Coordinating feeder and collector public transit lines for efficient MaaS services

Dr Konstantinos Gkiotsalitis
Assistant Professor
University of Twente

Center for Transport Studies (CTS)
Department of Civil Engineering
P.O. Box 217
7500 AE Enschede
The Netherlands
Email: k.gkiotsalitis@utwente.nl

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ABSTRACT
Coordinating the schedules of feeder and collector public transit lines can reduce the door-to-door travel times of passengers. With advances in smart mobility, mobility-as-a-service (MaaS) schemes allow passengers to book a combined ticket for all their trip legs. This detailed information about the origins and destinations of door-to-door trips offers the opportunity to coordinate the schedules of public transport lines to reduce the door-to-door passenger travel times. In this study, we model the coordination problem of feeder and collector lines by explicitly considering the regularity of the feeder lines and the transfer times of passengers. The coordination problem is modeled as a nonlinear non-convex problem and reformulated to an easy-to-solve convex optimization problem. We test the performance of our approach in a case study with feeder and collector lines in Singapore showing an improvement potential of 5-10% in door-to-door passenger travel times.

INTRODUCTION

The emergence of disruptive ride-hailing services provided by transportation network companies (TNCs) offers an alternative to travelers and traditional public transport captives (see Shaheen and Cohen (*1*)). This adds pressure to public transit to improve its operations and offer services with competitive door-to-door travel times Cohen (*2*). By improving their services, public transport authorities and operators can reverse the modal shift from public transport to car-based services, resulting in major benefits on pollution, congestion, and travel safety Rode et al. (*3*). This is imperative because even in countries with excessive use of public transit and active modes the vast majority of kilometers traveled are by car (e.g., in the Netherlands 8,000 out of the 11,000 kilometers traveled annually by the average person are in a car Statistics Netherlands (*4*)).

One of the main reasons for selecting car-based, door-to-door mobility services is their reduced travel times compared to public transit. Practical evidence from a meticulous investigation of the International Association of Public Transport (UITP) in sixty metropolitan areas suggests that the public transport ridership increases in line with the reduction of door-to-door travel times UITP (*5*). This requires the coordination of first/last-mile feeder lines with collector lines to reduce the travel times of door-to-door journeys conducted by public transit. This coordination can be supported by the emergence of Mobility-as-a-Service (MaaS) schemes that offer an opportunity to collect highly granular passenger demand data regarding the door-to-door trip(s) of passengers Kamargianni and Matyas (*6*). In past literature, MaaS services are designed based on the isolated economic interests of the service providers (e.g., first/last-mile operators, public transport, paratransit) that do not consider the overall door-to-door passenger travel times Kliewer et al., Shaheen and Chan, Jittrapirom et al., Geurs et al. (*7, 8, 9, 10*). To bridge this gap, this study focuses on adjusting the schedules of feeder lines to the schedules of collector lines to reduce the total door-to-door travel times of passengers that use a combination of public transport modes to complete their journeys.

Several works have examined the role of mobility-on-demand and the coupling of mass transit and last-mile services (see Basu et al. (*11*)). Car-based services, such as Uber, Lyft, BlaBlaCar, and bus-based options that include smart bookable last-mile options are examples of services that can be integrated with mass transit Hensher (*12*). Gambella et al. (*13*) incorporated Internet of Things (IoT) components to prove the necessity of real-time data for the efficient and seamless integration of public transport services inside on-demand mobility systems. Further, Pandey et al. (*14*) underscored the need for MaaS platforms to improve integration among competing first/last-mile services.

Despite the positive effect that can emerge from the integration of public transport services, traditional public transport planning typically seeks to optimize the in-vehicle passenger travel times and the waiting times at stops instead of the overall door-to-door travel times of passengers Ghoseiri et al., Yin et al., Gkiotsalitis and Maslekar, Gkiotsalitis and Cats (*15, 16, 17, 18*). However, this myopic scheduling approach disregards the negative effects of long transfer times between feeder/collector lines in the case of poor synchronization. The schedules of feeder lines are typically planned in isolation with objectives related to improving productivity and reducing operating costs without considering the door-to-door travel times of passengers that will need to transfer to a collector line Kliewer et al., Wang (*7, 19*). Thus, there is a lack of methodologies for the seamless integration of collector and feeder public transit lines for improving the door-to-door passenger travel times. Most works model the scheduling of public transport lines as an integer or mixed-integer mathematical program where the decision variables are the dispatching times of

the daily trips of the lines Fügenschuh, Chu, Gkiotsalitis and Van Berkum (*20, 21, 22*). Because of their discrete nature, such programs cannot be easily solved and many scheduling approaches try to produce robust schedules that can perform well in case of travel time and passenger demand disruptions to avoid numerous rescheduling(s) Tang et al., Gkiotsalitis and Alesiani (*23, 24*).

Using a baseline schedule developed at the tactical planning level, several works resort to dynamic rescheduling for adjusting to the travel time and passenger demand variations (see Gkiotsalitis and Van Berkum (*25*) for a detailed survey). Bly (*26*) used rescheduling on depleted bus services to provide equal headways for the buses in the schedule. Gkiotsalitis and Stathopoulos (*27*) proposed a rescheduling strategy that changes the dispatching times of bus trips to match the passenger demand of individuals who want to take part in joint leisure activities. Li et al. (*28*) modeled and solved the single depot bus rescheduling problem in pseudo-polynomial time using a parallel auction algorithm. In a follow-up work, Li et al. (*29*) showed that the bus rescheduling problem is NP-hard, and used a Lagrangian relaxation-based insertion heuristic for its solution.

