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Synchronization for Homogeneous and Heterogeneous Discrete-time Multi-agent Systems: A Scale-free Protocol Design

Donya Nojavanzadeh[1], Zhenwei Liu[2], Ali Saberi[1], Anton A. Stoorvogel[3]

1. School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164, USA
E-mail: donya.nojavanzadeh@wsu.edu; saberi@eecs.wsu.edu

2. College of Information Science and Engineering, Northeastern University, Shenyang 110819, China
E-mail: liuzhenwei@ise.neu.edu.cn

3. Department of Electrical Engineering, Mathematics and Computer Science, University of Twente, Enschede, The Netherlands
E-mail: A.A.Stoorvogel@utwente.nl

Abstract: This paper studies synchronization of homogeneous and heterogeneous discrete-time multi-agent systems. A class of linear dynamic protocol design methodology is developed based on localized information exchange with neighbors which does not need any knowledge of the directed network topology and the spectrum of the associated Laplacian matrix. The main contribution of this paper is that the proposed protocols are scale-free and achieve synchronization for arbitrary number of agents.

Key Words: Multi-agent systems, state synchronization, Discrete-time, Scale-free

1 Introduction

The synchronization problem of multi-agent systems (MAS) has attracted substantial attention during the past decade, due to the wide potential for applications in several areas such as automotive vehicle control, satellites/robots formation, sensor networks, and so on. See for instance the books [27] and [41] or the survey paper [24].

We identify two classes of multi-agent systems: homogeneous (i.e. agents are identical) and heterogeneous (i.e. agents are non-identical). State synchronization inherently requires homogeneous MAS. On the other hand, for a heterogeneous MAS generically, state synchronization cannot be achieved and focus has been on output synchronization. For homogeneous MAS state synchronization based on diffusive full-state coupling has been studied where the agent dynamics progress from single- and double-integrator dynamics (e.g. [25, 26]) to more general dynamics (e.g. [28, 34, 39]). State synchronization based on diffusive partial-state coupling has also been considered, including static design ([19] and [20]), dynamic design ([11], [29], [30], [33], [36]), and the design with localized communication ([3] and [28]). For MAS with discrete-time agents, earlier work can be found in [26, 13, 9, 4, 35] for essentially first and second-order agents, and in [15, 10, 12, 44, 43, 38, 37, 32, 31] for higher-order agents. Recently, scale-free collaborative protocol designs are developed for continuous-time heterogeneous MAS [22] and for homogeneous MAS subject to actuator saturation [18] and subject to input delays [17, 16].

In heterogeneous MAS, if the agents have absolute measurements of their own dynamics in addition to relative information from the network, they are said to be introspective, otherwise, they are called non-introspective. The output synchronization problem for agents with general dynamics has been studied in both introspective and non-introspective cases. For heterogeneous MAS with introspective right-invertible agents, [38] and [42] developed the output and regulated output synchronization results for discrete-time and continuous-time agents. Reference [14] provided regulated output consensus for both continuous- and discrete-time introspective agents. On the other hand, for heterogeneous MAS with non-introspective agents, [40, 6, 8] developed an internal model principle based design for output and regulated output synchronization. Reference [2] designed a static protocol design for MAS with non-introspective passive agents and [7] provided a purely distributed low-and high-gain based linear time-invariant protocol design for non-introspective homogeneous MAS with linear and nonlinear agents and for non-introspective heterogeneous MAS.

In this paper, we design **scale-free** collaborative protocols based on localized information exchange among neighbors for synchronization of homogeneous and heterogeneous discrete-time MAS. We study synchronization problem for discrete-time homogeneous MAS with non-introspective agents for both full- and partial-state coupling. Moreover, we deal with output and regulated output synchronization for heterogeneous discrete-time MAS with introspective agents. The protocol design is scale-free, namely:

- The design is independent of the information about communication networks such as a lower bound of non-zero eigenvalue of associated Laplacian matrix.
- The one-shot protocol design only depends on agent models and does not need any information about communication network and the number of agents.
- The synchronization is achieved for any MAS with any number of agents, and any communication network.

