Spat. Inf. Res.
https://doi.org/10.1007/s41324-021-00413-6

Comparison of surface water flow simulation over structured and unstructured grids

Nivedhitha Ajithkumar[1,2] · **Prabhakar Alok Verma**[1] · **Frank B. Osei**[2] ·
Hari Shankar[1]

Received: 12 October 2020 / Revised: 20 June 2021 / Accepted: 27 June 2021

Abstract Structured grids have drawbacks such as cell distortion and artefacts leading to numerical accuracy reduction during simulations. Spatial support size had proven its importance in various estimation, and this is where unstructured grid comes into action. Performing geostatistical simulation on unstructured grid requires accounting of support size effect. In this article, two methods have been used for accounting the support size effect, one is the classical fine-scale simulation approach and the other approach is using Discrete Gaussian Model (DGM). Each method is applied to generate simulated Digital Elevation Model (DEM) (as used in this study) and the resultant DEMs are studied to understand the effect of spatial support size on surface flow estimation of water. It was observed that due to regularisation in the data of unstructured grid the flow velocity and water surface elevation of the unstructured DEM gave a similar behaviour than the structured DEM. The minimum RMSE for water surface elevation is 0.83 m for unstructured DEM while the minimum RMSE for flow discharge is 0.38 m^3/s. The maximum coefficient of determination of flow channel velocity and water surface elevation is 0.709 and 0.86 respectively. The results suggested that the unstructured DEM generated using DGM approach shows a high correlation to reference DEM used in this study than the simulated structured DEM. The surface flow output it was inferred the flow variation in both structured and unstructured DEM is affected not only by the vertical resolution of DEM but also by the horizontal resolution.

Keywords Digital elevation models · Geostatistical simulation · Support size effect · Surface flow · Steady flow analysis · Unstructured grids

1 Introduction

Structured grids also known as regular grids are made up of identical blocks, which have uniform shape, size and orientation. Most geostatistical simulation have been performed over these grids since long time in the industries as they follow stratigraphy in corner point grids geometry. They were also convenient in optimizing algorithms of various kinds such as sequential simulation, Fast Fourier transform etc. [1, 2].

The drawbacks of structured grids include difficulty to deform the shape of regular grids which is generally known as the support size effect. Typically the shape of this grid has fixed area or volume [3, 4]. Therefore, if we had to add or remove any points from the grid it will affect the whole grid structure. Another disadvantage of the structured grids is that due to some artefacts, cell distortions are caused. These distortions might lead to many consequences like disturbing the symmetry of the grid which makes the numerical approximation no longer in centre of the volume element, thereby reducing the numerical accuracy of the grid [5–9]. For example, in hydrology, if we take a Digital

✉ Nivedhitha Ajithkumar
nivstarz20@gmail.com

Prabhakar Alok Verma
Prabhakar@iirs.gov.in

Frank B. Osei
f.b.osei@utwente.nl

Hari Shankar
hari5918phy@gmail.com

[1] Indian Institute of Remote Sensing, Dehradun, India

[2] Faculty of Geoinformation Science and Earth Observation, University of Twente, Enschede, The Netherlands

Elevation Model (DEM), the error produced in DEM will severely affect the ability to represent terrain, which indirectly affects the hydrological modelling. Quality of DEM is affected by many factors in which artefacts of grid cell size (resolution) is one among them. So if grid cell size (resolution) is decreased, quality of DEM decreases progressively [10–12]. In order to address these drawbacks of structured grids, unstructured grids were introduced.

In the last few decades, many new unstructured grid geometries have emerged such as tetrahedral meshes, Voronoi grids etc. They are mainly used in areas like hydrogeology for reservoir modelling and mining in petroleum industries. These newly emanated grids are more convenient to solve physical equations of flow and transport in permeable media [13–15]. Adaptive resolutions are enabled in building the models of unstructured grids, i.e., less important regions are coarser and for important regions are finer. For instance, a petroleum reservoir can be modelled with fine blocks in the vicinity of the wells in order to solve the flow equations with better accuracy, whereas the aquifer can be modelled with lower resolution in order to reduce the computation time [16].

The advantage of unstructured grid is that it solves complex structures in a short period. They are automated compared to regular grids, require less effort, and will generate full mesh under most situations. The unstructured grids were introduced as a practical alternative to regular grids for discretizing complex geometries. This increases the flexibility in the mesh and enabling the technique to add, delete, move mesh points and to enhance solution accuracy [17]. Most grids are used to discretize a reservoir, as it is easy to do numerical flow computation over grids. However, designing a grid structure to depict a reservoir structure is a demanding task as it is computationally complex to show the heterogeneous behaviour of the reservoir. The unstructured grid helps to solve this complexity as the generation of this grid can be constrained depending upon the flow simulator requirements [18].

There are various algorithm and simulation techniques on unstructured grid generation. Many research papers has implemented Direct Statistical Simulation technique to generate unstructured grids and also has used Discrete Gaussian Model (DGM) for the un-conditioning simulation to eliminate the artefacts imposed by the mesh, providing a full-size model of unstructured grids. [19–24]. In an article, the researcher has proposed truncated Gaussian modelling as a solution to the problem of geostatistical simulation on unstructured grids with support change effect. Even though unstructured grids existed earlier in grid generation, they are new generation grids in the domain of hydrology and petroleum industries. Thus, there are many theoretically proved simulation technique on unstructured grids and less practically applied research on these domains.

This paper addresses the issue of support size effect by using direct simulation technique on unstructured grids, shows the surface flow simulation on these grids and compare it with the structured grids.

