Proceedings of the 40th Chinese Control Conference
July 26-28, 2021, Shanghai, China

H_2 Almost State Synchronization of Homogeneous Multi-agent Systems–A Scale-free Design

Zhenwei Liu[1], Ali Saberi[2], Anton A. Stoorvogel[3], Donya Nojavanzadeh[2]

1. College of Information Science and Engineering, Northeastern University, Shenyang, P. R. China
E-mail: liuzhenwei@ise.neu.edu.cn

2. School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA, USA
E-mail: saberi@wsu.edu
E-mail: donya.nojavanzadeh@wsu.edu

3. Department of Electrical Engineering, Mathematics and Computer Science, University of Twente, Enschede, The Netherlands
E-mail: a.a.stoorvogel@utwente.nl

Abstract: This paper studies scale-free protocol design for H_2 almost state synchronization of homogeneous networks of non-introspective agents in presence of external disturbances. The necessary and sufficient conditions are provided by designing collaborative linear dynamic protocols. The design is based on localized information exchange over the same communication network, which does not need any knowledge of the directed network topology and the spectrum of the associated Laplacian matrix. Moreover, the proposed protocol is scalable and achieves H_2 almost synchronization with a given arbitrary degree of accuracy for any arbitrary number of agents.

Key Words: Multi-agent systems, H_2 almost output Synchronization, Scale-free protocols

1 Introduction

In recent decades, the synchronization problem for multi-agent systems (MAS) has attracted substantial attention due to the wide potential for applications in several areas such as automotive vehicle control, satellites/robots formation, sensor networks, and so on. See, for example, the books [19, 31, 6, 1] and references therein.

State synchronization inherently requires homogeneous networks (i.e. agents which have identical models). Therefore, in this paper we focus on homogeneous networks. So far, most work has focused on state synchronization based on diffusive full-state coupling, where the agent dynamics progress from single- and double-integrator dynamics (e.g. [14], [17], [18]) to more general dynamics (e.g. [20], [27], [29]). State synchronization based on diffusive partial-state coupling has also been considered, including static design (e.g. [12]), dynamic design ([4], [21], [22], [24], [26], [28]), and the design with additional communication ([7], [8], and [20]). Recently, scale-free collaborative protocol designs are developed for homogeneous and heterogeneous MAS [2, 13] and for MAS subject to actuator saturation [9].

Meanwhile, if the agents have absolute measurements of their own dynamics in addition to relative information from the network, they are said to be introspective, otherwise, they are called non-introspective. There exist some results about these two types of agents, for example, introspective agents ([5, 32], etc), and non-introspective agents ([3, 30], etc).

Synchronization and almost synchronization in presence of external disturbances are studied in the literature, where three classes of disturbances have been considered namely:

1) Disturbances and measurement noise with known frequencies.
2) Deterministic disturbances with finite power.
3) Stochastic disturbances with bounded variance.

For disturbances and measurement noises with known frequencies, it is shown in [34] and [35] that actually exact synchronization is achievable. This is shown in [34] for heterogeneous MAS with minimum-phase and non-introspective agents and networks with time-varying directed communication graphs. Then, this result is extended in [35] for non-minimum phase agents utilizing localized information exchange.

For deterministic disturbances with finite power, the notion of H_∞ almost synchronization is introduced by Peymani et.al for homogeneous MAS with non-introspective agents utilizing additional communication exchange [15]. The goal of H_∞ almost synchronization is to reduce the impact of disturbances on the synchronization error to an arbitrarily degree of accuracy (expressed in the H_∞ norm). This work was extended later in [16, 33, 36] to heterogeneous MAS with non-introspective agents and without the additional communication and for network with time-varying graphs. H_∞ almost synchronization via static protocols is studied in [23] for MAS with passive and passifiable agents. Recently, necessary and sufficient conditions are provided in [25] for solvability of H_∞ almost synchronization for homogeneous networks with non-introspective agents and without additional communication exchange. Finally, we developed a scale-free framework for H_∞ almost state synchronization for homogeneous network [10] utilizing suitably designed localized information exchange.

