ENHANCING SERVICE MATHEMATICS TEACHING THROUGH STRATEGIC ALIGNMENT

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ABSTRACT

Service mathematics teaching, such as calculus for engineering, needs to be aligned with the requirements of the departments it is servicing. Service mathematics courses can be subject to criticism if they are perceived to suffer from poor alignment. Designing such courses requires communication between the mathematics department and the engineering departments and this communication should remain ongoing as the needs of the students change or as teachers with different experience and mathematical preferences change.

In the Twente Educational Model the bachelor's degree is divided into twelve modules, each lasting one quarter of the academic year. In each module the students work in groups on projects and the project is supported by disciplinary units or skills development. Module design differs across and within departments, but the basic structure of supported project-based learning is ever present. The quarterly project reports provide insight into the work the students draw on to understand and complete their projects and offer an opportunity to determine whether the students have the mathematics knowledge and skills needed for their project assignments.

To investigate existing alignment and to seek ways of improving alignment we embarked on a project of distilling mathematics content from reports. While alignment was good in general certain points for improvement were apparent, both in the realm of content (for example important differential equations) and in key skills (for example interpretation of graphs). In this presentation I shall provide a brief synopsis of findings as well as present and reflect on the methodology of the project.

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1 INTRODUCTION

1.1 Service mathematics

Much of the mathematics taught at universities is taught to non-mathematics majors, such as students of engineering, economics and physical science. These co-called "service" courses are often taught by people who do not share the same disciplinary speciality as the students being taught. It is important that the mathematics included in such a course meet the needs, as far as is possible, of the discipline and the other courses the students will take and for which they will require mathematical knowledge and skill [1].

The need for alignment between the mathematics taught and the mathematics needed can be partially approached as a checklist of topics, for instance do we include complex numbers, multivariable calculus, compound interest and annuities and so forth, but the need for alignment is greater than a list of syllabus topics. Different disciplines have different discourses, different uses of (technical) language and different modes of engaging with the world. A mathematics course well aligned with the discipline of engineering contributes to development of an engineering identity [2,3].

Studies considering alignment between service mathematics courses and the disciplines they serve have been fruitfully carried out via interviews with and observations of lecturers [4] or students [5] or analysis of assessment for evidence of "transfer" [6]. The project-based learning practised at the University of Twente allows for an opportunity to compile a set of mathematical topics, skills and general processes not from the top down, but from the project reports, thereby by definition embedded in disciplinary context. Anecdotal evidence suggests that while alignment between the mathematics courses taught by the author and the programmes in which they are located is acceptable it could be improved. The study of which this paper forms part aims to improve alignment by utilising project reports as contextualised indicators of disciplinary mathematics requirements. The qualitiative method of thematic analysis was used to identify themes to inform the research questions.

1.2 Twente Educational Model

A bachelor's degree at the University of Twente is structured as a series of twelve thematic modules each lasting one quarter of an academic year. Each module consists of a cluster of courses or units as well as a project which the students complete in groups. An example is module 1 of Advanced Technology, Mechanics, where courses (or units) on mathematics, mechanics, laboratory practice and academic skills support a project modelling a dynamical system [7].

The degree to which the courses integrate with one another and with the project differs across modules. While the “ideal” of the Twente Educational Model (TEM) is that the courses in a module all support the project in some way in order to encourage an intrinsic motivation to learn [8] this is not always possible. Mathematics in particular can suffer from lack of alignment with any one particular module given that there is a certain body of calculus and linear algebra that is required to be taught across the first year and the technical areas in which it will be used may only be encountered by the students in a later module or year of study.

At the end of each module each group of students has to submit a report on the module’s project. The reports provide a valuable opportunity to see, in situ, what mathematical skills, concepts, techniques and processes (in this paper referred to collectively as “topics”) were employed by the students and hence to determine how well the mathematics courses are aligned with student and disciplinary need and whether the alignment could be improved by either minor or major changes. The method of thematic analysis was used to analyse and organise the data.

1.3 Research questions

- What mathematics skills, concepts, processes or techniques are observed in the project reports?
- To what extent are the mathematics courses aligned with the projects or do they support the projects?
- In what ways could the mathematics courses adapt or change to better align with or support the projects?

