SCHEDULING ON-DEMAND MINIBUSES CONSIDERING THE IN-VEHICLE CROWDING INCONVENIENCE DUE TO COVID-19

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ABSTRACT
On-demand minibus services have received significant attention during the COVID-19 pandemic because passengers are seeking alternatives outside the traditional public transport system. During the pandemic's peak, public transport ridership was reduced 70%-90% in several countries and many public transport users had to seek less crowded alternatives in demand responsive services. Scheduling the operations of demand responsive services requires to solve a standard Dial-A-Ride Problem (DARP). However, DARP problems do not consider the in-vehicle crowding as long as the capacity of the vehicles is not exceeded. To rectify this, this study proposes a new formulation that considers also the inconvenience of passengers due to the in-vehicle crowding levels. This study proposes a progressive penalization of the increase of in-vehicle crowding to account for social distancing inside on-demand minibus services. This is modeled with piecewise linear functions that map the inconvenience of passengers to the in-vehicle crowding levels. The proposed model is a MINLP and it is reformulated as a MILP that can be solved with branch-and-bound and linear programming. An alternative multi-objective optimization formulation is also introduced based on the ε-constraint method. This model is implemented in numerical experiments with benchmark DARP datasets to investigate the increases of the vehicle route costs when seeking to reduce the in-vehicle crowdedness.

Keywords: on-demand minibus scheduling, demand responsive, paratransit, social distancing, COVID-19.

INTRODUCTION

COVID-19 has significantly disrupted public transport operations with many passengers reducing their number of trips or shifting to alternative modes such as private vehicles, shared cars or on-demand minibus services (*1, 2*). At the beginning of new pandemic waves we occasionally have public transport ridership drops of 60%-90% in many cities in China, Iran, the United States, the United Kingdom, and Europe (*3–6*). The considerable drops in public transport ridership are influenced by the perceptions of passengers in terms of virus transmission risks, government guidelines and shifts in work practices (*7–9*). Travelers that do not use public transport because of the aforementioned reasons and, at the same time, do not own a private vehicle opt for alternative on-demand transport modes. In this study, we explicitly focus on on-demand minibuses that can offer alternatives to traditional public transport services, such as metro systems, light rail and fixed-line buses. Unlike traditional public transport services, on-demand minibuses have a limited number of seats (typically up to 8 seats) reducing the risk of virus transmission. In addition, passengers do not have to wait at crowded stations since they can be picked up by their home location.

We consider the on-demand minibus scheduling problem as the problem of defining the optimal routes of a number of available vehicles (minibuses) to accommodate the requests of travelers in a cost-efficient manner while taking into consideration different scenarios of social distancing. Minibuses are assumed to start from a terminal (depot) and they are assigned a number of passenger requests, where each passenger request includes a pick-up and a delivery operation. This problem definition extends the classic Dial-a-Ride Problems (DARP) and Pickup and Delivery Vehicle Routing Problems (PDVRP) that have been studied since the 1980s (*10, 11*).

This study addresses the on-demand minibus scheduling problem (MSP) where all passenger requests are known in advance (*12–15*). Our MSP aims at minimizing the total routing cost of all minibuses subject to full demand satisfaction and social distancing constraints. Because the resource availability (number of available minibuses) is limited, this might result in cases where it is not possible to accommodate all passenger demand. In particular, travelers are allowed to impose a time window on both their departure from their origin location and their arrival at their destination location (*16*). In addition, they can impose a maximum limit for their total ride time because if their trip takes too long they might opt for another means of transportation. Finally, the travel times and travel costs to traverse the network's edges (roads) are deterministic and the maximum route time of a minibus before returning back to its depot is fixed. The latter requirement is particularly important if the minibuses are electric vehicles and must return to the depot for charging (*17*).

The remainder of this study is structured as follows. Section 2 provides the literature review of DARP and MSP problems. Section 3 introduces the problem formulation for the MSP that considers social distancing. Section 4 reformulates the MSP to a mixed-integer linear program that can be solved with branch-and-bound and linear programming solvers. Section 5 offers a multi-objective optimization formulation of the problem that considers separately the vehicle costs and the in-vehicle crowding. Section 6 provides the numerical experiments considering different passenger sensitivities to social distancing. Finally, section 7 concludes our study and provides future research directions.

LITERATURE REVIEW

Dial-a-Ride Problem (DARP)

We start our literature review of the Dial-a-Ride Problem (DARP) that can be seen as a problem of scheduling vehicles to accommodate traveler requests. In its simplest form, DARP finds the lest costly route of a single vehicle (*18*). DARP extends the Pickup and Delivery Vehicle Routing Problem (PDVRP) in logistics by considering the ride times of passengers when traveling from their origin to their destination (*19, 20*). In DARP the travel requests consider the origin and destination locations of users, whereas in PDVRP the travel requests consider the pickup and delivery points for parcels.