Due to space limitation, the results pertaining to regulated output synchronization of heterogeneous MAS and some proofs (proofs of Theorem 2, 3) and numerical simulation are omitted. All of these are available in the full version of this paper [23].

Notations and definitions

Given a matrix $A \in \mathbb{R}^{m\times n}$, A^{T} denotes its conjugate transpose. A square matrix A is said to be Schur stable if all its eigenvalues are in the open unit disc. We denote by $\mathrm{diag}\{A_1, \ldots, A_N\}$, a block-diagonal matrix with $A_1, \ldots, A_N$ as its diagonal elements. $A \otimes B$ depicts the Kronecker prod-

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uct between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context.

To describe the information flow among the agents we associate a *weighted graph* $\mathcal{G}$ to the communication network. The weighted graph $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the weighted adjacency matrix with non negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. The *weighted in-degree* of a node i is given by $d_{in}(i) = \sum_{j=1}^{N} a_{ij}$. Similarly, the *weighted out-degree* of a node i, is given by $d_{out}(i) = \sum_{j=1}^{N} a_{ji}$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. A *directed tree* is a subgraph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. A *directed spanning tree* is a subgraph which is a directed tree containing all the nodes of the original graph. If a directed spanning tree exists, the root has a directed path to every other node in the tree [5].

For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$ [5]. Moreover, if the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane [1].

2 Homogeneous MAS with Non-introspective Agents

Consider a MAS composed of N identical linear time-invariant agents of the form,

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + Bu_i(k), \\ y_i(k) &= Cx_i(k), \end{aligned} \qquad (i = 1, \ldots, N) \qquad (1)$$

where $x_i(k) \in \mathbb{R}^n$, $u_i(k) \in \mathbb{R}^m$, $y_i(k) \in \mathbb{R}^p$ are respectively the state, input, and output vectors of agent i. Meanwhile, (1) satisfies the following assumption.

Assumption 1 *We assume that*

- *all eigenvalues of A are in the closed unit disk.*
- *(A, B, C) is stabilizable and detectable.*

The communication network provides each agent with a linear combination of its own outputs relative to that of other neighboring agents. In particular, each agent $i \in \{1, \ldots, N\}$ has access to the quantity,

$$\zeta_i(k) = \frac{1}{1 + d_{in}(i)} \sum_{j=1}^{N} a_{ij}(y_i(k) - y_j(k)), \qquad (2)$$

where $a_{ij} \geqslant 0$, and $a_{ii} = 0$ for $i, j \in \{1, \ldots, N\}$. The topology of the network can be described by a graph $\mathcal{G}$ with nodes corresponding to the agents in the network and edges given by the nonzero coefficients a_{ij}. In particular, $a_{ij} > 0$ implies that an edge exists from agent j to i. The weight of the edge equals the magnitude of a_{ij}. Next we write ζ_i as

$$\zeta_i(k) = \sum_{j=1}^{N} d_{ij}(y_i(k) - y_j(k)), \qquad (3)$$

where $d_{ij} \geqslant 0$, and we choose $d_{ii} = 1 - \sum_{j=1, j\neq i}^{N} d_{ij}$ such that $\sum_{j=1}^{N} d_{ij} = 1$ with $i, j \in \{1, \ldots, N\}$. Note that d_{ii} satisfies $d_{ii} > 0$. The weight matrix $D = [d_{ij}]$ is then a so-called, row stochastic matrix. Let $D_{in} = \text{diag}\{d_{in}(i)\}$ with $d_{in}(i) = \sum_{j=1}^{N} a_{ij}$. Then the relationship between the row stochastic matrix D and the Laplacian matrix L is

$$(I + D_{in})^{-1} L = I - D. \qquad (4)$$

We refer to (2) as *partial-state coupling* since only part of the states are communicated over the network. When $C = I$, it means all states are communicated over the network and we call it *full-state coupling*. Then, the original agents are expressed as

$$x_i(k+1) = Ax_i(k) + Bu_i(k) \qquad (5)$$

and $\zeta_i(k)$ is rewritten as

$$\zeta_i(k) = \sum_{j=1}^{N} d_{ij}(x_i(k) - x_j(k)). \qquad (6)$$

We define the set of graphs $\mathbb{G}^N$ for the network communication topology as following.

