Bus Holding of Electric Buses With Scheduled Charging Times

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***Abstract*— In high-frequency electric bus services, bus operators strive to increase the service regularity by minimizing the deviation between the planned and actual headways. In this pursue, bus operators apply corrective control strategies, such as bus holding(s) at control point stops. This study expands the traditional headway-based, bus holding models to cater for the planned arrival times of electric buses at the respective charging points. To this end, this study models - for the first time - the bus holding problem for electric buses considering the scheduled charging times in the objective function. Additionally, it introduces an analytic solution that can return an optimal holding decision in real-time. Our approach is tested using realistic data from bus line 15 in Amsterdam demonstrating a significant reduction of charging delays with only a marginal increase of the average passenger waiting times when compared to existing holding strategies. Its closed-form expression is suitable for real-time holding control and can be applied to obtain a reliable solution or perform stochastic optimization in the case of travel time uncertainty.**

***Index Terms*— Bus holding, service regularity, electric buses, real-time control, electric charging.**

I. Introduction

GLOBAL warming has accelerated the initiatives on the reduction of greenhouse gas (GHG) emissions. By the same token, several countries want to reduce their CO_2 output and limit their dependency on oil by shifting towards other sources of energy. To achieve this goal, bus operations play a major role. In their review of urban energy use and CO_2 emissions patterns in 84 cities around the globe, [1] found that the energy use per bus passenger is typically 2 to 3 times more than the energy used per tram, light rail or metro passenger.

This issue has attracted attention in Europe where several initiatives try to introduce a new generation of buses to modernize the $\simeq$ 40,000 buses registered in the EU [2]. One of the main reasons is that newer, "no-oi" propulsion technologies can significantly reduce the CO_2 emissions [3], [4]. For instance, the policy consultation paper of [5] showed that the CO_2 emissions measured in g/passengers*100 km can be reduced by up to 50 times when using electric buses. Besides, the lifecycle analysis by [6] showed that plug-in hybrid electric vehicles emit 50% less GHG compared to gasoline and diesel vehicles.

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Conventional electric buses are charged through traditional plug-in chargers [7] or wireless stationary charging [8]. Wireless charging is generally favorable in services with fixed routes, such as bus systems that operate public transit services [9]. The stationary point of charging can be a dedicated bus stop (e.g., the terminal or a dedicated charging location).

Electric buses typically have an almost identical charging capacity and fixed charging sites. Notwithstanding, planning the overall electric bus system is challenging given that a pool of electric buses might be in need of a limited number of charging sites [10]. This requires meticulous planning of the charging schedules of all operational buses which, oftentimes, takes into consideration the urban power network structure strength [11]. The latter is advisable because electric buses require significantly more energy than private electric vehicles, thus impacting the power grid [12]–[15].

Studies on planning the charging operations have demonstrated the importance of meeting the planned schedules for both the performance of the power grid and the efficiency of the bus operations. For instance, [16] proposed an algorithm for optimally managing a large number of plug-in hybrid electric vehicles charged at a municipal parking station. In addition, [17] proposed a mathematical model of controlled charging, which includes the operational guidelines of the bus company, the capacity, and the energy charges at the charging station.

While there is an extensive body of works on planning the charging at the charging station(s) and scheduling the bus timetables accordingly, the optimal charging plans can be disrupted during the actual operations due to the inherent uncertainty of bus travel times [18], [19]. For instance, exogenous factors such as traffic congestion and traffic light cycles might increase the waiting times of passengers and delay the arrival of the bus at the charging point [20]. The latter might result in the disruption of the charging schedules with several negative effects, such as: (i) charging with an increased energy price; (ii) delay or refuse of charging because the charging point is occupied by another bus; (iii) disruption of future bus dispatches/crew schedules; and (iv) excess strain on the power grid.

This motivates our work: we propose a bus holding control method with the dual objective of (a) minimizing the deviation between the actual and planned headways to improve the regularity of the bus operations, and (b) adhering to the pre-planned charging schedule by arriving at the charging point(s) on time. We note here that a charging schedule is the schedule of the arrival times of all electric buses operating in the course of one day at the respective charging point(s).

In this study, we will focus on bus holding which is the most prominent control method and does not lead to the refusal of passenger boardings [3], [21]. Bus holding has a similar effect as the decrease of traveling speed between bus stops and is implemented at particular control point stops where a bus is held if it has come too close to its preceding one [22]. Since we target urban environments with several electric buses, we note that in such context buses operate under high frequencies where the main objective is to adhere to the planned headways [23], [24].

The remainder of this paper is structured as follows: in section II, we provide a literature review on the corrective control method of bus holding. Section III provides our problem definition and our mathematical program. It also introduces an analytic bus holding solution that considers the service regularity and the scheduled charging times of electric buses. Section IV adjusts our analytic solution to obtain (i) a reliable bus holding solution, or (ii) a solution via stochastic optimization under the presence of travel time uncertainty. Section V demonstrates the solution of our mathematical program in a hypothetical scenario. In addition, it tests the performance of our proposed holding method against the closed-form bus holding method of [25] in a simulation environment of bus line 15 in Amsterdam. Section VI discusses the results of our work and its current limitations. Finally, section VII concludes our work and draws the future research directions.

II. Literature on Bus Holding

Passengers expect to experience waiting times at stops which are at most equal to the scheduled headways of the daily timetable [26]. Nonetheless, the travel time variability during the actual operations results in bus bunching and excessive passenger waiting [27], [28].

Control methods for bus bunching have been studied since the early 1970s [29]. Nevertheless, the bus bunching problem remains a prominent research topic because of its inherent complexity. References [29] and [30] introduced specific key performance indicators to monitor the average passenger waiting times at stops. To minimize the average waiting time of passengers, [30] considered one control point at which buses can be intentionally held.

Typical objectives of bus holding methods are headway adherence [31]–[33], headway regularity [28], [34], and the minimization of passenger waiting and in-vehicle times [35]–[37]. It should be noted here that, as a general practice, buses are not held at every stop because this will increase the passenger inconvenience. In contrast, bus holding is typically allowed at a pre-determined sub-set of important bus stops, known as control points [38]. Notwithstanding, holding buses at all stops might also have its benefits because this can result in reduced holding times given the increased number of potential control locations.

In bus holding, two different directions of research have emerged. One prominent direction models the bus holding problem as a rolling horizon problem where decisions about the holding times of a number of trips are made simultaneously [39], [40]. In this line of research, the bus holding problem is typically modeled as a multivariate mathematical program where collective decisions are made. The second direction is based on closed-form functions of bus arrival times that determine the holding times by considering the differences between the actual and target headways [25], [34], [41]–[44]. Works in the second direction prevent bus bunching from the onset and can return a globally optimal solution, unlike the more detailed multivariate mathematical programs of rolling horizon methods for which one should typically resort to heuristics.

A. Rolling Horizon Methods

Examples of works on bus holding in rolling horizons are the Ph.D. thesis of [45] and the works of [39], [46], [47]. Such works determine simultaneously the holding times of all buses that are expected to operate within a rolling horizon. The optimized holding times can be updated later in time when subsequent buses arrive at the control point.

References [39], [48] assumed that interstation travel times and passenger arrival rates are constant in rolling horizons with short time durations. The holding problem of all running buses in a rolling horizon was modeled as a quadratic program aiming at minimizing the total passenger waiting times. Reference [47] formulated a mathematical model to produce a plan of holding times that cater for expected changes in running times and demand. Its effectiveness was evaluated within a simulation environment.

