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The Dial-A-Ride Problem considering the in-vehicle crowding inconvenience due to COVID-19

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***Abstract*— The Dial-A-Ride Problem (DARP) has received significant attention during the COVID-19 pandemic. During the pandemic's peak, public transport ridership was reduced up to 90% in several countries and many public transport users had to seek less crowded alternatives in DARP services. Such alternatives are flexible modes that do not operate on fixed lines (i.e., on-demand minibuses, shared vehicles). However, the standard Dial-A-Ride Problem (DARP) does not consider the in-vehicle crowding as long as the capacity of the vehicle is not exceeded. To rectify this, this study proposes a new formulation of the DARP that considers also the inconvenience of passengers due to the in-vehicle crowding levels in the objective function of the problem. In our formulation, we consider a progressive penalization of the increase of in-vehicle crowding to account for social distancing. This is modeled with piecewise linear functions that map the inconvenience of passengers to the in-vehicle crowding levels. The proposed model is a MINLP and it is reformulated as a MILP that can be solved with branch-and-bound and linear programming. This model is implemented in numerical experiments with benchmark DARP datasets to investigate the increases of the vehicle route costs when seeking to reduce the in-vehicle crowdedness.**

Keywords: DARP; public transport; shared transport; COVID-19; social distancing.

I. Introduction

Public transport is one of the most disrupted sectors by the COVID-19 pandemic. In the works of [1]–[4] were reported ridership drops of 60%-90% in major cities in China, Iran, the United States, the United Kingdom, and Europe. Reductions can vary at different times of the pandemic and are influenced by the perceptions of passengers in terms of virus transmission risks, government guidelines and shifts in work practices [5]–[7]. Because of these reductions, public transport users who do not own a private vehicle have opted for the use of flexible, on-demand transport modes. In this study, we concentrate on on-demand services without fixed schedules that can be performed by a set of dedicated vehicles (i.e., shared vehicles or minibuses). The routes of these services can be organized based on the DARP problem in order to accommodate the requests of travelers in a cost-efficient manner.

DARP models were initially used in the early 1980s to plan the routes of on-demand buses based on telephone requests by users (typically young, elderly or travelers with disabilities in areas with low public transport accessibility). Over the years, however, the developed models have found applicability to on-demand minibuses that are booked via mobile phone apps, shared vehicles, city shuttles, and many more (see the surveys of [8], [9]).

This study addresses the *static* DARP where all transportation requests are known in advance [10]. This is in line with practical applications since *dynamic* DARPs rarely exist in a pure form because many transportation requests are often known when the planning starts [11]–[13]. DARP aims at minimizing the total routing cost of all vehicles subject to full demand satisfaction and side constraints. It is important to note that in DARP the fleet size is provided in advance and this might result in cases where it is not possible to accommodate all passengers.

In more detail, in the static version of DARP one can consider that all transportation requests are known beforehand. Namely, there is availability of a fleet of vehicles which are based at a single depot; users impose a time window on both their departure and arrival times at their origin and destination locations; and the travel times and travel costs to traverse the network's arcs (roads) are deterministic (no uncertainties). In addition, there is a maximum limit for the ride time of a user and the route time of a vehicle. It is worth noting that imposing a time window at the origin and destination points of a user is a contested issue. Although DARP models typically let users impose a time window on both their departure and arrival times, the service provider might be capable of improving the operational efficiency if this constraint is partially relaxed [14].

The simplest version of DARP is the single-vehicle DARP that was solved by [15] as a dynamic program. DARP is an extension of the pickup and delivery vehicle routing problem (PDVRP) in logistics [16], [17]. The main difference is that DARP considers the ride times of users when traveling from their origin to their destination points and tries to provide a trade-off between the user ride times and the vehicle route costs. In the PDVRP, the origin locations of users are considered as pickup points for parcels and the destination locations as delivery points. Following this convention, the origin locations of passengers can also be seen as pickup points and their destinations as delivery points.