Definition 1 *Let $\mathbb{G}^N$ denote the set of directed graphs of N agents which contains a directed spanning tree.*

If the graph $\mathcal{G}$ describing the communication topology of the network contains a directed spanning tree, then it follows from [26, Lemma 3.5] that the row stochastic matrix D has a simple eigenvalue at 1 with corresponding right eigenvector $\mathbf{1}$ and all other eigenvalues are strictly within the unit disc. Let $\lambda_1, \ldots, \lambda_N$ denote the eigenvalues of D such that $\lambda_1 = 1$ and $|\lambda_i| < 1$, $i = 2, \ldots, N$.

Obviously, state synchronization is achieved if

$$\lim_{k \to \infty} (x_i(k) - x_j(k)) = 0. \qquad (7)$$

for all $i, j \in 1, ..., N$.

In this paper, we also introduce a localized information exchange among protocols. In particular, each agent $i = 1, \ldots, N$ has access to the localized information, denoted by $\hat{\zeta}_i(k)$, of the form

$$\hat{\zeta}_i(k) = \sum_{j=1}^{N} d_{ij}(\rho_i(k) - \rho_j(k)) \qquad (8)$$

where $\rho_j(k) \in \mathbb{R}^n$ is a variable produced internally by agent j and to be defined in next sections.

We formulate the following problem for state synchronization of a homogeneous MAS based on localized information exchange.

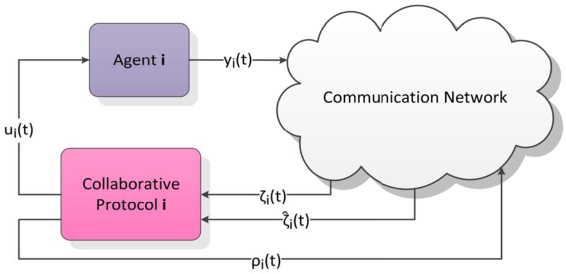

Figure 1: Architecture of Protocol 1 and 2

Problem 1 *Consider a MAS described by* (1) *and* (3) *satisfying Assumption 1. Let* $\mathbb{G}^N$ *be the set of network graphs as defined in Definition 1. Then the* ***scalable state synchronization problem based on localized information exchange*** *is to find, if possible, a linear dynamic controller for each agent* $i \in \{1, \dots, N\}$*, using only knowledge of the agents model, i.e.* (C, A, B)*, of the form:*

$$\begin{cases} x_{i,c}(k+1) = A_c x_{i,c}(k) + B_c \zeta_i(k) + C_c \hat{\zeta}_i(k), \\ u_i(k) \quad = F_c x_{i,c}(k), \end{cases} \tag{9}$$

where $\hat{\zeta}_i(k)$ *is defined as* (8) *with* $\rho_i = M_c x_{i,c}$*, and* $x_{i,c} \in \mathbb{R}^{n_i}$*, such that state synchronization* (7) *is achieved for all initial conditions.*

2.1 Protocol Design

We consider state synchronization problem of a homogeneous MAS for both cases of full- and partial-state coupling.

2.1.1 Full-state coupling

In this subsection, we consider state synchronization of MAS with full-state coupling. The design procedure is given in Protocol 1.