PDVRP and DARP are both NP-Hard optimization problems and there are no algorithms that can guarantee the solution of these problems to global optimality for large problem instances. For this reason, large DARP instances are solved with heuristics and metaheuristics that typically return good (but not globally optimal) solutions within a limited time. Heuristics and metaheuristics for the DARP problem can range from tabu search to regret-insertion heuristics (*10, 21–26*). DARP studies are discussed in our literature review because DARP has many common characteristics with the problem of on-demand minibus scheduling. In a typical DARP problem description we have a number of travelers willing to travel from their origin points to destination points by spending less than a maximum ride time and starting and/or finishing their trip within a predetermined time window. To provide a tangible example, a traveler declares his/her origin and destination locations, the time window for his/her pickup and delivery, and his/her maximum ride time. Given this input, DARP seeks to satisfy the requests of travelers in such a way that the route costs of the deployed vehicles are minimized (*10*).

It is finally worth noting that recent DARP formulations consider more objectives that go beyond the vehicle route costs. Such objectives include vehicle emissions (*27, 28*), staff workload (*29*), operational reliability (*30*) and in-vehicle occupancy rates (*31*). To the best of our knowledge, however, there are no works that consider social distancing and investigate its effect to operational costs.

On-demand Minibus Scheduling Problem (MSP)

We now consider the literature related to the on-demand Minibus Scheduling Problem (MSP). Unlike minibus services that operate in fixed lines (*32*), the on-demand MSP does not impose any fixed schedule requirements to the operational minibuses. That is, minibuses are not operating in fixed lines with specific stops, frequencies and timetables. Because of this, there are no requirements to solve problems related to network design (*33–35*), frequency setting (*36, 37*) and timetabling (*38–40*) when scheduling on-demand minibuses. In contrast, we have a pool of available minibuses at the depot and we seek to deploy them as efficiently as possible to satisfy the passenger requests. The passenger requests are similar to the travel requests described in DARP problems and include the origin and destination locations of passengers, their pickup and delivery time windows, and their maximum ride times. Under this context, the on-demand MSP is closer to the DARP problem rather than the typical scheduling problems of fixed-line public transport services.

In past studies, there have been analytical models for comparing on-demand bus services against fixed-line bus services (*41*). The performance of on-demand bus services has been also tested with agent-based simulation models (*42*). Kim and Schonfeld (*43*) compared the performances of the optimized solutions of fixed-line bus services and on-demand bus services in various scenarios. It is worth noting that on-demand bus or minibus services are also called *demand responsive* or *paratransit* services to differentiate them from conventional, fixed-line services. These scheduling of these on-demand services is fundamentally different than the scheduling of conventional (mini)bus services. As already discussed, our on-demand minibus scheduling problem is close to DARP problems. There are, however, other common problem descriptions for on-demand minibus scheduling problems. Namely, the Call-n-Ride scheduling problem in which drivers take requests for service directly on their cell phones and make all routing and scheduling decisions without a central dispatcher (*44*), the multi-depot DARP (*45*), and the High Coverage Point-to-Point Transit (HCPPT) problem that strives to reduce the number of transfers (*46*). From the aforementioned on-demand scheduling problems, the DARP problem description is the most common and this is why we reviewed the DARP works separately in our literature review.

Our study considers the case where minibuses are located at a depot and need to satisfy a number of passenger requests. Our problem description is closely linked to single-depot DARP problems, but we also consider the in-vehicle crowdedness to investigate the effect of different social distancing scenarios to the vehicle running costs. Our focus is explicitly on this aspect because during the pandemic passengers prefer less crowded vehicles and service providers have to follow the social distancing regulations of local governments. This study contributes in this direction by:

1. formulating the on-demand minibus scheduling problem as a mixed-integer nonlinear program (MINLP) by incorporating the in-vehicle crowdedness to the objective function of the problem with the use of piecewise linear functions.
2. reformulating the MINLP to an easier-to-solve mixed-integer linear program (MILP) that can be solved with branch-and-bound and linear programming.
3. implementing the proposed model to banchamrk problem instances in order to investigate the potential increases in the vehicle route costs when the social distancing requirements become more strict.