Reference [35] developed a mathematical program that incorporates vehicle capacity constraints. Their objective was to minimize the total travel times experienced by all passengers in the system resulting in a non-convex, quadratic objective function which cannot be always solved to global optimality. In a later work, [36] investigated two control policies applied within a rolling horizon framework: (i) vehicle holding, which can be applied at any stop, and (ii) holding combined with boarding limits, in which the number of passenger boardings at any stop can be restricted to increase operational speed. The respective mathematical programs were solved using MINOS on an Intel Core2 Duo @ 2.66 GHz with reported running times in the range of 3.8 s - 5.2 s.

Reference [37] utilized a dynamic objective function and a predictive model of the bus system to make decisions on bus holding and stop-skipping (known also as expressing). The uncertain passenger demand was included in the model as a disturbance. The resulting optimization problem was NP-Hard and was solved using an ad hoc implementation of a Genetic Algorithm. Reference [40] used also an algorithm from the area of evolutionary optimization to solve an NP-Hard program which suggests holding times that minimize the waiting times of passengers and account for regulatory constraints.

Reference [49] developed a mathematical control model for holding using real-time information of the locations of buses along a specified route. The model was solved with the simulated annealing metaheuristic. Reference [50] used the stochastic model developed by [51] to derive the trajectories of buses on a single route. Using Marguier's model, [50] developed a bus holding algorithm that is applied each time a bus arrives at the control point stop. To this end, Marguier's model was used to approximate the trajectories of all "upstream" buses. After that, the bus holding time was selected using a line search method because obtaining an analytic solution was not possible given the complexity of deriving the first-order conditions of the optimization problem.

Most of the above-mentioned models resort to (meta)heuristics to solve the respective multivariate mathematical programs because of their inherent complexity.

B. Closed-Form (Threshold-Based) Models

Typically, closed-form expressions are used to determine the holding times by considering the differences between the actual and the target headways. Reference [25] tested two of the most common closed-form expressions for bus holding: (i) the one-headway-based control where a bus is held at a control point stop if its time headway with its preceding bus is lower than a pre-defined threshold; and (ii) the two-headway-based control that considers the time headway of a bus with both its preceding and following bus.

Similarly, [52] set the holding time of a bus to zero if its predicted headway with its following bus is less than or equal to the minimum headway. When the actual vehicle headway is less than the prescribed minimum headway, the following vehicle will be delayed until the minimum headway requirement is satisfied.

Reference [41] proposed an adaptive control scheme that adjusts a bus cruising speed in real-time based on both its front and rear spacings. In line with other closed-form approaches, it had a simple and decentralized logic enabling to correct the effect of traffic disruptions in real-time.

Reference [34] proposed an analytic bus holding solution that changes the headway of each newly arrived bus at a control point stop to the weighted average of its former headway and the former headway of the trailing bus. This approach tends to re-equalize the headways after a disturbance. The major difference of [34] from the previously described works is that it merely normalizes the headways and does not adhere to a target headway value. Thus, it does not use a headway threshold to trigger the bus holding.

Finally, [43], [44] proposed a method consisting of identifying probabilistically the bus that will be the most delayed upon its arrival at a control point stop. Then, they held each preceding bus to prevent the lagging bus from departing with a big gap. Reference [22] also tested schedule-based and headway-based holding strategies where the solution was expressed as a closed-form expression of arrival times and scheduled headways. They tested the importance of setting a maximum holding time and a reliability buffer time in tram line 9 in The Hague.

C. Electric Bus Planning and Contribution

From the above studies, bus holding control methods focus overwhelmingly on improving the bus operations (whether this means maintaining the target headways, reducing the passenger waiting times or limiting the in-vehicle travel times). Therefore, we identify a main research gap: there is a lack of bus holding studies that consider the improvement of the service regularity and, at the same time, cater for the charging requirements of electric buses.

To the best of our knowledge, there are no past studies that develop bus holding models for electric buses. Past works on electric buses focus overwhelmingly on the scheduling of viable electric bus routes [53], [54], and the planning of the daily operations considering the availability of vehicles, the waiting times of passengers, and the charging costs [10], [55]–[57]. For instance, [57] developed a multi-objective particle swarm optimization algorithm for the vehicle scheduling problem of electric buses considering the smoothing of vehicle departure intervals, the minimization of the number of vehicles, and the reduction of total charging costs. In addition, [55] developed column-generation-based algorithms for the vehicle scheduling of electric buses considering both battery swapping and fast charging at a battery station. Recent works have also focused on planning decisions related to the selection of charging station locations, the number of chargers to be installed, and the recharging schedules (see [58]).

The aforementioned studies on route design, charging scheduling, and vehicle scheduling of electric buses are confined to the strategic and tactical planning levels. Thus, they cannot address abnormalities that emerge during the operational stage due to traffic congestion or changes in passenger demand. For this, real-time control measures – such as bus holding – are required to react to the disruptions during the actual operations and maintain the regularity of services.

This study tries to fill this research gap by integrating the planned charging times of electric buses to the bus holding decisions and incorporating both aspects in a mathematical model that can be applied at the operational stage. That is, our work uses as input the pre-defined bus routes, the daily timetables, and charging schedules when applying bus holding in real-time conditions.

The incremental contributions of this work to the state-of-the-art are:

- the development - for the first time - of a bus holding control model applicable to electric buses that accounts for the scheduled charging times and uncertain interstation travel times;
- the introduction of an analytic solution that has an intuitive interpretation and can return a bus holding suggestion in real-time;
- the investigation of its potential gain compared to state-of-the-art bus holding works that do not consider the charging schedules in a simulated scenario of bus line 15 in Amsterdam.

III. Problem Definition and Mathematical Program

A. Problem Definition

The bus holding decision of a bus trip i at a control point stop s is made when it has completed all its boardings/alightings and is ready to depart. In our problem, this holding decision should take into consideration not only the

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TABLE I
NOTATION

Sets	
$S = \langle 1, 2, \ldots \rangle$	ordered set of bus stops
$I = \langle 1, 2, \ldots \rangle$	ordered set of running bus trips
Indices	
s	bus stop
i	bus trip
Parameters	
H_0	target headway
c	holding control parameter, where $c \in \mathbb{R} : 0 \leq c \leq 1$
$T_{i,s}$	earliest possible time after which trip i can depart from stop s because it has completed its passenger boardings and alightings
$\mathbb{E}[t_{i,s}]$	expected travel time of trip i from stop s to the charging location
$\text{Var}[t_{i,s}]$	travel time variation of trip i from stop s to the charging location
ρ_i	scheduled charging time of bus trip i at the charging location. If a trip i does not require charging, $\rho_i = +\infty$
M	a very large number
Decision Variable	
$d_{i,s}$	determined departure time of trip i at stop s

regularity of the service, but also the planned charging time of the bus. As in the majority of works in real-time bus holding (see [28], [34], [41], [50]), we do not consider the effect of overcrowding when holding a bus. This is a reasonable assumption because the scheduling of the vehicle capacity is already performed at the tactical planning stage and includes a capacity buffer in case of increased passenger demand due to holding [59]. Besides, the vehicle capacity problem cannot be addressed in real-time control because it results in non-convex mathematical programs that do not have a globally optimal solution and are computationally intractable (see [36]).

Proceeding to a formal description of our model, we introduce the nomenclature in Table I.

At time instance $T_{i,s}$, bus trip i is ready to depart from the control point s. When we make a holding decision, our objective is to hold the bus trip i at stop s to maintain the target headway, H_0, with its preceding bus trip, $i-1$. At the same time, our holding decision, $d_{i,s} - T_{i,s} \geq 0$, should not result in a delay that will postpone the scheduled charging time, ρ_i, of bus trip i at the charging station. The bus holding decision considers the optimal option for the trip that is ready to depart from the control point stop (see [25]).