2 Methodology

2.1 Study area and data used

Study site for this research is Asan River Barrage, Dehradun, India (Fig. 1). Asan Barrage is situated in the confluence of eastern Yamuna canal and Asan River and having a surface area of 4 km^2 and its coordinates are 30.43° N, 77.66° E at the location Dakpathar in Dehradun. This Dam creates Asan Reservoir, which is also called as Dhalipur Lake. The spatial extent taken for the study lies between longitudes 77.55° E to 77.77° E, and latitudes from 30.34° N to 30.49° N. The overall elevation of that region varies from 335 to 935 m from mean sea level with mean elevation of 630 m.

CartoDEM Version-3 R1 having 30 m spatial resolution is used for simulation. CartoDEM having 10 m spatial resolution is used for validation (Fig. 2).

2.2 Unstructured grid generation

There are different shapes of unstructured grids like triangular, hexagon, thiessen polygon etc. and different methods like triangulation, tessellation are there to generate the unstructured grid. To generate the grids, there is need to define set of distributed points over the desired region of interest. The point data can be based on any parameter as it depends on the application of the grid. Since in this research, intended application of grid is for hydrology. So, slope is considered as a condition for grid generation. The sample points are taken in such a way that the slope of these points is greater than or equal to 3°. Voronoi polygon method is used to generate grid from point locations as shown in Fig. 3.

2.3 Change of support models

Unstructured grids constitute an important role in reservoir modelling, and geostatistical simulation on reservoir properties needs to take the change of support into consideration. Thus, in order to do simulation on unstructured grid the change of support has to be addressed. Many methods exist which address this support size effect issue. Some method includes Fine Scale Simulation followed by upscaling, Direct Block Simulation [5] and simulation using Discrete Gaussian Model [19, 20]. In this research,

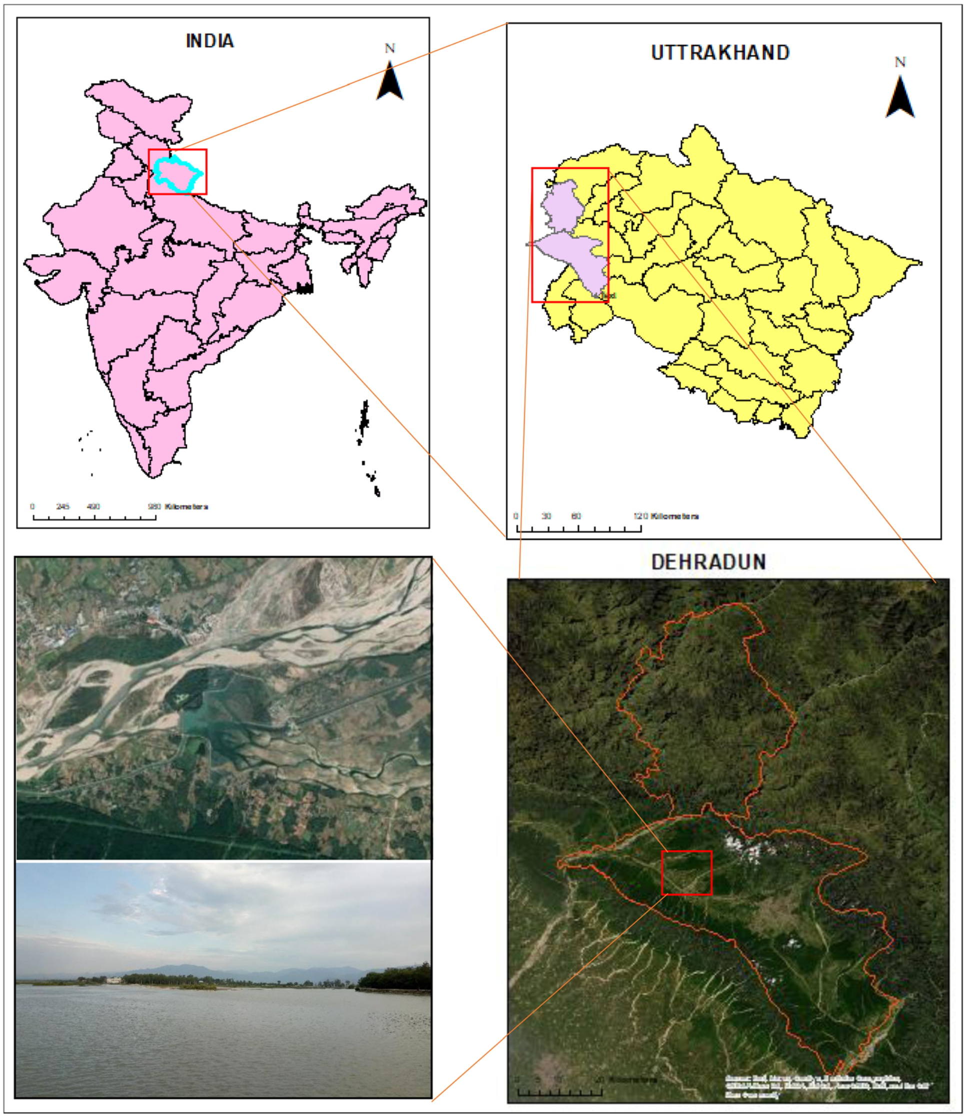

Fig. 1 The study Area located in India, Uttarakhand State, Dehradun district, Asan Barrage reservoir and its Satellite view from Google Earth