PROBLEM FORMULATION

Consider a network $G = (V, A)$ with V vertices and A arcs. An arc $a \in A$ is a direct connection between two vertices of the network and it can be seen as the fastest path between them. The vertices of the network are all origin and destination points of the passenger requests, including also the location of the depot. If we have multiple origin (or destination) points at the same physical location, then each point is represented by a different vertex. That is, a unique physical location might be represented by more than one vertex. The vertices of the network are denoted as $V = P \cup D \cup \{0, 2n+1\}$ where $P = \{1, 2, ..., n\}$ is the set of origin points of the user requests, $D = \{n+1, ..., 2n\}$ the set of destination points of the user requests, and 0, $2n+1$ are two copies of the depot. The network is encoded in such a way that the origin i of each user request corresponds to the destination $n+i$ and the origin P and destination D sets have the same number of vertices. If we have several passengers that travel from origin i to destination $n+i$ we do not need to generate multiple vertices for them. That is, we generate a new request with unique origin and destination vertices for every unique origin-destination pair.

Fig.1 provides a simple example with two passenger requests. The origin vertices 1 and 2 can be the same physical location. The same applies to the destination vertices 3 and 4. Vertices 3 and 4 are encoded in such a way that the picked-up passenger(s) from vertex 1 will be delivered to vertex 3 and from vertex 2 to vertex 4. Note also that vertices 0 and 5 are at the same physical location (the location of the depot). In addition, we have arcs $A = \{(0, j) : j \in P\} \cup \{(i, j) : i \in P \cup D, j \in P \cup D, i \neq j\} \cup \{(i, 2n+1) : i \in D\}$. These arcs connect directly any pair of pickup and delivery vertices. In addition, they connect directly vertex 0 with the possible origin points and vertex $2n+1$ with the possible destination points. This ensures that each minibus performs a tour:

it starts from the depot, it performs a number of traveler requests, and it returns to the depot.

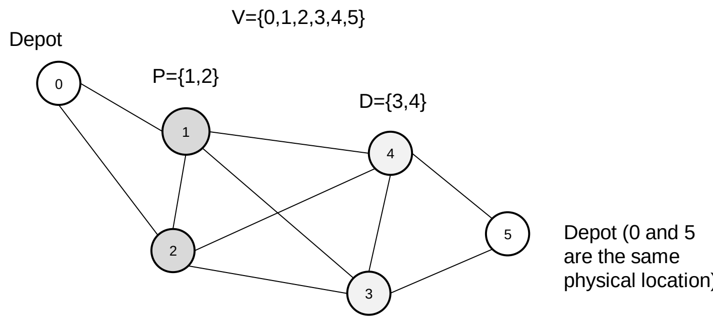

FIGURE 1 Network representation in the case of two traveler requests. Note that in vertices 1 and 2 we might have more than one passengers. respectively.

The main requirements of the on-demand minibus scheduling problem are:

- every route of a minibus $k \in K$ starts from the depot and ends at the depot;
- for every traveler request i, vertices i and $n+i$ belong to the same route and vertex $n+i$ is visited later than vertex i;
- the passenger load of any minibus k does not exceed its capacity, Q_k;
- the total duration of the route of any minibus k does not exceed a preset time bound T_k;
- the pickup (or delivery) of passengers at vertex i begins within a pre-defined time window $[e_i, l_i]$ and every minibus leaves the depot within a pre-defined time window $[e_0, l_0]$;
- the total ride time of any passenger does not exceed a pre-determined maximum value L.

Before proceeding to the formal presentation of the mathematical program, we introduce the parameters and variables of the model in Table 1.

TABLE 1 Nomenclature

Sets

G	$G=(V,A)$ is the network with vertices V and arcs A
P	set of origin points that have unique destination points $P=\{1,...,n\}$
D	set of destination points that have unique origin points $D=\{n+1,...,2n\}$
V	set of all vertices $V=P\cup D\cup\{0,2n+1\}$, where 0 and $2n+1$ are two copies of the depot
A	set of all arcs $A=\{(0,j):j\in P\}\cup\{(i,j):i\in P\cup D, j\in P\cup D, i\neq j\}\cup\{(i,2n+1):i\in D\}$
K	set of available minibuses

Parameters

c_{ij}^k	cost of traversing arc $(i,j)\in A$ with minibus k
t_{ij}	travel time of traversing arc $(i,j)\in A$
T_k	maximum allowed travel time by minibus k
$[e_i,l_i]$	time window of minimum and maximum amount of time related to serving vertex $i\in V\setminus\{2n+1\}$
d_i	time spent at request $i\in P\cup D$ to pickup or deliver passengers (service time). Note that $d_0=d_{2n+1}=0$
q_i	associated passenger load of vertex $i\in V$, where $q_0=q_{2n+1}=0$, $q_i\geq 0\ \forall i\in P$, $q_i=-q_{i-n}\ \forall i\in D$
L	maximum allowed ride time of a passenger
γ	a non-negative parameter indicating the relative importance of reducing the in-vehicle crowding levels compared to the vehicle travel costs
z_k	soft capacity of vehicle k that corresponds to the social distancing requirement (i.e., maintaining 1-meter distancing)
Q_k	hard capacity of vehicle k. This is the actual vehicle capacity

