

Comparison of Fairness based Coordinated Grid Voltage Control Methods for PV Inverters

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Abstract—Active power curtailment is a cost-effective technique for mitigating overvoltage issues as a consequence of distributed generation. However, many solutions treat prosumers at highly sensitive parts of the grid unfairly. The best solution to this problem is to explore the non-linear behavior between active power and voltage to find the fair amount of power that needs to be curtailed to satisfy grid codes. Current state-of-the-art techniques for power curtailment are computationally expensive and case-specific. In this work, a fair value for power curtailment is achieved analytically that is both computationally efficient and generic. The results obtained analytically are validated utilizing an iterative algorithm, which provides a near optimal value for fair power curtailment. The results demonstrate that for single-phase and balanced three-phase networks, analytical and iterative methods have similar control actions and there is a 50% increase in distributed generation infeed at sensitive parts of the grid.

Index Terms—Distributed generation, Fairness, Active power curtailment, Sensitivity

I. INTRODUCTION

Environmental concerns, rise in energy demand, finite nature of conventional energy sources and technological advancements have led to a paradigm shift in electric power generation from centralized to distributed generation (DG). This consists in incorporation of renewable energy sources (RES) at the distribution level. Although DG has several advantages, it constitutes different challenges for the stable operation of low voltage (LV) distribution networks, such as violation of grid constraints. Compared to other RES, solar photovoltaic (PV) systems are preferred in DG owing to advantages of low operation cost, maintenance cost, noiseless operation and easy installation. However, an increased penetration of PV systems leads to overvoltage, resulting in an upper limit for the maximum permissible PV integration [1]. Violation of voltage standards is one of the main concerns that emerges mainly during low load and high PV generation conditions.

Active power curtailment (APC) offers to be a more reliable and cost-effective technique to resolve violation of voltage standards [1]. APC involves regulation of distribution system voltage by curtailing power output of DG units. Curtailment algorithms such as the Droop control [2] and the Volt-Watt scheme [3] provide satisfactory voltage control. However, this leads to under-utilization of available solar power. Conse-

quently, it is of paramount importance to optimize power curtailment through intelligent control. Numerous methods are proposed in literature to reduce PV curtailment without violating voltage limits [4]–[6]. Although these techniques increase PV harvesting by altering different control parameters, prosumers at highly sensitive areas of the network are treated unfairly because power curtailed is more compared to prosumers near the substation transformer [2].

An efficient APC algorithm must ensure fair curtailment of power. The term 'fairness' for power curtailment can be defined taking into account different factors such as PV harvesting, financial benefits and energy export of prosumers. Tonkoski et al. [1] varied the droop parameters among PV prosumers in such a way that the amount of active power curtailed is shared equally among prosumers, but energy export to the grid is reduced compared to conventional droop control algorithms. A real power capping method based on adaptive optimal power dispatch for fair and optimal power curtailment is proposed in [7]. This algorithm focuses on fairness over a given period of time rather than each time instant. In [8] fairness is quantified in-terms of PV harvesting as well as financial benefits and concludes that fairness can be achieved but at the cost of increased power curtailment. [9] considers Jain's fairness index, with energy export, PV harvesting, and financial benefits as parameters defining fairness. Since trade-offs exist between different factors governing fairness, an ideal fair curtailment scheme does not exist and selection of these parameters is subjective.

To facilitate the development of voltage management strategies, numerical sensitivity based analysis have to be performed. Numerical approaches employ iterative algorithms like Newton Raphson load-flow (NRLF) [10] and perturb-and-observe method [11] to compute the sensitivity matrix. However, numerical methods are computationally expensive, complex, and deemed unsuitable. Analytical approaches address the issues of computational complexity and cost associated with numerical methods. Literature review indicates that there is hardly any work on analytical estimation of voltage sensitivity [12]–[15]. In [12], sensitivity based voltage regulation of medium voltage (MV) distribution network by controlling reactive power is proposed. Optimal reactive power control for voltage regulation of MV distribution system without

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considering the complex nature of nodal voltages is presented in [13], [14]. In [15], a probabilistic voltage sensitivity based analysis is performed, however it does not take into account the fairness among prosumers. In this work, the sensitivity of the network is found both iteratively and analytically to achieve fair PV harvesting among prosumers. The main contributions of this paper are:

- An iterative approach for fair coordinated voltage control.
- A novel and computationally efficient analytical based approach for near optimal and fair PV harvesting.
- Insight on how sensitivity is dependent on different physical parameters of the network.