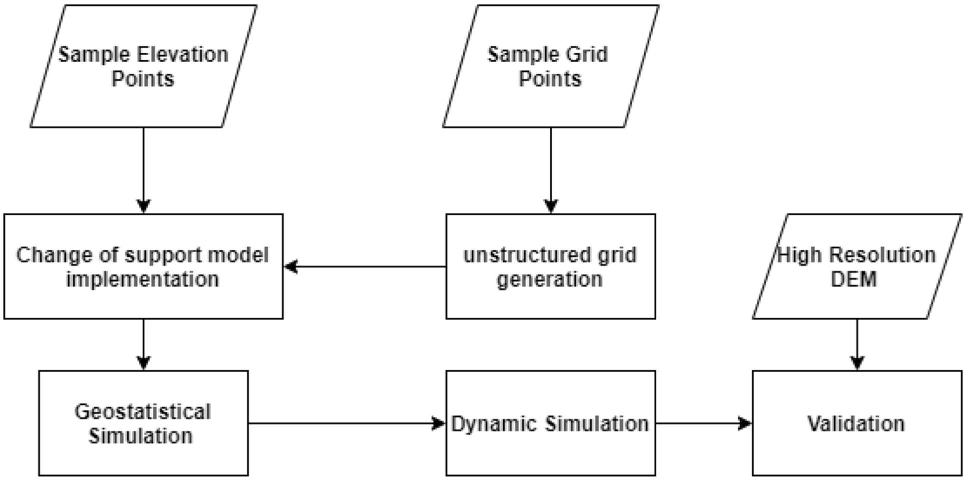

Fig. 2 Generic flow of the methodology

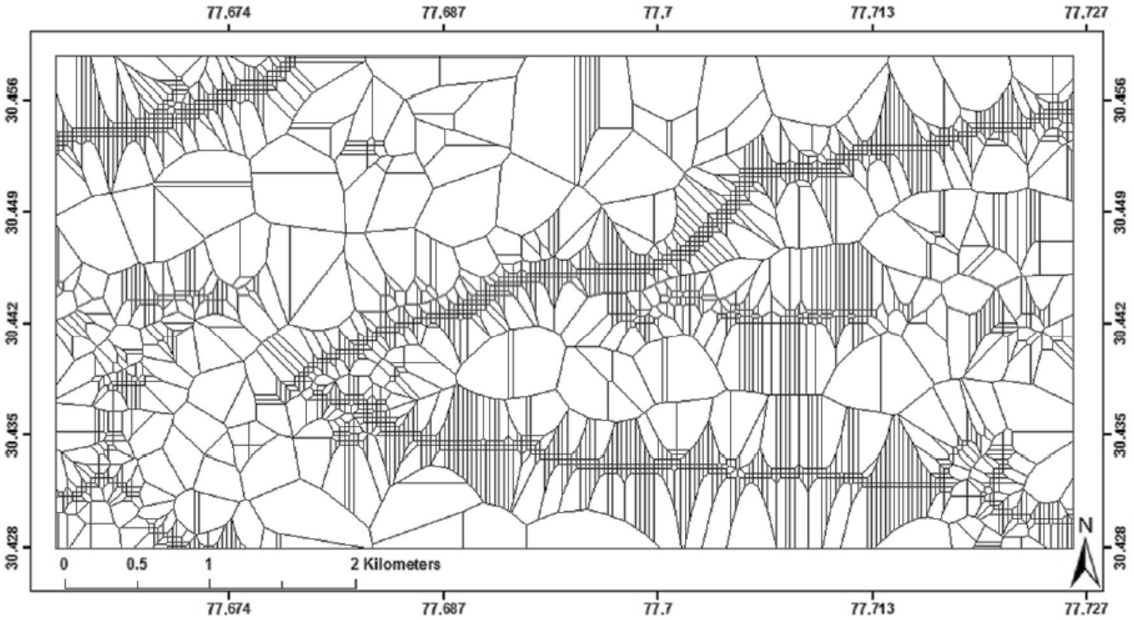

Fig. 3 Unstructured grid generated using Voronoi method

Fine Scale Simulation and Discrete Gaussian Model (DGM) are used.

2.3.1 *Fine-scale simulation approach*

This method is one of the classical approaches followed for simulation on unstructured grids. In this technique point support simulation is performed on auxiliary regular fine-scale grid and later the results are up-scaled to targeted unstructured grids. This classical approach in simulation assumes multigaussian spatial structure of the random function (Eq. 1).

$$Z(x) = \phi(Y(x)) \tag{1}$$

The assumption states that a non-linear function $Z(x)$ is a transform of a multivariate Gaussian random function $Y(x)$. In this technique using certain assumption, geostatistical simulation like Sequential Gaussian Simulation, spectral decomposition, turning band, direct sequential simulation is performed over the random function $Z(x)$. After performing the simulation on fine-scale grids, whose grid cells are considered as point support, the results are up-scaled on the target unstructured grid. The size of the fine scale grid should be equal to the smallest area of the unstructured grid generated Fig. 4.

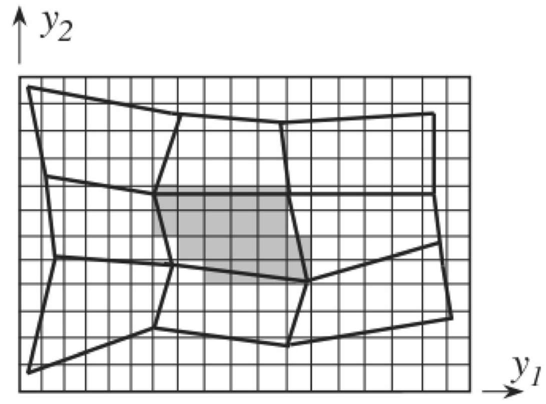

Fig. 4 Upscaling illustration, y1 and y2 representing the longitude and latitude respectively (adapted from [24])