Rescheduling has also been used for synchronizing services to reduce the transfer waiting times of passengers Gkiotsalitis and Cats, Gkiotsalitis and Cats (*30, 31*). Cevallos and Zhao (*32*) and Cevallos and Zhao (*33*) proposed simple perturbations that merely shift the pre-existing timetables to solve the aforementioned problem with a genetic algorithm. In addition, Coffey et al. (*34*) treated the synchronization problem as a demand-supply matching problem. In their approach, they rescheduled the timetables of public transport modes by matching the passenger demand expressed via journey planners with the public transport supply to reduce missed connections. Ceder et al. (*35*) and Gkiotsalitis et al. (*36*) developed deterministic and robust optimization models for reducing the transfer times among public transit lines. Notwithstanding, the aforementioned works do not consider the overall door-to-door travel times of passengers and merely focus on the reduction of transfer times.

To fill this research gap, this study models both the transfer times of passengers using multiple public transport modes and the waiting times of single-mode passengers to produce schedules that reduce the overall door-to-door travel times. In this pursuit, this study aims to provide an answer to the following research questions:

- how can we model the scheduling of feeder public transit lines to account for passengers' door-to-door travel times?

- can we derive a model to synchronize the schedules of feeder lines to the schedules of the collector lines? If yes, which is its computational complexity and can it be solved within a reasonable time?

- which is the improvement potential in terms of door-to-door passenger travel times and waiting times at stops when synchronizing the feeder line schedule to the schedule of a collector line?

By answering the aforementioned research questions, the main contributions of this study are the development of a novel, easy-to-solve model for coordinating the feeder and collector lines, and the investigation of the improvement potential in a case study in Singapore.

PROBLEM FORMULATION
To reduce the door-to-door travel times of passengers, we adjust the feeder line schedules to the schedules of the collector lines. The schedule of a collector line is typically considered being

fixed because it is difficult to adjust it Liang et al. (*37*). For instance, it is not trivial to adjust the schedules of train or metro lines, given the interconnections with other lines that use the same tracks Şahin (*38*). On the contrary, more flexible feeder lines (e.g., bus or minibus lines) can adjust their schedules to reduce the door-to-door travel times of passengers that use both feeder and collector lines as part of their trip legs. That is to say, the schedule adjustments of feeder modes is applied to find solutions that improve the service performance. In the schedule's adjustment, we allow the modification of the planned dispatching times of trips (e.g., trips can be dispatched earlier or later than originally planned).

The specific problem considered in this study is the problem of adjusting the baseline schedule of a feeder line to improve its synchronization with a collector line and reduce the door-to-door travel times of passengers. In this context, we define a feeder line as a public transit line that transfers a fraction of its passengers to a specific collector line.

This work makes a number of reasonable assumptions, such as:

(i) when rescheduling the feeder line, its baseline schedule can be modified by changing the dispatching times of its trips. Additional measures, such as introducing new trips, are not considered because this will affect the already planned vehicle and crew schedules resulting in requests for more vehicles and drivers that might not be available;

(ii) we do not allow bus trips from the feeder line to overtake one another by slowing down the following vehicle when an overtaking is about to occur (an assumption used in several seminal works, such as Xuan et al., Chen et al. (*39, 40*)).

(iii) the number of vehicles assigned to each line during the vehicle scheduling phase suffices to satisfy the expected passenger demand. This fleet size is determined with the use of optimal frequency setting methods at the tactical planning stage that precedes the rescheduling part of our work (see Hadas and Shnaiderman, Ferguson et al. (*41, 42*)).

Before proceeding to the description of the feeder/collector line synchronization problem, we introduce the following notation.

NOMENCLATURE

Sets

$N_f = \langle 1, 2, ..., n, ... \rangle$	is the set of all the planned daily trips of the feeder line
$N_m = \langle 1, 2, ..., m, ... \rangle$	is the set of all the planned daily trips of the main line
$S_f = \langle 1, 2, ..., s, ... \rangle$	is the ordered set of public transport stops of the feeder line
$S_m = \langle 1, 2, ..., s, ... \rangle$	is the ordered set of public transport stops of the main line
$B = S_f \cap S_m$	vector denoting all transfer stops between the feeder and the main line

Parameters

h^*	the ideal headway of the feeder line that should be maintained at all bus stops for attaining a regular service
$t_{n,s}$	the expected inter-station travel time of the feeder line trip $n \in N_f$ between stops $s-1$ and s, including the dwell time at stop $s-1$
δ_{min}	the pre-determined dispatching time of the first trip of the feeder line that prevents starting the daily operations before the bus drivers are available
δ_{max}	the pre-determined dispatching time of the last trip of the feeder line that prevents schedule sliding
ψ	the required layover time for the feeder line for performing two successive trips with the same vehicle
$\gamma_{n,s}$	the arrival time of trip $n \in N_m$ at stop s which cannot be rescheduled because it belongs to the main line
Y_{bnm}	0-1 parameter, where $Y_{bnm} = 1$ if trip $n \in N_f$ needs to synchronize its arrival times with trip $m \in N_m$ at stop $b \in B$, and 0 otherwise
$\Phi_{n,n'}$	0-1 parameter, where $\Phi_{n,n'} = 1$ if trips $n, n' \in N_f$ of the feeder line are operated by the same vehicle in a sequential order (e.g., n' after n), and 0 otherwise
M	a very large positive number

Variables

x_n	the (re)scheduled dispatching time of the nth trip of the feeder line
$a_{n,s}$	arrival time of trip $n \in N_f$ at stop s
$h_{n,s}$	inter-arrival headway between successive feeder line trips $n-1$ and n at stop s

The decision variables of our problem are the rescheduled dispatching times of the feeder line trips, x_n. Changing the dispatching times of trips modifies their expected arrival times at stops as follows:

$$a_{n,s} := x_n + \sum_{z=2}^{s} t_{n,z} \quad (\forall n \in N_f, \ \forall s \in S_f \setminus \{1\}) \tag{1}$$

where $t_{n,s}$ is the expected travel time of trip n from stop $z-1$ to stop z, including the dwell time

for boardings/alightings. We note that this travel time is stochastic in nature and in our simulation-based evaluation in section 3.2 we investigate the performance of rescheduled operations under travel time and dwell time variations. Because overtaking is not allowed, the arrival time of each feeder line trip $n \in N_f \setminus \{1\}$ at stop s should be greater than the arrival time of its previously dispatched trip $a_{n-1,s}$. That is to say, the dispatching order of trips is maintained upon their arrival at all stops and Eq.(1) is modified to:

$$a_{n,s} := \max \left(a_{n-1,s} \; ; \; x_n + \sum_{z=2}^{s} t_{n,z} \right) \qquad (\forall n \in N_f \setminus \{1\}, \; \forall s \in S_f \setminus \{1\}) \qquad (2)$$

which ensures that the dispatching order of feeder line trips is maintained. We further introduce a boundary condition for the first trip of the day which does not have a preceding trip,

$$a_{1,s} := x_1 + \sum_{z=2}^{s} t_{1,z} \qquad (\forall s \in S_f \setminus \{1\}) \qquad (3)$$