The rest of this paper is structured as follows: Section II motivates the problem and an iterative as well as analytical method for fair APC is explained. Section III validates the control algorithms detailed in Section II by simulating different scenarios for the network model. Section IV concludes the paper and suggests possible future work.

II. CONTROL ALGORITHMS

The issue of overvoltage in LV distribution network due to increased PV penetration and control algorithms to address this problem are described in this section. Let us consider the single line diagram shown in Fig. 1 consisting of 6 houses (H1-H6) connected to the LV side of distribution grid separated by distances x_1, x_2, x_3, x_1 and x_4 metres respectively. Each house consists of a base load and a grid tied PV system. The value of different parameters considered is given in Table I.

The resulting voltages at the point of common coupling (PCC) of different houses are shown in Fig. 2. It is observed that there is overvoltage at different house terminals, especially at houses that are located at the end of the feeder due to reverse flow of current and impedance of the cable.

This paper focuses on fairly addressing the issue of overvoltage through APC, by considering PV harvesting as the primary fairness factor. Thus, fairness is defined as, *providing the same percentage of power curtailment for all PV owners in a particular section of the electricity grid at every time instant*. Two different algorithms have been developed for such voltage control, which are detailed in sub-sections II-A and II-B. Moreover, both the methods are generic and can be used for any balanced radial distribution network.

A. Iterative Method for Active Power Curtailment

Precise APC in an LV distribution network is difficult due to the non-linear behaviour of electrical circuits. In this

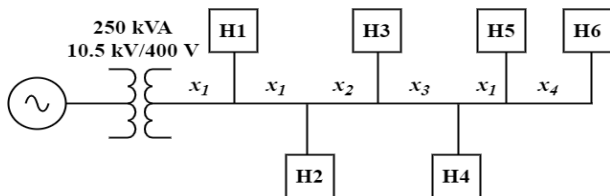


Fig. 1. Network model.

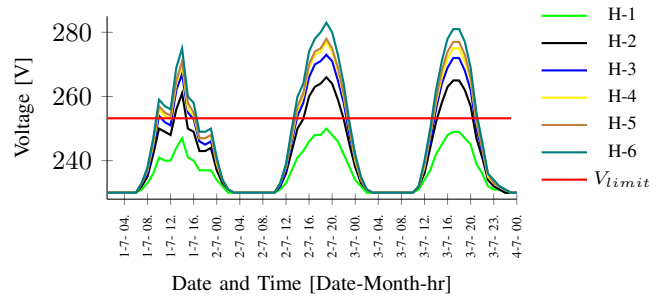


Fig. 2. Voltages at different PCC in single phase network.

algorithm, the perturb and observe method is used to find the sensitivity of the network and subsequently iteratively decide the fair amount of active power to be curtailed to control the voltage and ensure grid compliance.

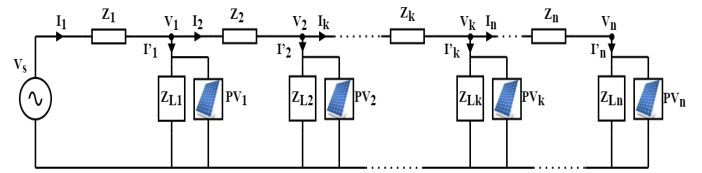


Fig. 3. Schematic diagram of a balanced LV distribution network

Referring to Fig. 3, suppose there is a change in voltage at observation node k . All parameters of the distribution network at a particular time instant t and the source voltage V_s is considered to be constant irrespective of the time (source acts as slack bus and all the other nodes as PQ bus). In this work we assume a balanced three phase LV grid and therefore omit phase indices and neutral point shifts. By sequentially perturbing the power at each node in the network we can find the change in voltage at a particular node due to the change in power at all the other nodes individually. The value of power that is perturbed by a node is defined by,

$$\Delta P_j(t) = P_j(t) - P_j(t-1) \quad (1)$$

Using this observation, the sensitivity of the network (ξ) at the non-linear region of interest can be found by,

$$\xi_{kj}(t) = \frac{\Delta P_j(t)}{\Delta V_{kj}(t)} \quad (2)$$

where, $\Delta V_{kj}(t)$ is the change in voltage at node k due to change in power at node j . The change in voltage at node k is the cumulative effect of change in power at all the nodes in the network. This change in voltage at time instant t is equated as,

$$\Delta V_k(t) = \sum_{j=1}^n \Delta V_{kj}(t) \quad (3)$$

where n is the total number of nodes in the distribution network. The ratio of change in voltage at node k due to

change in power at node j and change in power at all nodes is found as,

$$\Delta V_{k_j, ratio}(t) = \frac{\Delta V_{k_j}(t)}{\sum_{i=1}^n \Delta V_{k_i}(t)} \quad (4)$$