2.3.2 *Discrete Gaussian model*

The DGM was first attempted to integrate diffusion-type of random function by evaluating the change of support for showing the variation in probability distributive function (pdf) [21]. The author compared usual models and came to conclusion that "isofactorial" model is true for first order and multigaussian case is correct for second-order approximation. Initial model of DGM was proposed in literature by Matheron where he provides second order approximation for density of average values $Z(v)$, when support v is constant throughout the domain. Later studies were about the properties of DGM model and offers a streamlined method for deriving the change of support coefficient [22]. The application of DGM to geostatistical simulation to address the issue of support size effect was introduced. As mentioned by Matheron the model developed rely on multi-Gaussian random fields thus simulating using this model requires simulating realization of multivariate Gaussian random vectors. The block variance formulae (r_p^2) and covariance matrix formula $\left(Cov\left(Y_{v_p}Y_{v_q}\right)\right)$ to calculate DGM is given in Eq. 2 and 3.

$$r_p^2 = \frac{1}{|v_p||v_p|}\iint_{v} \rho(x', x)dxdx' \tag{2}$$

$$Cov\left(Y_{v_p}Y_{v_q}\right) = \frac{1}{r_p r_q}\frac{1}{|v_p||v_q|}\iint_{v_p v_q} \rho(x', x)dxdx' \tag{3}$$

This output of DGM is the correlation coefficient and covariance matrix for the multivariate Gaussian random vector $Y(\underline{x}_1), Y(\underline{x}_2), Y(\underline{x}_3)\ldots Y(\underline{x}_n)$. This can then be simulated using classical techniques, for instance Sequential Gaussian Simulation (SGS)) to get the unstructured data for this study (Fig. 5).

3 Results and discussion

3.1 Results generated for estimation of DEM using structured and unstructured grid

From Fig. 6, it can be observed that due to fine grid size, the gridded DEM that is generated gives a continuous and smoothened output surface. The mean value of the

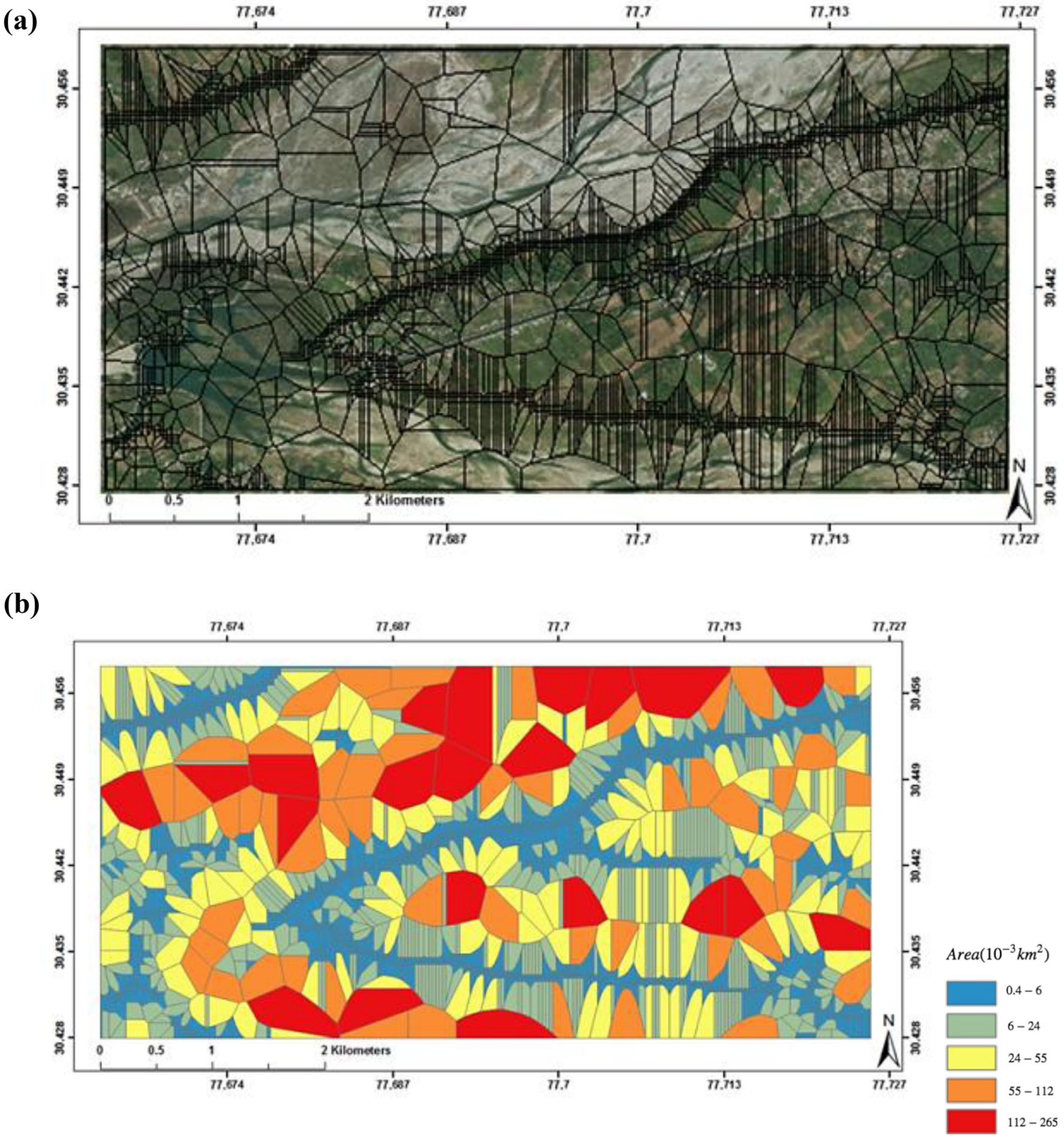

Fig. 5 shows the representation of unstructured gridded DEM generated with slope as base variable (**a**) representation showing the finer and coarser grid variation, (**b**) thematic map showing its area wise distribution of finer and coarser grids

