

MODELLING TRANSITIONS IN GRASS COVERS TO QUANTIFY WAVE OVERTOPPING EROSION

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Transitions in vegetated dike covers, such as geometry changes or roughness differences, are identified as weak spots in dikes for grass cover erosion by wave overtopping. Although several erosion models exist to model grass cover erosion on dikes, it is unclear how the effect of transitions on grass cover erosion must be included in these models. Therefore, we have developed a model approach to analyze the effects of transitions on grass cover erosion using field experimental data and to derive representative influence factors for one transition type. The model approach has been applied to the transition at the landward toe where the slope changes to a horizontal plane. The model approach is general applicable and can be transferred easily to other transitions. The derived factors can be used to improve predictions of dike cover erosion near transitions.

Keywords: wave overtopping, flood defense, grass cover, turbulence modeling, transitions

INTRODUCTION

A strong grass cover is essential for reliable flood defences that are subject to wave action. When waves overtop the dike, the overtopping water rushes over the crest and down the inner slope, leading to erosion of the grass cover layer (Schüttrumpf 2001, Van der Meer et al. 2010, Warmink et al. 2018). Dike covers are generally not smooth profiles, but they consist of different materials and geometries. Any change in material, geometry or roughness is called a transition. These transitions affect the strength of grass covers (because the continuous grass cover is interrupted) and they also affect the wave overtopping flow by increasing turbulence downstream of the transition. Therefore, transitions in vegetated dike covers are identified as weak spots in dikes for grass cover erosion by wave overtopping (Fig.1).

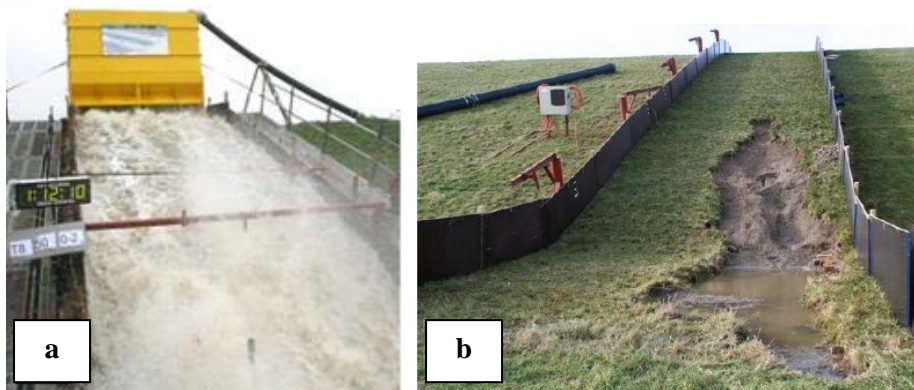


Figure 1. (a) Wave Overtopping Simulator test in The Netherlands (figure from ComCoast 2007). (b) Example of grass cover erosion at the landward toe after a wave overtopping test (figure courtesy Van der Meer, 2014).

Several models exist to model grass cover erosion at transitions. The Cumulative Overload Method (COM, developed by Hoffmans et al. 2018) is based on work of Dean et al. (2010) and compares the overtopping flow velocities to the critical flow velocity of the grass cover. The COM results in a damage number (D) for the simulated overtopping waves. Comparison of the calculated damage numbers with the damage categorization results in a safety assessment of the stability of the grass cover for wave overtopping. The categories for damage numbers are empirically derived from the Dutch wave overtopping tests at the Vechtdijk (Van der Meer et al. 2015).

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The analytical Grass-Erosion Model (GEM) combines a model for overtopping flow velocities with an erosion model. The analytical model by Van Bergeijk et al. (2019a) is adapted after Schüttrumpf and Oumeraci (2005) and results in maximum overtopping flow velocities along dike crests and landward slopes per simulated wave volume. Subsequently, the flow velocities along the dike profile are used as input for the erosion model of Hoffmans (2012) to calculate the cumulative erosion depths ($d(x)$) for all locations (x) along the dike profile for all overtopping wave volumes during a storm event.

