DUAL SELF-TUNING PARAMETER-ROBUST MINIMAX OUTPUT REGULATION OF A FIRST-ORDER PROCESS WITH ELLIPSOIDAL UNCERTAINTY

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Abstract. The parameters of a first-order process with known disturbance bounds are known to lie in an ellipsoidal region. At each sample, a static output feedback is designed which minimizes the maximum absolute output over the disturbance and parameter ranges. Then from the resulting measurements, the ellipsoid is updated according to a specific criterion. This criterion should be chosen for adequate performance of the resulting self-tuning regulator. It is shown that a dual criterion minimizing a weighted sum of ellipsoidal volume and control performance outperforms these separate criteria.

Keywords: self-tuning regulator, bounded disturbance, constrained parameters, membership functions, robust control, closed-loop identification, optimal estimation, duality, convergence analysis.

1. INTRODUCTION

Many practical processes suffer from parametric perturbances. Therefore recently much attention has been given to parameter-robust performance control (e.g. Malan et al., 1996). For such controllers, the worst-case performance increases for lower parameter uncertainty, which may be obtained by bound estimation (e.g. Walter and Piet-Lahanier, 1990). If control cannot wait for estimation, bound estimation and controller redesign should occur at each sample. This is called self-tuning parameter-robust performance control (e.g. Löhnberg and Van de Waal, 1994). It should not be confused with robust self-tuning control, that is a self-tuning certainty-equivalent controller which is robust to structural uncertainty.

Self-tuning parameter-robust control requires fast estimation of bounds, suitable for fast parameter-robust control design. For bound estimation, ellipsoidal bounding is faster than using an exact polytope. Moreover, it is more suitable for fast parameter-robust control than an orthotope. Specially for ellipsoidal parameter bounds, parameter-robust minimax output control (Löhnberg and Römer, 1995) can be designed fast. Therefore this combination is attractive.

Ellipsoidal bounding requires a criterion to choose between the infinite number of ellipsoids containing the real parameter range (e.g. Fogel and Huang, 1982). As such, it looks attractive to choose the ellipsoid that causes the best control performance when used for parameter-robust control design (Löhnberg and Schukkink, 1995). Unfortunately, like some adaptive certainty-equivalence controllers (Polderman, 1989), use of this criterion in adaptive robust control does not yield convergence in case only the controller parameters are identifiable but the process parameters are not.

Inspired by dual solutions for adaptive certainty-equivalence control (e.g. Wittenmark, 1975) and worst-case duality for polytopes (e.g. Graves and Veres, 1995), also for this situation a dual solution was sought which yields good control at the next sample, also decreasing the parameter uncertainty. The method is illustrated for a first-order process. Details have been described by Eisenberg (1995).

2. PROCESS, UNCERTAINTY AND CONTROL

2.1. Process

The method is illustrated for the first-order discrete-time LTI SISO *process*

$$y_{k+1} = -ay_k + bu_k + e_{k+1} =: \phi_{k+1}^T\theta + e_{k+1}, \quad (1)$$

where y_k, u_k and e_k are samples of *output, input* and white *disturbance* respectively at *sample number k*, where

$$|e_k| \le \delta \forall k \quad (2)$$

with known *bound* δ, and where the *regressor* and *process parameter vector* respectively are

$$\phi_{k+1} = \begin{bmatrix} y_k & u_k \end{bmatrix}^T, \quad (3)$$
$$\theta = \begin{bmatrix} -a & b \end{bmatrix}^T, \quad b \ne 0. \quad (4)$$

2.2. Regulator and controlled system

For simplicity, the process is regulated by the static output feedback

$$u_k = -\chi_k y_k, \quad (5)$$

where χ_k is the *controller parameter*. Substitution of control equation (5) into regressor (3) yields

$$\phi_{k+1} = \begin{bmatrix} y_k & -\chi_k y_k \end{bmatrix}^T =: -\mathrm{X}_k y_k, \quad (6)$$

for short notation introducing the *extended control parameter vector*

$$\mathrm{X}_k := \begin{bmatrix} -1 & \chi_k \end{bmatrix}^T. \quad (7)$$

Substitution of output feedback (7) into process model (1) yields controlled process equation

$$\begin{aligned} y_{k+1} &= -ay_k - b\chi_k y_k + e_{k+1} = c_k y_k + e_{k+1} \\ &= -\mathrm{X}_k^T\theta y_k + e_{k+1}, \end{aligned} \quad (8)$$

introducing the *closed loop pole*

$$c_k := -a - b\chi_k = -\mathrm{X}_k^T\theta. \quad (9)$$

A constant controller $\chi_k = \chi$, and hence by (10) $c_k = c$, only allows identification of the line

$$a = b\chi - c \quad (10)$$

and not of the process parameters a and b separately.