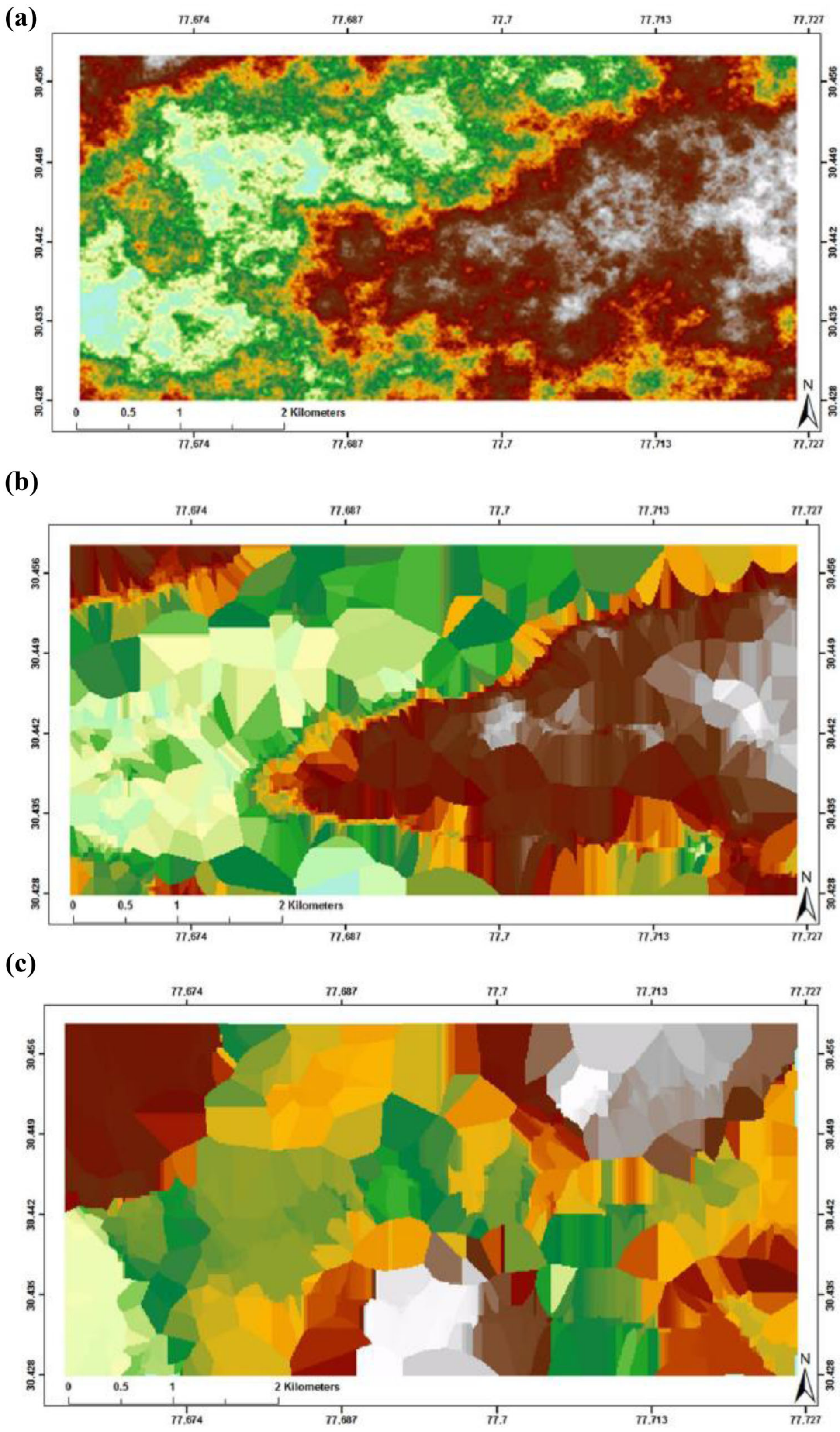

Fig. 6 The representation of DEMs generated using various grid techniques (**a**) DEM generated using kriging on structured grid, (**b**) DEM generated using upscaling method on unstructured Grid, (**c**) DEM generated using DGM method on unstructured grid

elevation, 419.64 m is in the region where river is flowing and the elevation increases as we move away from the river. In fine-scale simulation approach, the support size effect is addressed by performing regularization to the point support data. Thus, regularisation is performed by upscaling the structured simulated output to the generated unstructured grid. The resultant output of upscaling is the unstructured DEM and values of this DEM ranges from 392.50 to 451.76 m. It can be also seen from Fig. 6 that low elevated region with respect to surroundings is the region where river flows and slightly elevated areas are near the border regions which is separating the river and the non-river areas and high elevated values are in the urban land areas. In addition to that, it can also be noted that due to the structure of the grid and the upscaling procedure, the output produced shows a discrete variation in the values.

Figure 7 shows the semi-variogram of the structured and unstructured DEMs. The structured variogram is the point support variogram and the unstructured variogram is the block support variogram. It can be observed that, the variance of unstructured grid when compared to the structured grid has decreased while the range of the same has increased which means that the spatial variability of unstructured grid has decreased when compared to the structured point support grid.

Variogram being inversely proportional to correlation, the correlation increases as the size of the support increases. Figure 7b shows the value distribution based on the point support and block support generated using different approaches. It can be observed that the average mean value of the distribution is same while the variability among the values for different support varies.

