On the Extinction–Free Stabilization of Predator-Prey Dynamics

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***Abstract*—Scientists have long been attracted to mechanisms surrounding the predator–prey system. The Lotka–Volterra (LV) model is the most popular formalism used to investigate the dynamics of this system. LV equations present non-linear dynamics that exhibit periodic oscillations in both prey and predator populations. In practical situations, it is useful to stabilise the system asymptotically to a desired set point (population) wherein the two species coexist by fashioning specific control actions. This control strategy can be beneficial for problems that can arise when there is a risk of extinction of one of the species and human intervention must be planned. One natural and well-established theory for describing systems obeying energy balance laws is the port-Hamiltonian modeling, an extension of classical Hamiltonian mechanics to systems endowed with control and observation. The LV model can be formally represented as a non-linear mechanical oscillator employing the canonical equations of Hamilton. This special mathematical structure aids planning and designing efficient control actions. The proposed strategy employs a systematic procedure to efficiently plan biological control actions and bypass species extinction through asymptotic stabilisation of populations.**

***Index Terms*—Nonlinear control systems, biological system modeling, stability analysis.**

I. Introduction

AN INTERESTING phenomenon regarding population dynamics between predatory and prey fish was observed by the Italian marine biologist Umberto D'Ancona. His findings, which date back to the period after World War I, were that in the Adriatic Sea the percentage of predatory fish was decreasing with respect to their prey. This peculiar phenomenon attracted the curiosity of the biologist, because during the war fishing practices were considerably reduced. Hence, a decline in predatory fish was unexpected. In order to find a rational explanation to this event, D'Ancona asked his father-in-law, the mathematician Vito Volterra, to analytically model the evolution of the two competing species. This request attracted the interest and curiosity of the mathematician, who, in 1926, proposed a dynamic model to describe the predator–prey interaction of two species characterised by cyclic oscillations in the number of individuals [1]. The common interest of the two relatives in finding a scientific explanation for this phenomenon is unsurprising. In fact, population dynamics are at the intersection of various fields, viz. mathematics, biology, social science, and medicine [2]. It is interesting to observe that the same equations developed by Volterra to model a biological system were already independently derived by Alfred Lotka, in 1910, to describe a hypothetical autocatalytic chemical reaction in which chemical concentrations oscillate [3].

Subsequent experimental results showed that the so-called Lotka–Volterra (LV) equations can accurately predict the behaviour of different biological systems, e.g., bacteria [4], and the *snowshoe hare* (prey) and *Canadian lynx* (predator) [5]. Additionally, in order to analyse how perturbations affect the system, researchers have been studying how the presence of other significant species, along with factors such as food or disease, influence the predator–prey relation [6], [7], [8]. Many researchers have focused on changing the dynamics of the LV model by means of a proper controller. In fact, the design of a controller for this system can be used to solve significant problems, e.g., reducing the risk of extinction [9], [10], [11].

The LV model is appealing when modelling several relationships: resource–consumer [12], microbial competition for food [13], plant–herbivore [14], parasite–host [15], tumour cells (virus)–immune system [16], and susceptible–infectious interactions [17].

In addition to these practical aspects, the structure of the model is peculiar from a mathematical point of view, insofar as it is analogous to a nonlinear mechanical oscillator. In particular, from the beginning of the nineties, the underlying

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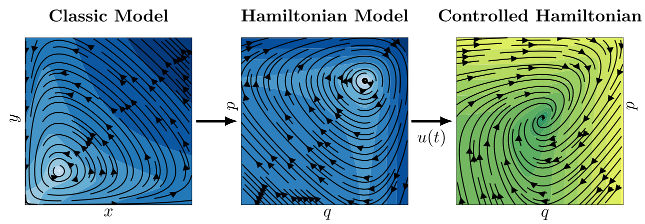


Fig. 1. Workflow of the control design proposed in this letter. First, the model is transformed in Hamiltonian form; in this reference frame extinction avoidance of species is implicitly guaranteed. Then, the stabilizing controller is derived in a *port–Hamiltonian* fashion.