DARP is an NP-Hard optimization problem and can be solved for small-sized problem instances with limited user requests. For this reason, there are several heuristic and metaheuristic approaches that try to return acceptable (but not globally optimal) solutions within a limited time. This work will not focus on developing heuristics for the DARP, but the interested reader can refer to the seminal works of [18]–[22] or the survey papers of [8], [23] for more

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information about such approaches that range from tabu search to regret-insertion heuristics.

In a typical DARP setting, we have a number of passengers at origin locations (pickup points) willing to travel at specific destination locations (delivery points). As a service provider, we seek to satisfy these transportation requests with our available vehicles so as to minimize the route costs of these vehicles. If the transportation requests cannot be satisfied given the fleet availability, DARP can be cast as the maximization of satisfied demand subject to vehicle availability. In this study, we treat problems where the available vehicles have sufficient capacity to satisfy the user requests because we seek to reduce further the crowding levels inside each vehicle.

Considering the objective functions of DARP models, the most common objective is to minimize the vehicle route costs [8]. Recent DARP formulations have also considered the vehicle emissions as well [24], [25]. Formulations that minimize the staff workload [26], or maximize the reliability of the system [27] and the occupancy rate [28] are also provided in past studies. To the best of our knowledge, however, there are no works that penalize the in-vehicle crowdedness to reduce the passenger inconvenience. This can be partially explained because the main objective of the transport service provider is to use the available fleet as cost-efficiently as possible. During the pandemic though, passengers prefer less crowded vehicles and there are also strict government regulations that mandate a minimum physical distancing in closed spaces, such as shared transport modes. This study contributes in this direction by proposing a DARP formulation that accounts for the in-vehicle crowdedness. The specific contributions of this study are:

1) the incorporation of the in-vehicle crowdedness to the objective function of the static DARP formulation with the use of piecewise linear functions.
2) the reformulation of the resulting problem from a mixed-integer nonlinear program (MINLP) to a mixed-integer linear program (MILP) that can be solved with branch-and-bound and linear programming.
3) the implementation of the model to a dataset of benchmark DARP instances in order to investigate the potential increases in the vehicle route costs when we modify the vehicle routes to reduce the in-vehicle crowdedness.

II. Problem Formulation

Let $G = (V, A)$ be a network with V vertices and A arcs. In this network the vertices are not always unique physical locations and an arc does not necessarily represent a specific road on the network. Each arc is a direct connection between two vertices and it can be comprised of several road sections. That is, an arc can be seen as the shortest or fastest path when traveling between two vertices without performing any intermediate stops. The vertices of the network are denoted as $V = P \cup D \cup \{0, 2n+1\}$ where $P = \{1, 2, ..., n\}$ is the set of pickup user requests, $D = \{n+1, ..., 2n\}$ the set of deliveries, and 0, $2n+1$ are two copies of the vertex that corresponds to the depot. The network is encoded in such a way that each pickup user request i will be delivered at $n+i$ and the pickup P and delivery D sets have the same number of vertices. In practice, this means that if in a physical location i we have two passengers that require to be delivered at two different locations, then we will generate two copies of vertex i (namely, i and $i+1$) and their delivery locations will be represented by vertices $n+i$ and $n+i+1$. If, however, both passengers at location i have the same destination, then we do not need to generate a copy of vertex i and their destination will be denoted as $n+i$. It follows that for every unique origin-destination pair we generate a new request in our network.

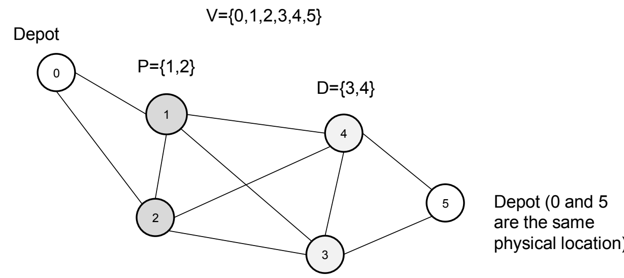

Fig. 1. DARP network representation in the case of two pickup requests. Note that in vertices 1 and 2 we might have more than one passengers. The picked up passengers from vertex 1 are delivered to vertex 3 and from vertext 2 to vertex 4, respectively.