Finally, not a research question but a reflection,

- How effective was the method of thematic analysis in suggesting ways of enhancing alignment?

2 METHODOLOGY

2.1 Thematic analysis

Thematic analysis is a qualitative method for identifying, analysing and reporting themes in data [9]. One begins with a data corpus, in this case the collection of all project reports, and chooses a data set within that corpus for analysis, in this case the data set is instances of use of procedural mathematics. Braun and Clarke [9] suggest a set of six phases for thorough thematic analysis. In this study the analysis was framed as “three passes through the data” where the first pass, reading the reports and making a concise list (per group) of anything of mathematical interest can be aligned with Braun and Clarke’s first two phases of familiarising oneself with the data and generating codes. The second pass, that of organising the data more meaningfully in a summary (per module) is also aligned with phase 2. The third pass, to identify and name themes, align with phases 3-5. Braun and Clarke’s phase 6 is to produce a report of which this paper represents a part.

In the context of mathematics as a part of engineering studies, Faulkner, Earl and Herman [10] use thematic analysis to better understand how engineering faculty view mathematical maturity, that is as skill at modelling and recognising the role of mathematics in underatanding the real world. Similarly Engelbrecht, Bergsten and Kågesten [11] employ thematic analysis of interviews with practising engineers to conclude that while both procedural fluency and conceptual understanding are valued in the workplace mathematics taught to engineering students should be more conceptually than procedurally oriented.

2.2 Context

The two programmes included in this study were Advanced Technology (AT) and Electrical Engineering (EE), specifically the four first-year modules. At the end of each module the students submit reports on projects on which they work in groups. As a member of the teaching team I am permitted access to those reports although I am not directly involved in the projects. No examples of student work or identifying information is included in this study.

2.3 Data collection and analysis

First pass: To read project reports and generate codes. The project reports were read and mathematical skills, processes, concepts and techniques (collectively "topics") that were observable in the students' work were recorded resulting in a short list per group. A procedural view of mathematics was explicitly taken here. For the purposes of this analysis mathematics was considered as a toolbox of procedures and techniques rather than mathematics as problem solving, or "thinking mathematically" [12,13]. Undoubtedly viewing the project reports through a different lens would reveal different and rich data, but the task at hand in the analysis discussed here was to determine indications of concrete curriculum content.

Second pass: To determine the most prevalent mathematical topics. The first pass produced topics clustered by group. In the second pass those topics were considered collectively and a summary was produced making note of the most prevalent topics in each module. Also in this second pass the content of the concurrent mathematics course was considered and whether the course in any way supported the project.

Third pass: To look at the entire data set and identify themes including but not limited to: topics that could be included in the mathematics courses to better support the projects, and topics that are already included that are clearly supporting the project.

3 RESULTS

3.1 Three passes through the data

The project reports from two programmes across four modules have been analysed. Table 1 lists the academic year, module, programme and broad project context.

Table 1. Project contexts by programme and module

Academic Year	Module	Programme	Project topic
2018-2019	4	AT	Accelerometer
2018-2019	4	EE	Antenna
2019-2020	1	AT	Dynamical system
2019-2020	1	EE	Sensor package
2019-2020	2	AT	Cooling system
2019-2020	2	EE	Solar inverter
2019-2020	3	AT	Materials for application
2019-2020	3	EE	Audio amplifier

AT: Advanced Technology
EE: Electrical Engineering

The first pass through the data revealed immediately that keeping a detailed and exhaustive count of mathematics topics would be impossible. The same engineering subject matter written about by two groups could and did include very different representations of mathematical processes. Sometimes a topic was so prominent that everyone included it (a system of differential equations, for instance) but on the whole a general sense of what the students found useful was all that could be recorded. A surprising finding of this first pass through the data was not content related but more stylistic. For instance a wide variety of Greek letters was used by both programmes in multiple modules, suggesting that the mathematics courses could display a similar prominence and thereby at least display a symbolic similarity with the technical programmes. Group numbers were recorded, but no names, each accompanied by a list of observed mathematical skills or techniques. An example:

Group 1

- Step function
- 2nd order DE
- Error estimates
- Estimation of quantities
- Manipulating symbolic expressions
- Differentials
- Damping
- Graphs

For the second pass through the data the points noted in the first pass were clustered into those that were present in all or many of the reports and those that appeared only once or twice and a module summary was drawn up. Thereafter the content of the concurrent mathematics course was considered and links were sought between the mathematics course and the summary, laying the groundwork for identifying themes. Alignment was classified as None, Weak, Good or Significant. Table 2 indicates that alignment was Significant in one case (AT module 1), Good in one case (EE module 1), Weak in two cases (AT and EE module 2) and None in four cases (AT and EE modules 3 and 4).

Table 2: Alignment of mathematics course with module project

Year	Module	Programme	Alignment
2018-2019	4	AT	None. The mathematics course is entirely linear algebra which plays no role in the project. Instead the project primarily uses differential equations.
2018-2019	4	EE	None. The mathematics course is entirely linear algebra which plays no role in the project. Instead the project relies heavily on vector calculus.
2019-2020	1	AT	Significant. A core topic of the mathematics course is differential equations which are used extensively in the project. Another core topic is vectors which are also important in the project.
2019-2020	1	EE	Good. A core topic of the mathematics course is vectors which are important in the project. Several project reports include limits at infinity, also a topic covered in the mathematics course.
2019-2020	2	AT	Weak. The project needed very little post-secondary school mathematics. An exception is integration in the form of single variable integration not requiring any special techniques.
2019-2020	2	EE	Weak. The project needed very little post-secondary school mathematics. An exception is partial differentiation which played a minor role in the project.
2019-2020	3	AT	None. The project needed very little post-secondary school mathematics. A possible exception is determining best-fit curves to data points.
2019-2020	3	EE	None. The project needed very little post-secondary school mathematics.

AT: Advanced Technology

EE: Electrical Engineering

The TEM ideal of the courses or units gathered together in a module all supporting the central project of that module is only apparent in module 1 for both AT and EE. This is not a problem, however, for two main reasons: First, the projects in the other modules are not using or needing mathematics which is not available to the students, that is there are no key topics that have been "omitted" to the impoverishment of the students' experience in the module. Secondly, the mathematics courses, in particular the three calculus courses, form a cohesive body of work that is drawn on generally. A good example is the third calculus course (included in module 3) which covers vector calculus which then forms a powerful tool for use in the EE module 4 project. If the four EE first-year modules form a "meta-module" [7], as it were, then the TEM structure is indeed apparent.

The third pass through the data was focused on identifying themes, or categories. The categories recognized as emerging from the data were: (1) mathematical topics or techniques that can relatively easily be changed or included in the syllabus, (2) mathematical topics or techniques it would be valuable to include, (3) mathematical topics or techniques that are already in the syllabus and were clearly useful in the projects, and (4) ideas related to style or appearance. This method of identifying themes [9] call "inductive" or "bottom up".

3.2 Categories

1. Topics for change

The "topics for change" are a collection of mathematical topics that were evident in the data but not present (or not prominently present) in the mathematics courses, yet could be included without much difficulty.They are an increased focus on graphing, differentials and error propagation, and specific examples of certain mathematical concepts, such as damping, Hooke's Law and the binomial theorem.

Graphs: Something that leaps out of the project reports is the heavy reliance on graphing. The students graph their experimental outputs both as continuous curves and as discrete data. They present the ideal curves that the models predict and compare their data graphs to those of the model. In some cases they determine best-fit straight lines (or other curves) to scattered data points and in all cases they need to interpret all the relevant graphs. Sinusoidal curves and the arctan function were particularly applicable functions and horizontal asymptotes made several appearances.

Differentials and error propagation. Differentials made an appearance across several modules as do the concepts of error and error propagation. The first calculus course, in module 1, begins with differential equations since the students need understanding of what differential equations are and some skill in solving them early on in the module, both AT and EE. Therafter the course deals with complex numbers (needed for second order differential equations) and vectors, another important topic. The course then closes with a cluster of topics generally related to differentiation, such as limits, continuity, tangent lines and extreme values. Differentials and error propagation would be a good fit at the end of the course.