The COM includes a load factor (α_M) and a strength factor (α_S) to account for the increase of the hydraulic load and decreases of the grass cover strength, respectively (Van der Meer et al. 2015). The COM is applied to derive representative load factors for revetment transitions, geometrical transitions and objects for the wave overtopping tests at Nijmegen and Millingen (Van Hoven et al. 2013). However, the derived load factors showed some variability between the test sections, so no generalizable load factors for a specific transition type yet exist.

The GEM does not yet consider load and strength factors to include the effects of transitions on grass cover erosion. However, this model contains a factor to account for the turbulence intensity which can be adapted for load increases near transitions. Van Bergeijk et al. (2019b) describes three alternative formulations for the turbulence intensity parameter (r_0) to account for the hydraulic load changes near transitions. These three formulations have been applied to the Afsluitdijk in the Netherlands, but they are not yet sufficiently validated.

The objective of this paper is to quantify the influence of transitions on grass cover erosion due to overtopping waves. A general framework is developed to determine the influence of transitions on grass cover erosion by wave overtopping (Frankena, 2019). The model approach uses the field observations of cover erosion during wave overtopping tests with the Wave Overtopping Simulator on grass-covered dikes in the Netherlands (Van der Meer et al. 2018). The model approach was applied to calibrate the influence of transitions for one type of transition: the landward toe. This transition was selected based on expert elicitation (Frankena, 2019) and sufficient availability of test data. Firstly, the grass cover strength is determined based on damage observations along the slope. Next, representative influence factors for the transitions are calibrated based on the observed damage at the transitions. Finally, the model analysis results in values for r_0 (GEM) and α_M (COM) that are representative for the load increase at the geometrical transition for each test section.

DATA AND MODELS

Wave overtopping simulator data

Since 2007, multiple tests with wave overtopping simulators have been performed in the Netherlands, Belgium, Vietnam and the United States. The wave overtopping simulator (WOS) was developed in the Netherlands for simulating overtopping waves on in-situ test sections (Van der Meer et al. 2018). During a test, the wave overtopping simulator releases a series of wave volumes at the dike crest that are representative for normative storm events. The Dutch wave overtopping simulator (Fig. 1a) was used during wave overtopping tests at nine locations in the Netherlands and two locations in Belgium from 2007 to 2015.

Table 1. Selected wave overtopping tests for which damage occurred at the landward toe transition. L_{cr} is length of the crest downstream of the WOS [m], L_{sl} is the total length of the slope [m], $\cot(\theta)$ is co-tangent of the landward slope, and grass quality (good, moderate, poor) was determined in the referred reports.					
Location – year (reference)	section	L_{cr} [m]	L_{sl} [m]	$\cot(\theta)$ [-]	grass type (quality)
Boonweg – 2008 (Bakker et al. 2008a)	1	2.5	27.0	2.9	Meadow (good)
	2	2.5	27.0	2.9	Meadow (good)
Kattendijke – 2008 (Bakker et al. 2008b)	1	2.0	15.0	3.0	Hay (good)
	2	2.5	17.5	3.0	Hay (good)
Afsluitdijk – 2009 (Bakker et al. 2009)	1	0.5	7.8	2.3	Meadow (good)
	2	0.7	7.5	2.3	Meadow (moderate)
Tholen – 2011 (Bakker et al. 2011)	1	2.0	14.0	2.3	Meadow (poor)

Various test conditions and transition types have been considered during the wave overtopping tests. Reports generally provide specific information per test location regarding test conditions, site characteristics and observed erosion. However, a summary of the wave overtopping tests that are performed worldwide is not available. Van der Meer (2014) provides a comprehensive overview of the wave overtopping tests in the Netherlands and Belgium. Table 1 shows the selected WOS tests for which damage occurred at the landward toe transition. All these tests were carried out for a significant wave

height (H_s) of 2.0 m and a peak wave period (T_p) of 5.7 s. The slope angles and slope lengths varied between the tests, as well as the grass quality (Table 1).