geometrical structure of the LV equations has been investigated in [18], [19], [20], where it became clear that such equations admit a non–trivial Hamiltonian formulation. Furthermore, as briefly suggested in [21], by a proper change of coordinates, the model resembles the canonical equations of classical mechanics. As first minor contribution of this letter, we review and derive in a systematic way these steps, which are only briefly mentioned in the literature. As major contribution, we introduce a new approach to control processes that can be modelled with LV equations by employing a relatively new but well–established branch of control theory: *port-Hamiltonian systems* [22], [23], [24], [25]. Previous attempts to stabilize food–chain systems in the port–Hamiltonian framework are present [26], [27]. The major difference of the proposed control strategy with respect to these approaches relies on the particular Hamiltonian model that is used. We take advantage of the aforementioned change of coordinates to resemble a canonical mechanical system. This aspect turns out to be crucial in designing the algorithm in a relatively simple way, in which no PDE matching conditions [26] nor LMIs [28] need to be solved.

The workflow on the control design proposed in this letter is graphically represented by Fig. 1.

II. Lotka–Volterra Equations: Modeling

The classical formulation of the LV model is the following autonomous dynamical system:

$$\begin{cases} \dot{x} = ax - bxy \\ \dot{y} = -cy + dxy, \end{cases} \tag{1}$$

where $x(t)$, $y(t) \in \mathbb{R}$ represent the time evolution of the populations of prey and predators, respectively. The positive parameters a, b, c, and d have the following meaning: a is the natural growth rate of the prey in absence of predators, b is the effect of predation on the prey, c is the natural death rate of the predators in absence of prey, and d is the efficiency and propagation rate of the predators in the presence of prey. The assumption of the LV model is that prey have infinite food resources and the only limitation to their increment is given by predation. The system presents two equilibrium points in the prey–predator space. These points can be classified through an initial stability analysis based on linearisation. The first point is the origin $[0, 0]$ and results in a *saddle point* that has an unstable eigendirection coincident with the x-axis, and a stable eigendirection coincident with the y-axis of the x–y plane (phase space). It follows that convergence toward a state in which both predators and preys are extinct is implicitly avoided by the autonomous dynamics of the system. The second equilibrium point, located at $[a/b, c/d]$, is *elliptic*. The eigenvalues of this point are complex. Hence, the linear classification is insufficient to ensure that the non-linear system will follow periodic orbits around the equilibria. However, the system is conservative with respect to a specific quantity, and, as a result, the trajectories follow a periodic trend. In fact, the LV model has the structure of a canonical Hamiltonian system [21]. The equations of motion of Hamiltonian mechanics, known as the *canonical equations of Hamilton*, have the following form:

$$\dot{q} = \partial_p H(q, p) \quad \dot{p} = -\partial_q H(q, p),$$

in which, generally, $p, q \in \mathbb{R}^n$ and the scalar Hamiltonian function $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is constant in time. Hence,

$$\dot{H} = \dot{q}^\top \partial_q H(q, p) + \dot{p}^\top \partial_p H(q, p) = 0.$$

The transformation of the LV to the structure of a Hamiltonian system is done by changing the variables. Indeed, we can divide the two equations in (1) by x and y, respectively, and replace $[q, p]$ with $[\ln(y), \ln(x)]$. This leads to

$$\begin{cases} \dot{q} = -c + de^p = \partial_p(-cp + de^p + \gamma(q)) \\ \dot{p} = a - be^q = -\partial_q(-aq + be^q + \mu(p)), \end{cases}$$

for any scalar functions $\gamma(q)$ and $\mu(p)$. Selecting $\gamma(q) = aq - be^q$ and $\mu(p) = -cp + de^p$ yields

$$\begin{cases} \dot{q} = \partial_p(-cp + de^p - aq + be^q) = \partial_p H(q, p) \\ \dot{p} = -\partial_q(-cp + de^p - aq + be^q) = -\partial_p H(q, p), \end{cases} \tag{2}$$

