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Scale-free collaborative protocol design for state synchronization of multi-agent systems in presence of unknown nonuniform and arbitrarily large communication delays☆,☆☆

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ABSTRACT

In this paper, we study state synchronization problem for homogeneous networks of multi-agent systems subject to unknown, nonuniform and arbitrarily large communication delays. A scale-free design framework utilizing localized information exchange has been adopted. The protocol design is solely based on agent models such that we do not need any information about the communication networks and the number of agents. Moreover, the necessary and sufficient solvability conditions are established.

1. Introduction

Cooperative control of multi-agent systems (MAS) has become a hot topic among researchers because of its broad application in various areas such as biological systems, sensor networks, automotive vehicle control, robotic cooperation teams and so on. See for example books [13,30,39] and [2] or the survey paper [28]. Two classes of multi-agent systems has been identified: homogeneous (i.e. agents are identical) and heterogeneous (i.e. agents are non-identical). State synchronization inherently requires homogeneous MAS.

In practical applications, the network dynamics are not perfect and may be subject to delays. Time delays may afflict system performance or even lead to instability. As discussed in [3], two kinds of delays have been considered in the literature: input delays and communication delays. Input delays encapsulate the processing time to execute an input for each agent, whereas communication delays can be considered as the time it takes to transmit information from an origin agent to its destination.

Some research has been done in the case of both constant and time-varying input delay, specifically with the objective of deriving an upper bound on the input delays such that agents can still achieve synchronization; see, for example [1,15,16,18,21,29,36,41,42]. In the case of communication delay, some research has been done; see [4,8,12,14,22–24,36,40,43]. Time-varying communication delays for a general multi-agent system have been considered in [33]. As it is well-known that in order to tolerate large communication delays one needs to preserve diffusiveness (namely to ensure the invariance of the synchronization manifold) as such, this can be achieved in two ways:

1. The first method is the standard state/output synchronization by regulating the states/outputs to a constant trajectory which is hugely utilized in the literature [3]. A notable phenomenon in this case is that the final consensus is constant where in many practical problems this would be the case; see for example [27].

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2. The second method for preserving diffusiveness in presence of communication delays is to consider delayed state/output synchronization which is introduced in [4–6,19,25] to allow non constant or dynamic desired output/state trajectory.

In this paper, we have considered state synchronization and as such we have utilized the first method for preserving the diffusiveness of the network by regulating the outputs to a constant synchronized trajectory.

It is worth to note that all of the existing literature as reviewed above require some knowledge of the communication network, commonly a bound on the spectrum of the Laplacian matrix and the number of agents where this data is explicitly utilized in the design of protocols. In particular, most of the existing protocols utilize the bound on the real part of the second smallest eigenvalue of the associated Laplcian matrix of the communication network which means the protocols are not scale-free. There is a current body of research that shows for a certain class of non-exhaustive graphs, the algebraic connectivity leads to zero as the size of the network increase, see [35]. Therefore, for the networks with protocols that are not scale-free if the size of the network increases the algebraic connectivity of the network tends to zero which leads to loss of synchronization (i.e., instability of the disagreement dynamic).

Recently, we have introduced a new generation of scale-free protocols for synchronization of homogeneous and heterogeneous MAS where the agents are subject to input saturation and input delays, see for example [17,20,26]. The *scale-free* protocol means the design is independent of the information about the associated communication graph or the size of the network, i.e., the number of agents. The main contribution of this paper is designing protocols for MAS subject to unknown, nonuniform, and arbitrarily large communication delays in *scale-free* framework such that:

- State synchronization is achieved by regulating the outputs of the agents to constant trajectories. The sufficient solvability condition is provided when the outputs are regulating to any arbitrary constant reference trajectory, while necessary and sufficient solvability conditions are established by restricting the constant reference trajectory to a set defined by the agent models.
- The protocol design is independent of any information about the associated communication graph and the size of the network and is designed solely based on the knowledge of the agent models.
- The proposed collaborative dynamic protocols can tolerate any unknown, nonuniform, and arbitrarily large communication delays.

Notations and preliminaries

We denote the set of real numbers by $\mathbb{R}$, non-negative real numbers by $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geq 0\}$ and the entire complex plane by $\mathbb{C}$. We denote the field of rational functions with real coefficients by $\mathbf{R}(s)$. By $rank_{\mathcal{K}}$ we denote the rank of a matrix whose entries are in the filed $\mathcal{K}$. We shall write *rank* only for the case when $\mathcal{K} = \mathbb{R}$, or $\mathcal{K} = \mathbb{C}$. Moreover, we use the term *normal rank* for $rank_{\mathcal{K}}$ whenever $\mathcal{K} = \mathbf{R}(s)$. Given a matrix $A \in \mathbb{R}^{n\times m}$, A^{T} denotes the transpose of A. Let $\mathbf{j}$ indicate $\sqrt{-1}$. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane. We denote by $diag\{A_1,\dots,A_N\}$, a block-diagonal matrix with $A_1,\dots,A_N$ as its diagonal elements. I_n denotes the n-dimensional identity matrix and 0_n denotes $n\times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context. For $\bar{A} \in \mathbb{C}^{n\times m}$ and $\bar{B} \in \mathbb{C}^{p\times q}$, the Kronecker product of $\bar{A}$ and $\bar{B}$ is defined as

$$\bar{A} \otimes \bar{B} = \begin{pmatrix} \bar{a}_{11}\bar{B} & \dots & \bar{a}_{1m}\bar{B} \\ \vdots & \ddots & \vdots \\ \bar{a}_{n1}\bar{B} & \dots & \bar{a}_{nm}\bar{B} \end{pmatrix}$$

where $[\bar{A}]_{ij} = \bar{a}_{ij}$. The following properties of the Kronecker product will be particularly useful,

$$(A\otimes B)(C\otimes D) = (AC)\otimes(BD),$$
$$\bar{A}\otimes(\bar{B}+\bar{C}) = \bar{A}\otimes\bar{B} + \bar{A}\otimes\bar{C}.$$

To describe the information flow among the agents we associate a *weighted graph* $\mathcal{G}$ to the communication network. The weighted graph $\mathcal{G}$ is defined by a triple $(\mathcal{V},\mathcal{E},\mathcal{A})$ where $\mathcal{V} = \{1,\dots,N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix with non negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j,i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1,\dots,i_k\}$ such that $(i_j,i_{j+1}) \in \mathcal{E}$ for $j = 1,\dots,k-1$. A *directed tree* is a subgraph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. The *root set* is the set of root nodes. A *directed spanning tree* is a subgraph which is a directed tree containing all the nodes of the original graph. If a directed spanning tree exists, the root has a directed path to every other node in the tree.

For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$ [9]. Moreover, if the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane [30].

Next, in the following, we recall the definitions of invariant zeros and right-invertibility of the linear time-invariant system Σ

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

Definition 1. $\lambda \in \mathbb{C}$ is called invariant zero of linear system Σ if

$$rank_{\mathbb{C}}\begin{pmatrix} \lambda I - A & -B \\ C & 0 \end{pmatrix} < \text{normal rank}\begin{pmatrix} sI - A & -B \\ C & 0 \end{pmatrix}$$

where by *normal rank* we mean the rank of a matrix with entries in the field of rational function $\mathbf{R}(s)$.

Definition 2. The linear system Σ is right-invertible if, given a smooth reference output $y_r(t)$, there exists an initial condition $x(0)$ and an input $u(t)$ that ensures $y(t) = y_r(t)$ for all $t \geq 0$.

Remark 1. The linear system Σ

- is right-invertible if and only if its transfer function matrix is a surjective rational matrix.
- is right-invertible if and only if the rank of $\begin{pmatrix} sI - A & -B \\ C & 0 \end{pmatrix} =$ $n + p$ for all but finitely many $s \in \mathbb{C}$.