Model description

The individual wave volumes (V_i) that were simulated during the wave overtopping tests were randomly generated from the probability exceedance distribution (Frankena, 2019). Multiple mean overtopping discharges (q_m) were simulated, varying from $q_m = 0.1$ l/s/m to $q_m = 75$ l/s/m following the discharges released during the wave overtopping tests. Representative storm conditions that were considered during the tests are the significant wave height ($H_s = 2.0$ m), the peak period ($T_p = 5.7$ s) and the total number of waves ($N_{ow} = 4596$). Table 2 presents the characteristics of the overtopping waves for the different mean overtopping discharges.

	$q_m=0.1$	$q_m=1$	$q_m=5$	$q_m=10$	$q_m=30$	$q_m=50$	$q_m=75$
R_c [m]	5.06	3.84	2.98	2.61	2.03	1.76	1.54
P_{ow} [%]	0.2	2.7	11.4	18.9	36.6	47	56
N_{ow} [-]	9	126	525	867	1683	2160	2574
V_{max} [l/m]	769	1222	2018	2697	4707	6387	8278

The hydraulic boundary conditions for both the COM and GEM model per simulated wave, V_i , are the overtopping flow velocity at the start of the crest ($U_{0,i}$, Van der Meer et al. 2015), the layer thickness at the start of the crest ($h_{0,i}$, Van der Meer et al. 2010) and the wave overtopping period ($T_{0,i}$, Hughes et al. 2012):

$$U_{0,i} = 4.5 \cdot V_i^{0.3} \quad (1)$$

$$h_{0,i} = 0.133 \cdot V_i^{0.5} \quad (2)$$

$$T_{0,i} = 3.9 \cdot V_i^{0.46} \quad (3)$$

Cumulative overload method

The cumulative overload method (COM) is introduced by Van der Meer et al. (2010) as an erosional index. The COM method is currently the standard tool used by the Dutch government to assess dike failure due to wave overtopping. In this model, the overtopping flow velocity that exceeds the critical flow velocity contributes to grass cover erosion. The amount of erosion is expressed as a damage number (D). The method, including the factors that account for flow acceleration on the slopes (α_a), the increase in load at transitions (α_M) and the decrease in cover strength at transitions (α_s) is (Hoffmans et al. 2018):

$$D = \sum_{i=1}^N (\alpha_M (\alpha_a \cdot U_{0,i})^2 - \alpha_s \cdot U_c^2) \quad (5)$$

$$\text{for: } U_{0,i} \geq U_c$$

where U_c is the critical flow velocity for the COM model. To account for acceleration of overtopping waves along the dike profile, the cumulative overload method includes the product of an acceleration factor α_a and the overtopping flow velocity at the start of the crest $U_{0,i}$. Generally, the acceleration factor is determined from a graph, based on the slope steepness $\cot(\theta)$ and the distance between the end of the dike crest and the location on the landward slope (Van der Meer et al. 2015).

The flow acceleration method by Van der Meer et al. (2015) is a simplification of the iterative model for overtopping flow velocities along the dike profile by Schüttrumpf and Oumeraci (2005). Also the analytical model of Van Bergeijk et al. (2019a) calculates the flow acceleration along the landward slope. For the model analysis with the COM, three methods are compared to account for acceleration of overtopping waves along the slope:

- 1) constant flow acceleration factor ($\alpha_{a,vdM}$: Van der Meer et al. 2015),
- 2) acceleration factor per wave volume ($\alpha_{a,SO}$: Schüttrumpf and Oumeraci, 2005) and
- 3) acceleration factor per wave volume ($\alpha_{a,vB}$: Van Bergeijk et al. 2019b).