Variables

x_{ij}^k	binary variable indicating whether arc $(i,j)\in A$ is traversed by vehicle k
u_i^k	the time at which vehicle k starts servicing vertex i
w_i^k	the passenger load of vehicle k upon leaving vertex i
r_i^k	the ride time of any user of request $(i,n+i)$

The objective of the on-demand minibus scheduling problem is to minimize the vehicle route costs:

$$\sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k$$

In this study, we also consider the increasing inconvenience of passengers when there are other passengers in the minibus. This reflects the impact of social distancing because passengers would prefer to board less crowded minibuses that reduce the virus transmission risk. This requirement is considered as another problem objective resulting in the generalized objective function:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k \in K} \sum_{i \in V} f(w_i^k)$$

where γ is a non-negative parameter indicating the relative importance of reducing the in-vehicle crowding levels compared to the vehicle route costs. In addition, $f : w_i^k \mapsto R_+$ is a function that maps the in-vehicle passenger loads w_i^k to the perceived passenger inconvenience. Function $f(w_i^k)$ reflects the discomfort of passengers with respect to social distancing. In the simpler case, one can assume a piecewise linear function of the form:

$$f(w_i^k) := \begin{cases} w_i^k \rho & 0 \leq w_i^k \leq z_k \\ z_k \rho + (w_i^k - z_k)\theta & z_k \leq w_i^k \leq Q_k \end{cases}$$

where $\rho, \theta \in \mathbb{R}_+$ and $\theta \ggg \rho$ indicating a sharp increase in passenger inconvenience when the in-vehicle crowding level exceeds the soft capacity z_k that corresponds to a regulatory suggestion. A graphical representation of this function is provided in Fig.2.

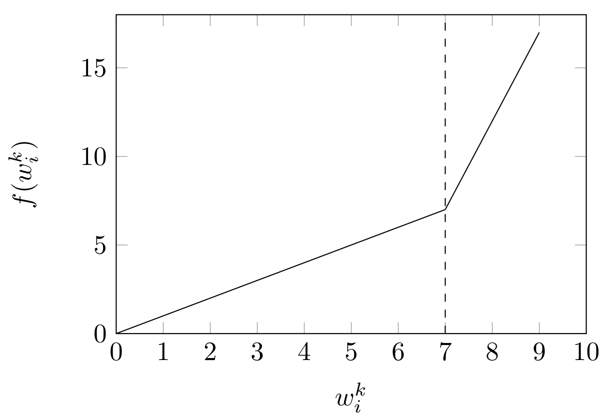

FIGURE 2 Example of the piecewise linear function $f(w_i^k)$ for $z_k = 7$ passengers, $Q_k = 9$ passengers, $\rho = 1$ and $\theta = 5$.

Note that the example piecewise linear function in Fig.2 is devised in such a way that there is already a passenger inconvenience even if we have one passenger in the vehicle because there is also a second person in it (the driver). If this is not considered, the aforementioned function can be simply replaced by the following piecewise linear function:

$$f(w_i^k) := \begin{cases} 0 & 0 \leq w_i^k \leq 1 \\ (w_i^k - 1)\rho & 1 \leq w_i^k \leq z_k \\ (z_k - 1)\rho + (w_i^k - z_k)\theta & z_k \leq w_i^k \leq Q_k \end{cases}$$

Based on the proposed objective function, the on-demand minibus scheduling problem that considers in-vehicle crowding levels is cast as:

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k\in K}\sum_{i\in V} f(w_i^k) \tag{1}$$

subject to:

$$\sum_{k\in K}\sum_{j:(i,j)\in A} x_{ij}^k = 1 \qquad \forall i \in P \tag{2}$$

$$\sum_{i:(0,i)\in A} x_{0i}^k = \sum_{i:(i,2n+1)\in A} x_{i,2n+1}^k = 1 \qquad \forall k \in K \tag{3}$$