Fig.1 provides an overview of a simple DARP network with two requests. Note that the pickup vertices 1 and 2 can be the same physical location. The same applies to the delivery vertices 3 and 4. Vertices 3 and 4 are encoded in such a way that the picked-up passenger(s) from vertex 1 will be delivered to vertex 3 and from vertex 2 to vertex 4. Note also that vertices 0 and 5 are at the same physical location (the location of the depot). In addition, we have arcs $A = \{(0, j) : j \in P\} \cup \{(i, j) : i \in P \cup D, j \in P \cup D, i \neq j\} \cup \{(i, 2n+1) : i \in D\}$. These arcs connect directly any pair of pickup and delivery vertices. In addition, they connect directly vertex 0 with the possible pickup points and vertex $2n+1$ with the possible delivery points. The latter is important to enforce that each vehicle starts from the depot and returns to the depot.

The main requirements of the DARP are:

- every vehicle route $k \in K$ starts from the depot and ends at the depot;
- for every request i, vertices i and $n+i$ belong to the same route and vertex $n+i$ is visited later than vertex i;
- the passenger load of any vehicle k does not exceed at any time the capacity of that vehicle, Q_k;
- the total duration of the route of any vehicle k does not exceed a preset time bound T_k;
- the pickup (or delivery) service at vertex i begins within a pre-defined time window $[e_i, l_i]$ and every vehicle leaves the depot within a pre-defined time window

$[e_0, l_0]$;
- the ride time of any passenger does not exceed a predetermined maximum value L.

Before proceeding to the formal presentation of the mathematical program, we introduce the parameters and variables of the model in Table I.

TABLE I
NOMENCLATURE

Parameters	
c_{ij}^k	cost of traversing arc $(i,j) \in A$ with vehicle k
t_{ij}	travel time of traversing arc $(i,j) \in A$
T_k	maximum allowed travel time by vehicle k
$[e_i, l_i]$	time window of minimum and maximum amount of time related to starting the service at vertex $i \in V \setminus \{2n+1\}$
d_i	time spent at request $i \in P \cup D$ to pickup or deliver passengers (service time). Note that $d_0 = d_{2n+1} = 0$
q_i	associated passenger load of vertex $i \in V$, where $q_0 = q_{2n+1} = 0$, $q_i \geq 0\ \forall i \in P$, $q_i = -q_{i-n}\ \forall i \in D$
L	maximum allowed ride time of a passenger
γ	a non-negative parameter indicating the relative importance of reducing the in-vehicle crowding levels compared to the vehicle travel costs
z_k	soft capacity of vehicle k that corresponds to the physical distancing requirement (i.e., maintaining 1-meter distancing)
Q_k	hard capacity of vehicle k. This is the actual vehicle capacity
Variables	
x_{ij}^k	binary variable indicating whether arc $(i,j) \in A$ is traversed by vehicle k
u_i^k	the time at which vehicle k starts servicing vertex i
w_i^k	the passenger load of vehicle k upon leaving vertex i
r_i^k	the ride time of any user of request $(i, n+i)$

The objective of the standard DARP is to minimize the vehicle route costs, $\sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k$. In this study, we also consider the increasing inconvenience of passengers when there are other passengers in the vehicle. During the pandemic, passengers would like to board less crowded vehicles and this is considered as another problem objective. We thus propose the following generalized objective function:

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k\in K}\sum_{i\in V} f(w_i^k)$$

where γ is a non-negative parameter indicating the relative importance of reducing the in-vehicle crowding levels compared to the vehicle route costs. In addition, $f : w_i^k \mapsto R_+$ is a function that maps the in-vehicle passenger loads w_i^k to the perceived passenger inconvenience.