Protocol 1: Scale-free collaborative protocol design for homogeneous MAS with full-state coupling

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We design dynamic collaborative protocols utilizing localized information exchange for agent $i \in \{1, \dots, N\}$ as

$$\begin{cases} \eta_i(k+1) = A\eta_i(k) + Bu_i(k) + A\zeta_i(k) - A\hat{\zeta}_i(k) \\ u_i(k) \quad = -K\eta_i(k), \end{cases} \tag{10}$$

where K is a matrix such that $A - BK$ is Schur stable and ρ_i is a variable produced internally by agent i and is chosen in (8) as $\rho_i = \eta_i$, therefore each agent has access to the following information:

$$\hat{\zeta}_i(k) = \sum_{j=1}^{N} d_{ij}(\eta_i(k) - \eta_j(k)). \tag{11}$$

meanwhile, ζ_i is defined in (6). The architecture of the protocol is shown in Figure 1, where $y_i(t) = x_i(t)$.

Our formal result is stated in the following theorem.

Theorem 1 *Consider a MAS described by* (5) *and* (6) *satisfying Assumption 1. Let* $\mathbb{G}^N$ *be the set of network graphs as defined in Definition 1.*

Then the scalable state synchronization problem based on localized information exchange as stated in Problem 1 is solvable. In particular, the dynamic protocol (10) *solves the state synchronization problem for any graph* $\mathcal{G} \in \mathbb{G}^N$ *with any number of agents* N.

Proof of Theorem 1: Firstly, let $\bar{x}_i(k) = x_i(k) - x_N(k)$ and $\bar{\eta}_i(k) = \eta_i(k) - \eta_N(k)$, we have

$$\begin{aligned} &\bar{x}_i(k+1) = A\bar{x}_i(k) + B(u_i(k) - u_N(k)), \\ &\bar{\eta}_i(k+1) = A\bar{\eta}_i(k) + B(u_i(k) - u_N(k)) \\ &\qquad\qquad + A(\bar{x}_i(k) - \bar{\eta}_i(k)) + \textstyle\sum_{j=1}^{N-1} \tilde{d}_{ij} A(\bar{x}_j(k) - \bar{\eta}_j(k)), \\ &u_i(k) - u_N(k) = -K\bar{\eta}_i(k). \end{aligned}$$

where $\tilde{D} = [\tilde{d}_{ij}] \in \mathbb{R}^{(N-1)\times(N-1)}$ with $\tilde{d}_{ij} = d_{ij} - d_{Nj}$. Then, we define

$$\bar{x}(k) = \begin{pmatrix} \bar{x}_1(k) \\ \vdots \\ \bar{x}_{N-1}(k) \end{pmatrix} \text{ and } \bar{\eta}(k) = \begin{pmatrix} \bar{\eta}_1(k) \\ \vdots \\ \bar{\eta}_{N-1}(k). \end{pmatrix}$$

Based on [23, Lemma 1], we have that eigenvalues of $\tilde{D}$ are equal to the eigenvalues of D unequal to 1. Then, we have the following closed-loop system

$$\begin{cases} \bar{x}(k+1) = (I \otimes A)\bar{x}(k) - (I \otimes BK)\bar{\eta}(k) \\ \bar{\eta}(k+1) = I \otimes (A - BK)\bar{\eta}(k) \\ \qquad\qquad + ((I - \tilde{D}) \otimes A)(\bar{x}(k) - \bar{\eta}(k)) \end{cases} \tag{12}$$

Let $e(k) = \bar{x}(k) - \bar{\eta}(k)$, we can obtain

$$\bar{x}(k+1) = (I \otimes (A - BK))\bar{x}(k) + (I \otimes BK)e(k) \tag{13}$$

$$e(k+1) = (\tilde{D} \otimes A)e(k) \tag{14}$$

We have that all eigenvalues of $\tilde{D}$ are in open unit disk. The eigenvalues of $\tilde{D} \otimes A$ are of the form $\lambda_i \mu_j$, with λ_i and μ_j eigenvalues of $\tilde{D}$ and A, respectively. Since $|\lambda_i| < 1$ and $|\mu_j| \leqslant 1$, we find $\tilde{D} \otimes A$ is asymptotically stable. Then we have $e_i(k) \to 0$ as $k \to \infty$.