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(n+i,j)\in A} x_{n+i,j}^k = 0 \qquad \forall i \in P, k \in K \tag{4}$$

$$\sum_{j:(j,i)\in A} x_{ji}^k - \sum_{j:(i,j)\in A} x_{ij}^k = 0 \qquad \forall i \in P \cup D, k \in K \tag{5}$$

$$u_j^k \geq (u_i^k + d_i + t_{ij}) x_{ij}^k \qquad \forall (i,j) \in A, k \in K \tag{6}$$

$$w_j^k \geq (w_i^k + q_j) x_{ij}^k \qquad \forall (i,j) \in A, k \in K \tag{7}$$

$$r_i^k = u_{n+i}^k - (u_i^k + d_i) \qquad \forall i \in P, k \in K \tag{8}$$

$$u_{2n+1}^k - u_0^k \leq T_k \qquad \forall k \in K \tag{9}$$

$$e_i \leq u_i^k \leq l_i \qquad \forall i \in V, k \in K \tag{10}$$

$$t_{i,n+i} \leq r_i^k \leq L \qquad \forall i \in P, k \in K \tag{11}$$

$$\max\{0, q_i\} \leq w_i^k \qquad \forall i \in V, k \in K \tag{12}$$

$$w_i^k \leq \min\{Q_k, Q_k + q_i\} \qquad \forall i \in V, k \in K \tag{13}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall i,j \in V, k \in K \tag{14}$$

Constraints (2) ensure that any origin point $i \in P$ will be directly connected with exactly one other vertex $j \in P \cup D$ by exactly one vehicle k. Constraints (3) and (5) ensure that the route of each minibus k starts from the depot 0 and ends at the depot $2n+1$. Constraints (4) ensure that each minibus k that serves request $i \in P$ will serve the corresponding destination vertex $n+i$. This ensures that the origin and destination points of a request are served by the same minibus.

Constraints (6) ensure that if arc $(i,j) \in A$ is served by minibus k, $x_{ij}^k = 1$, then the time at which minibus k starts servicing vertex j (denoted as u_j^k) is greater than or equal to u_i^k plus the travel time from i to j (denoted as t_{ij}) plus the service time at i (denoted as d_i). Constraints (9) ensure that minibus k will return to the depot before T_k. Constraints (10) ensure that each vertex $i \in V$ will be served within its predefined time window.

Constraints (8) ensure that the ride time of a passenger that uses minibus k to travel from origin i to destination $n+i$ (denoted as r_i^k) is equal to $u_{n+i}^k - (u_i^k + d_i)$. Constraints (11) ensure that the ride time of a passenger from i to $n+i$ who uses minibus k is less than the maximum allowed passenger ride time, L. It is also greater than or equal to the minimum travel time $t_{i,n+i}$ corresponding to a direct connection between i and $n+i$.

Constraints (7) ensure that if a vehicle k serves arc (i,j) the passenger load upon leaving vertex j is greater than or equal to the load at vertex i plus q_j. Constraints (14) ensure that the in-vehicle passenger load w_i^k upon leaving vertex i is at least $\max\{0, q_i\}$ and, at the same time, does not exceed the capacity of the minibus $\min\{Q_k, Q_k + q_i\}$.

Remark 3.1. *The mathematical program in Eqs.*(1)-(14) *is a mixed-integer nonlinear program (MINLP) because of the nonlinear constraints* (6)-(7) *where we have a multiplication between decision variables and the objective function that contains the conditional term* $f(w_i^k)$ *when adopting a piecewise linear expression. In the following section, we propose a problem reformulation to an easier-to-solve MILP.*

MILP REFORMULATION OF THE ON-DEMAND MINIBUS SCHEDULING PROBLEM

Note that constraints (6) and (7) are nonlinear. To linearize them, Cordeau (*47*) proposed to introduce parameters M_{ij}^k and Y_{ij}^k with very high positive values and replace (6)-(7) by constraints (15)-(16).

$$u_j^k \geq u_i^k + d_i + t_{ij} - M_{ij}^k(1 - x_{ij}^k) \quad \forall (i,j) \in A, k \in K \tag{15}$$

$$w_j^k = w_i^k + q_j - Y_{ij}^k(1 - x_{ij}^k) \quad \forall (i,j) \in A, k \in K \tag{16}$$

We also linearize the proposed piecewise linear function that considers the presence of a driver in the vehicle:

$$f(w_i^k) := \begin{cases} w_i^k \rho & 0 \leq w_i^k \leq z_k \\ z_k \rho + (w_i^k - z_k)\theta & z_k \leq w_i^k \leq Q_k \end{cases}$$

The piecewise linear function $f(w_i^k)$ is linearized as follows:

$$f(w_i^k) := z_k \rho \lambda_{i1}^k + (z_k \rho + (Q_k - z_k)\theta)\lambda_{i2}^k \tag{17}$$

$$\text{s.t.} \quad \zeta_{i1}^k + \zeta_{i2}^k = 1 \quad \forall i \in V, k \in K \tag{18}$$

$$\lambda_{i0}^k + \lambda_{i1}^k + \lambda_{i2}^k = 1 \quad \forall i \in V, k \in K \tag{19}$$

$$\zeta_{i1}^k \leq \lambda_{i0}^k + \lambda_{i1}^k \quad \forall i \in V, k \in K \tag{20}$$