Damping and Hooke's Law. Second order differential equations are a core topic in the first calculus course, however in the current curriculum damping is not covered. Simple harmonic motion with damping could be easily included in the syllabus, similarly Hooke's Law which is ubiquitous in the project reports could be included as a contextual example without any difficulty.

Binomial theorem. In the second calculus course we cover sequences and series in great detail. The students often struggle to see the relevance to any of their other work, such as calculus or their technical subjects. The binomial theorem, encountered a few times in the project reports, would be a relevant example to choose, along with the binomial series.

2. Valuable topics to include

The "valuable topics to include" category is a collection of topics that, if included in the mathematics courses, would improve alignment, but are not easily incorporated in the courses as they currently exist. Further thought is necessary to consider how to include them. There were three topics fitting this description. The first was the geometry of parabolas, specifically the role of the focal point. The second was the catenary, a curve with important real world applications and also an opportunity to refer to hyperbolic trigonometric functions. Finally there were the trio of square wave, triangular wave and sawtooth functions, all of great importance in EE module 2.

3. Relevant syllabus topics

The mathematical skills and techniques already included in the existing mathematics courses and obviously present in the project reports included single variable integration, partial derivatives, vector calculus and 1st and 2nd order differential equations. Also present were vectors (with the same notation we use in mathematics, which is interesting given the lack of global convention of vector notation [14]), complex numbers, limits at infinity, sigma notation and polar, spherical and cylindrical coordinates. As a teacher of engineers without herself having an engineering background, it is extremely helpful to see where the mathematics I teach is used.

4. Style elements

Symbolic usage in the reports revealed two common characteristics generally not shared with mathematics, that of using many Greek letters and manipulating large symbolic expressions with many constants or variables. The symbols used in mathematics tend to fall within a small set of Roman and Greek letters. To a student new to university and unfamiliar with seeing these symbols it can demystify them if they are encountered more ubiquitously than only in certain courses. The dot notation for time derivatives is widely used in the AT and EE project reports. This dot notation is not currently used in the mathematics courses but it is trivial to introduce and include.

The research questions were

- What mathematics skills, concepts, processes or techniques are observed in the project reports?
- To what extent are the mathematics courses aligned with the projects or do they support the projects?
- In what ways could the mathematics courses adapt or change to better align with or support the projects?

The first and second passes through the data answered the first question, the second pass answered the second question, and the third pass answered the third. The result is a strategically chosen collection of changes to the existing mathematics courses that will improve alignment. Continued communication within the module teaching teams remains important for many reasons, one of them being possible changes to the module project resulting in different needs for the mathematics courses.

3.3 Reflection on the methodology

It is important for a service mathematics course to be aligned with the needs of the discipline it is serving and the teaching and learning programme in which it is situated (such as these TEM modules) [15]. Typically to achieve that alignment either the mathematics is taught by people already embedded in the discipline or the two parties communicate over what is needed, perhaps using checklists of topics. The methodology of thematic analysis employed in this project is particularly suited to a pedagogy based on project-based learning, as TEM is. It takes a very particular and very practical view of the alignment of the mathematics course to the students' learning needs. A limitation of the method of using project reports is the significant differences between what different groups chose to include and hence the likelihood of something important not being apparent. As a mathematics lecturer without an engineering background I found this study extremely informative and educational. I have a much clearer idea of what the projects entail, what the students need and how mathematics can help. The next challenge will be to incorporate the changes discussed above.

4 SUMMARY AND ACKNOWLEDGMENTS

The study reported in this paper will be carried forward by communicating the findings of the thematic analysis with colleagues in the two disciplinary programmes in order to spur conversations of mutual benefit. Furthermore two initiatives are under development with the two disciplinary programmes: in advanced technology workshops are being planned linking calculus and mechanics, and in electrical engineering a project involving the creation of a suite of contextualised mathematics exercises is being planned.

Finally, in summary, the methodology of using students' written project reports as an indication of mathematics of importance in disciplinary context was successful and highly informative. The results of analysis will certainly bring about change in the mathematics courses to better align them with the needs of the projects, in line with the vision of the Twente Educational Model [8].

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