Grass erosion model

The grass-erosion model (GEM) is a combination of an analytical model for wave overtopping flow velocities along dike crests and landward slopes (Van Bergeijk et al. 2019a) and an erosion model (Hoffmans, 2012). This means that the combined model can be used to model the erosion depth along the dike profile for a number of overtopping wave volumes. The analytical model of Van Bergeijk et al. (2019a) provides two formulas for the maximum overtopping flow velocity for an overtopping wave along horizontal parts, e.g. dike crests and berms, and along slopes. The flow velocities along the dike profile are input for the erosion model by Hoffmans (2012) to calculate the erosion depth (d) along the dike profile per overtopping wave. The erosion model by Hoffmans (2012) has been adapted to account for variations in the hydraulic load (ω) and the grass cover strength (U_t) along the dike profile. The turbulence parameter (ω) depends on the depth-averaged turbulence intensity (r_0).

$$d(x) = \sum_{i=1}^N ((\omega(x)^2 \cdot U_i(x)^2 - U_t(x)^2) \cdot T_0 \cdot C_E) \quad (4)$$

$$\text{for: } \omega(x)^2 \cdot U_i(x)^2 \geq U_t^2$$

$$\text{where: } \omega(x) = 1.5 + 5 \cdot r_0(x) \quad (5)$$

where U_t is the threshold flow velocity for the GEM model at coordinate x [m/s] and C_E is the inverse strength parameter describing the erosion rate [s/m]. The final outcome of the GEM is an erosion depth along the dike slope which when subtracted from the original dike profile provides the dike profile after a storm.

Calibration approach

To quantify the effect of transitions on grass cover erosion, we calibrated the influence factors based on the seven wave overtopping tests shown in Table 1. For the GEM, the turbulence intensity factor (r_0) was calibrated and for the COM, the load factor (α_M) was calibrated. These calibrations were carried out in two steps. Firstly, a representative critical flow velocity was calibrated for all seven tests individually (Figure 2). For the COM, the model was run multiple times for a range of U_c values. For a location without a transition (the upper slope), the modelled damages numbers are compared to the observed damage category. The critical flow velocity value for which the modelled damaged number is closest to the observed damage category is selected as representative critical flow velocity U_c . For the GEM a similar approach was followed only the threshold velocity U_t was calibrated using the erosion depth.

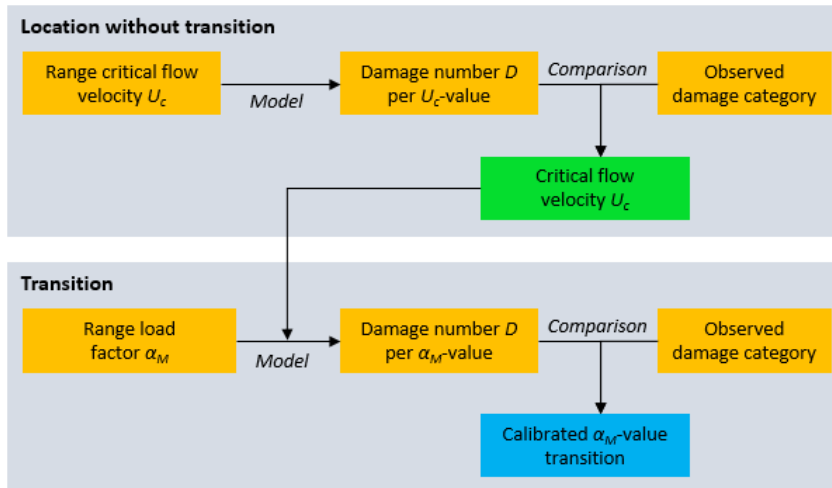


Figure 2. Overview of the approach for calibration of the critical flow velocity (U_c) and the influence factors for a transition on grass cover erosion using the COM. For the GEM a similar approach is followed. Figure from Frankena (2019).

In the second step of the calibration, the derived critical (COM) or threshold (GEM) velocities are applied for the location with a transition. The models are run multiple times with an α_M ranging between 1.0 and 2.0, with increments of 0.1 for the COM and the r_0 value ranging between 0.10 and 0.60 with increments of 0.05 for the GEM. The simulation that matched the observed erosion (GEM) or damage (COM) at the transition best was then selected. Figure 3 illustrates this calibration for the GEM for two different tests at Tholen (T4) and the Afsluitdijk (A2). In both cases, the simulation with a turbulence

intensity of 0.45 was closest to the measured erosion depth at the transition. For the COM a similar approach was followed only the damage number was calculated and compared to the observed damage criterion (Hoffmans et al. 2018).