In the case of stochastic disturbances with bounded variance, the concept of stochastic almost synchronization is introduced by [37] where both stochastic disturbance and disturbance with known frequency are present. The idea of stochastic almost synchronization is to reduce the stochastic RMS norm of synchronization error arbitrary small in the presence of colored stochastic disturbances that can be modeled as the output of linear time invariant systems driven by white noise with unit power spectral intensities. By augment-

This work is supported by the Nature Science Foundation of Liaoning Province, PR China under Grant 2019-MS-116, the Fundamental Research Funds for the Central Universities of China under Grant N2004014 and the United States National Science Foundation under Grant 1635184.

ing this model with agent model one can essentially assume that stochastic disturbance is white noise with unit power spectral intensities. In this case under linear protocols the stochastic RMS norm of synchronization error is the H_2 norm of the transfer function from disturbance to the synchronization error as such one can formulate the stochastic almost synchronization equivalently in a deterministic framework requiring to reduce the H_2 norm of the transfer function from disturbance to synchronization error arbitrary small. This deterministic approach is referred to as almost H_2 synchronization problem which is equivalent to stochastic almost synchronization problem. Recent work on H_2 almost synchronization problem is [25] which provided necessary and sufficient conditions for solvability of H_∞ almost synchronization for homogeneous networks with non-introspective agents and without additional communication exchange. Finally, H_2 almost synchronization via static protocols is also studied in [23] for MAS with passive and passifiable agents.

In this paper, we consider stochastic disturbances with bounded variance and we develop scale-free framework to solve H_2 almost state synchronization problem for homogeneous MAS. We design a class of linear parameterized dynamic protocols utilizing localized information exchange for both networks with full- and partial-state coupling. The linear dynamic protocol achieves H_2 almost state synchronization for any communication network with any number of agents which contains a spanning tree. The main contribution of this work is that the protocol design does not require any information about the communication network such as a lower bound of non-zero eigenvalue of the associated Laplacian matrix and the number of agents. It is worth to note that, so far in all the works of the literature on H_2 almost synchronization, the protocol design requires at least some knowledge of the communication network such as bounds on the spectrum of the associated Laplacian matrix and size of the network (i.e., the number of agents).

Notations and Background

Given a matrix $A \in \mathbb{R}^{m\times n}$, A^{T} and A^* denote transpose and conjugate transpose of A respectively while $\|A\|_2$ denotes the induced 2-norm (which has submultiplicative property). The $\mathrm{im}(\cdot)$ denote the image of matrix (vector). A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane. $A \otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context. For a deterministic continuous-time signal $v(t)$, the L_2 norm is defined by

$$\|v(t)\|_{L_2} = \left(\int_0^T v(t)^{\mathrm{T}} v(t) dt\right)^{\frac{1}{2}} \tag{1}$$

and its *Root Mean Square (RMS)* value is defined by

$$\|v(t)\|_{RMS} = \left(\lim_{T\to\infty} \frac{1}{T} \int_0^T v(t)^{\mathrm{T}} v(t) dt\right)^{\frac{1}{2}} \tag{2}$$

and for a stochastic signal $v(t)$ which is modeled as wide-sense stationary stochastic process, the $\|v(t)\|_{RMS}$ is given by

$$\|v(t)\|_{RMS} = (\mathbf{E}[v^{\mathrm{T}}(t)v(t)])^{\frac{1}{2}} \tag{3}$$

where $\mathbf{E}[\cdot]$ stands for the expectation operation. For stochastic signals that approach wide-sense stationarity as time t goes on to infinity (i.e., for asymptotically wide-sense stationary signals) (3) is rewritten as

$$\|v(t)\|_{RMS} = \left(\lim_{t\to\infty} \mathbf{E}[v^{\mathrm{T}}(t)v(t)]\right)^{\frac{1}{2}}. \tag{4}$$

For a continuous-time system having a $q \times l$ stable transfer function $G(s)$, the H_2 norm of $G(s)$ is defined as

$$\|G\|_{H_2} = \left(\frac{1}{2\pi} \mathrm{tr}\left[\int_{-\infty}^{+\infty} G(j\omega)G^*(j\omega)d\omega\right]^{\frac{1}{2}}\right).$$