The time instance $T_{i,s}$ when a bus trip i is ready to depart from control point stop s is presented in Fig.1. In the time-space diagram of Fig.1 we present the realized and expected trajectories of bus trip i and its preceding trip, $i-1$.

Bus trip i is held at stop s if it is closer to its preceding bus, $i-1$, than the target headway, H_0. That is, if $T_{i,s} - d_{i-1,s} < H_0$. If this is the case, we hold trip i for time $(d_{i-1,s} + H_0) - T_{i,s}$

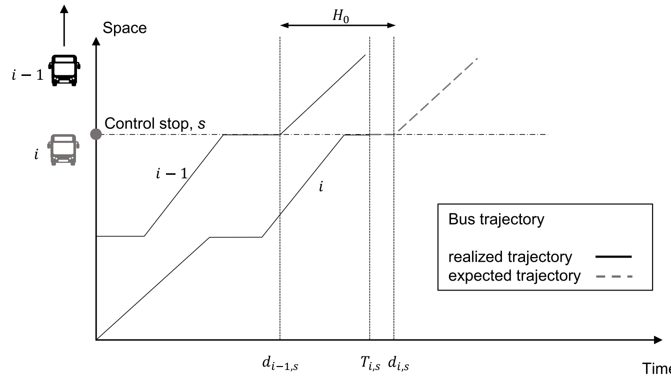

Fig. 1. Illustration of bus trajectories in a time-space diagram.

at the control point stop to eliminate the deviation between the actual and the target headway. In the opposite case where $T_{i,s} - d_{i-1,s} > H_0$, bus trip i will depart as soon as possible because its actual headway with the preceding bus trip $i-1$ is greater than H_0, indicating that it is left behind. This yields the control logic:

$$d_{i,s} = \begin{cases} d_{i-1,s} + H_0 & \text{if } T_{i,s} < d_{i-1,s} + H_0 \\ T_{i,s} & \text{otherwise} \end{cases} \tag{1}$$

where $d_{i,s}$ is the determined departure time of trip i from the control point s and $d_{i,s} - T_{i,s}$ the resulting holding time. The expected travel time of bus trip i from stop s to the charging location is $\mathbb{E}[t_{i,s}]$. If bus trip i needs to reach its charging location before time ρ_i for charging as planned, then:

$$d_{i,s} + \mathbb{E}[t_{i,s}] \leq \rho_i \tag{2}$$

In addition, trip i cannot depart prior to $T_{i,s}$ which is the time when trip i has completed the boardings/alighting at stop s. This yields:

$$T_{i,s} \leq d_{i,s} \tag{3}$$

Note that from the above constraints, the constraint of Eq.(3) is a physical, hard constraint and cannot be violated. Note also that constraints (1),(2) cannot be satisfied in all cases given the conflicting nature of constraints (1)-(3).

B. Mathematical Program

Since constraints Eq.(1),(2),(3) cannot be always satisfied, a hierarchy between soft and hard constraints should be established. Obviously, Eq.(3) is a hard constraint and should receive the highest priority because a bus cannot depart if it has not completed its boardings/alightings.

Lowest in the hierarchy is the equality constraint of Eq.(1) which determines the optimal bus departure from the control point and is more of an objective rather than a constraint. Therefore, we relax it by reformulating it as a problem objective. For this, we introduce the dummy variable:

$$\mu_{i,s} = \begin{cases} 1 & \text{if } T_{i,s} < d_{i-1,s} + H_0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

In addition, we introduce the objective function:

$$f(d_{i,s}) = \mu_{i,s}\big(d_{i,s} - (d_{i-1,s} + H_0)\big)^2 + (1-\mu_{i,s})\big(d_{i,s} - T_{i,s}\big)^2 \quad (5)$$

which returns the optimal value of $d_{i,s}$ when it is minimized. This value is equivalent to the value of the equality constraint of Eq.(1) if constraints Eq.(2) and (3) are satisfied. The proof of this claim is provided in Theorem 2 in the Appendix. Theorem 2 proves that the minimization of our objective function f returns the values of Eq.(1) provided that all other constraints are satisfied. Therefore, we can replace Eq.(1) with f and solve the following mathematical program to obtain the optimal holding solution of bus trip i at control point s:

$$\begin{aligned} (Q_{i,s}): \quad & \min_{d_{i,s}} \; f(d_{i,s}) \\ \text{s.t.:} \quad & d_{i,s} \in \mathcal{F} = \{d_{i,s} \mid d_{i,s} \text{ satisfies Eq. (2), (3)}\} \end{aligned} \quad (6)$$

Solving the mathematical program $(Q_{i,s})$ is equivalent to enforcing the (in)equality constraints Eq.(1),(2),(3) when constraints Eq.(2),(3) are satisfied for the optimal value of $d_{i,s}$. The advantage of mathematical program $(Q_{i,s})$ is that it still computes an optimal value for $d_{i,s}$ even if such solution does not result in meeting the target headway. That is, a trade-off between meeting the planned charging time and adhering to the target headway is established. Toward this end, the objective function $f(d_{i,s})$ will seek solutions close to $d_{i-1,s} + H_0$ if $\mu_{i,s} = 1$ and $T_{i,s}$ if $\mu_{i,s} = 0$ because any deviations from those values are progressively penalized with a squared penalty.

Finally, as we previously stated, there exist cases where the constraint of meeting the scheduled charging time, Eq.(2), and the hard constraint of Eq.(3) are conflicting. Thus, they cannot be satisfied simultaneously. Since Eq.(3) is a hard, physical constraint, we relax Eq.(2) which becomes a *soft* constraint and is allowed to be violated under certain circumstances.

Soft constraints are typically treated as penalty terms and are added to the objective function [60]. In this way, program $(Q_{i,s})$ that, under certain circumstances, has no feasible solution can be transformed to program $(\hat{Q}_{i,s})$. This is achieved by relaxing the inequality constraint of Eq.(2) and adding a penalty for its violation to the objective function. To this end, its relative importance is weighted by introducing a very large number $M \in \mathbb{R}_{\geq 0}$ according to the Big-M theory (see [61]). The scale of M depends on the problem at hand and is selected in such a way to ensure that the satisfaction of the charging constraint is prioritized over the reduction of the objective function value:

$$\begin{aligned} (\hat{Q}_{i,s}): \quad & \min_{d_{i,s}} \; f(d_{i,s}) + M \max(d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i, 0) \\ \text{s.t.:} \quad & d_{i,s} \in \mathcal{F} = \{ d_{i,s} \mid T_{i,s} \leq d_{i,s} \} \end{aligned} \quad (7)$$

Note that the "max" term of $M \max(d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i, 0)$ makes the new objective function of program $\hat{Q}_{i,s}$ non-smooth and the program cannot be always solved to global optimality. To rectify this, we implement the "max" penalty by introducing a new variable ν that, due to its bounds and the direction of optimization, will take the value $\max(d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i, 0)$ at the solution. The reformulated program is:

$$\begin{aligned} (\tilde{Q}_{i,s}): \quad & \min_{\nu, d_{i,s}} \; f(d_{i,s}) + M\nu \\ \text{s.t.:} \quad & d_{i,s} \geq T_{i,s} \\ & \nu \geq 0 \\ & \nu \geq d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i \end{aligned} \quad (8)$$

which can be solved to global optimality and has a unique solution, as shown in Theorem 3 in the Appendix.