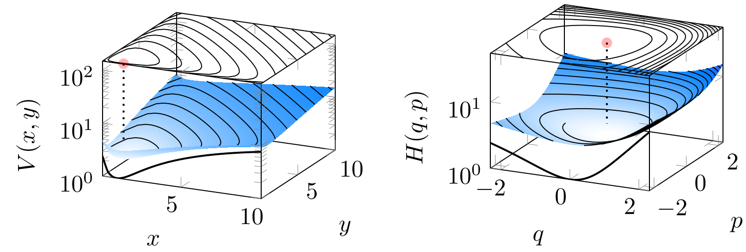
and, consequently, the Hamiltonian function results in $H(q, p) = -aq + be^q - cp + de^p$. In the case of classical mechanics, the Hamiltonian function physically represents the total energy of the system. In this case, it simply reflects the "conserved quantity" in time. Moreover, the equilibrium $[0, 0]$ has no finite correspondence in Hamiltonian coordinates. Hence, the possibility of extinction is automatically neglected by this transformation.

Remark 1: The derived Hamiltonian model for system (1) by means of the discussed change of coordinates yields a canonical interconnection structure, i.e., the system in the new variables looks exactly like the equations of motion in Hamiltonian mechanics. This constitutes a major difference with respect to the works [18], [19], [20], [26] investigating the Hamiltonian structure of the LV equations, in which the interconnection structure is represented by a *non constant* skew-symmetric operator, modulated by the state variables.

This particular structure implies the existence of the integral of motion $V(x, y)$ for the standard LV model:

$$V(x, y) = H(q, p)|_{[\ln(y), \ln(x)]} = -a\ln(y) - c\ln(x) + by + dx,$$

which demonstrates that the *motion* of the system in the x–y plane follows periodic trajectories coinciding with isolines of the integral of motion $V(x, y) = V_0$, where the value of V_0 is defined by the initial conditions at the starting time: $x(t_0) = x_0$, $y(t_0) = y_0$. Similarly, in the q–p plane the periodic motion happens on the closed curves corresponding to the level sets



(a) (b)

Fig. 2. Plots of $V(x, y)$ (a) and $H(q, p)$ (b). Coinciding with the level sets of V and H, foliations of phase-space (periodic) trajectories of the system. In both figures $a = b = c = d = 1$.

$H(q, p) = H_0$ given by the initial conditions: $H_0 = H(q_0, p_0)$. Therefore, the time evolution of the system consists of cyclic fluctuations of the two populations, for which predator populations follow variation in the prey population, and with certain dynamics that depend on the system parameters a, b, c and d. The biological interpretation of this oscillatory behaviour is that an abundance of hunters implies more killing of prey, that, in the long term, causes a consistent absence of food for predators and hence their decline. Consequently, the death of predators causes an increase in prey, and so on in cyclical alternates. The possibility of modelling the LV equations from a Hamiltonian perspective, allows us to apply to an ecological model the well-consolidated theory of *port-Hamiltonian systems*, widely exploited for the analysis and control of systems belonging to many physical domains in the control-theoretical community. It is easy to see that the equilibrium point in q–p is $[q^*, p^*] = [\ln(a/b), \ln(c/d)]$, which is also a minimum point of $H(q, p)$:

$$\partial H(q^*, p^*) = \mathbb{0}, \quad \partial^2 H(q^*, p^*) = \text{diag}\,[a, c] > 0.$$

In the same way, it can be shown that $[c/d, a/b]$ is a minimum for $V(x, y)$. Thus, for any $a, c > 0$, the Hessian of H is always positive-definite. As consequence the equilibrium of the LV model is always a minimum of the Hamiltonian function. A criticism of the LV model consists in its structural absence of stable attractors due to the presence of the conservativeness given by the Hamiltonian structure [21]. In fact, a generic perturbation, destroying the integral of motion where orbits lie, will dramatically change the behaviour of the system. $V(x, y)$ and $H(q, p)$ are shown in Fig. 2, which also illustrates the families of periodic orbits of the system in both their standard and the Hamiltonian forms corresponding to the isolines of V and H, respectively. The model's parameters have been set to $a = b = c = d = 1$.