If $V_{k, limit}$ is the voltage limit at node k and $V_k(t)$ is the voltage at node k at time t , then the required change in voltage at node k is, $\Delta V_k(t+1) = V_{k, limit} - V_k(t)$. Finally, the fair value of power to be curtailed for each node j is found as,

$$\Delta P_{j, fair} = \Delta V_{k_j, ratio}(t) \Delta V_k(t+1) \xi_{k_j}(t) \quad (5)$$

Algorithm 1 presents the pseudo-code for the iterative control algorithm. First, the power at each node is initialised according to the demand and generation at time t . Next, a load-flow analysis is performed to update the network voltages and currents. Then, if the voltage limit is violated at a node k , the sensitivity of all the nodes with respect to this node is found and the fair value of power to be curtailed among the prosumers is calculated. This procedure is repeated until all the node voltages are in compliance with the grid standards.

Algorithm 1 Control Algorithm.

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1: Initialize all the grid parameters
2:  $t = t + \Delta t$ 
3: Perform Load-flow analysis
4: for  $k \in \{1, 2, \dots, n\}$  do
5:   if  $V_k(t) > V_{k, limit}$  then
6:     for  $j \in \{1, 2, \dots, n\}$  do
7:       Calculate  $\xi_{k_j}(t)$ 
8:       Calculate  $\Delta P_{j, fair}$ 
9:        $P_j(t) = P_j(t) - \Delta P_{j, fair}$ 
10:    end for
11:    Go to line 3
12:  end if
13: end for
14: Go to line 2

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B. Analytical Method for Active Power Curtailment

As described in the previous subsection, a change in power at a node results in a change in voltage at all other nodes in a distribution network. Finding all node to node relations using the current state-of-the-art techniques are computationally expensive. This section presents a method to find a fair value (considering the definition of fairness) for change in power analytically, in order to efficiently and precisely change the voltage at a particular node.

Let us again consider the LV distribution network as presented in Fig. 3. Let us assume that the voltage $V_k(t)$ at node k needs to be controlled. The change in voltage at node k after each time interval Δt can also be calculated as:

$$\Delta V_k(t) = V_k(t) - V_k(t-1) \quad (6)$$

Let $I_j(t)$ be the current flowing through edge j at time t with impedance Z_{Lj} , $\forall j \in \{1, 2, \dots, n\}$ and V_s be the source voltage. Using Kirchoff's voltage law we obtain:

$$V_k(t-1) = V_s - \sum_{j=1}^k I_j(t-1) Z_{Lj} \quad (7)$$

Let $I_k(t)'$ be the net current flowing to the prosumer terminals at node k at time t . Therefore, using Kirchoff's current law, we find the current towards node k by:

$$I_k(t) = \sum_{j=k}^n I_j(t)' = \sum_{j=k}^n \left(\frac{P_j(t)}{V_j(t)} \right)^* \quad (8)$$

From which we obtain nodal voltage as:

$$V_k(t-1) = V_s - \sum_{i=1}^k \sum_{j=i}^n \left(\frac{P_j(t-1)}{V_j(t-1)} \right)^* Z_i \quad (9)$$

$$V_k(t) = V_s - \sum_{i=1}^k \sum_{j=i}^n \left(\frac{P_j(t-1) + \Delta P_j(t)}{V_j(t-1) + \Delta V_j(t)} \right)^* Z_i \quad (10)$$

Subtract (9) from (10) and since the change in voltage is small compared to node voltage, we can assume

$$\frac{\Delta V_j(t)}{(V_j(t-1) + \Delta V_j(t))(V_j(t-1))} \approx 0 \quad (11)$$

Thus, we obtain the change in voltage at node k as,

$$\Delta V_k(t) = \sum_{i=1}^k \sum_{j=i}^n \left(\frac{-\Delta P_j(t)}{V_j(t)} \right)^* Z_i \quad (12)$$