Linear system Σ is at most weakly unstable if all eigenvalues of A are in the closed left half plane. It should be noted that the set of at most weakly unstable agents contains stable agents, neutrally stable agents as well as weakly unstable agents. The related definitions and notations can be found in [10,31,37,38].

2. Problem formulation

Consider the multi-agent system composed of N identical general agents, which are denoted by Σ_i with $i \in \{1, \dots, N\}$,

$$\Sigma_i : \begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t) \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n, y_i(t) \in \mathbb{R}^p$, and $u_i(t) \in \mathbb{R}^m$ are the state, output and the input of agent i, respectively.

We need the following assumption.

Assumption 1. All eigenvalues of A are in closed left half plane, that is agents are at most weakly unstable.

Remark 2. Note that agents, satisfying Assumption 1, can be polynomially unstable, such as chain of integrators.

The network provides agent i with the following information

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})), \tag{2}$$

where $\tau_{ij} \in \mathbb{R}_{\geq 0}$ represents an unknown communication delay from agent j to agent i and $\tau_{ii} = 0$. In the above $a_{ij} \geq 0$ and $a_{ii} = 0$. This communication topology of the network, presented in (2), can be associated to a weighted graph $\mathcal{G}$ with each node indicating an agent in the network and the weight of an edge is given by the coefficient a_{ij}. The communication delay implies that it took τ seconds for agent j to transfer its state information to agent i.

Remark 3. It is worth to note that in this paper we have utilized the widely accepted formulation for communication delays which can be considered as the time it takes to transmit information from an origin agent to its destination [3]. To the authors best knowledge there are few papers such as [7,34] with another formulation for the communication delay where the same delay is imposed on agent i and agent j as

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}[y_i(t - \tau_{ij}) - y_j(t - \tau_{ij})]$$

while by modeling the communication delay in this way, namely the same delays for agent i and j, the complexity of preserving the diffusiveness is removed.

In terms of the coefficient of the associated Laplacian matrix L, $\zeta_i(t)$ can be represented as

$$\zeta_i(t) = \sum_{j=1}^{N} \ell_{ij} y_j(t - \tau_{ij}). \tag{3}$$

Meanwhile, Laplacian matrix L is expressed as

$$L = \begin{pmatrix} \ell_{11} & \ell_{12} & \cdots & \ell_{1N} \\ \ell_{21} & \ell_{22} & \cdots & \ell_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ \ell_{N1} & \ell_{N2} & \cdots & \ell_{NN} \end{pmatrix}.$$

Obviously, state synchronization is achieved if

$$\lim_{t\to\infty} \left(x_i(t) - x_j(t)\right) = 0 \quad \text{for all} \quad i, j \in \{1, \dots, N\}. \tag{4}$$

Our goal is to achieve state synchronization among all agents while the synchronized dynamics is equal to a constant trajectory. We assume that a nonempty subset $\mathscr{C}$ of the agents have access to their own output relative to the reference trajectory $y_r \in \mathbb{R}^p$. In other words, each agent has access to the quantity

$$\psi_i(t) = \iota_i(y_i(t) - y_r), \qquad \iota_i = \begin{cases} 1, & i \in \mathscr{C}, \\ 0, & i \notin \mathscr{C}. \end{cases} \tag{5}$$

Therefore, the information available for agent $i \in \{1, \dots, N\}$, is given by

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t - \tau_{ij})) + \iota_i(y_i(t) - y_r). \tag{6}$$

From now on, we will refer to the node set $\mathscr{C}$ as root set. For any graph with the Laplacian matrix L, we define the expanded Laplacian matrix as

$$\bar{L} = L + diag\{\iota_i\} = [\bar{\ell}_{ij}]_{N\times N} \tag{7}$$

which is not a regular Laplacian matrix associated to the graph, since the sum of its rows need not be zero. Meanwhile, it should be emphasized that $\bar{\ell}_{ij} = \ell_{ij}$ for $i \neq j$ in $\bar{L}$. Then, equation (6) can be rewritten as

$$\bar{\zeta}_i(t) = \sum_{j=1}^{N} \bar{\ell}_{ij}(y_j(t - \tau_{ij}) - y_r). \tag{8}$$

To guarantee that each agent can achieve the required regulation, we need to make sure that there exists a pass to each node starting with node from the set $\mathscr{C}$. Therefore, we denote the following set of graphs.

Definition 3. Given a node set $\mathscr{C}$, we denote by $\mathbb{G}_{\mathscr{C}}^{\mathbb{N}}$ the set of all directed graphs with N nodes containing the node set $\mathscr{C}$, such that every node of the network graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^{\mathbb{N}}$ is a member of a directed tree which has its root contained in the node set $\mathscr{C}$. Note that this definition does not require necessarily the existence of directed spanning tree.

Remark 4. From [11, Lemma 7] it follows for any $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^{\mathbb{N}}$ defined in Definition 3, the associated expanded Laplacian matrix $\bar{L}$ as defined by (7) is invertible and all the eigenvalues of $\bar{L}$ have positive real parts.

In this paper, we also introduce a localized information exchange among agents. In particular, each agent $i = 1, \dots N$ has access to the following information denoted by $\hat{\zeta}_i(t)$, of the form

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\xi_i(t) - \xi_j(t - \tau_{ij})) \tag{9}$$

where $\xi_j(t)$ is a variable produced internally by agent j and to be defined in next sections. Given that agents communicate $y_i(t)$ and $\xi_i(t)$ over the same communication networks, the communication delays τ_{ij} between agent j and agent i are the same in equations (8) and (9).

We formulate the following problem of state synchronization for networks subject to unknown, nonuniform and arbitrarily large communication delays utilizing linear scale-free collaborative protocols as follows.

Problem 1. Consider a MAS described by (1) and (8) and a given constant reference trajectory $y_r \in \mathbb{R}^p$. Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}_{\mathscr{C}}^{N}$. Then, the **scalable state synchronization problem based on localized information exchange utilizing collaborative protocols** for networks subject to unknown, nonuniform and arbitrarily large communication delays is to find, if possible, a linear dynamic protocol for each agent $i \in \{1, \dots, N\}$, using only knowledge of agent model, i,e. (A, B, C), of the form

$$\begin{cases} \dot{x}_{c,i}(t) = A_c x_{c,i}(t) + B_{c1}\bar{\zeta}_i(t) + B_{c2}\hat{\zeta}_i(t), \\ u_i(t) = F_c x_{c,i}(t), \end{cases} \tag{10}$$

where $\hat{\zeta}_i(t)$ is defined in (9) with $\xi_i(t) = H_c x_{c,i}(t)$ and $x_{c,i}(t) \in \mathbb{R}^{n_c}$ such that for any N, any graph $\mathscr{G} \in \mathbb{G}_{\mathscr{C}}^{\mathbb{N}}$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$ we achieve