III. Control of the Evolution

A. Definition of the Control Problem

We assume the ability to influence the behaviour of the system with some control actions, e.g., harvesting or repopulating the two species, promoting or suppressing the birth (or death) of the species. Let us assume that the control objective is to bring the ecological model to a certain set point in which the two species coexist. The controlled system is then described as

$$\begin{cases} \dot{q} = \partial_p H(p, q) + v(t) \\ \dot{p} = -\partial_q H(p, q) + w(t), \end{cases} \tag{3}$$

where the scalar functions v and w are the control inputs.

B. Passivity-Based Control and Stability of the System

Thanks to the special structure of the dynamic model of the considered system, it is suitable to use a control strategy that exploits an energy–based perspective. The strict minima of the energy function (i.e., the Hamiltonian function) correspond to a Lyapunov-stable equilibrium that can be asymptotically stabilised through a specific technique called *damping injection*. A brief introduction to passivity-based control of port-Hamiltonian systems is hereafter given [22], [23]. This methodology has been applied in several contexts which encompass classical Hamiltonian dynamics [29]. In general, a port-Hamiltonian system without dissipation is expressed as

$$\begin{cases} \dot{\xi} = J(\xi)\partial H(\xi) + g(\xi)u \\ \eta = g^\top(\xi)\partial H(\xi), \end{cases} \tag{4}$$

where ξ is the state of the system, u and η are the vectors of the inputs and the outputs, respectively. $H(\xi)$ is the Hamiltonian function, i.e., the energy storing function, $J(\xi)$ is a skew symmetric matrix representing the internal power of preserving interconnections, and $g(\xi)$ is a matrix describing the way that power coming from the external world is distributed into the system. The system defined in Eq. (4) is *lossless*, which means that the variations in time of the energy of the system are only due to the power flow through input and output ports:

$$\dot{H}(t) = \eta^\top(t)u(t).$$

In this condition, under local observability assumptions it is possible to asymptotically stabilise an equilibrium configuration corresponding to a local minimum point of the Hamiltonian function by the control law $u = -k\eta$ and $k > 0$. The power balance equation of controlled system becomes:

$$\dot{H}(t) = -k\eta^\top(t)\eta(t) \leq 0. \tag{5}$$

The damping injection adds to the lossless systems power dissipation. Unfortunately, it is common that the configuration to be stabilised does not correspond to a strict minimum of the Hamiltonian function. Therefore, in order to apply the damping injection control, it is necessary to introduce another controller, whose objective is to change the shape of the energy function of the controlled system. This will allow a strict minimum in the configuration of interest. This control strategy is called *energy shaping plus damping injection* and it consists of two steps:

- *Energy shaping:* Shape the energy of the plant by means of a proper control law able to assign a strict minimum in the desired configuration;
- *Damping injection:* Add dissipation via damping injection in order to asymptotically stabilise the desired configuration.

The rate of convergence of the state toward the minimum of $H(x)$ is determined by the amount of energy extracted from the system. In general, port-Hamiltonian systems admit lower bounded storage energy functions such that there exists $\zeta \in \mathbb{R}^+$ such that $H(\xi) > -\zeta$. Considering the port-Hamiltonian system described by Eq. (4) and the energy balance equation (5), the control problem of energy shaping for the lossless system can be formalised as follows.

Let ξ^* be a desired minimum of the energy function. Select a control $u = \beta(\xi) + v$ such that the closed-loop dynamics satisfy the new power balance equation:

$$\dot{H}^*(t) = z^\top(t)v(t), \tag{6}$$

where H^* is the desired energy function with a strict minimum in ξ^*, and z is the new output. This implies the design of a controller that modifies the energy function by changing its shape, but preserves the lossless property of the system.

C. Control Design for the Lotka–Volterra Model

Let $\xi = (q, p)$ and recall $H(p, q) = -aq + be^q - cp + de^p$. The port-Hamiltonian model of the LV equations is compactly rewritten in the form of Eq. (4) as follows:

$$\begin{cases} \dot{\xi} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \partial_\xi H + gu \\ \eta = g^\top \partial H \end{cases} \tag{7}$$

Let us first consider the energy-shaping part of the controller. The desired control action should maintain the lossless property of the system and, at the same time, place the global minimum of the Hamiltonian function, i.e., move the equilibrium point of the system in the desired set point.