Suppose the voltage at the k^{th} node, $V_k(t)$, needs to be controlled within limits by APC. In order to find a fair value of power to be curtailed, we need to find the influence of the change in power for all nodes individually. The influence of a change in power at node j alone on the voltage at node k can be found by substituting the value of $\Delta P_i(t) = 0, \forall i \in \{1, 2, \dots, n\}, i \neq j$ in equation (12). Doing so we obtain:

$$\Delta V_{k_j}(t) = \left(\frac{-\Delta P_j(t)}{V_j(t)} \right)^* Z_k' \quad (13)$$

where $Z_k' = \sum_{i=1}^k Z_i$. The change in voltage $\Delta V_k(t)$ by changing the power of all the nodes is as per equation (3). The ratio of change in voltage at desired node k due to node j is found by substituting (13) in (4). If the PV panels are of the same size and the power cables have the same specification throughout the network, the ratio of change in voltage at desired node k due to node j can be simplified by substituting $\Delta P_j = P, \forall j \in \{1, 2, \dots, n\}$, where P is a constant for all nodes as the curtailment is fair and PV sizes are the same. Let d_j be the distance from source to node j , then equation (4) is simplified as

$$\Delta V_{k_j, ratio}(t) \approx \frac{d_j}{\sum_{i=1}^k d_i} \quad (14)$$

Thus the ratio in equation (14) will be higher for nodes located at far end of the feeder. Hence, sensitivity for change in voltage with respect to power will be higher for these nodes. This is the reason for reduced energy export with fair power curtailment as nodes with a small sensitivity should also have the same high percentage of power curtailment.

Now, the sensitivity is found using equation (2). The fair value of active power that is curtailed or stored for later use from all the nodes to obtain a desired voltage $V_k(t + 1)$ at node k is found using equation (5). This method provides a precise value of power that needs to be curtailed to meet the grid voltage regulation. Moreover, the sensitivity of the entire network is computed in one iteration. The pseudo code for analytical method is similar to Algorithm 1, except that line 11 is omitted.

III. RESULTS

In this section, the performance of the iterative and analytical method based algorithms described in Section II are analysed for both single phase and balanced three phase LV distribution networks. A comparison to no control and existing droop control [1] is also carried out. The analysis is performed using the simulation and demonstration toolkit DEMkit [16]. Within this tool we have modelled a scenario as depicted in Fig. 3. For each house we consider, next to PV, also a base load. The base load is the same for each house and is generated using the Artificial Load Profile Generator [17]. The solar irradiance data obtained from [18] (data of the year 2014 from weather station Twenthe) is used to calculate the PV power output. The network is simulated for the first 3 days in July using the parameters in Table I.

TABLE I
SIMULATION PARAMETERS

Parameters	Quantity
Number of houses	6
Distance	$x_1 = 100$ m, $x_2 = 120$ m, $x_3 = 200$ m, $x_4 = 500$ m
Size of PV array (Single Phase)	H1,H2,H3,H4,H5,H6 = 32 m ²
Size of PV array (Three Phase)	H1,H2,H3,H4,H5,H6 = 160 m ²
Feeder cable characteristics	120 mm ² Al cable of impedance (0.253+0.08j) Ω /km
Time step, Δt	60 seconds

TABLE II
SIMULATION RESULTS FOR THREE PHASE NETWORK

Control	Combined PV power output (kW) on 2nd July 14:35	Energy Delivered in a day (kWhr)
No control	360	2436
Droop Control	204	1900
Fair Control	194	1643
Droop Control (House 6)	14.74	74
Fair Control (House 6)	30.75	120
Control Algorithm	Simulation Period	Execution Time
Iterative control	3 Days	616 seconds
Analytical control	3 Days	127 seconds

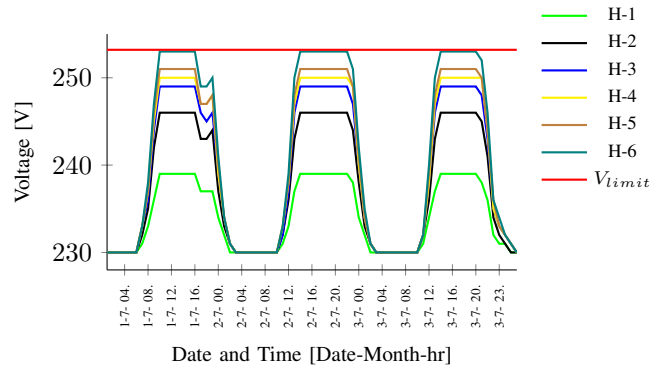


Fig. 4. Voltages at PCC for iterative and analytical methods (Single phase).