This results in the varying arrival times at stops depicted in Fig.1. Note that in Fig.1 we only show the varying arrival times of the trip of the feeder line, $a_{n,s}$, because we cannot modify the arrival times of the trip of the main collector line. Those arrival times vary subject to the rescheduled dispatching time, x_n, and the stochastic travel and dwell times $t_{n,s}$.
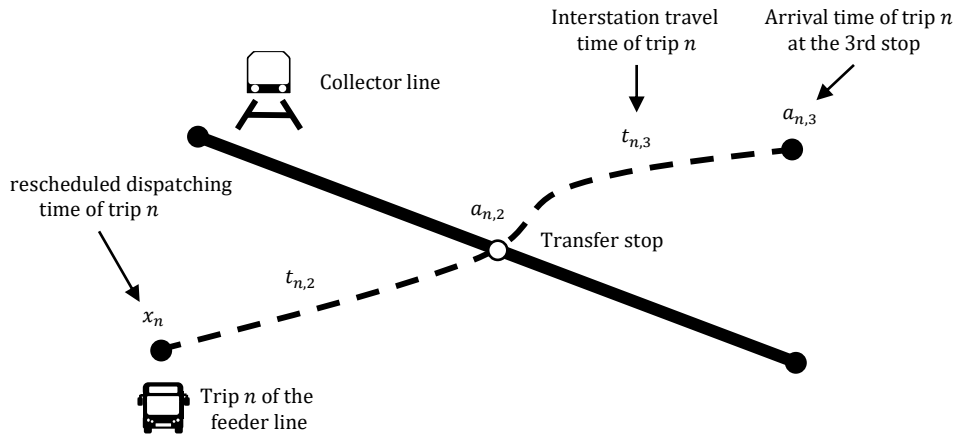

FIGURE 1 : Illustration of the varying arrival times of trip n that belongs to the feeder line based on the dispatching time decision x_n.

One of the main modeling contributions of our work is that we consider both the travel times of passengers that transfer from the feeder line to the collector line and the travel times of passengers that use only the feeder line. For this, we aim to synchronize the feeder line schedule with the schedule of the collector line, and, at the same time, maintain the regularity of the feeder line for the passengers that do not need to transfer.

To increase the regularity of a public transport service, the actual inter-arrival time headways[1] at bus stops should be as close as possible to their ideal values, h^*. Although the inter-arrival

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[1] the time headway between two consecutive trips $n, n+1$ at stop s is the headway between the front bumpers of the respective buses at the time of their arrival at stop s

headways of the *collector* lines do not depend on our decisions, the inter-arrival headways of the *feeder* lines are linked to the rescheduling of their trip dispatching times as follows:

$$h_{n,s} := a_{n,s} - a_{n-1,s} \qquad (\forall n \in N_f \setminus \{1\}, \ \forall s \in S_f \setminus \{1\}) \tag{4}$$

Rescheduling objectives
The aim of rescheduling the feeder line is to reduce the door-to-door travel times of public transit passengers that might use only the feeder line or a combination of the feeder and a collector line. Namely, passengers may use only one line or the combination of a feeder and a collector line to complete their origin-destination trip. Note that we do not consider origin-destination trips where passengers need to perform more than one transfer to arrive to their final destination because this is highly uncommon (see Kujala et al. (*43*)). To account for passengers' door-to-door travel times when scheduling the feeder line, we aim to minimize:

- their excessive waiting times when waiting at the feeder line stops;

- and their transferring waiting times from the feeder to the collector line.

That is, the aforementioned waiting times are used as proxies for the estimation of door-to-door passenger travel times since the in-vehicle travel times cannot be changed significantly with rescheduling (e.g., they depend on external factors such as congestion at peak hours, roadworks and accidents).

The excessive passenger waiting times are the main key performance indicator in high-frequency feeder line services in London, Singapore, and many other densely populated areas and are used to monitor the regularity of services Randall et al. (*44*). Note that in high-frequency services, the waiting time of a passenger of trip $n \in N_f$ at stop s is half the inter-arrival headway between trip n and trip $n-1$, $\frac{h_{n,s}}{2}$, because the passenger arrivals at stops are random Randall et al., Welding (*44, 45*). The reason behind this is that the high-frequency headways are so small that passengers do not plan their arrival times at stops based on the arrival times of buses/trains Muñoz et al. (*46*).

To reduce the excessive waiting times of the feeder line passengers, one should minimize the sum of the squared difference between the actual and the ideal headways:

$$f(h) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\frac{h_{n,s}}{2} - \frac{h^*}{2} \right)^2 \tag{5}$$

where $f(h)$ is the daily excessive waiting time of the feeder line passengers that indicates the service regularity and forms the *objective function* of our mathematical program that is formally expressed in Eqs.(8)-(14). Adopting this objective function, we assume that the last mile trips of the feeder line are within the catchment area of the collector line and the utility of the feeder line depends on its service reliability.