C. Analytic Solution

Theorem 3 proves that mathematical program $(\tilde{Q}_{i,s})$ has a unique solution which is also a globally optimal one. Solving program $(\tilde{Q}_{i,s})$ though with an off-the-shelf solver might result in high computational costs that are not acceptable when making a real-time holding decision. More importantly, these computational costs rise even further if we consider the uncertainty of interstation travel times that will require to solve program $(\tilde{Q}_{i,s})$ for an infinite (or at least a very large) number of possible interstation travel time realizations.

To address this issue, we introduce an analytic solution for program $(\tilde{Q}_{i,s})$ that can return an optimal bus holding time in real-time even when considering travel time uncertainty. This analytic solution is detailed in Theorem 1.

Theorem 1: The analytic solution of program $\tilde{Q}_{i,s}$ is

$$d_{i,s} = \begin{cases} \max\{T_{i,s}, \min\{\rho_i - \mathbb{E}[t_{i,s}], d_{i-1,s} + H_0\}\} & \text{for } \mu_{i,s} = 1 \\ T_{i,s} & \text{for } \mu_{i,s} = 0 \end{cases}$$

Proof: For $\mu_{i,s} = 1$, program $\tilde{Q}_{i,s}$ becomes

$$\begin{aligned} & \min_{\nu, d_{i,s}} \; (d_{i,s} - (d_{i-1,s} + H_0))^2 + M\nu \\ \text{s.t.:} \quad & d_{i,s} \geq T_{i,s} \\ & \nu \geq 0 \\ & \nu \geq d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i \end{aligned} \quad (9)$$

Due to constraint $d_{i,s} \geq T_{i,s}$, $d_{i,s}$ can only be greater or equal to $T_{i,s}$. Let $x \equiv d_{i,s}$ for simplifying the notation. Let also $g_1(x, \nu) = T_{i,s} - x \leq 0$, $g_2(x, \nu) = -\nu \leq 0$ and $g_3(x, \nu) = -\nu + x + \mathbb{E}[t_{i,s}] - \rho_i \leq 0$. Then, program $\tilde{Q}_{i,s}$ is equivalent to

$$\begin{aligned} & \min_{\nu, x} \; (x - (d_{i-1,s} + H_0))^2 + M\nu \\ \text{s.t.:} \quad & x, \nu \in \mathcal{F} = \{ x, \nu \mid g_1(x, \nu) \leq 0, \ldots, g_3(x, \nu) \leq 0 \} \end{aligned} \quad (10)$$

which seeks to minimize a function $\theta(x, \nu) \doteq (x - (d_{i-1,s} + H_0))^2 + M\nu$ in the *feasible region* $\mathcal{F}$. From the Karush-Kuhn-Tucker (KKT) conditions, (x^*, ν^*) minimize $\tilde{P}_{i,s}$, if and only if there exist dual variables (KKT multipliers $\lambda_1, \lambda_2, \lambda_3$) such that

(1) $\nabla \mathcal{L}(x^*, \nu^*, \lambda_1, \lambda_2, \lambda_3) = 0$
(2) $\lambda_j g_j(x, \nu) = 0, \quad \forall j \in \{1, 2, 3\}$ (complementary slackness)
(3) $\lambda_j \geq 0, \quad \forall j \in \{1, 2, 3\}$
(4) $(x^*, \nu^*) \in \mathcal{F}$

TABLE II
KKT MULTIPLIER VALUES AT EACH POTENTIAL CASE

Case	λ_1	λ_2	λ_3	Case	λ_1	λ_2	λ_3
I	$=0$	$=0$	$=0$	V	>0	$=0$	$=0$
II	$=0$	>0	$=0$	VI	>0	>0	$=0$
III	$=0$	$=0$	>0	VII	>0	$=0$	>0
IV	$=0$	>0	>0	VIII	>0	>0	>0

where

$$\mathcal{L}(x,\nu,\lambda_1,\lambda_2,\lambda_3) \doteq (x-(d_{i-1,s}+H_0))^2+M\nu+\lambda_1 g_1(x,\nu) + \lambda_2 g_2(x,\nu)+\lambda_3 g_3(x,\nu) \quad (11)$$

Interpreting the multipliers, each constraint $g_j(x,\nu)$ is active (binding) if $\lambda_j > 0$, because in that case $\lambda_j g_j(x,\nu) = 0 \Rightarrow g_j(x,\nu) = 0$. From the KKT conditions, we get the following system of equations

$$\begin{aligned}
&\partial\mathcal{L}/\partial x = 0 \Rightarrow 2(x^* - (d_{i-1,s}+H_0)) - \lambda_1 + \lambda_3 = 0\\
&\partial\mathcal{L}/\partial \nu = 0 \Rightarrow M - \lambda_2 - \lambda_3 = 0\\
&\lambda_1(T_{i,s} - x^*) = 0\\
&-\lambda_2 \nu^* = 0\\
&\lambda_3(-\nu^* + x^* + \mathbb{E}[t_{i,s}] - \rho_i) = 0\\
&\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0\\
&(x^*, \nu^*) \in \mathcal{F}
\end{aligned}$$

The above system of equations can be solved for 2^3 different cases, given the potential combinations of KKT multiplier values presented in Table II.

Let us consider case I ($\lambda_1 = \lambda_2 = \lambda_3 = 0$). Then, the system of equations of the KKT conditions has no feasible solution because we get that $M = 0$ which does not hold.

In case II, solving the above system of equations returns a unique solution $(x^*; \nu^*; \lambda_1; \lambda_2; \lambda_3) = (d_{i-1,s} + H_0; 0; 0; M; 0)$. This means that $x^* = d_{i-1,s} + H_0$.

In case III, there is no feasible solution because we get that $\lambda_3 = 2(d_{i-1,s} + H_0 - x^*)$ and, at the same time, $\lambda_3 = M$.

In case IV, solving the above system of equations returns a unique solution $(x^*; \nu^*; \lambda_1; \lambda_2; \lambda_3) = (\rho_i - \mathbb{E}[t_{i,s}]; 0; 0; M + 2(\rho_i - \mathbb{E}[t_{i,s}] - d_{i-1,s} - H_0); -2(\rho_i - \mathbb{E}[t_{i,s}] - d_{i-1,s} - H_0))$. This means that $x^* = \rho_i - \mathbb{E}[t_{i,s}]$ when $\lambda_3 > 0 \Leftrightarrow -2(\rho_i - \mathbb{E}[t_{i,s}] - d_{i-1,s} - H_0) > 0 \Leftrightarrow \rho_i - \mathbb{E}[t_{i,s}] < d_{i-1,s} + H_0$.

In case V, there is no feasible solution because we get that $M = 0$. In case VI, solving the above system of equations returns a unique solution $(x^*; \nu^*; \lambda_1; \lambda_2; \lambda_3) = (T_{i,s}; 0; 2(T_{i,s} - (d_{i-1} + H_0); M; 0)$. This means that $x^* = T_{i,s}$ when $\lambda_1 > 0 \Leftrightarrow 2(T_{i,s} - (d_{i-1,s} + H_0)) > 0 \Leftrightarrow T_{i,s} > d_{i-1,s} + H_0$.