A. Single phase network

Simulating the single phase network, it is found that both the analytical and iterative methods are capable of maintaining grid voltage within limits. Fig. 4 presents the voltage profile of the network after the execution of the analytical and iterative method, the results resembles for both the control algorithms. Fig. 5 shows the power curtailed at each time instant, which is identical for both the methods. It is also observed that the sensitivity of the network remains constant in a single phase network. Thus, finding the sensitivity of the network once by simulating the network for a day, the voltage at PCC of all the houses can be accurately controlled throughout the year with varied PV penetration.

B. Three phase network

In the case of three phase network, there are 6 different houses H1-H6. H1 and H2 are connected to phase 1, H3 and H4 are connected to phase 2, H5 and H6 are connected to phase 3. The distance between the houses are the same as in the previous case. When there is no control, it is observed that there is overvoltage at different houses during high PV penetration as shown in Fig. 6. In this case, the house near the substation transformer also experiences overvoltage due to the large size PV array. These overvoltage issues are resolved with both the iterative and analytical control algorithms which could maintain the PCC voltage of all the houses within the threshold value as shown in Fig. 7.

Table II summarizes the results obtained. Even-though the curtailment is fair among the prosumers, the total amount of energy delivered is reduced compared to the unfair curtailment techniques. Nevertheless, there is approximately 50% increase in PV power harvested by the prosumer at the sensitive part of the grid when the curtailment is fair. Although the PV curtailment performance offered by both control algorithms are similar, the analytical method takes less than one-fourth of the time consumed by the iterative method.

IV. CONCLUSION AND FUTURE RESEARCH

In this paper, to address overvoltage issue in LV distribution network due to increased PV penetration an analytical method based fair APC algorithm is introduced. The presented

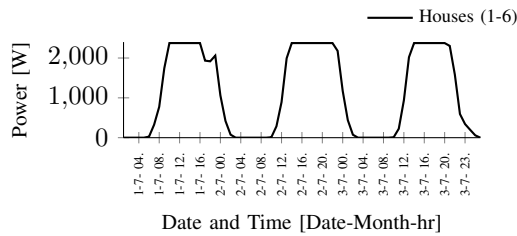


Fig. 5. PV power output with iterative and analytical methods (Single phase).

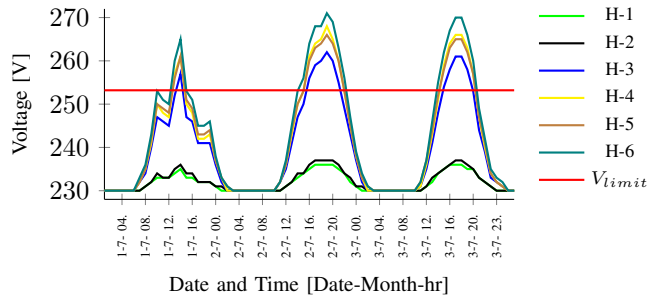


Fig. 6. Voltages at PCC without voltage control (Three phase).

analytical method is computationally efficient compared to the existing methods. Moreover, it provides an optimal and precise value of active power to be curtailed to satisfy the grid constraints. Results of simulation studies for both single and three phase networks demonstrate that the analytical method is faster and provides the same result as its iterative counterpart.

We have addressed the issue for the case of balanced PV production. However, many urban LV grids suffer from voltage unbalance problems. Extending and evaluating the presented control to resolve these unbalance issues remains future work. Also, due to the electrification, we aim to address undervoltage problems with the presented concept in future. Next to that, if we consider fairness and active power curtailment as independent problems, a more generic and detailed definition of fairness becomes possible.

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REFERENCES

- [1] R. Tonkoski, L. A. C. Lopes, and T. H. M. El-Fouly, "Coordinated active power curtailment of grid connected pv inverters for overvoltage prevention," *IEEE Transactions on Sustainable Energy*, vol. 2, no. 2, pp. 139–147, 2011.
- [2] R. Tonkoski and L. Lopes, "Impact of active power curtailment on overvoltage prevention and energy production of pv inverters connected to low voltage residential feeders," *Renewable Energy*, vol. 36, pp. 3566–3574, 12 2011.
- [3] M. Emmanuel, J. Giraldez, P. Gotseff, and A. Hoke, "Estimation of solar photovoltaic energy curtailment due to volt-watt control," *IET Renewable Power Generation*, vol. 14, no. 4, pp. 640–646, 2020.
- [4] O. Gagrira, P. H. Nguyen, W. L. Kling, and T. Uhl, "Microinverter curtailment strategy for increasing photovoltaic penetration in low-voltage networks," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 2, pp. 369–379, 2015.