By Parseval's theorem, $\|G\|_{H_2}$ can be equivalently be defined as

$$\|G\|_{H_2} = \left(\mathrm{tr}\left[\int_0^{+\infty} g(t)g^{\mathrm{T}}(t)dt\right]^{\frac{1}{2}}\right)$$

where tr(.) denotes the trace of the matrix, $g(t)$ is the weighting function or unit impulse (Dirac distribution) response matrix of $G(s)$, as such for single-input single-output system $\|G\|_{H_2} = \|g\|_{L_2}$. The H_2 norm of $G(s)$, can be interpreted as the RMS value of the output when the given system is driven by independent zero mean white noise with unit power spectral densities. Note that the H_2 norm of a stable transfer function $G(s)$ is finite if and only if it is strictly proper. The H_∞ norm of $G(s)$ is defined as

$$\|G\|_{H_\infty} := \sup_{\omega} \sigma_{\max}[G(j\omega)]$$

where $\sigma_{\max}$ is the largest singular value of $G(j\omega)$. Let $\omega(t)$ and $z(t)$ be energy signals which are respectively the input and the corresponding output of the given system. Then, the H_∞ norm of $G(s)$ turns out to coincide with its RMS gain, namely

$$\|G\|_{H_\infty} = \|G\|_{RMS\ gain} \sup_{\|\omega\|\neq 0} \frac{\|z\|_{RMS}}{\|\omega\|_{RMS}}$$

An important property of the H_∞ norm is that it is submultiplicative. That is for transfer functions G_1 and G_2, we have

$$\|G_1G_2\|_{H_\infty} \leqslant \|G_1\|_{H_\infty}\|G_2\|_{H_\infty}.$$

A *weighted graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighting matrix. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. A *directed tree* with root r is a subgraph of the graph $\mathcal{G}$ in which there exists a unique path from node r to each node in this subgraph. A *directed spanning tree* is a directed tree containing all the nodes of the graph.

For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$, i.e. a vector with all entries equal to 1. When graph contains a spanning tree, then it follows from [18, Lemma 3.3] that the Laplacian matrix L has a simple eigenvalue at the origin, with the corresponding right eigenvector $\mathbf{1}$, and all the other eigenvalues are in the open right-half complex plane.

2 Problem formulation

Consider a MAS composed of N identical linear time-invariant agents of the form,

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + E\omega_i, \\ y_i &= Cx_i, \end{aligned} \qquad (i = 1, \ldots, N) \tag{5}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^p$ are respectively the state, input, and output vectors of agent i, and $\omega_i \in \mathbb{R}^w$ is the external disturbance.

The communication network provides each agent with a linear combination of its own outputs relative to that of other neighboring agents. In particular, each agent $i \in \{1, \ldots, N\}$ has access to the quantity,

$$\zeta_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) = \sum_{j=1}^{N} \ell_{ij} y_j, \tag{6}$$

where $a_{ij} \geqslant 0$ and $a_{ii} = 0$ indicate the communication among agents while ℓ_{ij} denote the coefficients of the associated Laplacian matrix L. This communication topology of the network can be described by a weighted and directed graph $\mathcal{G}$ with nodes corresponding to the agents in the network and the weight of edges given by the coefficient a_{ij}.

The MAS (5) and (6) is referred to as MAS with full-state coupling when $C = I$, otherwise it is called MAS with partial-state coupling.

In this paper, we introduce an localized exchange of information among protocols. In particular, each agent $i = 1, \ldots, N$ has access to localized information, denoted by $\hat{\zeta}_i$, of the form

$$\hat{\zeta}_i = \sum_{j=1}^{N} a_{ij}(\xi_i - \xi_j) \tag{7}$$

where $\xi_j \in \mathbb{R}^n$ is a variable produced internally by agent j which will be appropriately chosen in the coming sections.

In this paper, we focus on scale-free stochastic almost state synchronization for MAS subject to external stochastic disturbances. The scale-free design framework does not require information of the communication topology and the size of the network.

We adopt a deterministic framework for stochastic almost state synchronization as explained in the Introduction that equivalently we focus on scale-free H_2 almost state synchronization problem for MAS subject to external stochastic disturbances. More specifically, reducing the stochastic RMS norm of synchronization error to any arbitrary degree of accuracy is equivalent to reducing the H_2 norm of the transfer function of synchronization error to the disturbance with desired arbitrary degree of accuracy.