Now, let us consider the waiting times of passengers when transferring from a feeder to a collector line. Reckon that B is the set with all transfer stops between the feeder and the collector line, where $Y_{bnm} = 1$ if trip $n \in N_f$ needs to synchronize its arrival time with trip $m \in N_m$ at the transfer stop $b \in B$, and $Y_{bnm} = 0$ otherwise. Ceder et al. (*35*) considers a perfect synchronization when trip $n \in N_f$ arrives at the transfer stop $b \in B$ exactly at the same time as trip $m \in N_m$. In this way, the transfer times of passengers are minimized when $a_{n,b} - \gamma_{m,b} = 0$, where $\gamma_{m,b}$ is the

arrival time of the trip of the collector line at stop s. Later studies by Eranki (*47*) and Ibarra-Rojas and Rios-Solis (*48*) proposed a more flexible scheme where the bus trip n is still considered synchronized if it arrives within a time range of $[0, \Delta t]$ seconds before the arrival of trip m at the transfer stop b. In our study, we follow the flexible synchronization scheme of Eranki (*47*) and Ibarra-Rojas and Rios-Solis (*48*) by modeling the required synchronizations at transfer stops as problem constraints:

$$0 \leq Y_{bnm} \left(\gamma_{m,b} - a_{n,b} \right) \leq \Delta t \quad (\forall n \in N_f \setminus \{1\}, \ \forall m \in N_m \setminus \{1\}, \ \forall b \in B) \quad (6)$$

Obviously, when $Y_{bnm} = 0$ the inequalities in Eq.(6) hold for any value of the arrival times $a_{n,b}$ and $\gamma_{m,b}$ because we are not required to synchronize the arrival times of those two trips.

Layover constraints
Typically, if one vehicle operates two successive trips of the same line, there should be a layover time by the time the first trip is finished until the next trip is dispatched (e.g., because of union contractual agreements drivers may need to take a short break before starting their next trip – see Berrebi et al. (*49*)). When rescheduling the dispatching times of our feeder line trips, one should factor in this layover time. This yields the inequality constraints:

$$\Phi_{n,n'} \left(x_{n'} - a_{n,|S_f|} \right) \geq \Phi_{n,n'} \psi \quad (\forall n, n' \in N_f) \quad (7)$$

That is, if trip n' is the next trip of trip n that is operated by the same vehicle (e.g., $\Phi_{n,n'} = 1$), then the dispatching time of trip n', $x_{n'}$, should be greater than the arrival time of trip n at the last stop, $a_{n,|S_f|}$, plus the required layover time ψ. It is worth noting that if trips n, n' are not operated by the same vehicle in a successive order, inequality (7) is satisfied because $\Phi_{n,n'} = 0$.

Model formulation
The following mathematical program summarizes the proposed feeder line synchronization problem that explicitly considers (i) the excessive waiting times of the feeder line, and (ii) the transferring waiting times.

$$
\begin{aligned}
(Q): \quad & \min_{h} \quad && f(h) && (8) \\
& \text{s.t.:} \quad && (x,a,h) \in \mathscr{F}(x,a,h) = \left\{ (x,a,h) \mid (x,a,h) \text{ satisfy Eqs.(2)-(7)} \right\} && (9) \\
& && x_1 \geq \delta_{min} && (10) \\
& && x_{|N_f|} \leq \delta_{max} && (11) \\
& && x_n \in \mathbb{R}_{\geq 0} && (12) \\
& && a_{n,s} \in \mathbb{R}_{\geq 0} && (13) \\
& && h_{n,s} \in \mathbb{R}_{\geq 0} && (14)
\end{aligned}
$$

Inequality constraints (10) and (11) ensure that (i) we dispatch the first feeder trip of the day when drivers and vehicles are available, and (ii) we dispatch the last trip of the day before the latest possible dispatching time, δ_{max}, to avoid schedule sliding. Program (Q) is a nonlinear (quadratic) mathematical program which is not convex because the constraint of Eq.(2) includes the non-convex term $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2}^{s} t_{n,z})$. Hence, it cannot be solved to global optimality with

numerical optimization methods and one should resort to heuristics. To rectify this, we propose a reformulation. Since the non-smooth term $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2}^{s} t_{n,z})$ results in a non-convex mathematical program:

- we replace the equality constraint $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2}^{s} t_{n,z})$ with the inequality constraints $a_{n,s} \geq a_{n-1,s}$ and $a_{n,s} \geq x_n + \sum_{z=2}^{s} t_{n,z}$

- we introduce the penalty term $M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2$ to the objective function which will force the value of $a_{n,s}$ to be equal to $\max(a_{n-1,s}; x_n + \sum_{z=2}^{s} t_{n,z})$ at the solution of the mathematical program. This is because $a_{n,s}$ is forced to be equal to $x_n + \sum_{z=2}^{s} t_{n,z}$ if $x_n + \sum_{z=2}^{s} t_{n,z} \geq a_{n-1,s}$ to avoid a major cost increase in our objective function because of the large value of M (for more details, refer to the Big M theory in Griva et al. (50)).

With this transformation, program (Q) is reformulated to the following program $(\tilde{Q})$ that has an equivalent solution:

$$
\begin{aligned}
(\tilde{Q}): \quad & \min_{h,a,x} \quad f(h) + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2 \\
& \text{s.t.:} \quad (x,a,h) \in \mathscr{F}(x,a,h) = \left\{ (x,a,h) \mid (x,a,h) \text{ satisfy Eqs.(3)-(7)} \right\} \\
& \qquad a_{n,s} \geq a_{n-1,s}, \ \forall n \in N_f \setminus \{1\}, \ \forall s \in S_f \setminus \{1\} \\
& \qquad a_{n,s} \geq x_n + \sum_{z=2}^{s} t_{n,z}, \ \forall n \in N_f \setminus \{1\}, \ \forall s \in S_f \setminus \{1\} \\
& \qquad x_1 \geq \delta_{min} \\
& \qquad x_{|N_f|} \leq \delta_{max} \\
& \qquad x_n \in \mathbb{R}_{\geq 0}, \ a_{n,s} \in \mathbb{R}_{\geq 0}, \ h_{n,s} \in \mathbb{R}_{\geq 0}.
\end{aligned}
\tag{15}
$$

Note that the equality constraint (2) in program (Q) is replaced by the two new inequality constraints and the objective function penalty $\sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2$.