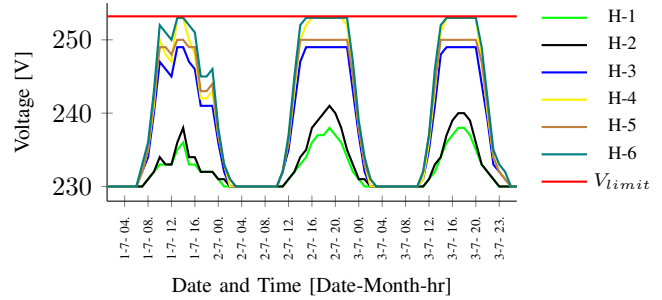


Fig. 7. Voltages at PCC with iterative and analytical methods (Three phase).

- [5] Q. Nguyen, H. V. Padullaparti, K. Lao, S. Santoso, X. Ke, and N. Samaan, "Exact optimal power dispatch in unbalanced distribution systems with high pv penetration," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 718–728, 2019.
- [6] T. S. Ustun, S. M. S. Hussain, and H. Kikusato, "Iec 61850-based communication modeling of ev charge-discharge management for maximum pv generation," *IEEE Access*, vol. 7, pp. 4219–4231, 2019.
- [7] S. Alyami, Y. Wang, C. Wang, J. Zhao, and B. Zhao, "Adaptive real power capping method for fair overvoltage regulation of distribution networks with high penetration of pv systems," *IEEE Transactions on Smart Grid*, vol. 5, no. 6, pp. 2729–2738, 2014.
- [8] A. Latif, W. Gawlik, and P. Palensky, "Quantification and mitigation of unfairness in active power curtailment of rooftop photovoltaic systems using sensitivity based coordinated control," *Energies*, vol. 9, no. 6, 2016. [Online]. Available: <https://www.mdpi.com/1996-1073/9/6/436>
- [9] M. Z. Liu, A. T. Procopiou, K. Petrou, L. F. Ochoa, T. Langstaff, J. Harding, and J. Theunissen, "On the fairness of pv curtailment schemes in residential distribution networks," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4502–4512, 2020.
- [10] D. K. Khatod, V. Pant, and J. Sharma, "A novel approach for sensitivity calculations in the radial distribution system," *IEEE Transactions on Power Delivery*, vol. 21, no. 4, pp. 2048–2057, 2006.
- [11] F. Tamp and P. Ciufu, "A sensitivity analysis toolkit for the simplification of mv distribution network voltage management," *IEEE Transactions on Smart Grid*, vol. 5, no. 2, pp. 559–568, 2014.
- [12] M. Brenna, E. Berardinis, F. Foiadelli, G. Sapienza, and D. Zaninelli, "Voltage control in smart grids: An approach based on sensitivity theory," *Journal of Electromagnetic Analysis and Applications*, vol. 2, no. 8, pp. 467–474, 2010.
- [13] N. B. Salim, T. Tsuji, T. Oyama, and K. Uchida, "Optimal reactive power control of inverter-based distributed generator for voltage stability insight using particle swarm optimization," *IEEE Transactions on Power and Energy*, vol. 137, no. 5, pp. 392–404, 2017.
- [14] B. Bakhshideh Zad, H. Hasanvand, J. Lobry, and F. Vallée, "Optimal reactive power control of dgs for voltage regulation of mv distribution systems using sensitivity analysis method and pso algorithm," *International Journal of Electrical Power Energy Systems*, vol. 68, pp. 52–60, 2015. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S014206151400773X>
- [15] S. Munikoti, B. Natarajan, K. Jhala, and K. Lai, "Probabilistic voltage sensitivity analysis to quantify impact of high pv penetration on unbalanced distribution system," *IEEE Transactions on Power Systems*, vol. 36, no. 4, pp. 3080–3092, 2021.
- [16] G. Hoogsteen, J. L. Hurink, and G. J. M. Smit, "Demkit: a decentralized energy management simulation and demonstration toolkit," in *2019 IEEE PES Innovative Smart Grid Technologies Europe (ISGT-Europe)*, 2019, pp. 1–5.
- [17] G. Hoogsteen, A. Molderink, J. L. Hurink, and G. J. M. Smit, "Generation of flexible domestic load profiles to evaluate demand side management approaches," in *2016 IEEE International Energy Conference (ENERGYCON)*, 2016, pp. 1–6.
- [18] KNMI, "Urgegevens van het weer in nederland." 2015. [Online]. Available: <http://www.knmi.nl/nederland-nu/klimatologie/uurgegevens>