In case VII, solving the above system of equations returns a unique solution $(x^*; \nu^*; \lambda_1; \lambda_2; \lambda_3) = (T_{i,s}; T_{i,s} - \rho_i + \mathbb{E}[t_{i,s}]; 2(T_{i,s} - d_{i-1,s} - H_0) + M; 0; M)$. This also means that $x^* = T_{i,s}$ when $\lambda_1 > 0 \Leftrightarrow 2(T_{i,s} - (d_{i-1,s} + H_0)) > 0 \Leftrightarrow T_{i,s} > d_{i-1,s} + H_0$. Finally, for case VIII there is no feasible solution because we get that $\mathbb{E}[t_{i,s}] = \rho_i$. Summarizing the above, for given parameter values $T_{i,s}, \rho_i, \mathbb{E}[t_{i,s}], d_{i-1,s}, H_0$ we have the following tree of options (Fig.2).

The solutions from Fig.2 can be succinctly written as

$$x^* = \max\{T_{i,s}, \min\{\rho_i - \mathbb{E}[t_{i,s}], d_{i-1,s} + H_0\}\} \text{ for } \mu_{i,s} = 1$$

and this completes the first part of our proof.

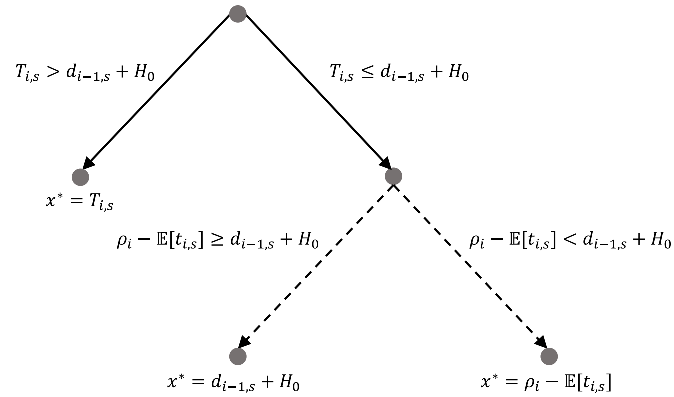

Fig. 2. Solutions of program $\tilde{Q}_{i,s}$ for $\mu_{i,s} = 1$.

Let us now consider the case where $\mu_{i,s} = 0$. Then, program $\tilde{Q}_{i,s}$ becomes

$$\begin{aligned}
\min_{\nu, d_{i,s}} \quad & (d_{i,s} - T_{i,s})^2 + M\nu\\
\text{s.t.:} \quad & d_{i,s} \geq T_{i,s}\\
& \nu \geq 0\\
& \nu \geq d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i
\end{aligned} \quad (12)$$

Note that this program is equivalent to the program of Eq.(9) with the only difference that $d_{i-1,s} + H_0$ is replaced by $T_{i,s}$. Since the program of Eq.(9) attains solution

$$x^* = \max\{T_{i,s}, \min\{\rho_i - \mathbb{E}[t_{i,s}], d_{i-1,s} + H_0\}\} \text{ for } \mu_{i,s} = 1,$$

the solution in the case where $\mu_{i,s} = 0$ becomes:

$$x^* = \max\{T_{i,s}, \min\{\rho_i - \mathbb{E}[t_{i,s}], T_{i,s}\}\} \text{ for } \mu_{i,s} = 0$$

Evidently, $\max\{T_{i,s}, \min\{\rho_i - \mathbb{E}[t_{i,s}], T_{i,s}\}\}$ is always equal to $T_{i,s}$ regardless whether $\rho_i - \mathbb{E}[t_{i,s}] < T_{i,s}$ or not. Hence, the second part of our proof is also complete. □

IV. SOLUTIONS UNDER TRAVEL TIME UNCERTAINTY

A. Finding a Reliable Solution

Until now, our bus holding solution considers the expected travel time, $\mathbb{E}[t_{i,s}]$, of bus trip i from the control point stop s to the charging station. Nevertheless, due to the interstation travel time uncertainty, our bus trip might arrive late at the charging station. A common approach to account for the inherent uncertainty of travel times is the computation of a *reliable* solution. In our search for a reliable solution, we seek a solution $d_{i,s}$ for which we can be confident that bus trip i will arrive at the charging point before time ρ_i. This can be achieved by considering the y-th percentile, t_s^y, of the total travel time from stop s to the charging point. The y-th percentile can be the 90th, 95th, 99th or another percentile according to the needs of the bus operator. It is defined as:

$$t_s^y : Pr(t_{i,s} \leq t_s^y) = \int_{-\infty}^{t_s^y} \phi(t_{i,s})\, dt_{i,s} = y\% \quad (13)$$

where $\phi(t_{i,s})$ is the probability density function of the random variable $t_{i,s}$. I.e., if y is the 95th percentile, then $t_s^{y=95}$: $Pr(t_{i,s} \leq t_s^y) = \int_{-\infty}^{t_s^y} \phi(t_{i,s})\ dt_{i,s} = 95\%$.

Consequently, the average travel time $\mathbb{E}[t_{i,s}]$ in program $\tilde{Q}_{i,s}$ can be substituted by the y-th percentile since it will only exceed that value at (100-y)% of the cases. With this substitution, the reliable solution is found by solving the following program:

$$\begin{aligned}(\tilde{P}_{i,s}): \min_{\nu, d_{i,s}} \quad & f(d_{i,s}) + M\nu \\ \text{s.t.:} \quad & d_{i,s} \geq T_{i,s} \\ & \nu \geq 0 \\ & \nu \geq d_{i,s} + t_s^y - \rho_i \end{aligned} \tag{14}$$

Note that program $\tilde{P}_{i,s}$ inherits the properties of $\tilde{Q}_{i,s}$ because its only difference is the replacement of $\mathbb{E}[t_{i,s}]$ with t_s^y. Thus, the *reliable* bus holding solution of trip i at control point s is:

$$d_{i,s} = \begin{cases} \max\{T_{i,s}, \min\{\rho_i - t_s^y, d_{i-1,s} + H_0\}\} & \text{for } \mu_{i,s} = 1 \\ T_{i,s} & \text{for } \mu_{i,s} = 0 \end{cases}$$

B. Stochastic Optimization

When searching for a reliable bus holding solution, we use the y-th percentile of the random variable $t_{i,s}$. This, however, might lead to a conservative solution that does not fully exploit the improvement potential of service regularity in cases that are close to the average. A potential remedy is to consider the stochastic nature of interstation travel times in the optimization phase and solve the bus holding problem as a stochastic optimization program.

To this end, let us assume that we can obtain the travel time variation Var$[t_{i,s}]$ from historical data. Then, the travel time of trip i from stop s to the charging station is a random variable $t_{i,s}$ that follows some probability distribution $t_{i,s} \sim \mathcal{X}(\bar{y}, \sigma)$ where $\bar{y} \equiv \mathbb{E}[t_{i,s}]$ and $\sigma^2 \equiv$ Var$[t_{i,s}]$. Note that the probability distribution, $\mathcal{X}$, can be derived from historical data. Past works [42], [62], have proposed the use of the normal distribution, $t_{i,s} \sim \mathcal{N}(\bar{y}, \sigma)$, along with the constraint of $t_{i,s} \geq t_{i,s}^{min}$ where $t_{i,s}^{min}$ is the lowest possible travel time under free flow conditions.