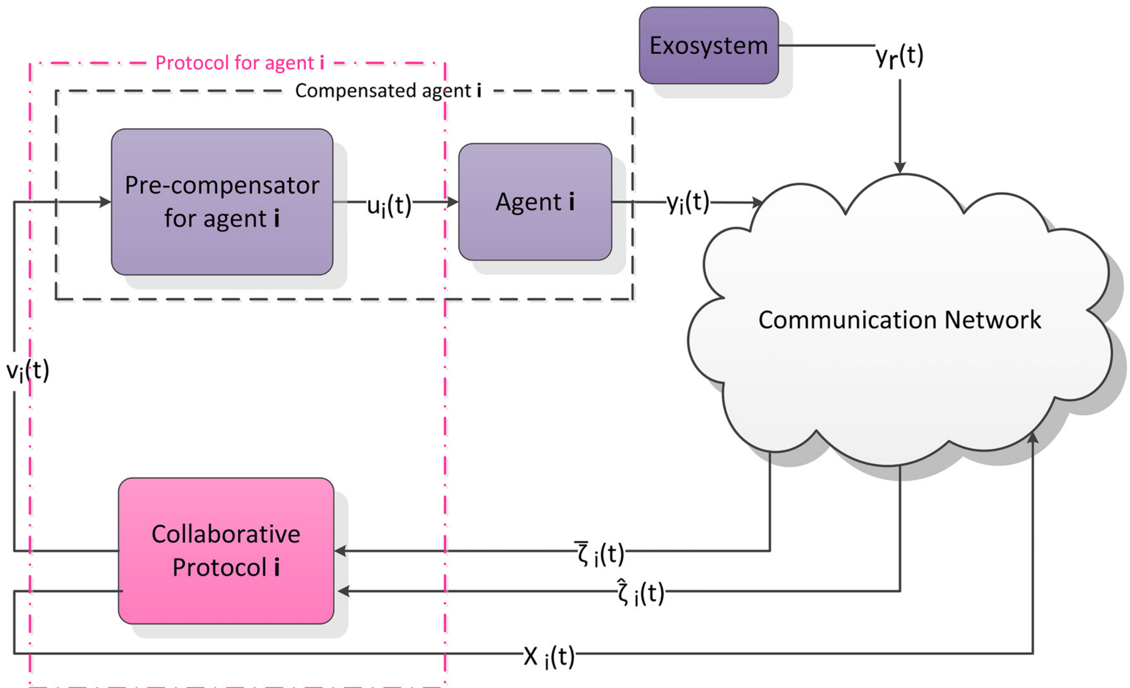

Fig. 1. Architecture of scale-free protocols.

(i) regulated output synchronization, i.e.,

$$\lim_{t\to\infty} (y_i(t) - y_r) = 0, \quad \text{for } i \in \{1, \dots, N\}, \tag{11}$$

(ii) state synchronization, i.e.,

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = 0, \quad \text{for all } i, j \in \{1, \dots, N\}. \tag{12}$$

3. Main results

Our main results are provided in the following two subsections. In the first subsection, we consider solvability of Problem 1 for any arbitrary given constant reference trajectory $y_r \in \mathbb{R}^p$. We show that if agents are right-invertible and have no invariant zeros at the origin Problem 1 is solvable for any arbitrary given constant reference trajectory and we provide protocol design for this class of agents. In the second subsection, we provide necessary and sufficient conditions for solvability of Problem 1. We identify a set $\mathscr{Y}_r \subseteq \mathbb{R}^p$ and we show that Problem 1 is solvable if and only if we restrict the constant reference trajectory to this set which obtained solely based on the knowledge of the agent models.

3.1. Solvability condition and protocol design for arbitrary constant reference trajectory $y_r \in \mathbb{R}^p$

In this subsection, we show that Problem 1 is solvable for any given arbitrary constant trajectory $y_r \in \mathbb{R}^p$ as long as the agents are right-invertible which has no invariant zeros at the origin. We design protocols for this class of agents. The architecture of the protocols is shown in Figure 1. As it is shown in the figure, the design consists of two steps. The first step is designing a pre-compensator for each agent to be able to regulate the states to a constant value (see [31, Chapter 2] for the classical output regulation problem in linear multivariable control). In the second step, we design collaborative protocols for the compensated agents to achieve state synchronization.

Step I: First we find an injective matrix V such that

$$\begin{pmatrix} A & BV \\ C & 0 \end{pmatrix} \tag{13}$$

is square and invertible. Such a matrix exists. To show that, we observe agent model described by (A, B, C) is right-invertible and has no invariant zeros at the origin, hence we have the matrix

$$\begin{pmatrix} A & B \\ C & 0 \end{pmatrix} \tag{14}$$

is full-row rank. Also, due to the detectability of (A, C), we have the first n columns of (14) are linearly independent. Therefore the existence of the injective matrix V is guaranteed.

Next, we consider the following regulator equations

$$\begin{pmatrix} A & BV \\ C & 0 \end{pmatrix}\begin{pmatrix} \Pi \\ \Gamma \end{pmatrix} = \begin{pmatrix} 0 \\ I \end{pmatrix}.$$

Since we have invertibility of (13), it implies that this equation has a unique solution. Meanwhile, invertibility of (13) means that

$$rank_{\mathbb{R}}\begin{pmatrix} A & BV\Gamma \\ C & 0 \end{pmatrix} = n + rank_{\mathbb{R}}\Gamma.$$

Then, we design the following precompensator for each agent of MAS (1).

$$\begin{aligned} \dot{p}_i(t) &= \begin{pmatrix} I & 0 \end{pmatrix} v_i(t), \qquad p_i(t) \in \mathbb{R}^r \\ u_i(t) &= \Gamma_1 p_i(t) + \begin{pmatrix} 0 & \Gamma_2 \end{pmatrix} v_i(t), \end{aligned} \tag{15}$$

where $v_i(t)$ is the input of the precompensator i, Γ_1 is injective and satisfies $Im V\Gamma = Im\Gamma_1$ with $r = rank_{\mathbb{R}}\Gamma$. Moreover, Γ_2 is chosen such that

$$\begin{pmatrix} \Gamma_1 & \Gamma_2 \end{pmatrix} \tag{16}$$

is square and invertible.

In order to design collaborative protocols we first obtain the compensated agents by combining (1) and (15) as

$$\begin{aligned}\dot{\bar{x}}_i(t) &= \bar{A}\bar{x}_i(t) + \bar{B}v_i(t)\\ y_i(t) &= \bar{C}\bar{x}_i(t)\end{aligned} \tag{17}$$

where

$$\bar{x}_i(t) = \begin{pmatrix} x_i(t) \\ p_i(t)\end{pmatrix}, \bar{A} = \begin{pmatrix} A & B\Gamma_1 \\ 0 & 0\end{pmatrix}, \bar{B} = \begin{pmatrix} 0 & B\Gamma_2 \\ I & 0\end{pmatrix}, \bar{C} = \begin{pmatrix} C & 0\end{pmatrix}.$$

We also need to verify stabilizability of $(\bar{A}, \bar{B})$ and detectability of $(\bar{A}, \bar{C})$. The stabilizability follows immediately from the invertibility of (16) and the stabilizability of (A, B). For detectability we need to verify that

$$rank_{\mathbb{C}}\begin{pmatrix} sI-A & -B\Gamma_1 \\ 0 & sI \\ C & 0\end{pmatrix} = n + r = n + rank_{\mathbb{R}}\Gamma_1$$

for all s in the closed right-half complex plane. If $s \neq 0$, then it immediately follows the detectability of (A, C). When $s = 0$, one have

$$rank_{\mathbb{R}}\begin{pmatrix} -A & -B\Gamma_1 \\ 0 & 0 \\ C & 0\end{pmatrix} = rank_{\mathbb{R}}\begin{pmatrix} -A & -BV\Gamma \\ C & 0\end{pmatrix} = n + rank_{\mathbb{R}}\Gamma_1.$$

Step II: In this step, the following linear collaborative protocol is designed for the compensated agents (17) as

$$\begin{cases} \dot{\hat{x}}_i(t) = \bar{A}\hat{x}_i(t) - \bar{B}K\hat{\zeta}_i(t) + F(\bar{\zeta}_i(t) - \bar{C}\hat{x}_i(t)) + \iota_i\bar{B}v_i(t) \\ \dot{\chi}_i(t) = \bar{A}\chi_i(t) + \bar{B}v_i(t) + \hat{x}_i(t) - \hat{\zeta}_i(t) - \iota_i\chi_i(t) \\ v_i(t) = -K\chi_i(t), \end{cases} \tag{18}$$

where matrices K and F are such that $\bar{A} - F\bar{C}$ and $\bar{A} - \bar{B}K$ are Hurwitz stable. In this protocol, agents communicate $\xi_i(t) = \chi_i(t)$, i.e. each agent has access to the localized information exchange

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\chi_i(t) - \chi_j(t - \tau_{ij})), \tag{19}$$

while $\bar{\zeta}_i(t)$ is defined via (8).