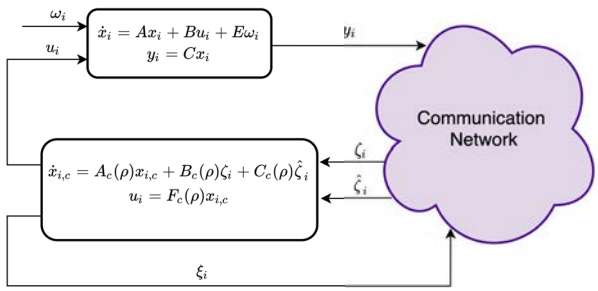

Figure 1: Architecture of scale-free collaborative protocol for H_2 almost state synchronization

We formulate the scale-free H_2 almost state synchronization problem of a MAS with localized information exchange.

Problem 1 *The* ***scale-free H_2 almost state synchronization problem with localized information exchange (scale-free H_2-ASSWLIE)*** *for MAS* (5) *and* (6) *is to find, if possible, a fixed linear protocol parameterized in terms of a scalar parameter ρ of the form*

$$\begin{cases} \dot{x}_{i,c} &= A_c(\rho)x_{i,c} + B_c(\rho)\zeta_i + C_c(\rho)\hat{\zeta}_i \\ u_i &= F_c(\rho)x_{i,c} \end{cases} \tag{8}$$

where $\hat{\zeta}_i$ is defined by (7)*, with $\xi_i = H_c x_{i,c}$ with $x_{i,c} \in \mathbb{R}^{n_c}$ such that for any number of agents N, and any communication graph $\mathcal{G}$ we have*

- *in the absence of the disturbance ω, for all initial conditions the state synchronization*

$$\lim_{t\to\infty}(x_i - x_j) = 0 \quad \textit{for all } i, j \in \{1, \ldots, N\} \tag{9}$$

  is achieved for any $\rho \geqslant 1$.
- *in the presence of the disturbance ω, for any $\gamma > 0$, one can render the H_2 norm from $\omega = \begin{pmatrix}\omega_1^T & \ldots & \omega_N^T\end{pmatrix}^T$ to $x_i - x_j$ less than γ by choosing ρ sufficiently large.*

The architecture of the protocol (8) *is shown in Figure 1.*

3 H_2 almost state synchronization: solvability conditions and protocol design

In this section, we will consider the H_2 almost state synchronization problem of a MAS for both cases of full- and partial-state coupling.

3.1 Full-state coupling

We design the Protocol1 on the following page with localized information exchanges. Then, we have the following theorem for scale-free H_2-ASSWLIE case.

Theorem 1 *Consider a MAS described by* (5) *and* (6)*, where $C = I$.*

1) *The scale-free H_2-ASSWLIE problem as stated in Problem 1 is solvable* ***if and only if***
   a) *(A, B) is stabilizable.*
   b) *all eigenvalues of A are in the closed left half plane.*
   c) *The graph $\mathcal{G}$, describing the communication topology of the network, contains a directed spanning tree.*
   d)

$$\mathrm{im}E \subseteq \mathrm{im}B. \tag{13}$$

Protocol 1: Full-state coupling

We design collaborative protocols for agent $i \in \{1, \ldots, N\}$ as

$$\begin{cases} \dot{\chi}_i = A\chi_i + Bu_i + \rho\zeta_i - \rho\hat{\zeta}_i, \\ u_i = -\rho B^{\mathrm{T}} P \chi_i \end{cases} \tag{10}$$

where ρ is a parameter satisfying $\rho \geqslant 1$ while P is the unique solution of algebraic Riccati equation

$$A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P + I = 0 \tag{11}$$

and ζ_i is defined by (6). The agents communicate $\xi_i = \chi_i$, therefore each agent has access to local information

$$\hat{\zeta}_i = \sum_{j=1}^{N} a_{ij}(\chi_i - \chi_j). \tag{12}$$

2) *The linear dynamic Protocol* 1 *solves scale-free H_2-ASSWLIE. In other words, for any number of agents N and any graph $\mathcal{G}$ in the absence of the disturbance ω, for any $\rho \geqslant 1$, the state synchronization* (9) *is achieved for any initial conditions while in the presence of the disturbance ω, for any $\gamma > 0$, the H_2 norm from ω to $x_i - x_j$ is less than γ by choosing ρ sufficiently large.*

To prove this theorem, we need the following lemma.