According to the above result, for (13) we just need to prove the stability of

$$\bar{x}(k+1) = (I \otimes (A - BK))\bar{x}(k). \tag{15}$$

given that $A - BK$ is Schur stable, (15) is asymptotically stable. Then, we will have

$$\lim_{k\to\infty} \bar{x}_i(k) = \lim_{k\to\infty} (x_i(k) - x_N(k)) \to 0$$

i.e. $x_i(k) \to x_j(k)$ as $k \to \infty$, which proves the result. ■

2.1.2 Partial-state coupling

In this subsection, we consider state synchronization of MAS with partial-state coupling. The design procedure is given in Protocol 2.

Protocol 2: Scale-free collaborative protocol design for homogeneous MAS with partial-state coupling

We propose the following dynamic protocol with localized information exchange for agent $i \in \{1,\dots,N\}$ as follows:

$$\begin{cases} \eta_i(k+1) = A\eta_i(k) + Bu_i(k) + A\hat{x}_i(k) - A\hat{\zeta}_i(k) \\ \hat{x}_i(k+1) = A\hat{x}_i(k) - BK\hat{\zeta}_i(k) + H(\zeta_i(k) - C\hat{x}_i(k)) \\ u_i(k) \quad = -K\eta_i(k) \end{cases} \tag{16}$$

where K is a matrix such that $A - BK$ is Schur stable and ρ_i is chosen as $\rho_i = \eta_i$ in (8) and with this choice of ρ_i, $\hat{\zeta}_i$ is given by:

$$\hat{\zeta}_i(k) = \sum_{j=1}^{N} d_{ij}(\eta_i(k) - \eta_j(k)). \tag{17}$$

meanwhile, ζ_i is defined in (3). The architecture of the protocol is shown in Figure 1.

Then, we have the following theorem for state synchronization for discrete-time MAS with partial-state coupling.

Theorem 2 *Consider a MAS described by* (1) *and* (3) *satisfying Assumption 1. Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

Then the scalable state synchronization problem based on localized information exchange as stated in Problem 1 is solvable. In particular, the dynamic protocol (16) *solves the state synchronization problem for any graph $\mathcal{G} \in \mathbb{G}^N$ with any number of agents N.*

Proof of Theorem 2: See [23]. ■

3 Heterogeneous MAS with Introspective Agents

In this section, we will study a heterogeneous MAS consisting of N non-identical linear agents:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u_i(k), \\ y_i(k) &= C_i x_i(k), \end{aligned} \qquad (i = 1,\dots,N) \tag{18}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^p$ are the state, input, output of agent i for $i = 1,\dots,N$.

The agents are introspective, meaning that each agent has access to its own local information. Specifically each agent has access to the quantity

$$z_i(k) = C_i^m x_i(k), \quad z_i \in \mathbb{R}^{q_i} \tag{19}$$

We also make the following assumption for the agents:

The communication network provides each agent with local information $\zeta_i(k)$ as (3).

Assumption 2 *For agents $i \in \{1,\dots,N\}$,*

1) *(A_i, B_i) is stabilizable.*
2) *(C_i, A_i) is detectable.*
3) *(C_i, A_i, B_i) is right-invertible*
4) *(C_i^m, A_i) is detectable.*

Remark 1 *Right-invertibility of a triple (C_i, A_i, B_i) means that given a reference output $y_r(t)$, there exists an initial condition $x_i(0)$ and an input $u_i(t)$ such that $y_i(t) = y_r(t)$ for all non-negative integers k. For example, every single-input single-output system is right-invertible, unless its transfer function is identically zero. The definition of right-invertibility can be found in [21].*

The heterogeneous MAS is said to achieve output synchronization if

$$\lim_{k\to\infty} (y_i(k) - y_j(k)) = 0, \quad \text{for } i,j \in \{1,\dots,N\}. \tag{20}$$

First, we formulate scalable output synchronization problem for heterogeneous networks as follows:

Problem 2 *Consider a heterogeneous MAS described by agent models* (18) *and local information* (19)*, satisfying Assumption 2 and associated network communication* (3)*. Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1. The **scalable output synchronization problem based on localized information exchange** is to find, if possible, a linear dynamic controller for each agent $i \in \{1,\dots,N\}$, using only knowledge of the agent model, i.e. (C_i, A_i, B_i), of the form:*

$$\begin{cases} x_{i,c}(k+1) = A_{i,c}x_{i,c}(k) + B_{i,c}\zeta_i(k) + C_{i,c}\hat{\zeta}_i(k) + D_{i,c}z_i(k), \\ u_i(k) \quad = E_{i,c}x_{i,c}(k) + F_{i,c}\zeta_i(k) + G_{i,c}\hat{\zeta}_i(k) + H_{i,c}z_i(k), \end{cases} \tag{21}$$

where $\hat{\zeta}_i(k)$ is defined as (8) *with $\rho_i = N_{i,c}x_{i,c}(k)$, and $x_{i,c}(k) \in \mathbb{R}^{n_i}$, such that for all initial conditions the output synchronization* (20) *is achieved for any graph $\mathcal{G} \in \mathbb{G}^N$ with any number of agents N.*

3.1 Output synchronization

In this section, we design protocols to solve scalable output synchronization problem as stated in Problem 2. After introducing the architecture of our protocol, we design the protocols through three steps.

3.1.1 Architecture of the protocol

Our protocol has the structure shown below in Figure 2.

As seen in the figure, the design methodology consists of two major modules.

- The first module is reshaping the dynamics of the agents to obtain the target model by designing pre-compensators following our previous results in [38].
- The second module is designing collaborate protocols for almost homogenized agents to achieve output synchronization.

3.1.2 Protocol design

For solving output synchronization problem for heterogeneous network of N agents (18), first we recall a critical lemma as stated in [38].

Lemma 1 *Consider the heterogeneous network of N agents* (18) *with local information* (19)*. Let Assumption 2 hold and let $\bar{n}_d$ denote the maximum order of infinite zeros of $(C_i, A_i, B_i), i \in \{1,\dots,N\}$. Suppose a triple (C, A, B) is given such that*

1) $\operatorname{rank}(C) = p$
2) *(C, A, B) is invertible of uniform rank $n_q \geq \bar{n}_d$, and has no invariant zeros.*

Then for each agent $i \in \{1,\dots,N\}$, there exists a pre-compensator of the form

$$\begin{cases} \xi_i(k+1) = A_{i,h}\xi_i(k) + B_{i,h}z_i(k) + E_{i,h}v_i(k), \\ u_i(k) \quad = C_{i,h}(k)\xi_i(k) + D_{i,h}v_i(k), \end{cases} \tag{22}$$

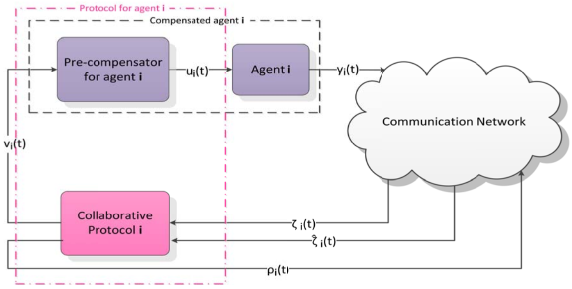

Figure 2: Architecture of Protocol 3

such that the interconnection of (18) *and* (22) *can be written in the following form:*

$$\begin{aligned} \bar{x}_i(k+1) &= A\bar{x}_i(k) + B(v_i(k) + d_i(k)), \\ y_i(k) &= C\bar{x}_i(k), \end{aligned} \tag{23}$$

where d_i is generated by

$$\begin{aligned} \omega_i(k+1) &= A_{i,s}\omega_i(k), \\ d_i(k) &= C_{i,s}\omega_i(k), \end{aligned} \tag{24}$$

for $i \in \{1,\ldots,N\}$, where $A_{i,s}$ is Schur stable.