The calibrated load factors for the COM are compared to the theoretical load factors from Hoffmans et al. (2018). They present an equation to compute the theoretical load factor, based on the angle of the landward slope (θ). This equation describes the load increase at geometrical transitions for the cumulative overload method:

$$\alpha_T = 1 + \sin(0.5\theta) \quad (6)$$

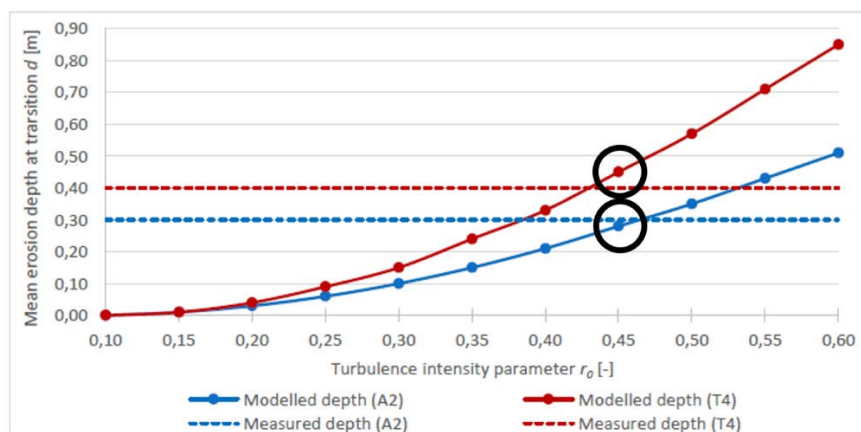


Figure 3. Calibration of the turbulence intensity parameter r_0 to quantify the influence of the geometrical transition on grass cover erosion. The modelled erosion depth (straight line) is compared to the measured mean erosion depth (dotted line) at the transition for test sections Afsluitdijk 2 (A2) and Tholen 4 (T4). Figure adapted after Frankena (2019).

RESULTS

Calibrated critical flow velocities

The overview of the calibrated critical flow velocities per test section for each method for the flow acceleration shows that calibrated U_c values differ between the used acceleration method (Table 3). The iterative method (Schüttrumpf and Oumeraci, 2005) results in the lowest critical flow velocity for each test section, while the constant Van der Meer et al. (2015) result in the highest critical flow velocities. The derived critical flow velocity is 0.5 m/s higher using the constant flow acceleration factor compared to the critical flow velocity that results from the analytical model. This shows that the flow acceleration methods according to Van der Meer et al. (2015) and Van Bergeijk et al. (2019b) result in comparable U_c -values.

The calibrated threshold flow velocities (U_t) for the GEM method are much larger than the calibrated critical velocity (U_c) for the COM. The ratio between the threshold flow velocity and the critical flow velocity ($U_t \approx 2.4 \cdot U_c$) can be related to the different criteria of erosion in both models. In the COM, the erosion criterium is defined as $\sqrt{\alpha_M} \cdot U > U_c$ with overtopping velocity U , while the erosion criterium for the GEM includes a turbulence parameter $\omega \cdot U > U_t$. This parameter is defined as $\omega = 1.5 + 5 \cdot r_0$. To derive the critical flow velocity, the transition is not considered, so $\alpha_M = 1.0$. The assumption $r_0 = 0.10$ along the landward slope results in $\omega = 2.0$. Combination of both equations results in $U_t = 2 \cdot U_c$, which means that the schematized hydraulic load in the grass-erosion model is twice as large as the schematized hydraulic load in the cumulative overload method, using the same flow acceleration method.