This newly introduced term in the DARP's objective function can be defined based on the passenger perceptions and the tolerance of the mobility service provider. For instance, if maintaining a low number of in-vehicle passengers is important due to governmental regulations, then γ can receive a high value. Function $f(w_i^k)$ can be defined based on the discomfort of passengers with respect to physical distancing. In the simpler case, one can assume a piecewise linear function of the form:

$$f(w_i^k) := \begin{cases} w_i^k \rho & 0 \leq w_i^k \leq z_k \\ z_k\rho + (w_i^k - z_k)\theta & z_k \leq w_i^k \leq Q_k \end{cases}$$

where $\rho, \theta \in \mathbb{R}_+$ and $\theta \ggg \rho$ indicating a sharp increase in passenger inconvenience when the in-vehicle crowding level exceeds the soft capacity z_k that corresponds to a regulatory suggestion (i.e., a capacity that allows for 1-meter in-vehicle distancing).

Note that the piecewise linear function is devised in such a way that there is already a passenger inconvenience even if we have one passenger in the vehicle because there is also a second person in it (the driver). Based on the proposed objective function, the DARP that considers in-vehicle crowding is cast as:

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k\in K}\sum_{i\in V} f(w_i^k) \tag{1}$$

subject to:

$$\sum_{k\in K}\sum_{j:(i,j)\in A} x_{ij}^k = 1 \quad \forall i \in P \tag{2}$$

$$\sum_{i:(0,i)\in A} x_{0i}^k = \sum_{i:(i,2n+1)\in A} x_{i,2n+1}^k = 1 \quad \forall k \in K \tag{3}$$

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(n+i,j)\in A} x_{n+i,j}^k = 0 \quad \forall i \in P, k \in K \tag{4}$$

$$\sum_{j:(j,i)\in A} x_{ji}^k - \sum_{j:(i,j)\in A} x_{ij}^k = 0 \quad \forall i \in P \cup D, k \in K \tag{5}$$

$$u_j^k \geq (u_i^k + d_i + t_{ij})x_{ij}^k \quad \forall (i,j) \in A, k \in K \tag{6}$$

$$w_j^k \geq (w_i^k + q_j)x_{ij}^k \quad \forall (i,j) \in A, k \in K \tag{7}$$

$$r_i^k = u_{n+i}^k - (u_i^k + d_i) \quad \forall i \in P, k \in K \tag{8}$$

$$u_{2n+1}^k - u_0^k \leq T_k \quad \forall k \in K \tag{9}$$

$$e_i \leq u_i^k \leq l_i \quad \forall i \in V, k \in K \tag{10}$$

$$t_{i,n+i} \leq r_i^k \leq L \quad \forall i \in P, k \in K \tag{11}$$

$$\max\{0, q_i\} \leq w_i^k \quad \forall i \in V, k \in K \tag{12}$$

$$w_i^k \leq \min\{Q_k, Q_k + q_i\} \quad \forall i \in V, k \in K \tag{13}$$

$$x_{ij}^k \in \{0,1\} \quad \forall i,j \in V, k \in K \tag{14}$$

The constraints are adapted from the classic three-index formulation of the static DARP (see [29]). Constraints (2) ensure that any pickup point $i \in P$ will be directly connected with exactly one other vertex $j \in P \cup D$ by exactly one vehicle k. Constraints (3) and (5) ensure that the route of each vehicle k starts from the depot 0 and ends at the depot $2n+1$. Constraints (4) ensure that each vehicle k that serves request $i \in P$ will serve the corresponding delivery vertex $n+i$. That is, the pickup and delivery points of a request are served by the same vehicle.

Constraints (6) ensure that if arc $(i,j) \in A$ is served by vehicle k, $x_{ij}^k = 1$, then the time at which vehicle k starts servicing vertex j (denoted as u_j^k) is greater than or equal to u_i^k plus the travel time from i to j (denoted as t_{ij}) plus the service time at i (denoted as d_i). Constraints (9) ensure that vehicle k will return to the depot before T_k. Constraints (10) ensure that each vertex $i \in V$ will be served within its predefined time window.