2.3. Certainty minimax output regulation

In case of parameter certainty, that is known process parameter vector θ, a simple practical regulation criterion is to minimize the maximum absolute output value over the disturbance range, which with closed loop equation (8) can be written as the *certainty cost function*

$$J(\theta, \mathrm{X}_k) = \max_{|e_{k+1}| \le \delta} |y_{k+1}| = |\mathrm{X}_k^T\theta||y_k| + \delta. \quad (11)$$

Because disturbance bound δ is known, and because y_k does not depend on χ_k, minimization of cost function (11) by *optimal certainty controller* χ^* is equivalent to minimizing the *simplified cost function*

$$J_S(\theta, \mathrm{X}) = |\mathrm{X}^T\theta| = |c| \quad (12)$$

being <1 for stability.

This cost is minimized by the *optimal certainty controller vector* $\mathrm{X}^*(\theta)$ fulfilling

$$|\mathrm{X}^{*T}(\theta)\cdot\theta| = 0. \quad (13)$$

Geometrically this is equivalent to

$$\mathrm{X}^{*T}(\theta) \perp \theta.$$

2.4. Process parameter uncertainty region

At sample k, the process parameter vector θ is bounded by the known *a-priori ellipsoidal region*

$$E_k = \left\{ \theta \middle| \left(\theta - \hat{\theta}_k\right)^T \mathbf{P}_k^{-1} \left(\theta - \hat{\theta}_k\right) \le 1 \right\}, \quad (14)$$

where $\hat{\theta}_k$ is the *a-priori ellipsoidal center*, and $\mathbf{P}_k$ is the symmetric positive definite *a-priori ellipsoidal matrix* representing the ellipsoidal size and orientation. For the example process,

$$\hat{\theta}_k = \begin{bmatrix} -\hat{a}_k \\ \hat{b} \end{bmatrix}, \quad \mathbf{P}_k = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}. \quad (15)$$

2.5. Parameter-robust minimax output control

The parameter-robust control criterion is to minimize the *worst-case cost function*

$$J_W(E_k, \mathrm{X}_k) = \max_{\theta \in E_k} J_S(\theta, \mathrm{X}_k) \quad (16)$$

with $J_S(\theta, X_k)$ (12) having its maxima over $\theta \in E_k$ at the parameter values (Löhnberg and Schukkink, 1995)

$$\theta_{1,2} = \hat{\theta}_k \pm \frac{\mathbf{P}_k X_k}{\sqrt{X_k^T \mathbf{P}_k X_k}}. \tag{17}$$

Substitution of these parameter values (17) into worst-case cost function (16) yields with (12)

$$J_W(E_k, X_k) = \left| X_k^T \hat{\theta}_k \right| + \sqrt{X_k^T \mathbf{P}_k X_k}\,, \tag{18}$$

minimized by the *parameter-robust controller*

$$X_k^*(E_k) = \underset{X_k}{\arg\min}\, J_W(E_k, X_k)\,. \tag{19}$$

This results in the *minimum worst-case control cost*

$$\begin{aligned} J_{C,k} &= J_W\left(E_k, X_k^*(E_k)\right) \\ &= \left| X_k^{*T}(E_k)\hat{\theta}_k \right| + \sqrt{X_k^{*T}(E_k)\mathbf{P}_k X_k^*(E_k)} \\ &= J_{\hat{\theta},k} + J_{\mathbf{P},k}\,, \end{aligned} \tag{20}$$

with the *center certainty equivalent (CCE) controller* $X_k^*\left(\hat{\theta}_k\right)$ fulfilling

$$J_{\hat{\theta},k} = \left| X_k^{*T}\left(\hat{\theta}_k\right)\hat{\theta}_k \right| = 0\,, \tag{21}$$

often being found as the solution of (19), then resulting in a remaining *cost from the uncertainty*

$$J_{C,k} = J_{\mathbf{P},k} = \sqrt{X_k^{*T}\left(\hat{\theta}_k\right)\mathbf{P}_k X_k^*\left(\hat{\theta}_k\right)}\,. \tag{22}$$