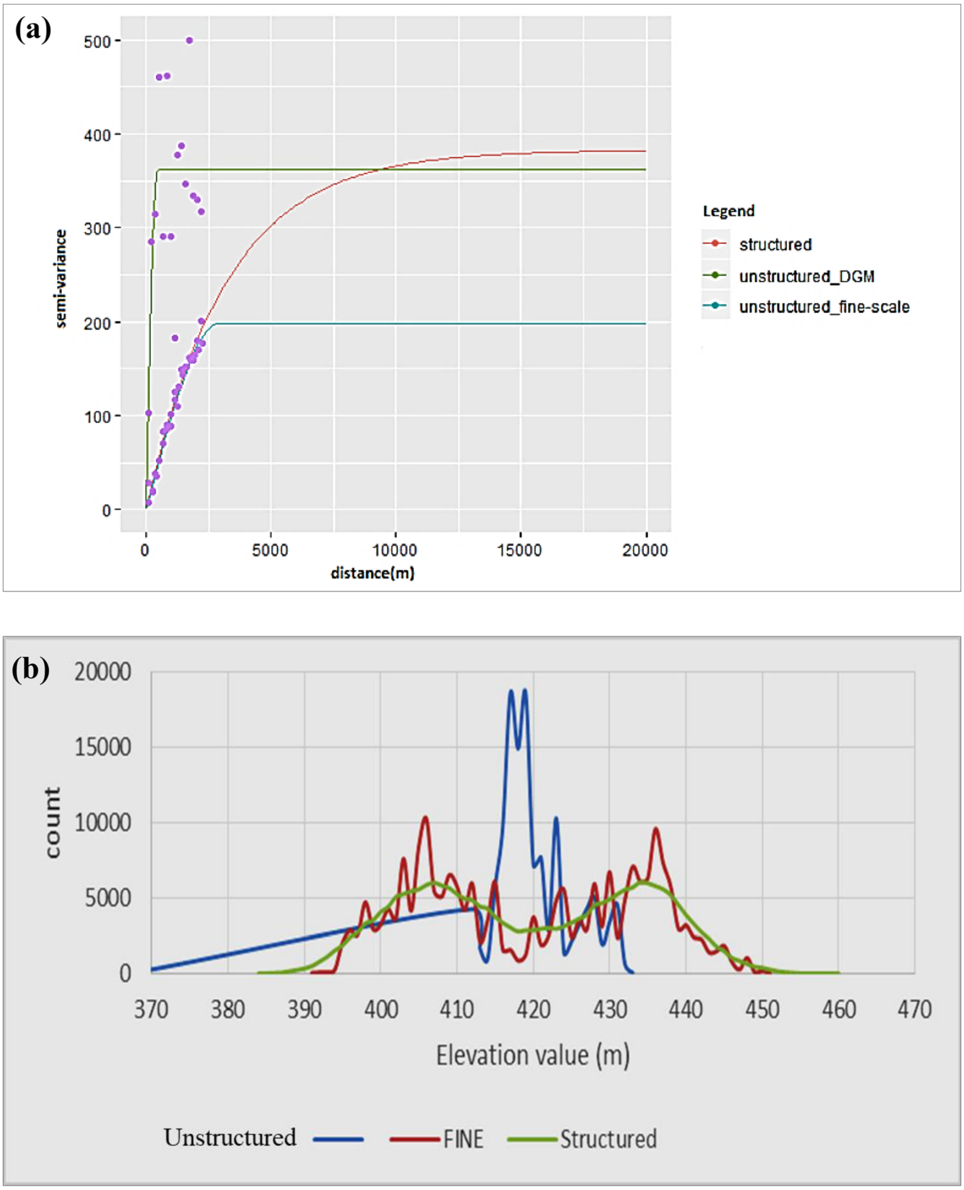

Fig. 7 represents the comparison between point support and block support (**a**) Semi-variogram showing variation between the DEM generated using structured grid and unstructured grid, (**b**) frequency distribution of point support (Structured) and Block support (Unstructured and Fine)