Lemma 1 ([10]) *Let a Laplacian matrix $L \in \mathbb{R}^{N\times N}$ be given associated with a graph that contains a directed spanning tree. We define $\bar{L} \in \mathbb{R}^{(N-1)\times(N-1)}$ as the matrix $\bar{L} = [\bar{\ell}_{ij}]$ with $\bar{\ell}_{ij} = \ell_{ij} - \ell_{Nj}$. Then the eigenvalues of $\bar{L}$ are equal to the nonzero eigenvalues of L.*

Proof of Theorem 1: Due to space limitation the proof is omitted and provided in [11, Proof of Theorem 1]. ■

3.2 Partial-state coupling

In this subsection, we will consider H_2 almost state synchronization via partial-state coupling. Firstly, we design Protocol2. Then, we have the following theorem for MAS via partial-state coupling.

Protocol 2: Partial-state coupling

We design collaborative protocols for agent $i \in \{1, \ldots, N\}$ as

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i - \rho BB^{\mathrm{T}}P\hat{\zeta}_i + \delta^{-2}Q_\rho C^{\mathrm{T}}(\zeta_i - C\hat{x}_i) \\ \dot{\chi}_i = A\chi_i + Bu_i + \rho\hat{x}_i - \rho\hat{\zeta}_i \\ u_i = -\rho B^{\mathrm{T}}P\chi_i, \end{cases} \tag{14}$$

where $P > 0$ is the unique solution of (11). Since $(A, E, C, 0)$ is minimum-phase and left invertible, then for any $\rho \geqslant 1$, there exists $\delta > 0$ small enough such that $Q_\rho > 0$ is the unique solution of

$$Q_\rho A^{\mathrm{T}} + AQ_\rho + EE^{\mathrm{T}} - \delta^{-2}Q_\rho C^{\mathrm{T}}CQ_\rho + \rho^2 Q_\rho^2 = 0. \tag{15}$$

In this protocol, agents communicate $\xi_i = \chi_i$, i.e. each agent has access to localized information (12), while ζ_i is defined by (6).

Theorem 2 *Consider a MAS described by* (5) *and* (6).

1) *The scale-free H_2-ASSWLIE problem as stated in Problem 1 is solvable* ***if and only if***
   a) *(A, B) are stabilizable and (C, A) are detectable.*
   b) *all eigenvalues of A are in the closed left half plane.*
   c) *$(A, E, C, 0)$ is minimum phase and left invertible.*
   d) *the graph $\mathcal{G}$, describing the communication topology of the network, contains a directed spanning tree.*
   e) $\mathrm{im}E \subseteq \mathrm{im}B$ *(i.e.* (13)*).*
2) *The linear dynamic Protocol* 2 *solves scale-free H_2-ASSWLIE, for any number of agents N and any graph $\mathcal{G}$ such that in the absence of disturbance ω, for any $\rho \geqslant 1$, the state synchronization* (9) *is achieved for any initial conditions and in the presence of disturbance ω, for any $\gamma > 0$, the H_2 norm from ω to $x_i - x_j$ is less than γ by choosing ρ sufficiently large.*

Proof of Theorem 2: Due to space limitation the proof is omitted and provided in [11, Proof of Theorem 2]. ■

Remark 1 *We would like to emphasize that Protocol* 1 *and* 2 *are designed solely based on agent models i.e. (A, B, C) and do not need any information about the communication graph and size of the network (i.e number of agents N) as such they are* ***scalable****. On the other hand, the parameter ρ in protocol designs is used to reduce the impact of disturbance on synchronization error. However, when we want a better disturbance rejection level (smaller H_2 norm), we need to increase ρ. It worth to mention that in the absence of disturbance we can choose ρ arbitrarily.*

Remark 2 *It is worth noting that the formulation in Problem 1 does not specify how the matrices of the protocol should evolve with parameter ρ. However, our designs, as given in Protocol 1 and 2, provide an explicit solution from which the protocol matrices can be derived. The structure of the protocols are independent of the parameter ρ; thus, one may develop the structure at one stage and tune the parameter ρ later so as to obtain the desired degree of accuracy. Due to continuity in ρ, tuning may be even carried out online. Hence, the method is a one-shot design and is not iterative.*

4 Numerical example

In this section we will illustrate the effectiveness of our protocol design with a numerical example for H_2 state synchronization with partial-state coupling. Consider agent models (5) with

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad E = B.$$

For this agent model, we obtain Protocol 2, by solving algebraic Riccati equations (11) and (15) for three values of $\rho = 4$, $\rho = 6$ and $\rho = 10$ (The Riccati equation (15) is solved with $\delta = 0.0004$).