Table 3. Calibrated critical (U_c [m/s]) and threshold (U_t [m/s]) flow velocities for the COM and the GEM per test section using three flow acceleration methods for the COM: $\alpha_{a,vdM}$, $\alpha_{a,SO}$ and $\alpha_{a,vB}$

	Boonweg 1	Boonweg 2	Kattendijke 1	Kattendijke 2	Afsluitdijk 1	Afsluitdijk 2	Tholen 1
$U_c(\alpha_{a,vdM})$	10.0	10.0	8.5	8.5	7.5	6.0	6.5
$U_c(\alpha_{a,SO})$	7.5	7.5	6.5	6.5	6.0	5.0	5.0
$U_c(\alpha_{a,vB})$	9.5	9.5	8.0	8.0	7.0	5.5	6.5
U_t	22	22	19	19	17	14	16

Calibrated load factors due to transitions

Despite that different critical flow velocities were derived using the three different flow acceleration methods, the calibrated load factors (α_M) per test section are quite similar between different flow acceleration methods (Figure 3). According to the theory, the load factor is high for steep slopes, while a mild slope results in a moderate load factor (Hoffmans et al. 2018). Relatively steep slopes were considered at Afsluitdijk 1 and 2 and Tholen 4 with $\cot(\theta) = 2.3$, while mild slopes were present at Boonweg 1 and 2 with $\cot(\theta) = 2.9$ and at Kattendijke 1 and 2 with $\cot(\theta) = 3.0$. The calibrated values for the turbulence intensity parameter r_0 agree to the theory with $r_0 = 0.25$ for mild slopes (Boonweg 1 & 2 and Kattendijke 1 & 2) and $r_0 = 0.45$ for relatively steep slopes (Afsluitdijk 2 and Tholen 4). The calibrated load factor and turbulence intensity for Afsluitdijk 1 case are unexpectedly low. A detailed analysis of the wave overtopping tests at Afsluitdijk 1 showed that the erosion rate decreased during the overtopping tests. The grass top layer (5-10 cm) was eroded at low overtopping discharge. This led only to initial damage, but not to failure of the top layer (20 cm erosion depth). The clay layer below was able to resist much larger overtopping discharges until failure occurred. In the calibration, this resulted in a relatively high critical velocity ($U_c = 7.5$) and subsequently, in a low calibrated value of the load factor. This suggests that for minor damage, the derivation of critical velocities and load factors is less reliable.

Comparison of the calibration results to the theory by Hoffmans et al. (2018) (Eq. 6) shows that almost all calibrated load factors exceed the theoretical values. The theoretical load factors as function of the slope steepness underestimate the influence of the geometrical transition on the hydraulic load on the grass cover, according to the data from the wave overtopping tests.

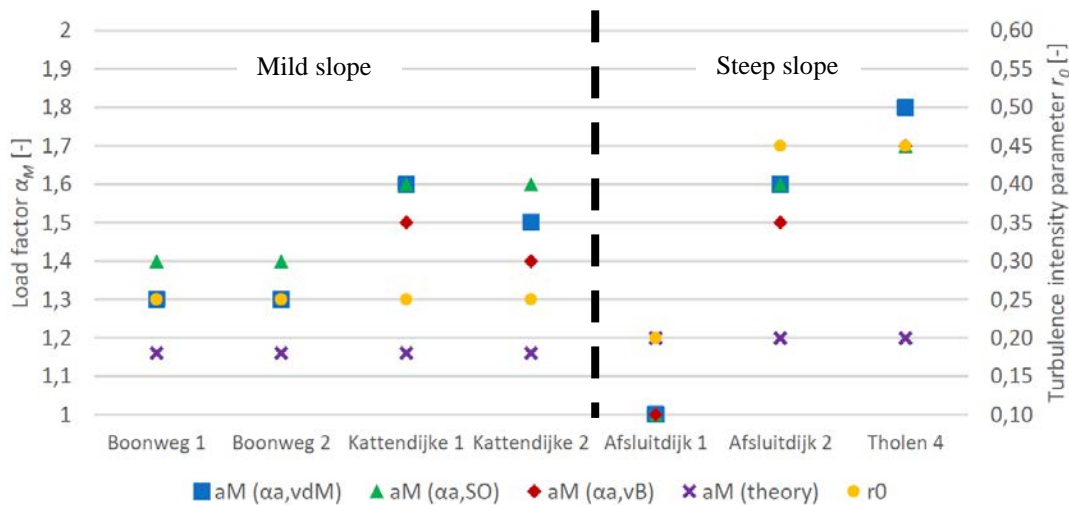


Figure 3. Calibrated load factors α_M (left vertical axis) for three flow acceleration methods and turbulence intensity r_0 (right vertical axis) per test section. The calibrated load factors are compared to the theoretical load factor based on steepness of the landward slope $\cot(\theta)$ (Eq. 6). Figure adapted from Frankena (2019).