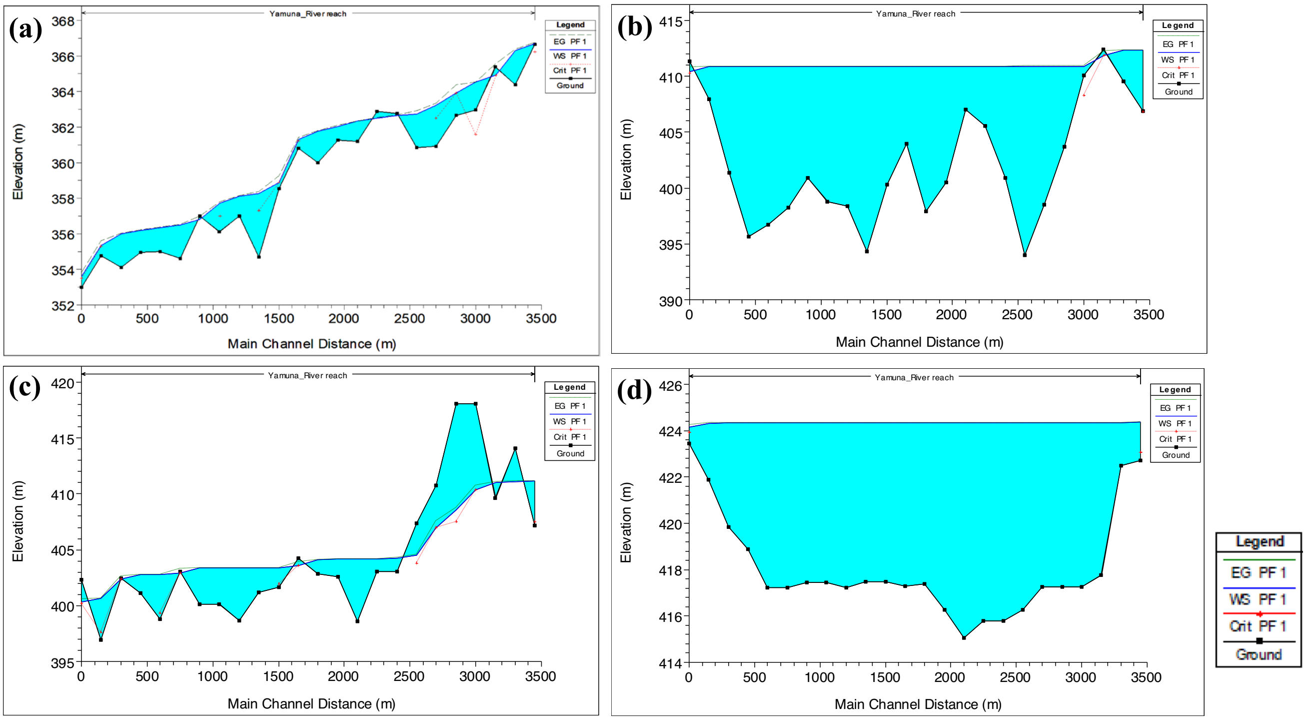

Fig. 8 Water Surface profile created for different DEMs, denoting the variation of surface flow in different generated DEMs (**a**) Reference DEM, (**b**) Structured DEM, (**c**) Fine scale unstructured DEM, (**d**) DGM unstructured DEM, (EG—Elevation, WS—Water Surface, Crit—Critical, PF—Profile)

3.2 Results generated for estimation of surface flow analysis

Figure 8 shows the water surface profile for different DEMs. The water surface profile is plotted between the elevation and the main channel distance. The profiles are generated at each cross-section by creating an energy grade line, a water surface profile, and a critical profile line. Figure 8a shows the water surface profile for the Reference DEM, which was taken for validating the other outputs. Figure 8b is the surface plot for structured gridded DEM. Figure 8c shows the Elevation vs Channel Distance plot for unstructured Grid generated using fine-scale simulation, and Fig. 8d is the surface plot for the Unstructured grid generated using DGM approach. It can be observed that Fig. 8c gives a similar result, having high elevation in upstream and low elevation in downstream, to that of reference DEM surface profile. It can also be observed that in these in reference DEM, the surface elevation is a gradually decreasing while in unstructured DEM of fine-scale simulation, the water surface elevation at from upstream till a distance of 3000 m there is a steep decrease and then the water surface decreases gradually. But in case of structured DEM, the water surface elevation almost looks flat and it can also be observed that the figure shows the depth to be deeper in the structured DEM. In the case of unstructured DEM of DGM, the difference between the water surface profile and the ground is too large and stating that the river is too deeper. It is noted that the elevation difference for the unstructured grid (fine-scale simulation approach) has almost near to same value to that of the reference DEM. This can be added to the behavior of the water surface profile from Fig. 8a and c that the Fine-scale Unstructured DEM gives similar water surface elevation to that of reference DEM. Similarly, the difference in elevation for the structured grid and the unstructured DGM is very less, stating that the water level is flat without much roughness in the flow.

3.3 Validation

Steady flow analysis is done by updating the elevation values of different elevation models in the geometric data. As per the observation done it can be noted that due to the difference in the elevation values of each DEM, the velocity of the flow in the channel also varies. From this, it can be said that performing geostatistical simulation by addressing the support size effect gives a high impact on the flow simulation due to the variation in the values. Comparative analysis was performed to find which DEM gives an output similar to the high-resolution reference DEM. Furthermore, in order to validate the results of the flow simulations, the outputs are compared with the reference DEM. It can be noted the all the 3 generated DEMs,

Table 1 Statistical comparison for surface water elevation (m)

DEM	Mean elevation (m)	RMSE (m)	Standard deviation (m)	Correlation coefficient	R^2
Structured	424.47	0.87	1.20	0.84	0.72
Unstructured DEM—Fine scale approach	417.42	0.89	4.47	0.90	0.82
Unstructured—DGM approach	436.57	0.83	1.13	0.93	0.86

Table 2 Statistical comparison for channel velocity (m/s)

DEM	RMSE (m/s)	Standard deviation (m/s)	Correlation coefficient	R^2
Structured	0.65	1.25	0.64	0.42
Unstructured DEM—Fine scale approach	0.38	0.95	0.46	0.21
Unstructured—DGM approach	0.58	0.95	0.84	0.70

give the RMSE in a similar range with not much variation in the value. The coefficient of determination also gives similar results. But among the values obtained the DEM generated using DGM approach gives high correlation with the reference DEM, compared to the structured DEM. This can be observed from Tables 1 and 2 that the DGM generated DEM gives better results compared to other generated DEM. As Digital Elevation Model is a primary parameter in flow analysis, the variation of flow is reasonable due to the variability in the value of each DEM. It is also to be noted that this flow variation is also affected if the support change effect occurs. It can be stated that the DEM structure influence on flood modelling, denoting that the vertical as well as the horizontal accuracy of the DEM influence the flood modelling.