Constraints (8) ensure that the ride time of a passenger that uses vehicle k to travel from origin i to destination $n+i$ (denoted as r_i^k) is equal to $u_{n+i}^k - (u_i^k + d_i)$. Constraints (11) ensure that the ride time of a passenger from i to $n+i$ who uses vehicle k is less than the maximum allowed passenger ride time, L. It is also greater than or equal to the minimum travel time $t_{i,n+i}$ corresponding to a direct connection between i and $n+i$.

Constraints (7) ensure that if a vehicle k serves arc (i,j) the passenger load upon leaving vertex j is greater than or equal to the load at vertex i plus q_j. Constraints (14) ensure that the in-vehicle passenger load w_i^k upon leaving vertex i is at least $\max\{0, q_i\}$ and, at the same time, does not exceed the vehicle capacity $\min\{Q_k, Q_k + q_i\}$.

III. Reformulation to a MILP

Note that constraints (6) and (7) are nonlinear. To linearize them, Cordeau [29] proposed to introduce parameters M_{ij}^k and Y_{ij}^k with very high positive values and replace (6)-(7) by constraints (15)-(16).

$$u_j^k \geq u_i^k + d_i + t_{ij} - M_{ij}^k(1 - x_{ij}^k) \quad \forall (i,j) \in A, k \in K \tag{15}$$

$$w_j^k = w_i^k + q_j - Y_{ij}^k(1 - x_{ij}^k) \quad \forall (i,j) \in A, k \in K \tag{16}$$

Let us also linearize the proposed piecewise linear function that considers the presence of a driver in the vehicle:

$$f(w_i^k) := \begin{cases} w_i^k \rho & 0 \leq w_i^k \leq z_k \\ z_k\rho + (w_i^k - z_k)\theta & z_k \leq w_i^k \leq Q_k \end{cases}$$

The minimization of the piecewise linear function $f(w_i^k)$ can be linearized as follows:

$$\min \ z_k\rho\lambda_{i1}^k + (z_k\rho + (Q_k - z_k)\theta)\lambda_{i2}^k \tag{17}$$

$$\text{s.t } \zeta_{i1}^k + \zeta_{i2}^k = 1 \quad \forall i \in V, k \in K \tag{18}$$

$$\lambda_{i0}^k + \lambda_{i1}^k + \lambda_{i2}^k = 1 \quad \forall i \in V, k \in K \tag{19}$$

$$\zeta_{i1}^k \leq \lambda_{i0}^k + \lambda_{i1}^k \quad \forall i \in V, k \in K \tag{20}$$

$$\zeta_{i2}^k \leq \lambda_{i1}^k + \lambda_{i2}^k \quad \forall i \in V, k \in K \tag{21}$$

$$\zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\} \quad \forall i \in V, k \in K \tag{22}$$

$$w_i^k = z_k\lambda_{i1}^k + Q_k\lambda_{i2}^k \quad \forall i \in V, k \in K \tag{23}$$

$$\lambda_{i0}^k, \lambda_{i1}^k, \lambda_{i2}^k \in \mathbb{R}_+ \quad \forall i \in V, k \in K \tag{24}$$

where $\zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\}$ and $\lambda_{i0}^k, \lambda_{i1}^k, \lambda_{i2}^k \in \mathbb{R}_+$ are newly added decision variables.

This results in the reformulated DARP:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \gamma \sum_{k \in K} \sum_{i \in V} \left(z_k\rho\lambda_{i1}^k + (z_k\rho + (Q_k - z_k)\theta)\lambda_{i2}^k \right) \tag{25}$$

$$\text{s.t. Eqs. } (2) - (5), (8) - (24) \tag{26}$$

$$x_{ij}^k \in \{0,1\} \quad \forall i,j \in V, k \in K \tag{27}$$

The reformulated problem is a MILP. If we drop the integrality constraints (22) and (27) related to $\zeta_{i1}^k, \zeta_{i2}^k$ and x_{ij}^k, the continuous relaxation of the MILP is a linear program (LP) that can be solved to global optimality in polynomial time.