We formulate the following theorem.

Theorem 1. *Consider a MAS described by (1) and (8) where (A, B) is stabilizable and (A, C) is detectable. Assume Assumption 1 is satisfied. Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}^N_{\mathscr{C}}$.*

*Then, the scalable state synchronization problem utilizing localized information exchange via linear dynamic protocol as stated in Problem 1 is solvable for any $y_r \in \mathbb{R}^p$ **if** the system represented by (A, B, C) is right-invertible and has no invariant zeros in the origin. More specifically, under these conditions, for any given constant reference trajectory $y_r \in \mathbb{R}^p$, protocol (18) and (15) achieves scalable state synchronization for any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$ and any graph $\mathscr{G} \in \mathbb{G}^N_{\mathscr{C}}$ with any size of the network N.*

To obtain the result of Theorem 1, we need the following lemmas where Lemma 1 is a classical result for the stability of linear time-delayed system (see [32,43]) and Lemma 2 has been used in the literature for consensus of MAS in the presence of delay.

Lemma 1. *Consider a linear time-delay system*

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{20}$$

where $x(t) \in \mathbb{R}^n$ and $\tau_i \in [0, \bar{\tau}]$ with $\bar{\tau} > 0$. Assume that $A + \sum_{i=1}^m A_i$ is Hurwitz stable. Then, (20) is asymptotically stable for $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$ if

$$\det\left[\mathbf{j}\omega I - A - \sum_{i=1}^{m} e^{-\mathbf{j}\omega\tau_i}A_i\right] \neq 0, \tag{21}$$

for all $\omega \in \mathbb{R}$, and for all $\tau_1, \ldots, \tau_N \in [0, \bar{\tau}]$.

Lemma 2. *[43, Lemma 1] Let α be a lower bound for the eigenvalues of $\bar{L}$. Then, for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$, $(i, j = 1, \cdots, N)$ and all $\omega \in \mathbb{R}$, the real part of all eigenvalues of $\bar{L}_{j\omega}(\tau)$ will be larger than or equal to α, where*

$$\bar{L}_s(\tau) = \begin{pmatrix} \bar{\ell}_{11} & \ldots & \bar{\ell}_{1k}e^{-s\tau_{1k}} & \ldots & \bar{\ell}_{1N}e^{-s\tau_{1N}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\ell}_{k1}e^{-s\tau_{k1}} & \ldots & \bar{\ell}_{kk} & \ldots & \bar{\ell}_{kN}e^{-s\tau_{kN}} \\ \vdots & \ldots & \vdots & \ddots & \vdots \\ \bar{\ell}_{N1}e^{-s\tau_{N1}} & \ldots & \bar{\ell}_{Nk}e^{-s\tau_{Nk}} & \ldots & \bar{\ell}_{NN} \end{pmatrix} \tag{22}$$

is the expanded Laplacian matrix in the frequency domain and τ denotes a vector consisting of all $\tau_{ij} (i \neq j)$ with $i \in \{1, \ldots, N\}$.

Proof of Theorem 1.. We need to show that protocol (18) and (15) solves Problem 1. First, we show that there exists a $\bar{\Pi}$ such that $\bar{A}\bar{\Pi} = 0$ and $\bar{C}\bar{\Pi} = I$. Let W be such that $\Gamma_1 W = V\Gamma$, in that case it is easy to verify that we can choose

$$\bar{\Pi} = \begin{pmatrix} \Pi \\ W \end{pmatrix}.$$

Let $\tilde{x}_i(t) = \bar{x}_i(t) - \bar{\Pi}y_r$, we have

$$\begin{cases} \dot{\tilde{x}}_i(t) &= \bar{A}\tilde{x}_i(t) + \bar{B}v_i(t) \\ y_i(t) - y_r &= \bar{C}\tilde{x}_i(t) \end{cases} \tag{23}$$

and by defining

$$\tilde{x}(t) = \begin{pmatrix} \tilde{x}_1(t) \\ \vdots \\ \tilde{x}_N(t) \end{pmatrix}, \quad \chi(t) = \begin{pmatrix} \chi_1(t) \\ \vdots \\ \chi_N(t) \end{pmatrix}$$

we have the following closed-loop system in frequency domain as:

$$\begin{cases} s\tilde{x} = (I \otimes \bar{A})\tilde{x} - (I \otimes \bar{B}K)\chi \\ s\hat{x} = (I \otimes \bar{A})\hat{x} - (\bar{L}_s(\tau) \otimes \bar{B}K)\chi + (\bar{L}_s(\tau) \otimes F\bar{C})\tilde{x} - (I \otimes F\bar{C})\hat{x} \\ s\chi = (I \otimes (\bar{A} - \bar{B}K))\chi - (\bar{L}_s(\tau) \otimes I)\chi + \hat{x} \end{cases} \tag{24}$$

where $\bar{L}_s(\tau)$ is defined in Lemma 2. Let $\delta = \tilde{x} - \chi$, and $\bar{\delta} = (\bar{L}_s(\tau) \otimes I)\tilde{x} - \hat{x}$, then we have

$$\begin{aligned} s\bar{\delta} =& s(\bar{L}_s(\tau) \otimes I)\tilde{x} - s\hat{x} \\ =& (\bar{L}_s(\tau) \otimes \bar{A})\tilde{x} - (\bar{L}_s(\tau) \otimes \bar{B}K)\chi \\ & - (I \otimes \bar{A})\hat{x} + (\bar{L}_s(\tau) \otimes \bar{B}K)\chi - (\bar{L}_s(\tau) \otimes F\bar{C})\tilde{x} + (I \otimes F\bar{C})\hat{x} \\ =& (I \otimes \bar{A})((\bar{L}_s(\tau) \otimes I)\tilde{x} - \hat{x}) - (I \otimes F\bar{C})((\bar{L}_s(\tau) \otimes I)\tilde{x} - \hat{x}) \\ =& \big(I \otimes (\bar{A} - F\bar{C})\big)\bar{\delta} \end{aligned}$$

and

$$\begin{aligned} s\delta =& s\tilde{x} - s\chi \\ =& (I \otimes \bar{A})\tilde{x} - (I \otimes \bar{B}K)\chi \\ & - (I \otimes (\bar{A} - \bar{B}K))\chi + (\bar{L}_s(\tau) \otimes I)\chi - ((\bar{L}_s(\tau) \otimes I)\tilde{x} - \bar{\delta}) \\ =& (I \otimes \bar{A})(\tilde{x} - \chi) - (\bar{L}_s(\tau) \otimes I)(\tilde{x} - \chi) + \bar{\delta} \\ =& (I \otimes \bar{A} - \bar{L}_s(\tau) \otimes I)\delta + \bar{\delta} \end{aligned}$$