3. UNCERTAINTY REGION UPDATE

3.1. Set of possible a-posteriori ellipsoids

By process equation (1) with bounds (2), y_{k+1} and ϕ_{k+1} bound the feasible region for θ between the hyperplanes

$$\begin{aligned} H_{k+1} &= \left\{ \theta \middle| y_{k+1} - \delta \le \phi_{k+1}^T \theta \le y_{k+1} + \delta \right\} \\ &= \left\{ \theta \middle| \left(y_{k+1} - \phi_{k+1}^T \theta\right)^T \delta^{-2} \left(y_{k+1} - \phi_{k1}^T \theta\right) \le 1 \right\}. \end{aligned} \tag{23}$$

The *intersection* of H_{k+1} and prior ellipsoid E_k (14)

$$U_{k+1} := E_k \cap H_{k+1} \tag{24}$$

is contained in an infinite number of potential *a-posteriori ellipsoids*. Of these, the ones which cross the crossings between E_k and H_{k+1} are characterized by (Fogel and Huang, 1982)

$$\begin{aligned} &E_{k+1}(q) = \\ &\left\{ \theta \middle| \begin{aligned} &\left(\theta - \hat{\theta}_k\right)^T \mathbf{P}_k^{-1} \left(\theta - \hat{\theta}_k\right) \\ &+ q\left(y_{k+1} - \phi_{k+1}^T \theta\right)^T \delta^{-2} \left(y_{k+1} - \phi_{k+1}^T \theta\right) \le 1 + q \end{aligned} \right\} \\ &= \left\{ \theta \middle| \left(\theta - \hat{\theta}_{k+1}(q)\right)^T \mathbf{P}_{k+1}^{-1}(q) \left(\theta - \hat{\theta}_{k+1}(q)\right) \le 1 \right\}, \\ &q \ge 0 \end{aligned} \tag{25}$$

with *a-posteriori ellipsoid center and matrix*

$$\hat{\theta}_{k+1} = \hat{\theta}_k + q\mathbf{P}_t \phi_{k+1} R^{-1} v_{k+1}, \quad R = \delta^2, \tag{26}$$

$$\mathbf{P}_{k+1} = \mathbf{P}_t \left(1 + q - \frac{q v_{k+1}^2}{R + q G_{k+1}} \right), \tag{27}$$

$$G_{k+1} = \phi_{k+1}^T P_k \phi_{k+1}, \quad v_{k+1} = y_{k+1} - \phi_{k+1}^T \hat{\theta}_k, \tag{28}$$

$$\mathbf{P}_t = \mathbf{P}_k - q \frac{\mathbf{P}_k \phi_{k+1} \phi_{k+1}^T \mathbf{P}_k}{R + q G_{k+1}}. \tag{29}$$

In new ellipsoid equation (25), *update factor* q should be chosen adequately. It is illustrated by figure 1 that $q = 0$ indicates that the old ellipsoid E_k is kept, whereas for $q = \infty$ the new ellipsoid degenerates to the hyperplanes H_{k+1}.