Proceeding further, we prove that our reformulated program $(\tilde{Q})$ can be solved to global optimality.

Theorem 2.1. *A local minimizer of $(\tilde{Q})$ is also a global minimizer.*

Proof. A local minimizer of $(\tilde{Q})$ is also a global minimizer if the objective function is convex and the feasible region is a convex set. The feasible region in $(\tilde{Q})$ is defined by linear inequalities

and is a polyhedron (thus, it is also a convex set). Further, we prove that the objective function in $\tilde{Q}$ is convex with respect to h, a, x. Let $\tilde{f}(h,a,x) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\frac{h_{n,s}}{2} - \frac{h^*}{2} \right)^2 + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M \left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z}) \right)^2$. To prove the convexity of our objective function, $\tilde{f}(h,a,x)$, we introduce functions $g_{n,s}(h) := \left(\frac{h_{n,s}}{2} - \frac{h^*}{2} \right)^2$ and $k_{n,s}(a,x) := M(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z}))^2$. Then, $\tilde{f}(h,a,x)$ can be written as:

$$\tilde{f}(h,a,x) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} g_{n,s}(h) + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} k_{n,s}(a,x)$$

Function $g_{n,s}(h)$ is convex with respect to h for any $n \in N_f \setminus \{1\}$, $s \in S_f \setminus \{1\}$ because

$$\frac{\partial^2 g_{n,s}}{\partial h^2} = \frac{1}{2} > 0 \quad \forall n \in N_f \setminus \{1\}, \ \forall s \in S_f \setminus \{1\}$$

Similarly, function $k_{n,s}(a,x)$ is convex with respect to (a,x) for any $n \in N_f \setminus \{1\}$, $s \in S_f \setminus \{1\}$ because its Hessian matrix is positive semi-definite. Namely, the Hessian of $k_{n,s}(a,x)$ is:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 k}{\partial a^2} & \frac{\partial^2 k}{\partial a \partial x} \\ \frac{\partial^2 k}{\partial x \partial a} & \frac{\partial^2 k}{\partial x^2} \end{bmatrix} = \begin{bmatrix} 2M & -2M \\ -2M & 2M \end{bmatrix}$$

which is positive semi-definite since the leading principal minor $\det(\mathbf{H}) = 4M - 4M = 0$ is non-negative. Therefore, $\tilde{f}(h,a,x)$ is convex as the sum of the convex functions $g_{n,s}(h)$, $k_{n,s}(a,x)$, and this completes our proof. □

CASE STUDY
Current status of the operations and theoretical improvement potential
Our case study is feeder bus line 302 in Singapore operating from Choa Chu Kang Interchange and looping at Choa Chu Kang Crescent. It serves Choa Chu Kang Way, Choa Chu Kang Street and Yew Tee MRT. The feeder bus line is connected with the collector Mass Rapid Transit (MRT) North-South line at station Yew Tee MRT (NS5). Hence, the arrival times of the feeder line trips at stop Yew Tee MRT should be adjusted to the arrival times of the main MRT North-South line, while minimizing the excessive waiting times of feeder line passengers.

Feeder line 302 serves residential blocks, schools and public amenities along Choa Chu Kang Way, Choa Chu Kang St, Choa Chu Kang Dr, Choa Chu Kang North and Choa Chu Kang Cres, connecting them to Choa Chu Kang Town Centre and Yew Tee MRT. Its primary area of service is Choa Chu Kang Neighbourhoods 5 and 6. The route serves schools such as Kranji Secondary School, De La Salle School and Unity Primary School. It also serves community amenities such as Limbang Shopping Centre, Yew Tee Point and Yew Tee Community Centre. High capacity articulated buses are deployed on a daily basis because of high demand from residents. It serves $S_f = \langle 1, 2, ..., 22 \rangle$ stops and is a circular line (Fig.2).
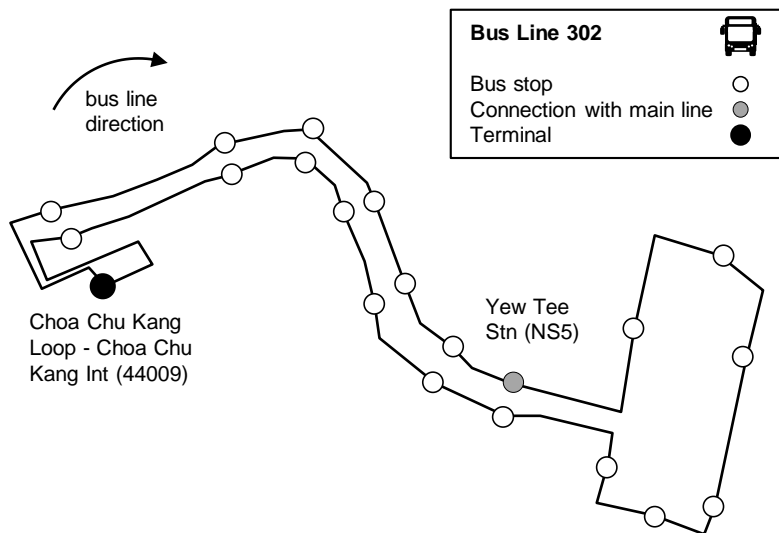
FIGURE 2 : Topology of feeder bus line 302 and location of the transfer stop Yee Tee MRT (NS5) that connects it to the main MRT north-south line.

In feeder line 302 operate 245 bus trips on weekdays. Its route length is 8.1km and its total trip travel time ranges between 35 to 40 minutes among peak/off-peak hours. Bus line 302 is selected as our main case study because:

- it is a feeder line connected to a collector MRT line;

- it is one of the seven high-frequency bus lines in Singapore monitored in terms of excessive passenger waiting times Leong et al. (*51*). Hence, maintaining its regularity is important because regular operations receive monetary incentives from the transport authority (up to 3000$ for every 0.1 minute improvement in regularity at the end of each month).

The ideal headways of feeder line 302 differ across peak/off-peak hours, as presented in Table 1.