We create two homogeneous MAS with different number of agents and different communication topologies to show the designed protocol is scale-free, i.e. it is independent of information of the communication network and the number of agents N.

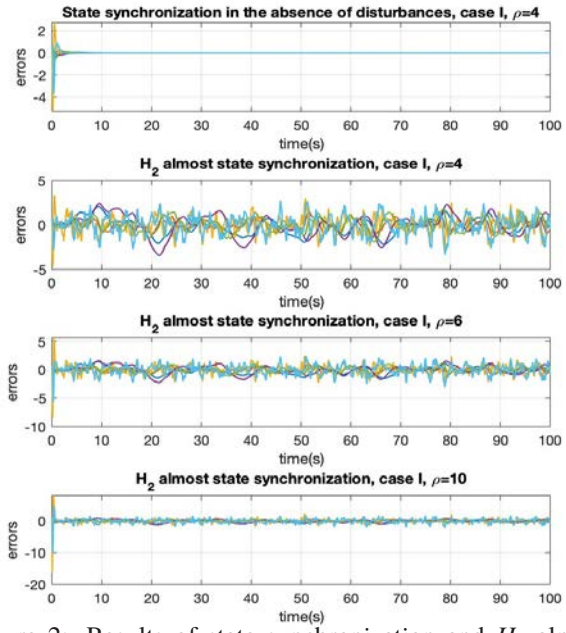

Figure 2: Results of state synchronization and H_2 almost state synchronization for the MAS with $N = 3$

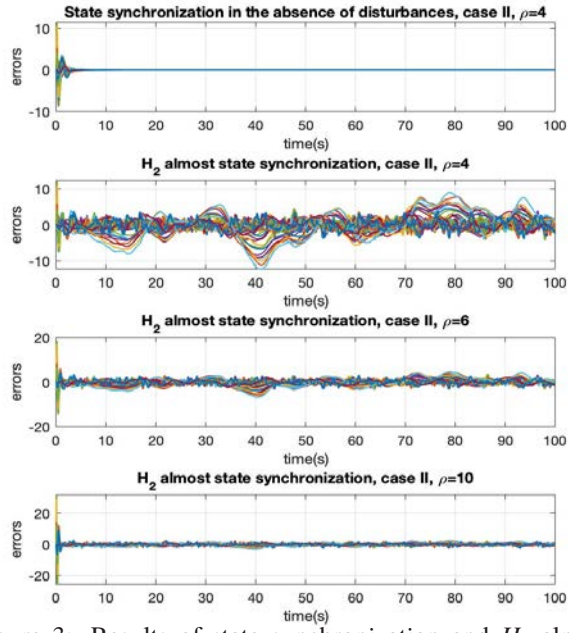

Figure 3: Results of state synchronization and H_2 almost state synchronization for the MAS with $N = 20$

- *Case I*: In this case, we consider MAS with 3 agents and communication topology $\mathcal{A}_1$, with $a_{21} = a_{32} = 1$. The result of exact state synchronization in the absence of disturbance and H_2 almost state synchronization in presence of white noises with unit power spectral densities for $i = 1, \cdots, N$ are shown in Figure 2. The results show that by increasing ρ, one can decrease the impact of disturbances on synchronization error.
- *Case II*: Next, we consider a MAS with 20 agents and associated adjacency matrix $\mathcal{A}_2$, with $a_{16} = a_{21} = a_{32} = a_{43} = a_{54} = a_{65} = a_{76} = a_{87} = a_{98} = a_{10,9} = a_{11,10} = a_{12,11} = a_{13,12} = a_{13,20} = a_{14,13} = a_{15,14} = a_{15,6} = a_{16,15} = a_{17,16} = a_{18,17} = a_{19,18} = a_{20,18} = 1$. Figure 3 shows the results for exact state synchronization in the absence of disturbance with $\rho = 4$ and H_2 almost state synchronization results in presence of disturbances ω_i equal to white noises with unit power spectral densities for $i = 1, \cdots, N$ with $\rho = 4$, $\rho = 6$, and $\rho = 10$.

The simulation results show that the protocol design is independent of the communication graph and is scale free so that we can achieve H_2 almost state synchronization with one-shot protocol design, for any graph with any number of agents. The simulation results also show that by increasing the value of ρ, almost state synchronization is achieved with higher degree of accuracy.