Proof: The proof of Lemma 1 is given in [38, Appendix A.1] by explicit construction of the pre-compensator (22). ■

Remark 2 *We would like to make several observations:*

1) *The property that the triple (C,A,B) is invertible and has no invariant zero implies that (A,B) is controllable and (C,A) is observable.*
2) *The triple (C,A,B) is arbitrarily assignable as long as the conditions are satisfied. In particular, one can choose the eigenvalues of A in arbitrary desired place.*

Lemma 1 shows that we can design a pre-compensator based on local information z_i to transform the nonidentical agents to almost identical models given by (23) and (24). The compensated model has the same model for each agent except for different exponentially decaying signals d_i in the range space of B, generated by (24).

Now, we design collaborative protocols to solve the scalable output synchronization problem as stated in Problem 2 in three steps. The design procedure is given in Protocol 3.

Then, we have the following theorem for output synchronization of heterogeneous MAS. The proof is given in [23].

Theorem 3 *Consider a heterogeneous MAS described by agent models* (18) *and local information* (19) *satisfying Assumption 2 and associated network communication* (3) *and* (25). *Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

Then the scalable output synchronization problem based on localized information exchange as stated in Problem 2 is solvable. In particular, the dynamic protocol (26) *solves the scalable output synchronization problem for any graph $\mathcal{G} \in \mathbb{G}^N$ with any number of agents N.*

Protocol 3: Scale-free collaborative protocol design for output synchronization of heterogeneous MAS

- *step 1:* First, we choose design parameters (C,A,B) such that
  1) $\operatorname{rank}(C) = p$
  2) (C,A,B) is invertible of uniform rank $n_q \geq \bar{n}_d$, and has no invariant zeros.
  3) eigenvalues of A are in closed unit disc.

  Next, given (C,A,B) and agent models (18) we design pre-compensators as (22) for $i \in \{1,\ldots,N\}$ using Lemma 1. Then, by applying (22) to agent models we get the compensated agents as (23) and (24).
- *step 2:* In this step, we design dynamic collaborative protocols based on information exchange for compensated agents (23) and (24) as follows.

$$\begin{cases} \eta_i(k+1) = A\eta_i(k) + Bv_i(k) + A\hat{x}_i(k) - A\hat{\zeta}_i(k) \\ \hat{x}_i(k+1) = A\hat{x}_i(k) - BK\hat{\zeta}_i(k) + H(\zeta_i(k) - C\hat{x}_i(k)) \\ v_i(k) = -K\eta_i(k), \end{cases}$$

  where K and H are matrices such that $A-BK$ and $A-HC$ are Schur stable, ζ_i is defined in (3) while ρ_i is chosen as $\rho_i = \eta_i$ in (8) which yields: (3) $\hat{\zeta}_i$ is given by:

$$\hat{\zeta}_i(k) = \sum_{j=1}^{N} d_{ij}(\eta_i(k) - \eta_j(k)). \tag{25}$$

- *step 3:* Finally, we combine the designed collaborative protocol for homogenized network in step 2 with pre-compensators in step 1 to get our protocol as:

$$\begin{cases} \xi_i(k+1) = A_{i,h}\xi_i(k) + B_{i,h}z_i(k) - E_{i,h}K\eta_i(k), \\ \hat{x}_i(k+1) = A\hat{x}_i(k) - BK\hat{\zeta}_i(k) + H(\zeta_i(k) - C\hat{x}_i(k)) \\ \eta_i(k+1) = (A-BK)\eta_i(k) + A\hat{x}_i(k) - A\hat{\zeta}_i(k) \\ u_i(k) = C_{i,h}\xi_i(k) - D_{i,h}K\eta_i(k), \end{cases} \tag{26}$$

  where H and K are matrices chosen in step 2.

The architecture of the protocol is shown in Figure 2.

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