TABLE 1 : Ideal headway of feeder bus line 302 at different times of the day

Period	Ideal (target) Headway
05:30-08:30	4 min
08:30-19:00	5 min
After 19:00	8 min

Additionally, Fig.3 presents the expected inter-departure travel times of the trips of the feeder bus line based on historical data.
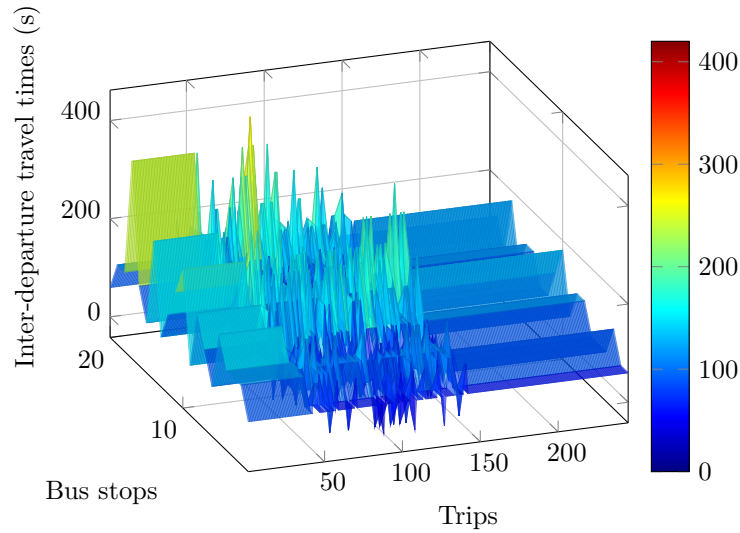
FIGURE 3 : Expected inter-departure travel times between bus stops for the 245 daily trips of the feeder line.

Considering the collector MRT line, the north-south MRT line operates 45km with 27 stations and is a high capacity line with 6-car fully automatic trains. We present its topology in Fig.4, including the transfer station with the feeder line (Yew Tee Stn). Train headways are 2 to 3 minutes during the peak hours of 7am to 9am and about 5 to 7 minutes during off-peak times.ïż£
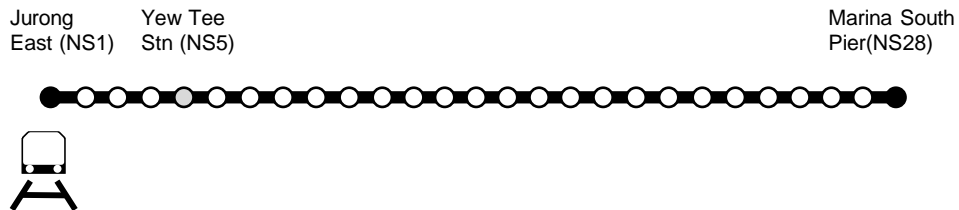

FIGURE 4 : Topology of the North-South MRT line

Passengers transferring from the feeder bus line 302 to the main MRT North-South line at stop NS5 have the following average transfer times at different times of the day, according to the original schedules of the operations (see Fig.5).
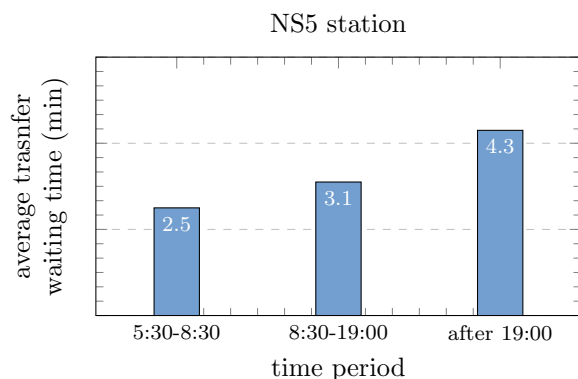
FIGURE 5 : Average transfer waiting times from the feeder bus line 302 to the North-South MRT line at different time periods of the day.

In addition, the passenger excessive waiting times of bus line 302 are measured at twelve control point stops (see Fig.6). Note that the excessive waiting time is the mean value of the square root of Eq.(5) that computes the squared deviation among actual and ideal headways.
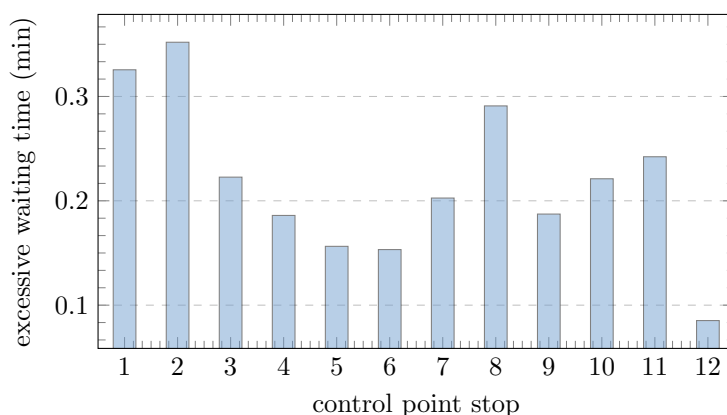

FIGURE 6 : Excessive waiting times of feeder bus line 302 at different control point stops during the daily operations. The overall excessive waiting times after aggregating the regularity from all control point stops is 0.21 min.

With our proposed method, one can compute improved door-to-door passenger travel times by solving the reformulated convex program $(\tilde{Q})$. The convex program can be solved with numerical optimization methods, such as sequential quadratic programming or the interior point methods Boggs and Tolle, Mehrotra (*52, 53*). In this work, we program $(\tilde{Q})$ in Python 3.6 and solve it to global optimality with the commercial optimization solver Gurobi (8.1.1). Because of the convex formulation of $(\tilde{Q})$, the optimization solver is able to converge to a globally optimal solution within seconds.