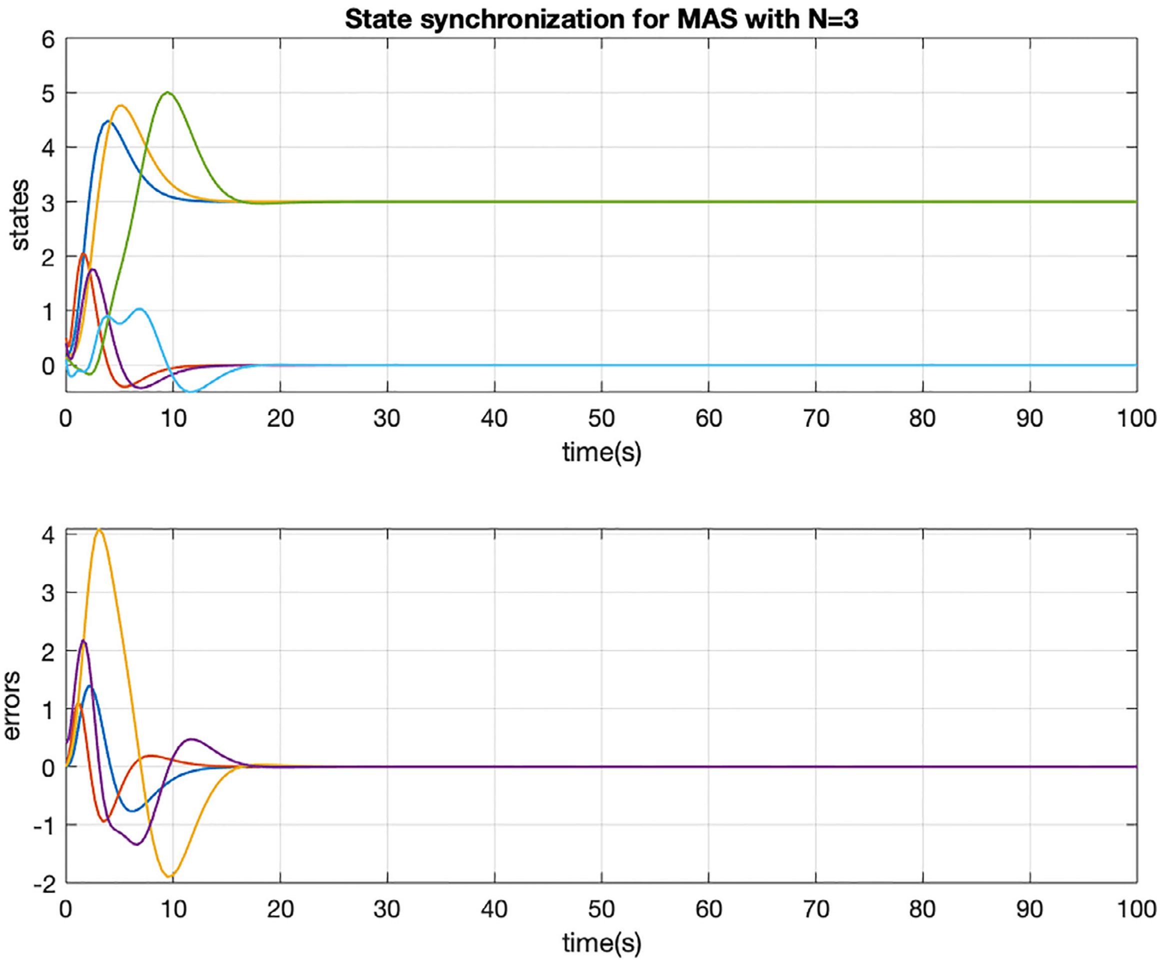

Fig. 2. State synchronization of MAS with right-invertible agents and $N = 3$.

Thus, we obtain

$$\begin{cases} s\tilde{x} = (I \otimes (\bar{A} - \bar{B}K))\tilde{x} + (I \otimes \bar{B}K)\delta \\ s\delta = (I \otimes \bar{A} - \bar{L}_s(\tau) \otimes I)\delta + \bar{\delta} \\ s\bar{\delta} = \big(I \otimes (\bar{A} - F\bar{C})\big)\bar{\delta} \end{cases} \tag{25}$$

We need to show the asymptotic stability of (25) for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$. Since $\bar{A} - F\bar{C}$ is stable, then we have $\bar{\delta}$ is stable. As such asymptotic stability of (25) is implied by asymptotic stability of the following reduced system.

$$\begin{pmatrix} s\tilde{x} \\ s\delta \end{pmatrix} = \begin{pmatrix} I \otimes (\bar{A} - \bar{B}K) & I \otimes \bar{B}K \\ 0 & I \otimes \bar{A} - \bar{L}_s(\tau) \otimes I \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \delta \end{pmatrix} \tag{26}$$

Following Lemma 1, we prove the stability of (26) in two steps. In the first step, we prove the stability in the absence of communication delays and in the second step we prove the stability of (26) by checking condition (21).

1. When there is no communication delay in the network, the stability of system (25) is equivalent to asymptotic stability of the matrix

$$\begin{pmatrix} I \otimes (\bar{A} - \bar{B}K) & I \otimes \bar{B}K \\ 0 & I \otimes \bar{A} - \bar{L} \otimes I \end{pmatrix}. \tag{27}$$

According to Remark 4, since eigenvalues $\lambda_1, \ldots, \lambda_N$ of $\bar{L}$ has positive real part, we have

$$(T \otimes I)(I \otimes \bar{A} - \bar{L} \otimes I)(T^{-1} \otimes I) = I \otimes \bar{A} - \bar{J} \otimes I \tag{28}$$

for a non-singular transformation matrix T, where (28) is upper triangular Jordan form with $\bar{A} - \lambda_i I$ for $i = 1, \cdots, N$ on the diagonal. Since $\bar{A}$ has all eigenvalues in the closed left half plane, $\bar{A} - \lambda_i I$ is stable. Therefore, all eigenvalues of $I \otimes \bar{A} - \bar{L} \otimes I$ have negative real part. Then, since we have $I \otimes \bar{A} - \bar{L} \otimes I$ is Hurwitz stable, we just need to prove the stability of

$$\dot{\tilde{x}}(t) = I \otimes (\bar{A} - \bar{B}K)\tilde{x}(t) \tag{29}$$

which $\bar{A} - \bar{B}K$ is Hurwitz stable. Therefore, we can obtain the asymptotic stability of (25), i.e.,

$$\lim_{t \to \infty} \tilde{x}_i(t) \to 0.$$

It implies that $x_i(t) - \Pi y_r \to 0$, i.e. $x_i(t) \to x_j(t)$.

2. Next, in the light of Lemma 1, the closed-loop system (26) is asymptotically stable for all communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$, $(i \neq j)$ if

$$\det \left[\mathbf{j}\omega I - \begin{pmatrix} I \otimes (\bar{A} - \bar{B}K) & I \otimes \bar{B}K \\ 0 & I \otimes \bar{A} - \bar{L}_{\mathbf{j}\omega}(\tau) \otimes I \end{pmatrix} \right] \neq 0 \tag{30}$$

for all $\omega \in \mathbb{R}$ and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$. Condition (30) is satisfied if the matrix

$$\begin{pmatrix} I \otimes (\bar{A} - \bar{B}K) & I \otimes \bar{B}K \\ 0 & I \otimes \bar{A} - \bar{L}_{\mathbf{j}\omega}(\tau) \otimes I \end{pmatrix} \tag{31}$$

does not have any eigenvalue on the imaginary axis and any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$. In the light of Lemma 2, we have that all eigenvalues of $\bar{L}_{\mathbf{j}\omega}(\tau)$ have positive real part

