10 agents and communication network with associated adjacency matrix $\mathcal{A}_2$, where $a_{21}=a_{5,10}=a_{32}=a_{43}=a_{54}=a_{65}=a_{76}=a_{87}=a_{98}=a_{10,9}=a_{15}=1$. Communication delays are equal to $\tau_{54}=2$, $\tau_{65}=1$, $\tau_{98}=3$, and the rest of delays between links equal to zero.

Note that for both networks $p=1$ and $\bar{n}_d=2$, which is the degree of infinite zeros of (C_3,A_3,B_3). It is obtained that $n_q=2$. We choose the target model as

$$A=\begin{pmatrix}1&1\\0&1\end{pmatrix}, B=\begin{pmatrix}0\\1\end{pmatrix}, C=\begin{pmatrix}1&0\end{pmatrix}.$$

We design precompensators, stated in Step II, as follows.

$$u_i=\begin{pmatrix}1\\0\end{pmatrix}v_i+\begin{pmatrix}0&0&-1&0\\1&0&-1&1\end{pmatrix}z_i$$

for $i=1,8$, and

$$u_i=\begin{pmatrix}1\\0\end{pmatrix}v_i+\begin{pmatrix}-1\\-1\end{pmatrix}z_i$$

for $i=2,4,7$, and finally

$$u_i=v_i+2z_i$$

for $i=5,6,10$.

By choosing matrices $K=\begin{pmatrix}0.5&1\end{pmatrix}$ and $H^{\mathrm{T}}=\begin{pmatrix}1&0.5\end{pmatrix}$, we obtain the following collaborative protocols for the compensated agents $i=1,\ldots,N$:

$$\begin{cases}\hat{x}_i(t+1)=\begin{pmatrix}0&1\\-0.5&1\end{pmatrix}\hat{x}_i(t)-\begin{pmatrix}0&0\\0.5&1\end{pmatrix}\hat{\zeta}_i(t)\\ \qquad+\begin{pmatrix}1\\0.5\end{pmatrix}\bar{\zeta}_i(t)+\frac{\iota_i}{2+D_{\text{in}}}\begin{pmatrix}0\\1\end{pmatrix}v_i(t),\\ \chi_i(t+1)=\begin{pmatrix}1&1\\0&1\end{pmatrix}\chi_i(t)+\begin{pmatrix}0\\1\end{pmatrix}v_i(t)\\ \qquad+\begin{pmatrix}1&1\\0&1\end{pmatrix}\Big(\hat{x}_i(t)-\hat{\zeta}_i(t)-\frac{\iota_i}{2+D_{\text{in}}}\chi_i(t)\Big),\\ v_i(t)=-\begin{pmatrix}0.5&1\end{pmatrix}\chi_i(t).\end{cases}\tag{52}$$

Note that similar to the case of continuous-time, protocol (52) does not require any information about the communication graph and the number of agents. The designed protocol only depends on the discrete-time target model, (A,B,C).

The simulation results are shown in Figs. 5 and 6 for discrete-time MAS with $N=5$ and 10, respectively. All the outputs of agents converge to $y_r=5$. We observe that our one-shot-designed protocol (52) is scale-free and can achieve output synchronization for any heterogeneous MAS with any communication networks, any number of agents N, and any unknown, non-uniform, and arbitrary large communication delays.

V. Conclusion

In this paper we have proposed scale-free protocol design utilizing localized information exchange for regulated output synchronization of continuous- and discrete-time heterogeneous networks subject to unknown, nonuniform and arbitrarily large communication delays. The proposed *scale-free* protocols were designed solely based on the knowledge of agent models without utilizing any information about the communication network such as bounds on the eigenvalues of

Laplacian matrix associated with the communication graph and the size of the network. It is worth noting that considering the arbitrarily time-varying communication delay in the *scale-free framework* is the subject matter of our future work.

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