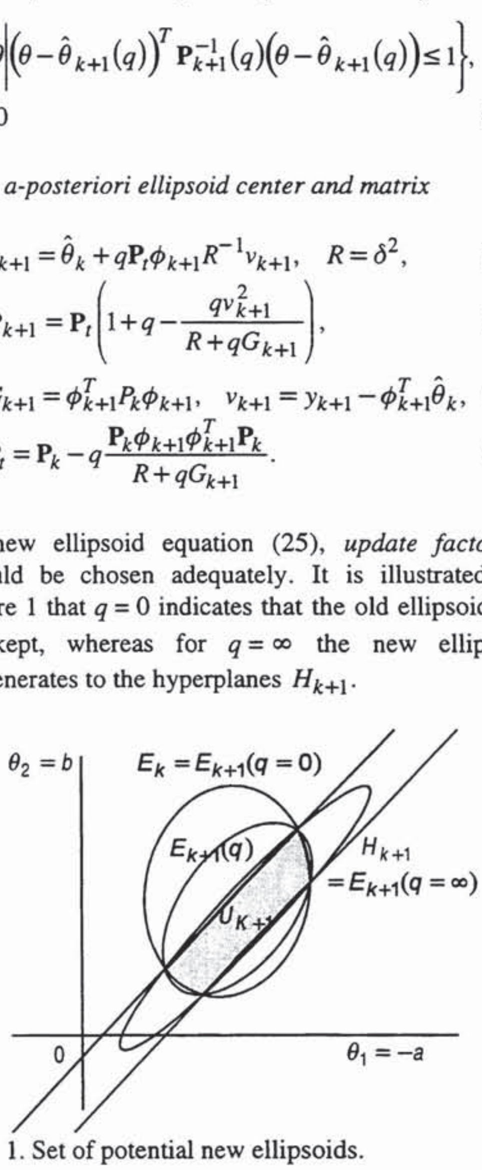

Fig. 1. Set of potential new ellipsoids.

A unique choice of q requires a criterion, for which the most relevant candidates are treated in the next sections.

3.2. Minimum-volume q

Fogel and Huang (1982) suggested to select the value of q which minimizes the *ellipsoidal volume*

$$V_{k+1}'(q) = \pi\sqrt{\left|\mathbf{P}_{k+1}(q)\right|} = \pi\sqrt{V_{k+1}(q)}\,, \tag{30}$$

which is equivalent to minimizing the *ellipsoidal determinant*

$$V_{k+1}(q)=\left|\mathbf{P}_{k+1}(q)\right|=\det\left(\mathbf{P}_{k+1}(q)\right). \tag{31}$$

This yields the *minimum volume q*

$$q_V^*=\arg\min_{q\geq 0}\left|\mathbf{P}_{k+1}(q)\right|, \tag{32}$$

resulting in *minimum ellipsoidal determinant*

$$V_{k+1}^V=V_{k+1}\left(q_V^*\right) \tag{33}$$

with inherently decreasing value

$$0\leq V_{k+1}^V\leq\left|\mathbf{P}_{k+1}(0)\right|=V_k^V. \tag{34}$$

3.3. Optimum performance control q

The most logical identification criterion for control would be to select q such that at the next sample the worst case control cost (20) would be minimized. This yields the *optimal performance control q*

$$\begin{aligned} q_C^* &= \arg\min_{q\geq 0} J_{C,k+1}(q) \\ &= \arg\min_{q\geq 0}\left\{J_{\hat{\theta},k+1}(q)+J_{\mathbf{P},k+1}(q)\right\}. \end{aligned} \tag{35}$$

3.4. Duality q

A compromise between identification and control would be a *dual cost* similar to the stochastic one by Wittenmark (1975)

$$J_{D,k+1}(q,\lambda_{k+1})=J_{C,k+1}(q)+\lambda_{k+1}V_{k+1}(q), \tag{36}$$

where the *parameter uncertainty weight* λ_{k+1} is designed below to decrease the volume V_k. This yields the *duality q*

$$\Rightarrow q_D^*=\arg\min_{0\leq q\leq q_V^*} J_{D,k+1}(q), \tag{37}$$

where $q\leq q_V^*$ can be shown to guarantee

$$V_{k+1}(q)\leq V_{k+1}(0)\equiv V_k. \tag{38}$$

4. ADAPTIVE CONVERGENCE

4.1. Self-tuning regulator convergence problem

Self-tuning coupling of parameter bound estimation and parameter-robust control should converge to reasonable control in spite of poor excitation in closed-loop. The convergence is analyzed and illustrated for a simple example.

4.2. Example process, uncertainty and CE control

The self-tuning behavior is illustrated for the arbitrary unstable process with parameter vector $\theta=[3.5\quad 0.4]^T$, disturbance uniformly distributed with disturbance amplitude bound $\delta=0.5$, initial output $y_0=10$, and initial ellipsoid with $\hat{\theta}_0=[2\quad 2]^T$ and $\mathbf{P}_0=6\mathbf{I}$. This yields the initial CCE controller

$$\chi_0=\frac{-\hat{a}_0}{\hat{b}_0}=\frac{2}{2}=1 \tag{39}$$

with initial closed-loop pole

$$c_0=-a-b\chi_0=3.5-0.4\cdot 1=3.1>1, \tag{40}$$

yielding an unstable controlled system needing adaptation.