$$\zeta_{i2}^k \leq \lambda_{i1}^k + \lambda_{i2}^k \quad \forall i \in V, k \in K \tag{21}$$

$$\zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\} \quad \forall i \in V, k \in K \tag{22}$$

$$w_i^k = z_k \lambda_{i1}^k + Q_k \lambda_{i2}^k \quad \forall i \in V, k \in K \tag{23}$$

$$\lambda_{i0}^k, \lambda_{i1}^k, \lambda_{i2}^k \in \mathbb{R}_+ \quad \forall i \in V, k \in K \tag{24}$$

where $\zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\}$ and $\lambda_{i0}^k, \lambda_{i1}^k, \lambda_{i2}^k \in \mathbb{R}_+$ are newly added decision variables. This results in the reformulated mathematical program:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k \in K} \sum_{i \in V} \Big(z_k \rho \lambda_{i1}^k + (z_k \rho + (Q_k - z_k)\theta)\lambda_{i2}^k \Big) \tag{25}$$

$$\text{s.t. Eqs. } (2)-(5), (8)-(24) \tag{26}$$

$$x_{ij}^k \in \{0,1\} \quad \forall i,j \in V, k \in K \tag{27}$$

The reformulated problem is a MILP. If we drop the integrality constraints (22) and (27) related to $\zeta_{i1}^k, \zeta_{i2}^k$ and x_{ij}^k, the continuous relaxation of the MILP is a linear program (LP) that can be solved to global optimality.

Lemma 4.1. *The reformulated program in Eqs.*(28)-(27) *can be solved to global optimality when dropping its integrality constraints.*

Proof. Without the integrality constraints, we have a continuous relaxation of the problem. The remaining constraints are equality and inequality constraints of affine functions. Each equality constraint can be easily substituted by a pair of two inequality constraints resulting in two *halfspaces*. Thus, all constraints can be formulated as inequality constraints of affine functions and this forms a *polyhedron*, which is the intersection of a finite number of (closed) halfspaces resulting in a convex set. In addition, the linearized objective function is convex (and concave). Thus, the problem is convex and any locally optimal solution is a globally optimal one. □

To find the globally optimal solution of the MILP expressed in Eqs.(28)-(27) one can apply a branch-and-bound algorithm (*48*) that allows $x_{ij}^k, \zeta_{i1}^k, \zeta_{i2}^k$ to take real values in the closed interval [0,1]. When branching, some of these variables receive fixed $\{0,1\}$ values and some others are free to take real values from the interval [0,1]. The performance of each node that emerges from branching is calculated by solving a linear program with simplex or the Ellipsoid method and the branch-and-bound algorithm terminates when finding the optimal solution with $x_{ij}^k, \zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\}$.

MULTI-OBJECTIVE OPTIMIZATION FORMULATION AND PARETO OPTIMALITY

Our MILP expressed in Eqs.(28)-(27) is a single-objective optimization problem that considers the in-vehicle crowding that reflects the discomfort of passengers as a compensatory term, $f(w_i^k)$:

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k\in K}\sum_{i\in V}\Big(z_k\rho\lambda_{i1}^k + (z_k\rho + (Q_k - z_k)\theta)\lambda_{i2}^k\Big) \tag{28}$$

This implies that we use a non-negative parameter indicating the relative importance of reducing the in-vehicle crowding levels compared to the vehicle travel costs, γ. If the value of this compensatory term cannot be accurately defined, the problem can be expressed as a multi-objective optimization problem (MOOP) using two separate objective functions:

$$f_1(x) = \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k \tag{29}$$

and

$$f_2(\lambda) = \sum_{k\in K}\sum_{i\in V}\Big(z_k\rho\lambda_{i1}^k + (z_k\rho + (Q_k - z_k)\theta)\lambda_{i2}^k\Big) \tag{30}$$

Our multi-objective optimization problem involves two conflicting objectives and has a set of Pareto optimal solutions (*49*). The Pareto optimal solutions (or non-dominated solutions) form a Pareto front. Our solution approach finds the Pareto optimal solutions, which balance the vehicle route costs and the in-vehicle crowding costs. To solve the MOOP one can use the ε-constraint method adapted from Haimes et al. (*50*). The ε-constraint method requires one of the objective functions in Eqs.(29) and (30) to be set as a constraint. The objective that is set equal to a constraint needs to be less than ε, where ε is a scalar. Using the ε-constraint method we transform objective function (30) to the constraint of Eq.(31).

$$f_2(\lambda) \leq \varepsilon \tag{31}$$

With this reformulation, the MOOP is transformed to a MILP that can be formally written as:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{32}$$

$$\text{s.t. Eqs. } (2)-(5),(8)-(24),(31) \tag{33}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall i,j \in V, k \in K \tag{34}$$