The energy-shaping controlled system assumes the following form:

$$\begin{cases} \dot{q} = -c + de^p - \beta_1(p) \\ \dot{p} = a - be^q + \beta_2(q) \end{cases} \tag{8}$$

If β_1 does not depend on q, and β_2 does not depend on p, the lossless property of the system is preserved. The new *shaped* Hamiltonian function, which maintains the system in the form of Eq. (7), is the following:

$$\begin{aligned} H^*(q, p) = -cp + de^p - \int_0^p \beta_1(\varphi)d\varphi \\ - aq + be^q - \int_0^q \beta_2(\psi)d\psi. \end{aligned}$$

Therefore, the new equilibrium configurations of the system correspond to states that nullify $\partial_\xi H^*$:

$$\partial_\xi H^* = \begin{bmatrix} \partial_q H(q, p) \\ \partial_p H(q, p) \end{bmatrix} = \begin{bmatrix} -a + be^q - \beta_2(q) \\ -c + de^p - \beta_1(p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A simple way to regulate the system to a fixed point in the phase space is achieved by fixing β_1 and β_2 to a constant value, the new Hamiltonian will have a unique minimum in

$$q = \ln\left(\frac{\beta_2 + a}{b}\right), \quad p = \ln\left(\frac{\beta_1 + c}{d}\right). \tag{9}$$

Now, let $[x^*, y^*]$ be the desired set point of the system, which can be expressed in the q–p coordinate system as $[q^*, p^*] = [\ln(y^*), \ln(x^*)]$. By setting $[q^*, p^*]$, the minimum point of the shaped Hamiltonian can be straightforwardly obtained by replacing q and p with the coordinates of the desired set point in Eq. (9). This leads to the following constant energy-shaping control actions β_1 and β_2:

$$\beta_1 = dx^* - c, \quad \beta_2 = by^* - a. \tag{10}$$

Thus, the substitution of β_1 and β_2 in Eq. (8) leads to the energy-shaped controlled system:

$$\begin{cases} \dot{q} = -dx^* + de^p \\ \dot{p} = by^* - be^q \end{cases},$$

and the corresponding new shaped Hamiltonian H^* is

$$H^*(q, p) = -dx^*p + de^p - by^*q + be^q,$$

which is bounded from below and has a minimum in $[\ln(y^*), \ln(x^*)]$.

Note that the existence of the logarithms in Eq. (9) is always guaranteed by this choice of β_1, β_2. In fact, x^* and y^* must be positive by definition.

It can also be noticed that the Hamiltonian might not be zero at the minimum point, though it is certainly lower–bounded:

$$H^*(q, p) \geq H^*(\ln(y^*), \ln(x^*)) = \zeta,$$

and, therefore, without any loss of generality, it is worth redefining H^* as $H^* - \zeta$. We can select, for example, as output of the controlled system the prey's per-individual birth rate: $z = \frac{\dot{x}}{x} = \dot{p}$. It is now possible to design the damping injection controller. The control's law can be defined as:

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H^* \\ \partial_p H^* \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} v \\ z = \dot{p} = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H^* \\ \partial_p H^* \end{bmatrix}. \end{cases}$$

Consequently, the damping injection control action will be given by $v = -kz$, which yields to the closed-loop system:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -k & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H^* \\ \partial_p H^* \end{bmatrix}.$$

It is possible to conduct a stability analysis of the equilibrium point $[\ln(y^*), \ln(x^*)]$ for both the uncontrolled ($v = 0$) and controlled systems via Lyapunov's second method. Taking as Lyapunov function candidate the Hamiltonian H^*, it holds that

$$H^*(q, p) > 0 \quad \forall q \neq q^*, \forall p \neq p^*, \quad H^*(q^*, p^*) = 0.$$