4.3. Self-tuning minimum-volume

Eisenberg (1995) showed the self-tuning minimum-volume method of section 3.2 to converge with the CCE controller if after a finite number of samples the ellipsoid does not alter any more. This assumption held in all simulations because new hyperplanes ceased to intersect the previous ellipsoid.

For the example, figure 2 shows a fast decrease in the ellipsoidal volume as expected from the minimum-volume criterion.

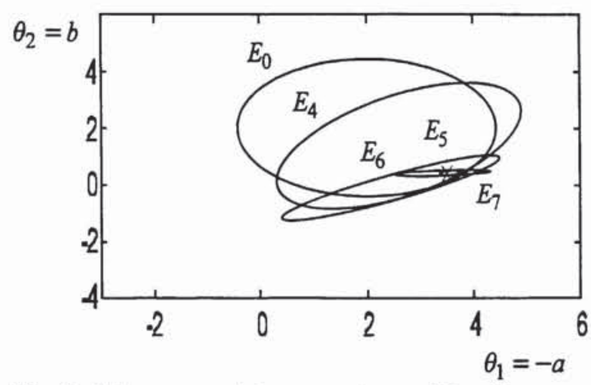

Fig. 2. Subsequent minimum-volume ellipsoids. X = process.

Because robust control performance was not optimized in this choice of q, figure 3 shows a relatively large output value at sample 5.

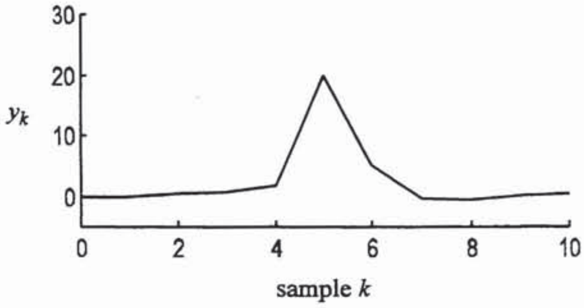

Fig. 3. Output samples for minimum-volume criterion.

4.4. Self-tuning optimum performance

Self-tuning optimum performance adaptation using the optimum performance control q of section 3.3 showed two problems. The first one was that the performance optimization according to (35) was too complicated to carry out on-line.

The second problem was the possibility that $q_C^* = \infty$, so $E_k = H_k$. Then definition (19) of optimal parameter-robust extended controller vector $X_k^*(E_k)$ yields $X_k^* \perp H_k$. Because according to (6), closed-loop regressor ϕ_{k+1} has the direction of X_k^*, also the new hyperplanes $H_{k+1} \perp X_k^*$ and therefore have the same direction as H_k. This is illustrated in figure 4 together with the solution described below. So $q_C^* = \infty$ remains, keeping worst-case cost (16) $J_W(E_k, X_k) = \infty$.

This situation always occurred when replacing new controller $X_{k+1}^*(q)$ by old controller X_k^*, which did converge off-line (Löhnberg and Schukkink, 1995).

Both problems can be solved by taking the CCE controller $X_{CCE,k}$ minimizing the next uncertainty cost (22). Figure 4 shows that because $X_{CCE,k} \perp \hat{\theta}_{k+1}$, $X_{CCE,k}$ is not perpendicular to H_{k+1} and hence $q \neq \infty$ unless H_{k+1} is symmetric relative to $\theta = 0$.

Because of the CCE controller,

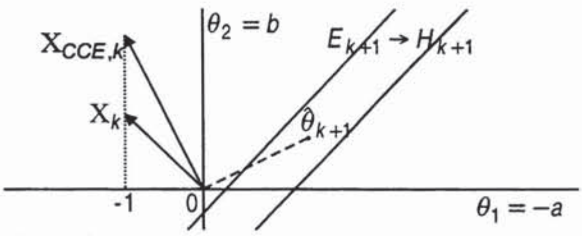

Fig. 4. Optimal and CCE controller.

$$\begin{aligned} q_C^* &= \arg\min_{q \geq 0} J_{\mathbf{P},k+1}^2(q) \\ &= \arg\min_{q \geq 0} X_{k+1}^{*T}(q) \mathbf{P}_{k+1}(q) X_{k+1}^*(q) \\ &\Rightarrow 0 \leq J_{C,k+1}\left(q_C^*\right) \leq J_{C,k}, \; q_C^* < \infty, \; X_k^* < \infty, \quad (41) \end{aligned}$$

which still may stop adapting at an unstable system.