5 Conclusion

In this paper, we studied H_2 almost state synchronization of homogeneous networks of non-introspective agents. A parameterized scale-free linear dynamic protocol, parameterized in scalar ρ, was developed using localized information exchange over the same communication network and solely based on agent models. In particular, in the absence of disturbance, we achieved synchronization for any $\rho > 1$ and in the presence of disturbance we achieved almost state synchronization for a given arbitrary degree of accuracy by choosing ρ sufficiently large. Despite all the existing results, our design methodology was scale-free so that we did not need any information about the communication network such as bounds on the associated Laplacian matrix and the number of agents. The expanded version of this paper is available at [11].

References

[1] F. Bullo. *Lectures on network systems*. Kindle Direct Publishing, 2019.

[2] D. Chowdhury and H. K. Khalil. Synchronization in networks of identical linear systems with reduced information. In *American Control Conference*, pages 5706–5711, Milwaukee, WI, 2018.

[3] H.F. Grip, T. Yang, A. Saberi, and A.A. Stoorvogel. Output synchronization for heterogeneous networks of non-introspective agents. *Automatica*, 48(10):2444–2453, 2012.

[4] H. Kim, H. Shim, J. Back, and J. Seo. Consensus of output-coupled linear multi-agent systems under fast switching network: averaging approach. *Automatica*, 49(1):267–272, 2013.

[5] H. Kim, H. Shim, and J.H. Seo. Output consensus of heterogeneous uncertain linear multi-agent systems. *IEEE Trans. Aut. Contr.*, 56(1):200–206, 2011.

[6] L. Kocarev. *Consensus and synchronization in complex networks*. Springer, Berlin, 2013.

[7] X. Li, Y. C. Soh, and L. Xie. A novel reduced-order protocol for consensus control of linear multiagent systems. *IEEE Trans. Aut. Contr.*, 64(7):3005–3012, 2019.

[8] Z. Li, Z. Duan, G. Chen, and L. Huang. Consensus of multi-agent systems and synchronization of complex networks: A unified viewpoint. *IEEE Trans. Circ. & Syst.-I Regular papers*, 57(1):213–224, 2010.

[9] Z. Liu, A. Saberi, A. A. Stoorvogel, and D. Nojavanzadeh. Global and semi-global regulated state synchronization for homogeneous networks of non-introspective agents in presence of input saturation. In *Proc. 58th CDC*, pages 7307–7312, Nice, France, 2019.

[10] Z. Liu, A. Saberi, A.A. Stoorvogel, and D. Nojavanzadeh. H_∞ almost state synchronization for homogeneous networks of non-introspective agents: a scale-free protocol design. *Automatica*, 122:109276 (1–7), 2020.

[11] Z. Liu, A. Saberi, A.A. Stoorvogel, and D. Nojavanzadeh. Scale-free H_2 almost state synchronization for homogeneous networks of non-introspective agents. Available: arXiv:2006.01890, 2020.

[12] Z. Liu, M. Zhang, A. Saberi, and A. A. Stoorvogel. State synchronization of multi-agent systems via static or adaptive nonlinear dynamic protocols. *Automatica*, 95:316–327, 2018.

[13] D. Nojavanzadeh, Z. Liu, A. Saberi, and A. A. Stoorvogel. Output and regulated output synchronization of heterogeneous multi-agent systems: A scale-free protocol design using no information about communication network and the number of agents. In *American Control Conference*, pages 865–870, Denver, CO, 2020.

[14] R. Olfati-Saber and R.M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Aut. Contr.*, 49(9):1520–1533, 2004.

[15] E. Peymani, H.F. Grip, and A. Saberi. Homogeneous networks of non-introspective agents under external disturbances - H_∞ almost synchronization. *Automatica*, 52:363–372, 2015.

[16] E. Peymani, H.F. Grip, A. Saberi, X. Wang, and T.I. Fossen. H_∞ almost ouput synchronization for heterogeneous networks of introspective agents under external disturbances. *Automatica*, 50(4):1026–1036, 2014.

[17] W. Ren. On consensus algorithms for double-integrator dynamics. *IEEE Trans. Aut. Contr.*, 53(6):1503–1509, 2008.

[18] W. Ren and R.W. Beard. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Trans. Aut. Contr.*, 50(5):655–661, 2005.