4 Conclusion

The overall objective of this article was divided into 2 parts. One was to generate an adaptive unstructured grid for the hydrological model and to address the support size effect issue and the other one is to do surface flow simulation on the resulted unstructured grid and compare it with the structured grid. The unstructured grid which was generated using slope as base variable resulted in such a way that the grids represent the boundary of the river and the urban land in finer blocks and the other areas in coarser grid blocks. The resultant grid shows a clear distinguishing structure between river and land thus, making it an adaptive structure for hydrological model. The support size effect is addressed using two different approaches on the generated unstructured grid. It was observed that the variability of the block support is decreased compared to the point support due to regularisation of the grids. The output of the geostatistical simulation, after addressing the support size effect, is used as the elevation input data (DEM) for the flow simulation. It was observed that in surface flow simulation, the variation in support has impact on the velocity of flow in the channel as there is the difference in flow velocity through the channel when the DEM is structured and unstructured. In the flow simulation model, not only with the vertical comparison (in the elevation values) of DEMs, but also in horizontal comparison (area) of the DEM, the values gave abrupt change in the flow of the water across the channel. The resultant output of the flow simulation when validated with the reference DEM in this study showed that the DEM generated using DGM model gives more accurate results as it has the minimum RMSE value and maximum coefficient of determination when compared to other DEMs. Moreover, the variation in the flow simulation suggests that the change in grid structure of DEM, horizontal or vertical, both impact the simulated flow of water on the land. The study can be further improved by using different data like LIDAR – to see how the DEM generated using that data is impacting the optical data. Moreover, the application can be further improvised by implementing it on dynamic simulation model and understanding the support size effect when flood simulation is applied.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Zaytsev, V., Biver, P., Wackernagel, H., & Allard, D. (2016). Change-of-Support models on irregular grids for geostatistical

simulation. *Mathematical Geosciences, 48*(4), 353–369. https://doi.org/10.1007/s11004-015-9614-x

2. Dusserre, M. G., Garbolino, M. E., Jaber, M. A., Guarnieri, M. F., & Karim, M. H. (2016). *Stochastic methods for modeling uncertainties on unstructured meshes*. PSL Research University.
3. Bergamaschi, L. (2005). Geostatistics in Hydrology: Kriging Interpolation. *Course Mater*, 2005. http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Geostatistics+in+Hydrology+:+Kriging+interpolation#6.
4. Hengl, T. (2006). Finding the right pixel size. *Computers & Geosciences, 32*(9), 1283–1298. https://doi.org/10.1016/j.cageo.2005.11.008
5. Manchuk, J. (2010). *Geostatistical modeling of unstructured grids for flow simulation*. University of Alberta.
6. Braun, P., Molnar, T., & Kleeberg, H. B. (1997). The problem of scaling in grid-related hydrological process modelling. *Hydrological Processes, 11*(9), 1219–1230. https://doi.org/10.1002/(SICI)1099-1085(199707)11:9%3c1219::AID-HYP553%3e3.3.CO;2-J
7. Fields, R., et al. (1996). Geological applications of automatic grid generation tools for finite elements applied to porous flow modeling from: Numerical grid generation in computational fluid dynamics and from. *Methods, 1996*, 1–9.
8. Loseille, A. (2017). Unstructured mesh generation and adaptation. In *Handbook of numerical methods for hyperbolic problems—Applied and modern issues*. Elsevier.
9. Mavriplis, D. J. (1997). Unstructured grid techniques. *Annual Review of Fluid Mechanics, 29*(1), 473–514. https://doi.org/10.1146/annurev.fluid.29.1.473
10. Usul, N., & Pasaogullari, O. (2004). Effect of map scale and grid size for hydrological modelling. *GIS and remote sensing in hydrology, water resources and environment, 289*, 91–100.
11. Yakar, M., Yilmaz, H. M., & Yurt, K. (2010). The effect of grid resolution in defining terrain surface. *Experimental Techniques, 34*(6), 23–29. https://doi.org/10.1111/j.1747-1567.2009.00553.x
12. Zhou, Q., Pilesjö, P., & Chen, Y. (2011). Estimating surface flow paths on a digital elevation model using a triangular facet network. *Water Resources Research, 47*(7), 1–12. https://doi.org/10.1029/2010WR009961
13. Biver, P., Zaytsev, V., Allard, D., & Wackernagel, H. (2017). Geostatistics on unstructured grids, theoretical background and applications. *Geostatistics Val., 2016*, 449–458.
14. Dusserre, M. G., Garbolino, M. E., Jaber, M. A., Guarnieri, M. F., & Karim, M. H. (2016). de l ' Université de recherche Paris Sciences et Lettres.
15. Manchuk, J., Leuangthong, O., & Deutsch, C. V. (2005). Direct geostatistical simulation on unstructured grids. In O. Leuangthong & C. V. Deutsch (Eds.), *Geostatistics Banff 2004* (pp. 85–94). Dordrecht: Springer.
16. Manchuk, J. G., & Deutsch, C. V. (2009). *Algorithms for unstructured grids* (Vol. 2009, pp. 1–8). http://www.ccgalberta.com.
17. Balan, B., & Schlumberger, G. (1997, June). Structured versus Unstructured (PEBI) Grids in Reservoir Modeling and Simulation. *Soagi.Ru*, pp. 37998–37998. http://soagi.ru/Free_article/StructuredversusUnstructured.pdf.
18. Zaytsev, V., Biver, P., Wackernagel, H., & Allard, D. (2016). Geostatistical simulations on irregular reservoir models using methods of nonlinear geostatistics.
19. Delfiner, P. (1999). *Geostatistics: Modeling spatial uncertainty* (1st ed.). Wiley.
20. Delfiner, P. (2012). *Geostatistics: Modeling spatial uncertainty* (2nd ed.). Wiley.
21. Matheron, G. (1995). Change of support for diffusion-type random functions. *Journal of the International Association for Mathematical Geology, 17*(2), 137–165. https://doi.org/10.1007/BF01033151
22. Emery, X. (2007). Change of support for estimating local block grade.
23. Sahid, A., Nurrohman, W., & Hadi, M. P. (2018). An investigation of digital elevation model (DEM) structure influence on flood modelling. *IOP conference series: earth and environmental science, 148*, 012001. https://doi.org/10.1088/1755-1315/148/1/012001
24. Durlofsky, L. J. (2005). Upscaling and gridding of fine scale geological models for flow simulation. In *8th international forum on reservoir simulation Iles Borromees, Stresa, Italy* (Vol. 2024, pp. 1–59). https://doi.org/10.1002/er.3246.