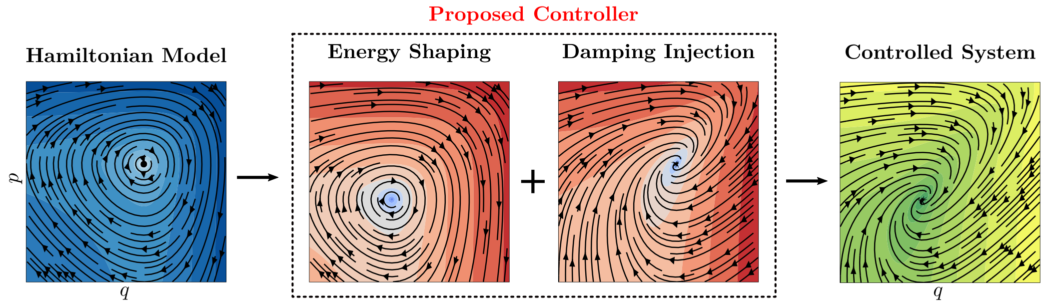


Fig. 3. Qualitative effects of the proposed controller on the Hamiltonian vector field. While the energy shaping "shifts" the equilibrium (i.e., the minimum of *H*) to the desired set point preserving the losslessness property of the system, the damping injection asymptotically stabilizes the flows, guaranteeing global convergence to the equilibrium without altering its location.

Regarding the system in which only energy shaping is applied but not damping injection, i.e., the one without a damping injection control action, the Hamiltonian is expected to be constant in time. In fact,

$$\dot{H}^* = (\partial H^*)^\top \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \partial H^* = 0.$$

Thus, the uncontrolled system is stable, but not asymptotically stable, and characterised by a periodic trend around the desired set point. Rather, the controlled system should present a monotonically decreasing Hamiltonian thanks to the dissipation effect of the damping injection controller:

$$\dot{H}^* = (\partial H^*)^\top \begin{bmatrix} -k & 1 \\ -1 & 0 \end{bmatrix} \partial H^* = -k(\partial_q H^*)^2 \leq 0, \ \forall q, p.$$

Therefore, the controlled system is Lyapunov-stable. To prove the asymptotic stability we can use LaSalle's invariance principle [30]. In fact,

$$\dot{H}^* = 0 \quad \Leftrightarrow \quad q, p \in \Lambda = \{q, p \mid q = q^* = \ln(y^*)\},$$

and the largest *invariant subset* of Λ is the point $[q^*, p^*]$, which, consequently, is asymptotically stable, as is the controlled system. The effect of the controller on the Hamiltonian LV's vector field is shown by Fig. 3.

Remark 2: The proposed approach, relying on the dynamics in Hamiltonian canonical form, leads to a simpler, yet effective, control design compared to [22], [27], [28]. This is due to the fact that, in $p-q$ coordinates, where the underlying interconnection structure is not state–dependent, the energy shaping can be carried out "by hand" without solving any matching PDE. Finally, as in canonical coordinates the *extinction state* $[x, y] = [0, 0]$ is not mapped in any finite state, the risk of accidental annihilation of the species during the control phase is automatically avoided.

IV. Simulation Experiments

The autonomous models (1) and (2) were simulated using the *Livermore* solver for non-linear ordinary differential equations (LSODE) of `solve_ivp`, from the `SciPy` library of *Python*.[1] To validate the effectiveness of the controller, simulations were performed with parameters $a = b = c = d = 1$. It is important to underline that the time scale of the simulation does not influence the analysis presented in this letter. In fact, it strongly depends on the context of the system modelled. For example, in the case of fish, the time scale might be *years*, whereas in the case of bacteria it might be *hours*. Likewise, in the case of a chemical reaction, the time scale will be *minutes* or *seconds*.

[1]The whole code to reproduce all the experiments presented in this letter is available at https://github.com/massastrello/Lotka-Volterra-Control.