This introduces randomness to program $\tilde{Q}_{i,s}$ which is transformed into a stochastic optimization program:

$$\begin{aligned}(P_{i,s}): \min_{\nu, d_{i,s}} \quad & f(d_{i,s}) + M\nu \\ \text{s.t.:} \quad & d_{i,s} \geq T_{i,s} \\ & \nu \geq 0 \\ & nu \geq d_{i,s} + t_{i,s} - \rho_i \\ & t_{i,s} \sim \mathcal{X}(\bar{y}, \sigma) \\ & t_{i,s} \geq t_{i,s}^{min} \end{aligned} \tag{15}$$

In general, the stochastic optimization program $(P_{i,s})$ can be solved with iterative approximation methods that try to minimize the expected value $\mathbb{E}[f(d_{i,s}) + M\nu]$. An example is the Sample Average Approximation (SAA) method [63] that uses a combination of sampling and deterministic optimization to solve $P_{i,s}$.

In more detail, SAA tries to approximate the value of $\mathbb{E}[f(d_{i,s}) + M\nu]$ with the use of Monte Carlo sampling. In this pursue, the problem is optimized for a very large number of realizations $(t_{i,s})_{1=1}^r$ and each time is solved deterministically (as in $\tilde{Q}_{i,s}$). Evidently, solving this problem in real-time for a very large number of realizations $(t_{i,s})_{1=1}^r$ is not possible if one uses an off-the-self solver. To rectify this, our analytic solution can be used to solve the stochastic optimization problem in real-time by merely evaluating the following solutions:

$$d_{i,s} = \begin{cases} \max\{T_{i,s}, \min\{\rho_i - t_{i,s}, d_{i-1,s} + H_0\}\} & \text{for } \mu_{i,s} = 1 \\ T_{i,s} & \text{for } \mu_{i,s} = 0 \end{cases}$$

for each $(t_{i,s})_{1=1}^r$.

TABLE III
PARAMETER VALUES OF THE IDEALIZED SCENARIO

Parameter	Value	Unit
$d_{i-1,s}$	1000	s
$T_{i,s}$	1500	s
H_0	600	s
$\mathbb{E}[t_{i,s}]$	3000	s
M	9000	-

TABLE IV
HOLDING DECISIONS FOR DIFFERENT VALUES OF ρ_i

ρ_i	Analytic Solution	Solving $\tilde{Q}_{i,s}$ with CPLEX		
	$d_{i,s}$	ν	$d_{i,s}$	iterations until convergence
4800 s	1600 s	0 s	1600 s	19
4600 s	1600 s	0 s	1600 s	15
4550 s	1550 s	0 s	1550 s	15
4500 s	1500 s	0 s	1500 s	9
4200 s	1500 s	300 s	1500 s	9

V. NUMERICAL EXPERIMENTS

A. Demonstration

To describe the mechanism behind our control logic, we perform a small demonstration. In our demonstration, we use an idealized scenario to manifest the solution of our model and its underlying control logic. In our idealized scenario, trip i arrives at control stop s and completes its boardings/alightings at time $T_{i,s} = 1500$s. The parameters of our scenario are presented in Table III.

From Table III it is evident that $T_{i,s} < d_{i-1,s} + H_0$, and thus $\mu_{i,s} = 1$.

Let us now display the decisions of our proposed control logic for different values of planned charging times, ρ_i. To perform this task, we report (i) the value of our analytic solution, and (ii) the value of the solution of our mathematical program $\tilde{Q}_{i,s}$. Mathematical program $\tilde{Q}_{i,s}$ is solved in a general-purpose computer with Intel Core i7-455 7700HQ CPU @ 2.80GHz and 16 GB RAM using CPLEX 12.8. The results are summarized in Table IV.

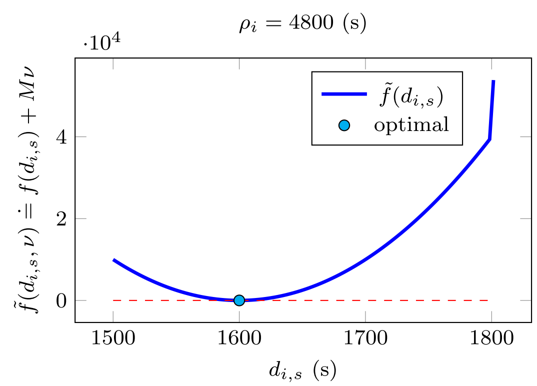

Fig. 3. Performance of the objective function in the region $\mathcal{F} = \{\ \nu = 0, d_{i,s} \geq T_{i,s}\ \}$ for $\rho_i = 4800$ s.

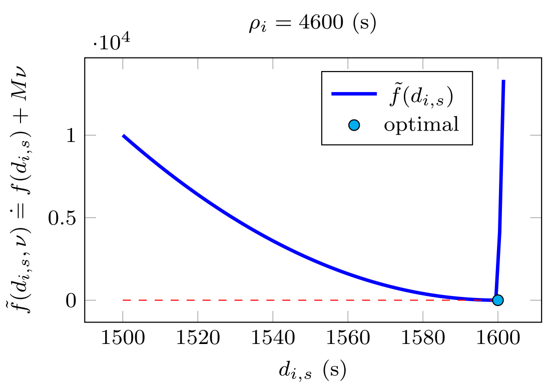

Fig. 4. Performance of the objective function in the region $\mathcal{F} = \{\ \nu = 0, d_{i,s} \geq T_{i,s}\ \}$ for $\rho_i = 4600$ s.

For $\rho_i = 4800$ s, we plot the objective function of program $\tilde{Q}_{i,s}$ in the region $\mathcal{F} = \{\ d_{i,s} \geq T_{i,s}\ \}$ (see Fig.3). From Fig.3 it is evident that the optimal solution is $d_{i,s} = 1600$ s. Note that for $d_{i,s} > 1800$ s, $\nu > 0$ because $d_{i,s} + t_s^y - \rho_i > 0$ and the objective function is penalized by the product $M\nu$.

For $\rho_i = 4600$ s, we also plot the objective function of program $\tilde{Q}_{i,s}$ in the region $\mathcal{F} = \{\ d_{i,s} \geq T_{i,s}\ \}$ (see Fig.4). For $d_{i,s} > 1600$ s, the objective function cost exhibits a sharp increase because the constraint $\nu \geq d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i$ is activated and the objective function is penalized by the large value of $M\nu$.

For $\rho_i = 4550$ s, the constraint $\nu \geq d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i$ is active (where active means that $\nu = d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i$ for $d_{i,s} = 1550$ s). Hence, $d_{i,s} = 1550$ s is the optimal solution of program $\tilde{Q}_{i,s}$ instead of the solution $d_{i,s} = 1600$ s that does not consider the adherence to the planned charging time.

Now, for $\rho_i = 4500$ s, we have an active constraint $\nu = d_{i,s} + \mathbb{E}[t_{i,s}] - \rho_i$ when $d_{i,s} = T_{i,s} = 1500$ s. Finally, for $\rho_i = 4200$ s, bus i is already delayed and cannot meet its scheduled charging time because, even if $d_{i,s}$ admits its minimum possible value, $T_{i,s} + \mathbb{E}[t_{i,s}] > \rho_i$. Therefore, the solution of $\tilde{Q}_{i,s}$ is indeed $d_{i,s} = 1500$ s with $\nu = 300$ s.

The above analysis demonstrates that our analytic solution is equivalent to the solution of program ($\tilde{Q}_{i,s}$) in every possible scenario.