The rescheduled dispatching times of the feeder line resulting from the solution of our program ($\tilde{Q}$) are presented in Fig.7. Fig.7 shows the rescheduled dispatching times of every daily trip. Note that in Fig.7 many of the trips of the feeder line are dispatched between 6:00-8:00 and 16:00-20:00, corresponding to the respective peak periods.
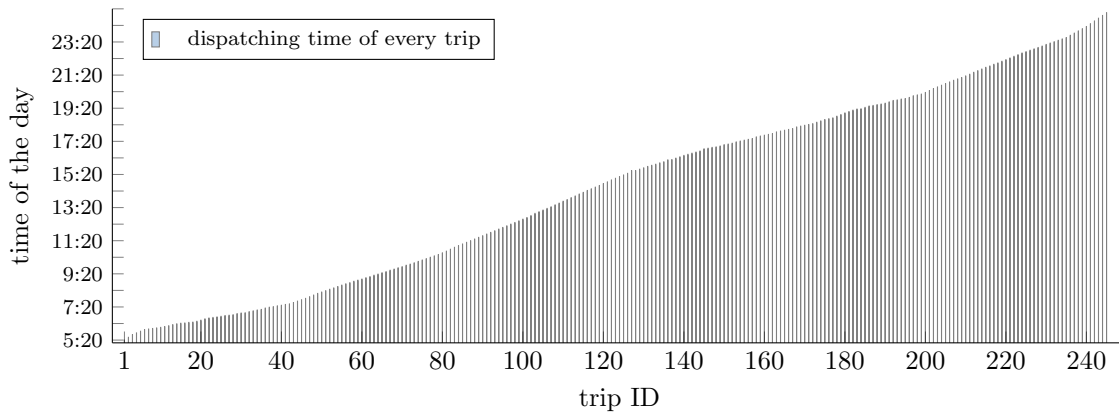

FIGURE 7 : Rescheduled dispatching times of the feeder line for the 245 trips of the day.

After rescheduling the dispatching times of the feeder line based on our globally optimal solution, the average transfer waiting times are presented in Fig.8.
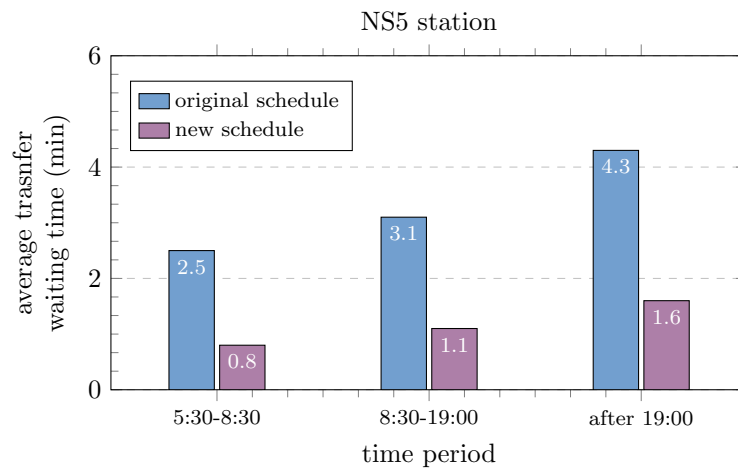

FIGURE 8 : Average transfer waiting times from the feeder bus line 302 to the North-South MRT line with the original schedule and after rescheduling.

Additionally, the excessive waiting times of the feeder line passengers are presented in Fig.9.
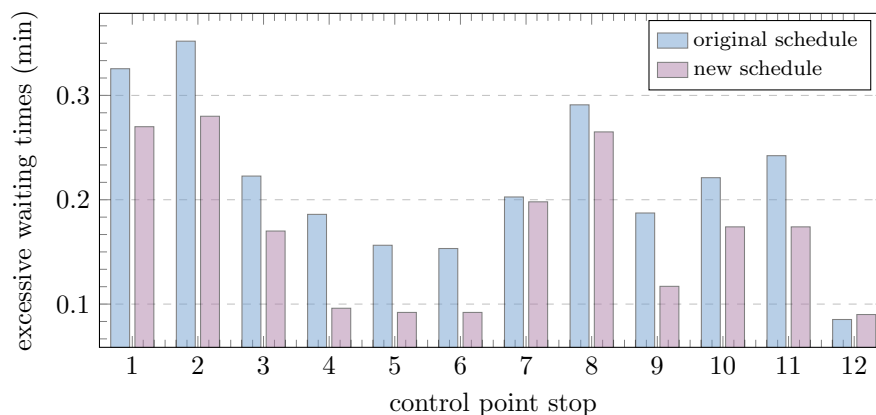
FIGURE 9 : Excessive waiting times of feeder bus line 302 at different control point stops with the original and the new schedule.

As it can be seen from Fig.9, the overall theoretical improvement after applying the new schedule is at the level of 25% demonstrating a significant improvement potential.

Simulation-based evaluation

After showing the theoretical gain based on the expected travel times, we perform a simulation-based evaluation to compare the performance of our proposed coordination-based schedule (CS) presented in Fig.7 against the following baseline strategies:

- Do-nothing strategy (DN). DN refers to applying the original schedule of the feeder line.

- Regularity-based strategy (RB). RB refers to the case where the schedule of the feeder line is adjusted to improve its regularity, without considering the transfer synchronization and the waiting times at transfer stops (see Cats (*54*)). To derive such schedule, one should solve program ($\tilde{Q}$) without considering the synchronization constraint (6).

The selected evaluation metrics assess the door-to-door travel times of passengers and the regularity of the feeder mode. This study uses the following three evaluation metrics:

- Average transfer waiting times of passengers, which indicate the coordination level between the feeder and the main public transport line;

- Excessive waiting times at the bus stops of the feeder line;

- The overall door-to-door travel times of passengers.

The values of the aforementioned evaluation metrics are calculated after applying the DN schedule, the RB schedule, and our proposed CS schedule in 500 Monte Carlo simulation experiments. Each simulation experiment samples the inter-departure travel times of trips and the passenger demand from probability distributions fitted to the historical travel time and passenger demand observations. Using the simulations, the travel times and the passenger demand are

stochastic and can take any value when they are sampled from the fitted probability distributions. This enables us to investigate the potential of schedule synchronization in realistic scenarios because synchronized transfers might have limited effect on reducing the door-to-door travel times if there is no demand for transfers.