To find the globally optimal solution of the MILP expressed in Eqs.(25)-(27) one can apply a branch-and-bound algorithm [30] that allows $x_{ij}^k, \zeta_{i1}^k, \zeta_{i2}^k$ to take real values in the closed interval [0,1]. When branching, some of these variables receive fixed $\{0,1\}$ values and some others are free to take real values from the interval [0,1]. The performance of each node that emerges from branching is calculated by solving a linear program with simplex or the Ellipsoid method and the branch-and-bound algorithm terminates when finding the optimal solution with $x_{ij}^k, \zeta_{i1}^k, \zeta_{i2}^k \in \{0,1\}$.

IV. Numerical Experiments

The focus of the numerical experiments is to study the impact of reducing the in-vehicle crowding levels due to COVID-19 on the vehicle route costs of a DARP service. In the numerical experiments, we use the well-known DARP instances from [31]. From this dataset, we focus on the instances b2-16, b2-20 and b2-24 that contain 2 vehicles and 16, 20 and 24 requests, respectively. Note that a network with n requests includes $2n+2$ vertices. Namely, for $n = 16$ requests we have pickup vertices $P = \{1, ..., 16\}$, delivery vertices $D = \{17, ..., 32\}$, and vertices 0 and 33 denoting the depot. At each pickup vertex $i \in P$ we pickup q_i passengers who are delivered to $n+i$ such that $q_{n+i} = -q_i$.

The proposed DARP that considers the in-vehicle crowding inconvenience of passengers is programmed in Python 3.7 and it is solved by the commercial solver Gurobi 9.0.3

that employs branch-and-bound and dual simplex. The implementation of the algorithms is performed in a general-purpose i7 2.8GHZ processor with 16GB RAM.

We first start with the b2-16 network with the 2 vehicles and the 16 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ passengers for $k \in \{1, 2\}$ and the maximum route duration for each vehicle is $T_k = 480$. In addition to these parameter values that are provided from the test instance, we assume that the soft capacity that corresponds to the recommended social distancing is $z_k = 3$ passengers for each $k \in K$, $\rho = 5$ and $\theta = 100$. Note that we use a very high value for θ to over-penalize crowding levels beyond the recommended distancing level.

Table II provides the results for this test instance when considering different importance levels for the passenger crowding, γ. In more detail, when $\gamma = 0$ we solve the classic DARP without considering the inconvenience of passengers due to in-vehicle crowding. As γ increases its value, we are permitting the increase of the vehicle route costs in order to reduce the in-vehicle crowding levels. In the first column of the table, we report the values of γ that correspond to each experiment. In the second column, we report the vehicle route costs for this value of γ. In the third column, we report the passenger costs related to their inconvenience due to in-vehicle crowding. We let the solver run for 5 minutes for each value of γ. If the solver is capable of finding a solution before the 5-minute time limit, then this is a globally optimal solution. If not, we report the estimated optimality gap provided by the solver in the fourth column of the table. Note that this gap is just an estimation provided by the solver based on the difference between the incumbent solution that meets the integrality constraints and the lower bound. The actual gap might be much smaller because the lower bound solution does not meet some of the integrality constraints. This is why we call this gap "maximum possible optimality gap".

TABLE II
PERFORMANCES OF THE SOLUTIONS OF THE B2-16 NETWORK FOR DIFFERENT VALUES OF γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	309.406	n/a	11.10%
0.03	309.612	3335	11.94%
0.05	309.612	3335	13.95%
0.25	326.033	3075	0.00%
0.75	326.033	3075	0.00%
1	326.033	3075	0.00%
100	326.033	3075	0.00%

Note that for $\gamma = 0$ we do not report the passenger-related costs because they are not taken into consideration in the optimization process. As one might have expected, the passenger-related costs decrease when γ increases. This is a result of placing more emphasis on the reduction of passenger-related costs. At the same time, for larger values of γ the vehicle costs increase until reaching 326.033 for $\gamma = 0.25$. After this point, there is an equilibrium and we cannot reduce further the in-vehicle crowding levels by increasing the route costs of the two vehicles. In summary, when moving from $\gamma = 0.03$ to $\gamma = 0.25$ we increase the vehicle costs by 5.3% in order to reduce the passenger costs related to crowding by 7.8%.