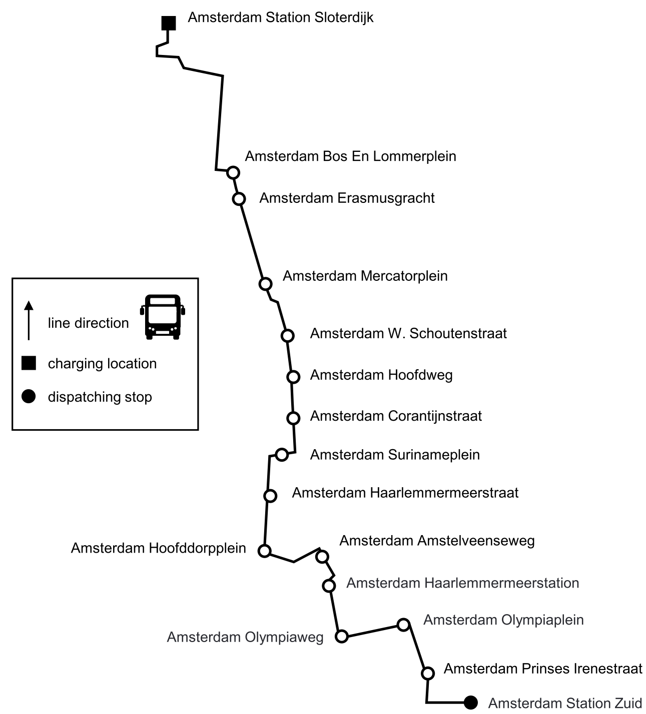

Fig. 5. Bus line 15 in Amsterdam.

B. Simulation-Based Evaluation of Our Control Logic

Holding a bus every time it arrives a control point stop is a local-level decision that might have broader impacts on the entire chain of running trips. For this reason, we investigate the impact of such decisions.

To evaluate our control logic in a systematic manner, we build a simulation of bus line 15 operating in Amsterdam. The first stop of bus line 15 is Amsterdam Station Zuid and the last stop is Amsterdam Station Sloterdijk. Bus line 15 serves 16 bus stops, as presented in Fig.5. The total trip duration for this line is approximately 26 minutes and it is operated by electric buses.

We consider a short time period of the day from 8 am to 9 am with 7 bus trips. We select this period because it is the morning peak resulting in high frequencies. Trips start from Amsterdam Station Zuid (stop 1) and complete their service at Amsterdam Station Sloterdijk (stop 16). When in scheduled service, the buses will be recharged with seven rapid chargers at Sloterdijk station. During the night, the batteries are fully recharged with slow chargers at another location. For the purpose of our simulation-based evaluation, we assume that buses can be held at any intermediate bus stop $\langle 1, 2, 3, \ldots, 15\rangle$.

Bus trips $i = \{1, 2, \ldots, 7\}$ are dispatched every 8 minutes and the target headway H_0 is 8 min, or, equivalently, $H_0 = 480$ s. In addition, our control logic computes a *reliable* holding solution every time a bus trip is ready to depart from an intermediate bus stop considering the 95th percentile of the total travel time from stop s to the charging point, t_s^y.

In the current implementation of our simulation system, we do not explicitly model signal control and other traffic. Instead, we consider the interstation bus travel times (speed) as random variables that follow a probability distribution. This simulation approach has been widely used for validation purposes in bus holding control studies because the selection of an appropriate travel time distribution is able to capture the effects of signal control and traffic congestion [34], [64].

In past literature, normal and lognormal distributions are commonly used to model interstation travel times [25], [44], [65]. In this study, we assume a normal distribution as in [25]. Therefore, the realization of each interstation travel time from stop j to stop $j+1$ is $t_{i,j} \sim \mathcal{N}(\mathbb{E}[t_{i,j}], \text{Var}[t_{i,j}])$ where $\mathbb{E}[t_{i,j}]$ is the expected interstation travel time and $\text{Var}[t_{i,j}]$ its variance for the respective j (see Table V). Given that a sampled travel time from a normal distribution can assume a negative value, we bound the lower value of the interstation travel time $t_{i,j}$ by $t_{i,j}^{min} \in \mathbb{R}_{>0}$, where $t_{i,j}^{min}$ is the minimum possible travel time under free flow conditions. That is to say, in our simulation scenario the interstation travel times, $\tilde{t}_{i,j}$, are sampled from the following probability distribution:

$$\tilde{t}_{i,j} = \max\{t_{i,s}^{min}, t_{i,j} \sim \mathcal{N}\big(\mathbb{E}(t_{i,j}), \text{Var}(t_{i,j})\big)\} \tag{16}$$

Furthermore, our simulation tests assume that passenger arrivals at each bus stop follow a Poisson process, as in [25], [66]. This is a reasonable assumption because several studies have shown that passengers do not coordinate their arrivals at bus stops with the expected arrival times of buses in high-frequency services [50], [67]. In addition, as in [59], [67], the hourly passenger boarding and alighting rates in the simulation tests are assumed to be constant.

In the simulation analysis, four performance measures are used to evaluate the general effects of our control strategy that seeks to minimize the deviation between the actual and the target headways while meeting the charging schedules. These are: (i) the average passenger waiting times, (ii) the average bus travel time, (iii) the missed scheduled charging(s), and (iv) the overall charging delay.

The average passenger waiting times are calculated according to the well-known formula of [66]:

$$\mathbb{E}[W] \doteq \frac{\mathbb{E}[H]}{2} + \frac{\text{Var}[H]}{2\mathbb{E}[H]} \tag{17}$$

where $\mathbb{E}[W]$ is the average passenger waiting time and $\text{Var}[H]$ the headway variance at the control point stop. Table V presents the expected travel times of trips from intermediate bus stops $\langle 1, 2, 3, \ldots, 15 \rangle$ to the charging location, $\mathbb{E}[t_{i,s}]$, their variance, $\text{Var}[t_{i,s}]$, and the 95th percentile, t_s^y. Those values are derived after analyzing two months of historical automated vehicle location (AVL) data from [68].

In addition, the scheduled dispatching and charging times with the rapid chargers at Sloterdijk station are presented in Table VI.

Then, we perform a comparative analysis using as a benchmark the control logic of [25]. In the comparative analysis, the control logic of [25] and the control logic proposed in this study are applied in the same simulation scenario. Similar to the vast majority of works in bus holding, the control logic of [25] is deterministic and does not consider the scheduled charging times of electric buses. The logic of [25] is summarized as:

$$d_{i,s} = \begin{cases} d_{i-1,s} + H_0 & \text{if } T_{i,s} < d_{i-1,s} + cH_0 \\ T_{i,s} & \text{otherwise} \end{cases} \tag{18}$$

TABLE V
TRAVEL TIMES FROM CONTROL POINT STOPS TO THE CHARGING LOCATION IN MINUTES

bus stop	average $\mathbb{E}[t_{i,s}]$	st. deviation $\sqrt{\text{Var}[t_{i,s}]}$	95th percentile t_s^y
1	26.01	4.41	34.20
2	25.00	4.36	33.01
3	22.91	4.35	31.01
4	22.04	4.47	30.55
5	20.01	4.37	28.20
6	18.97	4.41	27.20
7	16.88	4.22	24.86
8	15.13	4.31	23.20
9	14.03	4.38	22.20
10	11.54	3.10	16.11
11	10.74	2.02	14.09
12	10.41	2.18	13.81
13	8.39	2.17	11.81
14	5.96	1.09	7.89
15	5.07	0.91	6.65

TABLE VI
SCHEDULED DISPATCHING AND CHARGING TIMES OF THE 7 SIMULATED TRIPS

Trip, i	Dispatching time	Scheduled charging time, ρ_i
1	8:04	8:32
2	8:12	8:40
3	8:20	8:48
4	8:28	8:56
5	8:36	9:04
6	8:44	9:12
7	8:52	9:20

where $c \in \mathbb{R} : 0 \leq c \leq 1$ and is a holding control parameter (for our evaluation, we set $c = 1$).