After applying the three schedules to 500 simulated daily operations, the results of the daily average transfer waiting times are presented in Fig.10.
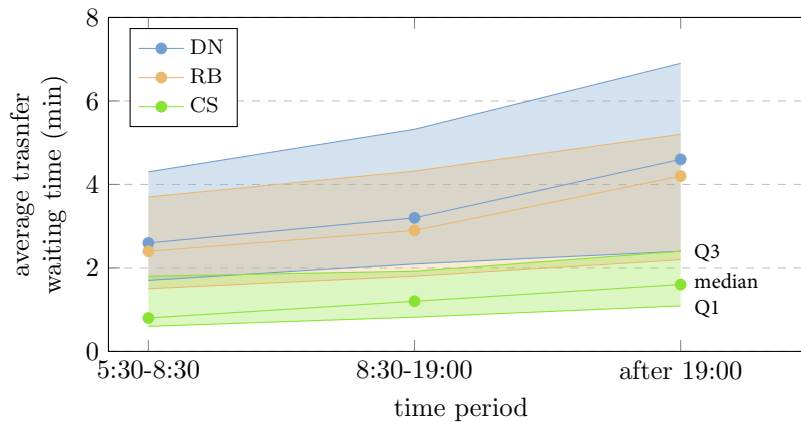

FIGURE 10 : Median and interquartile range of the average transfer waiting time when using the DN, RB, and CS schedules in 500 Monte Carlo simulations of the daily operations.

The dots in Fig.10 present the median value of the average passenger transfer waiting times in the 500 simulations for the three different time periods of the day. The lower and upper lines present the interquartile range (IQR=$Q_3 - Q_1$) between the 25th and the 75th percentile of the average transfer waiting times over the 500 days. Note that there is significant improvement on the average transfer waiting times when using our schedule. To investigate how this improvement is translated into a reduction of door-to-door passengers travel times, in Fig.11 we present the average door-to-door passenger travel times after 500 simulations together with the excessive waiting times and transfer times.
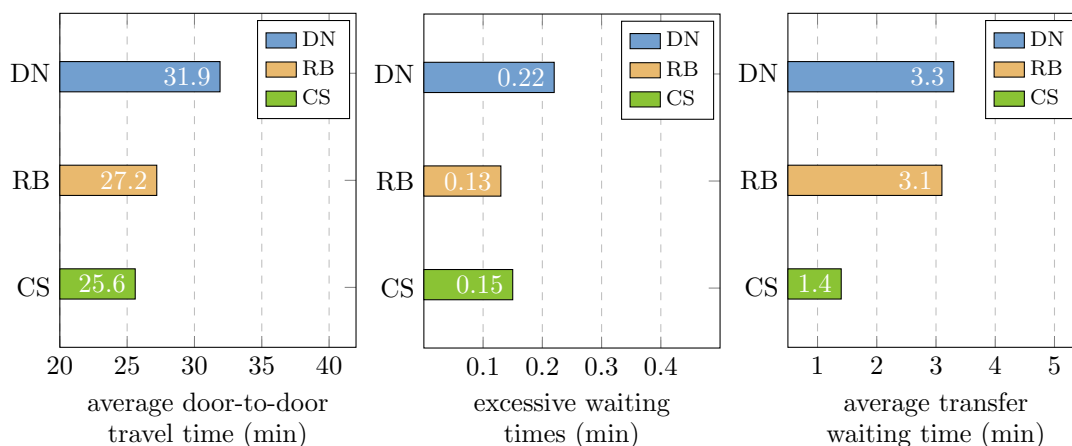
FIGURE 11 : Median of the service regularity of the feeder bus line when using the DN, RB, and CS schedules in 500 Monte Carlo simulations of the daily operations.

From Fig.11, one can note that our proposed schedule (CS) improved the average transfer waiting times by 54% when applied in 500 simulated daily operations. However, the improvement of the overall door-to-door travel times when applying our schedule is only 6%. This is the case because (i) the transfer times are a small part of the total door-to-door passenger travel times, and (ii) not all passengers are transferring from the feeder to the collector line. We finally note that the RD schedule, which does not consider the transfer times from the feeder to the collector line, results in a 13% improvement on the excessive travel times of *feeder* line passengers.

CONCLUDING REMARKS

This study introduced a mathematical model for reducing the door-to-door travel times of passengers that use up to two public transit modes to complete their origin-destination journey. This model reschedules the dispatching times of trips that belong to the feeder line and, after its proposed reformulation, it can be solved to global optimality. To reduce the door-to-door travel times when demand is uncertain, we use the excessive waiting times at regular stops and the waiting times at transfer stops as door-to-door travel time indicators. To evaluate the performance of our approach in realistic operations, we used a case study in Singapore with a feeder and collector line showing an improvement potential of 6% in terms of door-to-door passenger travel times. This improvement can increase if there is more demand for transferring from the feeder to the collector line, or reduce if there is limited demand for transfers. For this, the public transit operators might consider relaxing the synchronization constraint at time periods when the demand for transfers is low, and vice versa.

To facilitate the reproduction and extension of our work, we hereby report its main limitations:

(a) our work can be applied when the number of vehicles assigned to the feeder and collector lines at the tactical planning stage suffices to satisfy the passenger demand;

(b) our work can be applied to reduce the door-to-door travel times of passengers that use up to two public transit modes for completing their origin-destination trips.

Our model provides a first step towards coordinating feeder and collector lines to improve the efficiency of MaaS schemes. One can extend it further with the use of highly granular transfer demand data derived from combined public transit tickets purchased via MaaS providers Kamargianni and Matyas (*6*). In addition, it can be expanded outside the realm of public transit by considering bike-sharing or car-sharing services as feeder modes. This expansion can be further examined in future research where the transferring demand can also be part of the problem formulation.

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