[19] W. Ren and Y.C. Cao. *Distributed coordination of multi-agent networks*. Communications and Control Engineering. Springer-Verlag, London, 2011.

[20] L. Scardovi and R. Sepulchre. Synchronization in networks of identical linear systems. *Automatica*, 45(11):2557–2562, 2009.

[21] J.H. Seo, J. Back, H. Kim, and H. Shim. Output feedback consensus for high-order linear systems having uniform ranks under switching topology. *IET Control Theory and Applications*, 6(8):1118–1124, 2012.

[22] J.H. Seo, H. Shim, and J. Back. Consensus of high-order linear systems using dynamic output feedback compensator: low gain approach. *Automatica*, 45(11):2659–2664, 2009.

[23] A. A. Stoorvogel, A. Saberi, Z. Liu, and D. Nojavanzadeh. H_2 and H_∞ almost output synchronization of heterogeneous continuous-time multi-agent systems with passive agents and partial-state coupling via static protocol. *Int. J. Robust & Nonlinear Control*, 29(17):6244–6255, 2019.

[24] A. A. Stoorvogel, A. Saberi, and M. Zhang. Solvability conditions and design for state synchronization of multi-agent systems. *Automatica*, 84:43–47, 2017.

[25] A.A. Stoorvogel, A. Saberi, M. Zhang, and Z. Liu. Solvability conditions and design for H_∞ and H_2 almost state synchronization of homogeneous multi-agent systems. *European Journal of Control*, 46:36–48, 2019.

[26] Y. Su and J. Huang. Stability of a class of linear switching systems with applications to two consensus problem. *IEEE Trans. Aut. Contr.*, 57(6):1420–1430, 2012.

[27] S.E. Tuna. LQR-based coupling gain for synchronization of linear systems. Available: arXiv:0801.3390v1, 2008.

[28] S.E. Tuna. Conditions for synchronizability in arrays of coupled linear systems. *IEEE Trans. Aut. Contr.*, 55(10):2416–2420, 2009.

[29] P. Wieland, J.S. Kim, and F. Allgöwer. On topology and dynamics of consensus among linear high-order agents. *International Journal of Systems Science*, 42(10):1831–1842, 2011.

[30] P. Wieland, R. Sepulchre, and F. Allgöwer. An internal model principle is necessary and sufficient for linear output synchronization. *Automatica*, 47(5):1068–1074, 2011.

[31] C.W. Wu. *Synchronization in complex networks of nonlinear dynamical systems*. World Scientific Publishing Company, Singapore, 2007.

[32] T. Yang, A. Saberi, A.A. Stoorvogel, and H.F. Grip. Output synchronization for heterogeneous networks of introspective right-invertible agents. *Int. J. Robust & Nonlinear Control*, 24(13):1821–1844, 2014.

[33] M. Zhang, A. Saberi, H. F. Grip, and A. A. Stoorvogel. $\mathcal{H}_\infty$ almost output synchronization for heterogeneous networks without exchange of controller states. *IEEE Trans. Control of Network Systems*, 2(4):348–357, 2015.

[34] M. Zhang, A. Saberi, and A. A. Stoorvogel. Regulated output synchronization for heterogeneous time-varying networks with non-introspective agents in presence of disturbance and measurement noise with known frequencies. In *American Control Conference*, pages 2069–2074, Chicago, IL, 2015.

[35] M. Zhang, A. Saberi, and A. A. Stoorvogel. Synchronization for heterogeneous time-varying networks with non-introspective, non-minimum-phase agents in the presence of external disturbances with known frequencies. In *Proc. 55th CDC*, pages 5201–5206, Las Vegas, NV, 2016.

[36] M. Zhang, A. Saberi, A. A. Stoorvogel, and P. Sannuti. Almost regulated output synchronization for heterogeneous time-varying networks of non-introspective agents and without exchange of controller states. In *American Control Conference*, pages 2735–2740, Chicago, IL, 2015.

[37] M. Zhang, A.A. Stoorvogel, and A. Saberi. Stochastic almost regulated output synchronization for time-varying networks of nonidentical and non-introspective agents under external stochastic disturbances and disturbances with known frequencies. In M.N. Belur, M.K. Camlibel, P. Rapisarda, and J.M.A. Scherpen, editors, *Mathematical control theory II*, volume 462 of *Lecture Notes in Control and Information Sciences*, pages 101–127. Springer Verlag, 2015.