To reduce the comparison bias of the comparative analysis, we run 1,000 Monte Carlo simulations with repeated random sampling. For this reason, at each simulation we sample the interstation travel times using Eq.(16) and apply the respective control logic given the underlying status of the simulated operations.

The average values of the four performance measures after using the respective holding control measures at each simulation are summarized in Table VII. The improvement (deterioration) of the performance indicators when applying our proposed control logic instead of the control logic of [25] is: (i) 1.05% deterioration of the average passenger waiting times, (ii) 4.54% improvement of the average trip travel times, and (iii) 55.1% improvement of the overall charging delays.

VI. DISCUSSION

A. Evaluation Results

As shown in Table VII, the average passenger waiting time is increased by 1.05% when applying our control logic. Therefore, passengers will have to wait 2.7 seconds more (on average). This is expected because, unlike past studies that seek to minimize the average passenger waiting times [25], [28], [43], our control logic prioritizes the adherence to the charging schedule.

From our evaluation, one can observe that the deterioration of average passenger waiting times is minimal compared to the potential gains from arriving at the charging points

TABLE VII
AVERAGE PERFORMANCE OF THE TWO CONTROL LOGICS IN 1,000 SIMULATION SCENARIOS

Key Performance Indicator	Control Logic of [25]	Proposed Control Logic
Average Passenger Waiting Time (s)	254.4	257.1
Average Trip Travel Time (s)	1716	1638
Missed Chargings	3	1
Overall Charging Delay (s)	74.59	33.49

on time (i.e., the improvement of charging delays stands at 55.1%). This is one of the main findings of our evaluation and underlines the potential benefit of using our proposed control logic instead of bus holding methods that do not account for the scheduled charging times. To summarize, our bus holding approach reduces significantly the charging delays of electric buses without increasing the average passenger waiting times.

In addition, our control logic has the following indirect positive effects:

- the total trip travel time is reduced by 4.54% (on average). Hence, the in-vehicle travel times of passengers are also reduced;
- bus trips are completed earlier; thus, the dispatches of future trips operated by the same buses are not postponed. This alleviates the negative effects of "schedule sliding" which is a key issue of bus holding approaches [65].

The aforementioned indirect effects are crucial, and other simulation studies, such as [69], have also made it explicit that one should consider the trade-off between passenger wait times and other factors.

B. Limitations

To facilitate the reproducibility of our work, we state the main limitations of our control logic. Those limitations are:

- it can be only applied to high-frequency bus lines that operate under regularity-based schemes;
- it is suitable for correcting the effects of mild disruptions to the service regularity. In the case of severe disruptions, bus operators should consider more radical measures such as changes in the planned service provision and resource allocation;
- as in the vast majority of bus holding works [25], [28], [39], [50], our control logic is suitable in the context where service supply (capacity) is sufficient to ensure that there are no passengers who are unable to board due to overcrowding.

VII. CONCLUSION

This work provided a control logic for bus holding of electric buses. The consideration of the scheduled charging times of electric buses added another dimension to the traditional bus holding problem and this resulted in a novel mathematical program with a convex, quadratic objective function and linear inequality constraints.

Our control logic is proved to have an analytic solution (closed-form expression of departure times, the headway threshold and the scheduled charging time). After carrying out a systematic simulation-based analysis, the following conclusions have been drawn from our case study of bus line 15 in Amsterdam:

- the charging time delay(s) can be reduced by up to 55% with a minimal trade-off of an 1.05% increase in passenger waiting times;
- restraining the holding times due to the charging constraints can improve the total trip travel times by 4.5% and limit schedule sliding effects.

In future research, our approach can be expanded in a wide range of problems involving electric vehicles. For instance, with the proper modifications, future research can expand our method to railway operations that operate under regularity-based schemes.

Other advances could be the expansion towards using a two-headway-based logic to consider the headway with the preceding and the following bus when making a holding decision, and the incorporation of time-varying charging costs in the objective function.

APPENDIX

Theorem 2: Provided that constraints Eq.(2) *and* (3) *are satisfied, the minimizer of* $f(d_{i,s})$ *is equal to* $d_{i-1,s} + H_0$ *if* $T_{i,s} < d_{i-1,s} + H_0$ *and* $T_{i,s}$ *if* $T_{i,s} \geq d_{i-1,s} + H_0$.

Proof: Assuming that Eq.(2) and (3) are satisfied, the global optimum of function f over its domain $[0, +\infty)$ is $d_{i,s}$ such that $f(d_{i,s}) \leq f(x),\ \forall x \in [0, +\infty)$. Given that the devised function f is a smooth quadratic function with continuous first and second-order derivatives for all feasible $x \in [0, +\infty)$, it has a stationary point when $\frac{\partial f}{\partial x} = 0 \Rightarrow 2\mu_{i,s}(x - (d_{i-1,s} + H_0)) + 2(1 - \mu_{i,s})(x - T_{i,s}) = 0$. The stationary point when $T_{i,s} < d_{i-1,s} + H_0$ is $2 \cdot 1(x - (d_{i-1,s} + H_0)) + 2 \cdot 0(x - T_{i,s}) = 0 \Rightarrow x = (d_{i-1,s} + H_0)$. When $T_{i,s} \geq d_{i-1,s} + H_0$, we have $\mu_{i,s} = 0$ and the stationary point becomes $x = T_{i,s}$. The second-order derivative of f is $\frac{\partial^2 f}{\partial x^2} = 2\mu_{i,s} + 2(1-\mu_{i,s}) = 2 > 0$; thus the stationary point is the minimizer of f in $[0, +\infty)$. Hence, the minimizer of f is $d_{i-1,s} + H_0$ if $T_{i,s} < d_{i-1,s} + H_0$ and $T_{i,s}$ if $T_{i,s} \geq d_{i-1,s} + H_0$. $\square$

Theorem 3: A local minimizer of $(\tilde{Q}_{i,s})$ *is also its unique global minimizer.*

Proof: A local minimizer of $(\tilde{Q}_{i,s})$ is the unique global minimizer of $(\tilde{Q}_{i,s})$ if the objective function is strictly convex and the feasible region is a convex set. The feasible region is defined by linear inequalities and is a polyhedron (thus, it is also a *convex set*). Further, we prove that the objective function $f(d_{i,s}) + M\nu$ is strictly convex with respect to $d_{i,s}, \nu$. Let $\tilde{f}(d_{i,s}, \nu) \doteq f(d_{i,s}) + M\nu$. Let also $x \equiv d_{i,s}$ for simplifying the notation. Then, the *Hessian* matrix of $\tilde{f}$ reads:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \tilde{f}(x,\nu)}{\partial x^2} & \frac{\partial^2 \tilde{f}(x,\nu)}{\partial x \partial \nu} \\ \frac{\partial^2 \tilde{f}(x,\nu)}{\partial \nu \partial x} & \frac{\partial^2 \tilde{f}(x,\nu)}{\partial \nu^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

To prove the strict convexity of $\tilde{f}$, we should prove that the Hessian matrix is positive definite. That is, $\mathbf{z}^\intercal\mathbf{H}\mathbf{z}$ is positive for every column vector $\mathbf{z} \in \mathbb{R}^2 \setminus \{0, 0\}$. This yields

$$\mathbf{z}^\intercal\mathbf{H}\mathbf{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix} \mathbf{H} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} (2z_1) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 2z_1^2$$

which is positive for any possible values of $z_1, z_2 \in \mathbb{R} \setminus \{0\}$. Thus, $\tilde{f}$ is strictly convex and this completes our proof. □

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