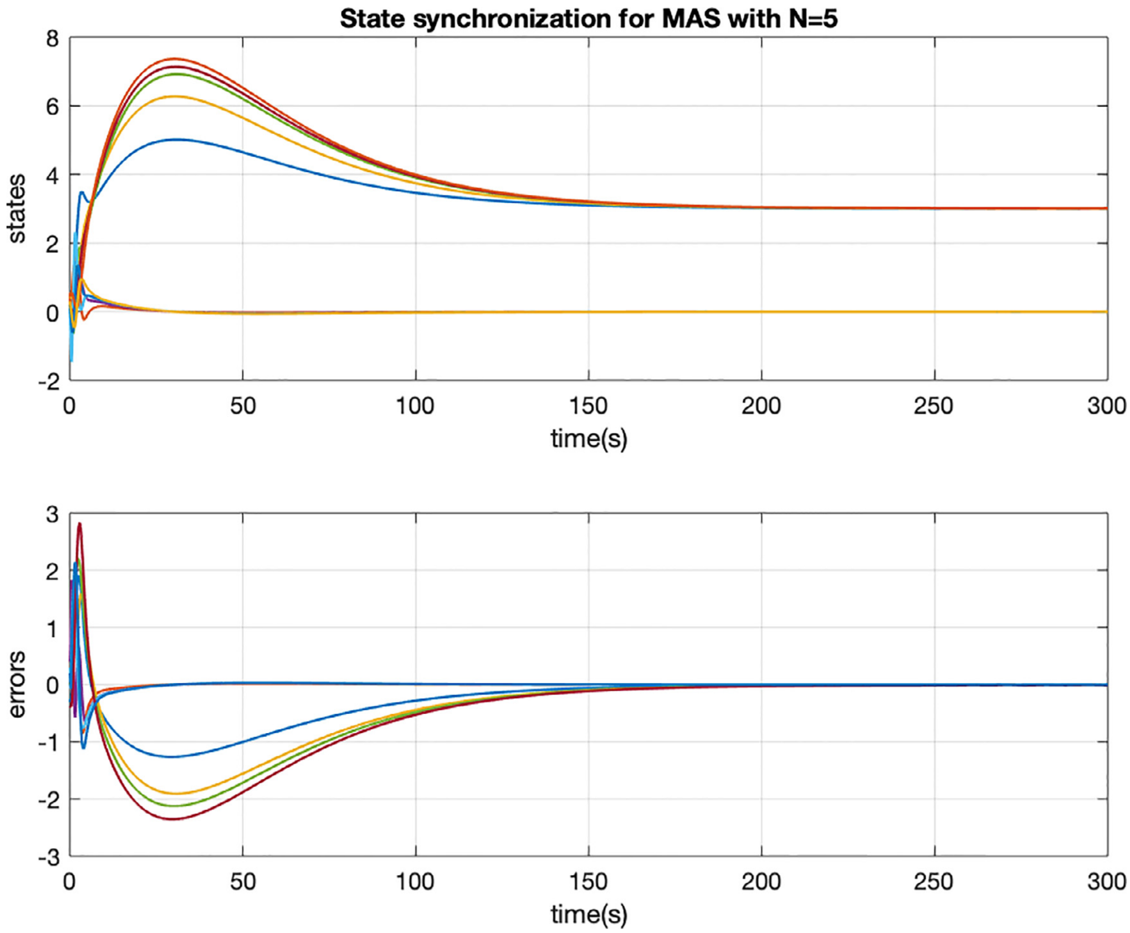

Fig. 3. State synchronization of MAS with right-invertible agents and $N=5$.

for any τ_{ij}. Therefore

$I\otimes\bar{A}-\bar{L}_{\mathbf{j}\omega}(\tau)\otimes I$

has all negative real part eigenvalues. It implies that all eigenvalues of matrix (31) have negative real parts, i.e. matrix (31) does not have any eigenvalue on the imaginary axis for all $\omega\in\mathbb{R}$ and any communication delays $\tau_{ij}\in\mathbb{R}_{\geq 0}$ $(i\neq j)$. Thus we have

$\tilde{x}_i(t)\to 0$ i.e. $x_i(t)\to\Pi y_r$

which means the synchronization $x_i(t)\to x_j(t)$ is achieved.

□

3.2. Necessary and sufficient solvability conditions and protocol design

In this subsection, we provide necessary and sufficient conditions for solvability of Problem 1. We define a set $\mathscr{Y}_r\subseteq\mathbb{R}^p$ solely based on the knowledge of the agent models and we show that problem 1 is solvable if and only if the constant reference trajectory belongs to this set. In this case, the agent models can be general and non right-invertible. We begin first by defining set $\mathscr{Y}_r$ as following.

$$\begin{aligned}\mathscr{Y}_r &= \left\{y\in\mathbb{R}^p \middle| \begin{pmatrix}0\\ y\end{pmatrix}\in Im\begin{pmatrix}A & B\\ C & 0\end{pmatrix}\right\}\\ &=\{y\in\mathbb{R}^p|\exists x\in\mathbb{R}^n, u\in\mathbb{R}^m: Ax+Bu=0, Cx=y\}.\end{aligned}$$

Note that $\mathscr{Y}_r=\mathbb{R}^p$ if (A,B,C) is right-invertible and without invariant zeros in the origin.

Next, for a given $y_r\in\mathscr{Y}_r$, we provide protocol design which has the same architecture as the previous subsection. The first step is designing a pre-compensator for each agent and the second step is designing collaborative protocols for the compensated agents to achieve state synchronization.

Step I: Let R be an injective matrix such that $\mathscr{Y}_r=ImR$. In this case, we can find the matrices Π and Γ such that:

$$\begin{pmatrix}0\\ R\end{pmatrix}=\begin{pmatrix}A & B\\ C & 0\end{pmatrix}\begin{pmatrix}\Pi\\ \Gamma\end{pmatrix} \tag{32}$$

and

$$rank_{\mathbb{R}}\begin{pmatrix}A & B\Gamma\\ C & 0\end{pmatrix}=n+rank_{\mathbb{R}}\Gamma. \tag{33}$$

Given that (A,C) detectable, the first n columns of $\begin{pmatrix}A & B\Gamma\\ C & 0\end{pmatrix}$ are linearly independent. If (33) is not satisfied, then there exist x and v such that

$$\begin{pmatrix}A & B\Gamma\\ C & 0\end{pmatrix}\begin{pmatrix}x\\ v\end{pmatrix}=0$$

with $B\Gamma v\neq 0$, $v\perp\ker\Gamma$ and $v^{\mathrm{T}}v=1$. On the other hand, we have

$$\begin{pmatrix}A & B\\ C & 0\end{pmatrix}\begin{pmatrix}\Pi-xv^{\mathrm{T}}\\ \Gamma(I-vv^{\mathrm{T}})\end{pmatrix}=\begin{pmatrix}0\\ R\end{pmatrix}$$

It shows that $\tilde{\Pi}=\Pi-xv^{\mathrm{T}}$ and $\tilde{\Gamma}=\Gamma(I-vv^{\mathrm{T}})$ also satisfy the above equation but with $rank_{\mathbb{R}}\tilde{\Gamma}<rank_{\mathbb{R}}\Gamma$. Recursively, we can