To find the Pareto optimal solutions, one has to give different values to ε. The pseudo code of the ε-constraint algorithm for solving the multi-objective optimization model is provided in algorithm 1. This algorithm starts with an initial value of ε and solves the aforementioned MILP. This results in solution $\hat{x}$, which is used to calculate the f_2. The new value of ε is then set to $f_2(\zeta,\lambda)$ + Δ, where Δ is a small number ($\Delta \leq 0.01$). This process continues until the stated problem becomes infeasible. The initial value of ε is set equal to the minimal possible crowding level. Finally, the dominated solutions are filtered from all found solutions, such that only Pareto optimal solutions remain.

Algorithm 1: ε-constraint method for solving the MOOP

Initialize set $sols \leftarrow \emptyset$ containing the Pareto optimal solutions;
$\varepsilon \leftarrow 0$;
while $\min\{f_1(x) \mid Eqs.\ (2)-(5),(8)-(24),(31)\}$ *is feasible* **do**
    $\hat{x} \leftarrow \text{argmin}\{f_1(x) \mid \text{Eqs. } (2)-(5),(8)-(24),(31)\}$;
    $sols \leftarrow sols \cup \hat{x}$;
    $\varepsilon \leftarrow f_2(\lambda) + \Delta$;
end
Filter dominated solutions in *sols*;
return: *sols*

NUMERICAL EXPERIMENTS

The focus of the numerical experiments is to study the impact of reducing the in-vehicle crowding levels due to COVID-19 on the vehicle route costs of an on-demand minibus service. In the numerical experiments, we use the well-known DARP instances from (*51*). From this dataset, we focus on the instances b2-16, b2-20 and b2-24 that contain 2 vehicles and 16, 20 and 24 requests, respectively. Note that a network with n requests includes $2n+2$ vertices. Namely, for $n = 16$ requests we have pickup vertices $P = \{1,...,16\}$, delivery vertices $D = \{17,...,32\}$, and vertices 0 and 33 denoting the depot. At each pickup vertex $i \in P$ we pickup q_i passengers who are delivered to $n+i$ such that $q_{n+i} = -q_i$.

Our proposed minibus scheduling formulation that considers the in-vehicle crowding inconvenience of passengers is programmed in Python 3.7 and it is solved by the commercial solver

Gurobi 9.0.3 that employs branch-and-bound and dual simplex. The implementation of the algorithms is performed in a general-purpose i7 2.8GHZ processor with 16GB RAM.

We first start with the b2-16 network with the 2 vehicles and the 16 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ passengers for $k \in \{1,2\}$ and the maximum route duration for each vehicle is $T_k = 480$. In addition to these parameter values that are provided from the test instance, we assume that the soft capacity that corresponds to the recommended social distancing is $z_k = 3$ passengers for each $k \in K$, $\rho = 5$ and $\theta = 100$. Note that we use a very high value for θ to over-penalize crowding levels beyond the recommended distancing level.

Table 2 provides the results for this test instance when considering different importance levels for the passenger crowding, γ. In more detail, when $\gamma = 0$ we solve the classic problem without considering the inconvenience of passengers due to in-vehicle crowding. As γ increases its value, we are permitting the increase of the vehicle route costs in order to reduce the in-vehicle crowding levels. In the first column of the table, we report the values of γ that correspond to each experiment. In the second column, we report the vehicle route costs for this value of γ. In the third column, we report the passenger costs related to their inconvenience due to in-vehicle crowding. We let the solver run for 5 minutes for each value of γ. If the solver is capable of finding a solution before the 5-minute time limit, then this is a globally optimal solution. If not, we report the estimated optimality gap provided by the solver in the fourth column of the table. Note that this gap is just an estimation provided by the solver based on the difference between the incumbent solution that meets the integrality constraints and the lower bound. The actual gap might be much smaller because the lower bound solution does not meet some of the integrality constraints. This is why we call this gap “maximum possible optimality gap".

TABLE 2 Performances of the solutions of the b2-16 network for different values of γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	309.406	n/a	11.10%
0.03	309.612	3335	11.94%
0.05	309.612	3335	13.95%
0.25	326.033	3075	0.00%
0.75	326.033	3075	0.00%
1	326.033	3075	0.00%
100	326.033	3075	0.00%

Note that for $\gamma = 0$ we do not report the passenger-related costs because they are not taken into consideration in the optimization process. As one might have expected, the passenger-related costs decrease when γ increases. This is a result of placing more emphasis on the reduction of passenger-related costs. At the same time, for larger values of γ the vehicle costs increase until reaching 326.033 for $\gamma = 0.25$. After this point, there is an equilibrium and we cannot reduce further the in-vehicle crowding levels by increasing the route costs of the two vehicles. In summary, when moving from $\gamma = 0.03$ to $\gamma = 0.25$ we increase the vehicle costs by 5.3% in order to reduce the passenger costs related to crowding by 7.8%.