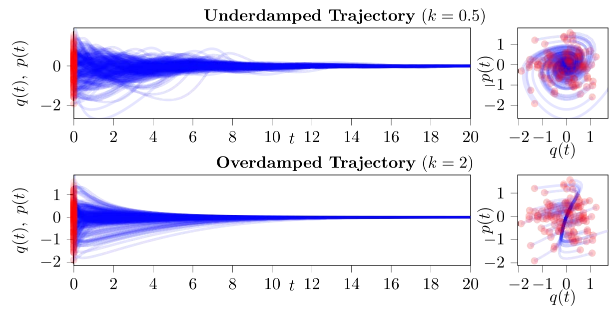


Fig. 4. Time evolution and phase-space trajectories of the controlled system. In the underdamped case, the state converges to the set point oscillating around it. In the overdamped case, no oscillations are observed.

First, a total of 100 initial conditions $[q_0, p_0]$ have been sampled from a bivariate Gaussian distribution with standard deviation 0.75. The desired set point has been chosen as the natural minimum of H ($[x^*, y^*] = [1, 1]$). For each initial condition, the system has been integrated for a total of 20*s* with $k = 0.5$ (*underdamped* case) and $k = 2$ (*overdamped* case). Results are shown in Fig. 4. It can be noticed how, with the higher dissipation rate, trajectories converges to the minimum of H without oscillations.

To extensively explore how the (arbitrary) choice of the positive scalar k affects the behavior of the system, 1000 simulation with the same initial condition $[q_0, p_0] = [1, 1]$ and random k between 0.01 and 2 have been carried out. Results are presented in Fig. 5.

V. Discussion

In this letter, the equilibrium of species population for an LV system is obtained by employing a novel port-Hamiltonian representation of the equations. Simulations were conducted to demonstrate the effectiveness of the derived controller. The results show that, if it is only possible to observe the prey's birth or death rate and only two control actions can be performed (i.e., on the tracked variable and on the predators, for example by harvesting them or encouraging their

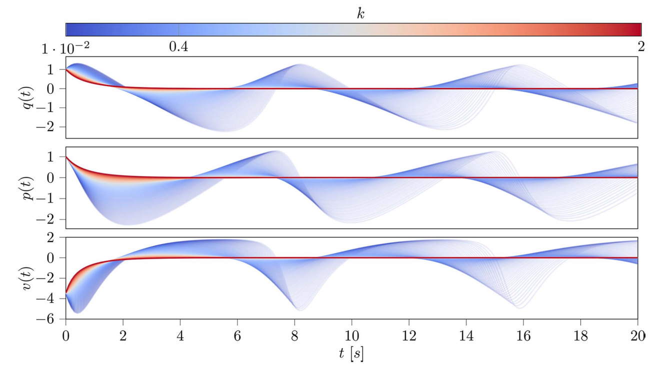


Fig. 5. State trajectories and damping injection control effort for different values of k.

reproduction), then it is indeed possible to control the system and reach and maintain a desired stable equilibrium for the populations of the two species. One of the most interesting aspects of this formulation of the problem is that, since the control is designed for the Hamiltonian form of the LV model, we do not need to know the number of prey individuals to stabilise the output. Rather, only their per-individual birth rates must be estimated, and this is simple to obtain in practical cases, e.g., with bacteria. Additionally, the proposed port-Hamiltonian control scheme ensures that there will never be the extinction of one or both species, i.e., that is not possible to bring q and p to infinity.

The proposed scheme could be beneficial to many applications, as any possible modification made to an ecosystem based on the scheme would never lead to irreversible states. On the other hand, this might be considered a limitation in other applications, e.g., killing a colony of a specific bacteria species. (It has been experimentally verified that certain species pairs of bacteria, placed in the same environment, match the LV dynamics [4].) Furthermore, it is worth underlining that the energy shaping control efforts, β_1 and β_2, are constant in time. The aim of the energy shaping controller is to bring the equilibrium point of the system to the desired set point while preserving the energy balance. In order to place the minimum of the Hamiltonian function in the desired set point perfectly, exact knowledge is needed of the model's parameters a, b, c, and d. However, in practice, only experimental estimates of these quantities are available. The designed controller relies on combined passivity properties of the LV model and the port-Hamiltonian controller, inheriting robustness properties of the system. This means that the control task is achieved also for small perturbations of the system parameters. A quantitative analysis of the admissible perturbations as well as the implementation of a set up to perform experiments for the described system is part of the future activities involving this letter.

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