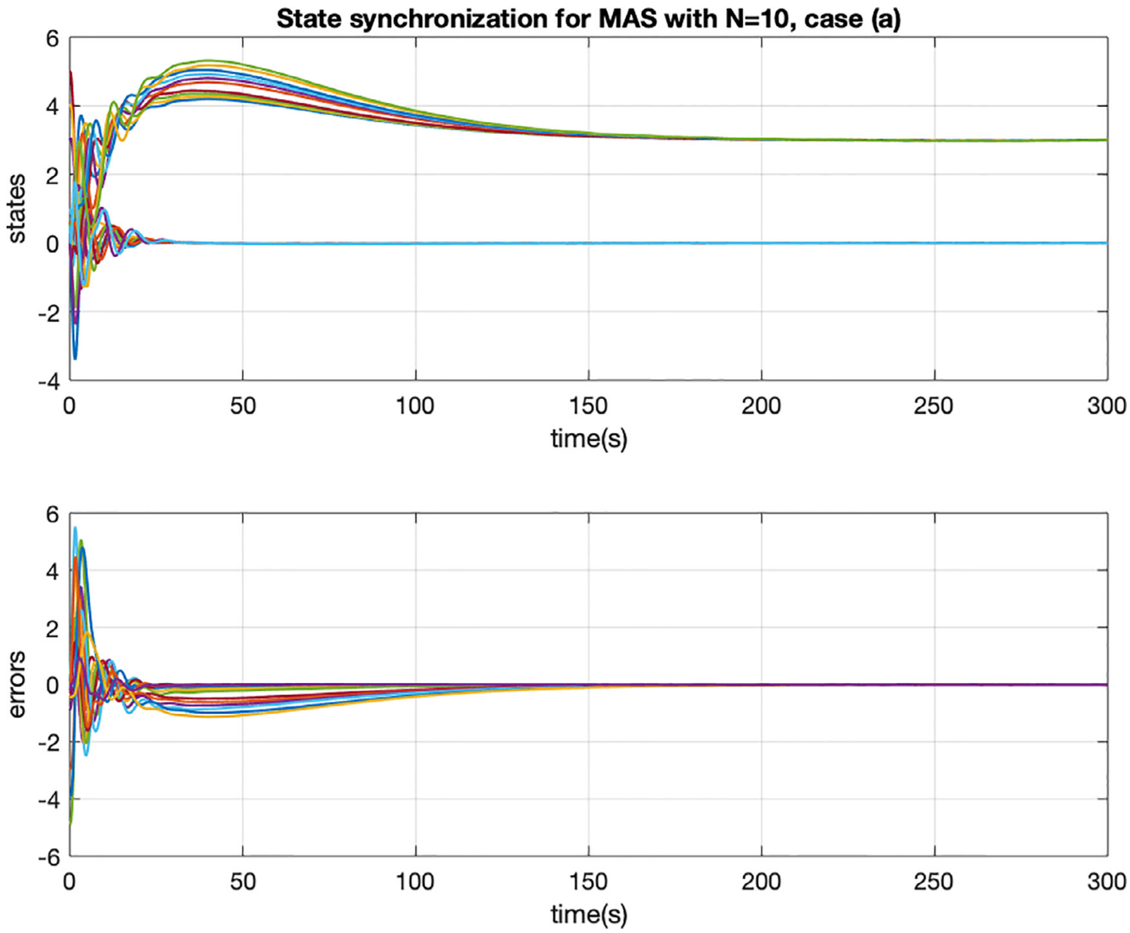

Fig. 4. State synchronization of MAS with right-invertible agents and $N=10$ and communication delays as stated in case (3) (a).

find a solution of (32) such that the rank condition (33) is satisfied.

Then, similar to precompensator design in Section 3.1, with Γ obtained as above, we have the following precompensator.

$$\dot{p}_i(t) = \begin{pmatrix} I & 0 \end{pmatrix} v_i(t), \qquad p_i(t) \in \mathbb{R}^r$$
$$u_i(t) = \Gamma_1 p_i(t) + \begin{pmatrix} 0 & \Gamma_2 \end{pmatrix} v_i(t), \tag{34}$$

where Γ_1 and Γ_2 are chosen such that $Im\Gamma_1 = Im\Gamma$ and

$$\begin{pmatrix} \Gamma_1 & \Gamma_2 \end{pmatrix} \tag{35}$$

is square and invertible.

In order to design collaborative protocols we first obtain the compensated agents by combining (1) and (34) as

$$\dot{\bar{x}}_i(t) = \bar{A}\bar{x}_i(t) + \bar{B}v_i(t)$$
$$y_i(t) = \bar{C}\bar{x}_i(t),$$

where

$$\bar{x}_i = \begin{pmatrix} x_i \\ p_i \end{pmatrix}, \bar{A} = \begin{pmatrix} A & B\Gamma_1 \\ 0 & 0 \end{pmatrix}, \bar{B} = \begin{pmatrix} 0 & B\Gamma_2 \\ I & 0 \end{pmatrix}, \bar{C} = \begin{pmatrix} C & 0 \end{pmatrix}.$$

We need to verify the stabilizability and detectability of the compensated system. The stability follows immediately from (35) and the stabilizability of (A, B), and for detectability we need to verify that

$$rank_{\mathbb{C}} \begin{pmatrix} sI - A & -B\Gamma_1 \\ 0 & sI \\ C & 0 \end{pmatrix} = n + v$$

for all s in the closed right-half complex plane. For $s \neq 0$, this immediately follows form the detectability of (C, A). For $s = 0$, we have

$$rank_{\mathbb{R}} \begin{pmatrix} -A & -B\Gamma_1 \\ 0 & 0 \\ C & 0 \end{pmatrix} = rank_{\mathbb{R}} \begin{pmatrix} -A & -B\Gamma \\ C & 0 \end{pmatrix} = n + rank_{\mathbb{R}}\Gamma_1 = n + v.$$

Since $rank_{\mathbb{R}}\Gamma = rank_{\mathbb{R}}\Gamma_1$ and $rank_{\mathbb{R}}\Gamma_1 = r$ (since Γ_1 is injective), we can obtain $(\bar{C}, \bar{A})$ is detectable.

Step II: In this step, we design collaborative protocol for the compensated agents similar to collaborative protocol designed in Section 3.1.

$$\begin{cases} \dot{\hat{x}}_i(t) = \bar{A}\hat{x}_i(t) - \bar{B}K\hat{\zeta}_i(t) + F(\bar{\zeta}_i(t) - \bar{C}\hat{x}_i(t)) + \iota_i\bar{B}v_i(t) \\ \dot{\chi}_i(t) = \bar{A}\chi_i(t) + \bar{B}v_i(t) + \hat{x}_i(t) - \hat{\zeta}_i(t) - \iota_i\chi_i(t) \\ v_i(t) = -K\chi_i(t), \end{cases} \tag{36}$$

where matrices K and F are such that $\bar{A} - F\bar{C}$ and $\bar{A} - \bar{B}K$ are Hurwitz stable. In this protocol, the agents communicate $\xi_i(t) = \chi_i(t)$, i.e. each agent has access to localized information exchange $\hat{\zeta}_i(t)$ and $\bar{\zeta}_i(t)$ defined by (19) and (8), respectively.