To provide more details of the solution changes, in Table 3 we report the solution when the in-vehicle crowding levels are ignored ($\gamma = 0$), and the solution when we prioritize the in-vehicle crowding levels ($\gamma = 100$). Note that for $\gamma = 0$ the first vehicle serves way more vertices in order to reduce the vehicle route costs. For $\gamma = 100$ the vehicle routes change considerably and we observe a balance between vehicles 1 and 2 with respect to the number of visiting vertices. Note that each vehicle starts from the depot empty and returns to it empty. In addition, when a vehicle serves request $i \in P$, it serves also the delivery vertex $n+i$, where $n = 16$.

TABLE 3 Sequence of visited vertices by each one of the two vehicles in the b2-16 network for $\gamma = 0$ and $\gamma = 100$, respectively.

$\gamma = 0$		$\gamma = 100$	
Vehile 1	Vehicle 2	Vehile 1	Vehicle 2
0	0	0	0
9	10	10	9
25	26	26	25
8	1	2	8
2	17	18	24
18	13	16	6
16	11	32	22
24	29	3	1
6	27	19	17
22	14	5	13
32	30	21	29
3	33	7	33
19		12	11
5		28	27
21		23	15
7		14	31
12		30	4
23		33	20
15			33
28			
31			
4			
20			
33			

Let us now consider the b2-20 network with 2 vehicles and 20 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ and the maximum route duration for each vehicle is $T_k = 600$. We also assume $z_k = 3$, $\rho = 5$ and $\theta = 100$. Table 4 provides the results for this test instance when considering different importance levels for the passenger crowding, γ.

TABLE 4 Performances of the solutions of the b2-20 network for different values of γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	332.638	n/a	0.00%
0.03	333.644	7550	0.00%
0.05	333.644	7550	0.00%
0.25	350.612	7330	0.00%
0.75	350.612	7330	0.00%
1.00	350.612	7330	0.00%
100	350.612	7330	0.00%

In the b2-20 network we have a reduction of the passenger costs related to crowding for $\gamma = 0.25$. After that, the passenger costs cannot be improved further. Until that point, for a 5% increase of the vehicle route costs we had a 2.9% reduction of the passenger-related costs.

Finally, let us consider the b2-24 network with 2 vehicles and 24 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ and the maximum route duration for each vehicle is $T_k = 720$. We also assume $z_k = 3$, $\rho = 5$ and $\theta = 100$. Table 5 provides the results for this test instance when considering different importance levels for the passenger crowding, γ.

TABLE 5 Performances of the solutions of the b2-24 network for different values of γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	445.422	n/a	15.75%
0.03	452.508	5950	11.43%
0.05	451.097	5950	8.93%
0.25	460.253	5825	2.18%
0.75	460.253	5825	0.00%
1	460.253	5825	0.00%
100	460.253	5825	0.00%

In the b2-24 network we have a reduction of the passenger costs related to crowding for $\gamma = 0.25$. After that, the passenger costs cannot be improved further. Until that point, for a 1.7% increase of the vehicle route costs we had a 2.1% reduction of the passenger-related costs.

CONCLUSION

This study proposed a minibus scheduling formulation that considers the in-vehicle crowdedness as an additional problem objective. The proposed formulation resulted in a MINLP. We considered a piecewise linear function to over-penalize crowding levels that exceed a regulatory limit and we offered a MILP reformulation of the original MINLP. This reformulation allows the solution of the problem with commercial solvers.

In our numerical experiments, we investigated the potential effect of reducing the vehicle crowdedness to the increase of the vehicle route costs. Interestingly, there was a balanced increase of the vehicle route costs for a reduction in the in-vehicle crowding levels. Another important observation is that because in the minibus scheduling problem the fleet availability is fixed, if there are many user requests we cannot always reduce the in-vehicle crowding levels considerably. For the test instances of b2-20 and b2-24 we observed a reduction of in-vehicle crowding by 2-3% even if we consider this as the first priority (for $\gamma = 100$). There are two reasons for this: first, we must serve all requests with the available vehicles. Second, our formulation requires from a single vehicle to pickup all customers by any pickup point at once. This might create crowding problems when having many passengers at a specific pickup point.

In future research, the flexibility of reducing the in-vehicle crowding levels can be investigated further by allowing the increase of the number of available vehicles based on well-defined trade-offs or allowing vehicles to serve only part of the passenger demand at a pickup point.

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