Let us now consider the b2-20 network with 2 vehicles and 20 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ and the maximum route duration for each vehicle is $T_k = 600$. We also assume $z_k = 3$, $\rho = 5$ and $\theta = 100$. Table III provides the results for this test instance when considering different importance levels for the passenger crowding, γ.

TABLE III
PERFORMANCES OF THE SOLUTIONS OF THE B2-20 NETWORK FOR DIFFERENT VALUES OF γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	332.638	n/a	0.00%
0.03	333.644	7550	0.00%
0.05	333.644	7550	0.00%
0.25	350.612	7330	0.00%
0.75	350.612	7330	0.00%
1.00	350.612	7330	0.00%
100	350.612	7330	0.00%

In the b2-20 network we have a reduction of the passenger costs related to crowding for $\gamma = 0.25$. After that, the passenger costs cannot be improved further. Until that point, for a 5% increase of the vehicle route costs we had a 2.9% reduction of the passenger-related costs.

Finally, let us consider the b2-24 network with 2 vehicles and 24 requests. In this test instance, the maximum allowed ride time of a passenger is $L = 45$, the nominal capacity of each vehicle is $Q_k = 6$ and the maximum route duration for each vehicle is $T_k = 720$. We also assume $z_k = 3$, $\rho = 5$ and $\theta = 100$. Table IV provides the results for this test instance when considering different importance levels for the passenger crowding, γ.

TABLE IV
PERFORMANCES OF THE SOLUTIONS OF THE B2-24 NETWORK FOR DIFFERENT VALUES OF γ.

γ	Vehicle costs	Passenger-related costs due to crowding	Maximum possible optimality gap
0	445.422	n/a	15.75%
0.03	452.508	5950	11.43%
0.05	451.097	5950	8.93%
0.25	460.253	5825	2.18%
0.75	460.253	5825	0.00%
1	460.253	5825	0.00%
100	460.253	5825	0.00%

In the b2-24 network we have a reduction of the passenger costs related to crowding for $\gamma = 0.25$. After that, the passenger costs cannot be improved further. Until that point, for a 1.7% increase of the vehicle route costs we had a 2.1% reduction of the passenger-related costs.

V. Conclusion

This study proposed a DARP formulation that considers the in-vehicle crowdedness as an additional problem objective. The proposed formulation resulted in a MINLP. We considered a piecewise linear function to over-penalizes crowding levels that exceed a regulatory limit and we offered a MILP reformulation of the original MINLP. This reformulation allows the solution of the problem with commercial solvers.

In our numerical experiments, we investigated the potential effect of reducing the vehicle crowdedness to the increase of the vehicle route costs. Interestingly, there was a balanced increase of the vehicle route costs for a reduction in the in-vehicle crowding levels. Another important observation is that because in DARP the fleet availability is fixed, if there are many user requests we cannot always reduce the in-vehicle crowding levels considerably. For the test instances of b2-20 and b2-24 we observed a reduction of in-vehicle crowding by 2-3% even if we consider this as the first priority (for $\gamma = 100$). There are two reasons for this: first, we must serve all requests with the available vehicles. Second, DARP requires from a single vehicle to pickup all customers by any pickup point at once. This might create crowding problems when having many passengers at a specific pickup point.

In future research, the flexibility of reducing the in-vehicle crowding levels can be investigated further by allowing the increase of the number of available vehicles based on well-defined trade-offs or allowing vehicles to serve only part of the passenger demand at a pickup point.

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