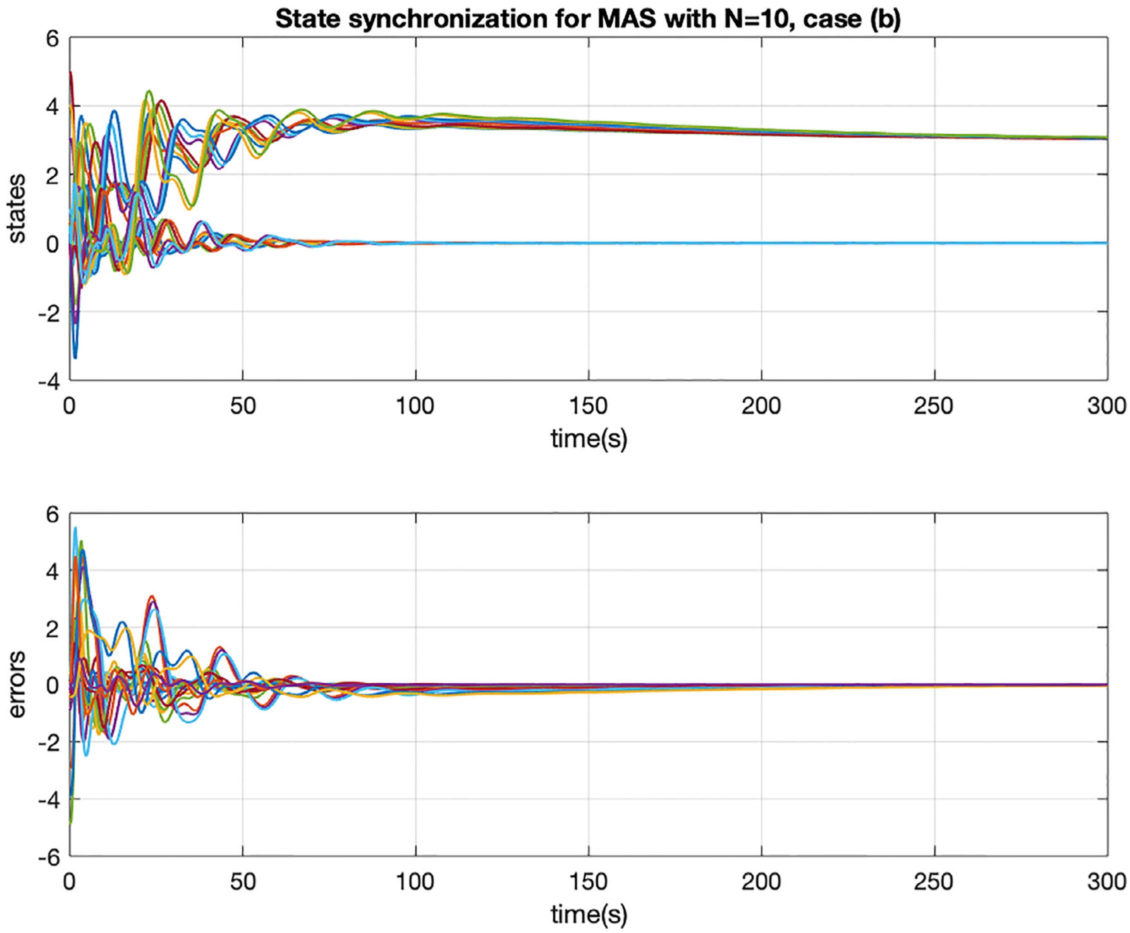

Fig. 5. State synchronization of MAS with right-invertible agents and $N = 10$ and communication delays as stated in case (3) (b).

Then, we have the following theorem.

Theorem 2. *Consider a MAS described by (1) and (8) where (A, B) is stabilizable and (A, C) is detectable. Assume Assumption 1 is satisfied. Let a set of nodes $\mathscr{C}$ be given which defines the set $\mathbb{G}^N_{\mathscr{C}}$.*

*Then, the scalable state synchronization problem with localized information exchange via linear dynamic protocol as stated in Problem 1 is solvable **if and only if** $y_r \in \mathscr{Y}_r$. More specifically, for any $y_r \in \mathscr{Y}_r$, protocol (36) and (34) achieves scalable state synchronization for any communication delays $\tau_{ij} \in \mathbb{R}_{\geq 0}$ $(i \neq j)$ and any graph $\mathscr{G} \in \mathbb{G}^N_{\mathscr{C}}$ with any size of the network N.*

Proof of Theorem 2..

1. Necessity: In order agents track a constant reference trajectory signal y_r, there must exists x_0 and u_0 such that
$$\begin{pmatrix} A & B \\ C & 0 \end{pmatrix}\begin{pmatrix} x_0 \\ u_0 \end{pmatrix} = \begin{pmatrix} 0 \\ y_r \end{pmatrix} \tag{37}$$
Clearly, such x_0 and u_0 exist only if y_r belongs to the set $\mathscr{Y}_r$, that proves the necessary condition.
2. *Sufficiency:* For the sufficiency, we need to show that protocol (36) and (34) solves Problem 1. The proof is exactly the same as proof of Theorem 1 except for the choice of $\bar{\Pi}$ and $\tilde{x}$. In this case, we choose $\tilde{x}_i(t) = \bar{x}_i(t) - \bar{\Pi} z$ where z is such that $y_r = Rz$. Moreover, we set
$$\bar{\Pi} = \begin{pmatrix} \Pi \\ W \end{pmatrix}$$
where W is such that $\Gamma_1 W = \Gamma$. It is then easily seen that $\bar{A}\bar{\Pi} = 0$ and $\bar{C}\bar{\Pi} = R$.

□

4. Numerical examples

The aim of this section is to show the scalability and effectiveness of our protocol design via numerical examples. To show the scalability, we consider three networks with different communication graphs, different number of agents and will show that we can achieve state synchronization with our one-shot-designed protocol. We also illustrate that our protocol can tolerate arbitrarily large communication delays.

Consider the agents model (1) as
$$\begin{cases} \dot{x}_i(t) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x_i(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i(t), \\ y_i(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x_i(t) \end{cases}$$

Since our agents include eigenvalues at the origin, we do not need to design pre-compensators. By choosing matrix K and F as $K = F^{\mathrm{T}} = \begin{pmatrix} 3 & 2 \end{pmatrix}$, we have the following protocol,
$$\begin{cases} \dot{\hat{x}}_i(t) = \begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix} \hat{x}_i(t) - \begin{pmatrix} 0 & 0 \\ 3 & 2 \end{pmatrix} \hat{\zeta}_i(t) + \begin{pmatrix} 3 \\ 2 \end{pmatrix} \bar{\zeta}_i(t) + \iota_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i(t) \\ \dot{\chi}_i(t) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \chi_i(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i(t) + \hat{x}_i(t) - \hat{\zeta}_i(t) - \iota_i \chi_i(t) \\ u_i(t) = -\begin{pmatrix} 3 & 2 \end{pmatrix} \chi_i(t) \end{cases} . \tag{38}$$

Given that agent models are right-invertible, we can regulate the states to any arbitrary constant trajectory. In all the following cases we choose $y_r = 3$.

1. Firstly, we consider a MAS with 3 agents, $N = 3$ and communication network with associated adjacency matrix $\mathcal{A}_1$, where $a_{21} = a_{32} = 1$. Communication delays are chosen as $\tau_{32} = 2\,\text{sec}$ and the rest are equal to zero. The results of state synchronization via protocol (38) are presented in Fig. 2.
2. Next, we consider a MAS with 5 agents, $N = 5$, and communication network with associated adjacency matrix $\mathcal{A}_2$, where $a_{13} = a_{21} = a_{25} = a_{32} = a_{35} = a_{43} = a_{54} = 1$. Communication delays are chosen as $\tau_{13} = 0.2\,\text{sec}$, $\tau_{32} = 1\,\text{sec}$, $\tau_{35} = 0.5\,\text{sec}$ and the rest are equal to zero. The simulation results are shown in Fig. 3.
3. Finally, we consider a MAS with 10 agents and communication network with associated adjacency matrix $\mathcal{A}_3$, where $a_{21} = a_{5,10} = a_{32} = a_{43} = a_{54} = a_{65} = a_{76} = a_{87} = a_{98} = a_{10,9} = a_{15} = 1$. In this case, we also show that we can achieve state synchronization for any unknown, nonuniform and arbitrarily large communication delays. Therefore, we consider two cases as following.
   (a) The simulation results in the case that communication delays are $\tau_{54} = 2.5, \tau_{65} = 1\,\text{sec}$, $\tau_{98} = 3\,\text{sec}$ and the rest are equal to zero, are presented in Fig. 4.
   (b) In the second case, we consider the 10 nodes MAS when the delays are equal $\tau_{54} = 4, \tau_{65} = 6\,\text{sec}$, $\tau_{98} = 8\,\text{sec}$ and the rest are equal to zero. The simulation results are shown in Fig. 5.

   According to Figs. 4 and 5, we observe that state synchronization is achieved regardless of values of the communication delays.

5. Conclusion

In this paper we have proposed scale-free protocol design utilizing localized information exchange for state synchronization of homogeneous networks subject to unknown, nonuniform and arbitrarily large communication delays. The necessary and sufficient solvability conditions also has been provided. The proposed *scale-free* protocols were designed solely based on agent models without utilizing any information about the communication network such as bounds on the Laplacian matrix associated to the communication graph and the size of the network. It is worth noting that considering the arbitrarily time-varying communication delay in the *scale-free framework* is the subject matter of our future work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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