DISCUSSION

This paper presents a method to calibrate load factors that can be applied to a range of transition types. The influence factor is able to incorporate the effect of transitions on dike cover erosion in any model, but calibration is needed for all types of transitions separately. Therefore, accurate measurements of dike cover erosion (including their temporal evolution) and measurements of the hydraulic load are needed. Further research using detailed numerical models can potentially extend the range of transitions that are currently not physically tested, but accurate numerical modelling of dike cover erosion is still challenging.

The value of the turbulence intensity seems quite large with a default value larger than 1.5 (Eq. 5), so further research is needed to better quantify the load. Accurate measurements and modelling of the turbulence intensity in overtopping flows is needed to provide insight in physically more realistic values of the loads. Until then, values for critical or threshold flow velocities are calibrated values for which the physical meaning remains largely unknown.

Several models exist to model grass cover erosion. Bomers et al. (2018) applied a CFD-simulation to the grass-erosion model to study the influence of a road on top of a dike on grass cover erosion. Furthermore, Aguilar-López et al. (2018) used CFD-simulations for a probabilistic assessment method to determine the influence of an asphalt road on top of a dike to the probability of failure locally and for

an entire dike profile. The computation time of a CFD-simulation of a single wave is relatively large, in the order of hours for a single overtopping wave. These numerical models are therefore useful to better understand and quantify the hydraulic loads at transitions, but are not (yet) suitable to simulate dike cover erosion during a storm. Van Bergeijk et al. (2020a) developed an OpenFoam model to simulate the forces at transitions under wave overtopping and applied this model to the Afsluitdijk case (Van Bergeijk et al. 2020b). However, until now only transitions in roughness and the effect of the slope angle on hydraulic forces are studied. A next step is to compare the simulated turbulence at the landward toe of the dike to the calibrated turbulence intensity and load factors to further validate the values calibrated in this study.

Research has taught us that there are many uncertainties in the assessment of dike erosion. Numerical models are being developed and are promising to quantify the additional loads due to transitions (e.g. Van Bergeijk et al. 2020a). In this research, but also in earlier studies (e.g. Hoffmans et al. 2018), the critical threshold velocity to initiate erosion is often calibrated based on flume experiments. Also the erosion rate parameter C_E was assumed constant in this study due to lack of reliable information. This shows that accurate predictions of wave overtopping erosion are still a challenge that need to be addressed in future research.

CONCLUSIONS

The aim of this research was to quantify the influence of transitions on grass cover erosion due to overtopping waves. The developed model approach was used to derive an influence factor for a single transition type: the transition from the landward slope to a horizontal plane for both the Cumulative Overload Method (COM) and the analytical Grass-Erosion Model (GEM).

We conclude that:

- The turbulence intensity (r_0) for the GEM is approximately 0.25 for mild slopes (1:3) and 0.45 for steep slopes (1:2.3) and quite constant for the seven tested cases.
- The load influence factor (α_M) for the COM ranges between 1.3-1.6 for mild slopes (1:3) and 1.5-1.8 for steep slopes (1:2.3) for the seven tested cases. These factors are larger than expected based on theory ($\alpha_M \approx 1.2$).
- The calibrated threshold velocity (U_t) for the GEM method is approximately a factor 2 larger than the calibrated critical velocity (U_c) for the COM method. This may suggest that both U_t and U_c are empirical parameters with limited physical meaning.
- Depth-dependency of dike cover erosion is currently not included in both models. The Afsluitdijk 1 case showed that extending the erosion equation by explicitly including the strength of the clay layer underneath the grass sod enables more accurate estimates of dike cover strength.

Finally, we recommend application of the calibration approach to other transition types to determine representative influence factors for each transition type using both the cumulative overload method and the analytical grass -erosion model.

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