4.5. Self-tuning dual

The constant ellipsoid obtained in section 4.4 can be prevented by not only penalizing the next control cost but also the next volume in dual cost (36).

Adaptation is assured if

$$J_{D,k+1}\left(q_V^*, \lambda_{k+1}\right) < J_{D,k+1}\left(0, \lambda_{k+1}\right). \quad (42)$$

According to dual cost definition (36) with $V_{k+1}(0) = V_k$, condition (42) is equivalent to

$$\lambda_{k+1} > \frac{J_{C,k+1}(0) - J_{C,k+1}\left(q_V^*\right)}{V_k - V_{k+1}\left(q_V^*\right)} =: \lambda_{\min,k+1}, \quad (43)$$

requiring

$$V_k - V_{k+1}\left(q_V^*\right) > \varepsilon, \quad \varepsilon > 0, \quad (44)$$

which always occurred. Otherwise one should take $q_D^* = 0$. Inequality (43) can be assured by taking

$$\lambda_{k+1} = \lambda_{\min,k+1} + \alpha, \quad \alpha > 0. \quad (45)$$

If $q_V = 0$, then take $E_{k+1} = E_k$, and if $\lambda_{\min,k+1} < 0$, choose $\lambda_{\min,k+1} = 0$. The resulting λ_{k+1} appeared to remain bounded.

For the example and $\alpha = 0.01$, figure 5 shows a slower decrease in the ellipsoidal volume relative to minimum-volume ellipsoids of figure 2.

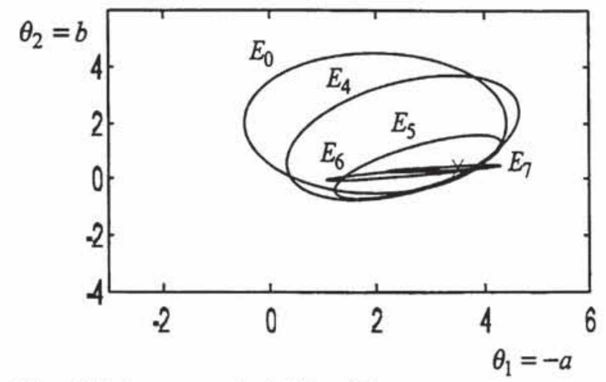

Fig. 5. Subsequent dual ellipsoids;
X = process.

Because robust control performance is part of the dual criterion, figure 6 shows lower output values than the minimum-volume output of figure 3.

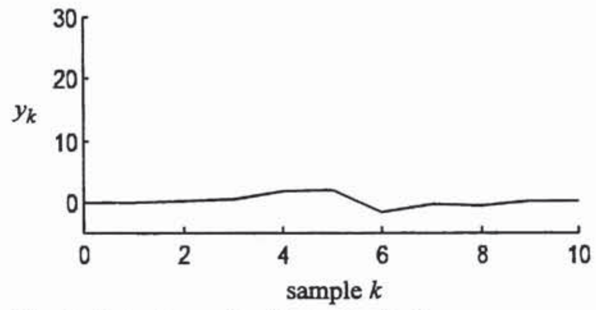

Fig. 6. Output samples for dual criterion.

5. CONCLUSIONS

This paper showed sub-optimal solutions to the problem that optimal self-tuning control minimizing the control cost at the next sampling instant would be complicated and yield a constant controller causing degenerating hyperplanes by non-identifiability.

It was shown that on the one hand minimizing the ellipsoidal volume decreases the uncertainty but yields poor control, and that on the other hand minimizing the certainty-equivalent control cost is simple but yields poor identification.

In contrast, the compromise dual cost yields adequate control and identification.

Although it would be elegant to prove that the parameter uncertainty weight is bounded, no counter-examples were found in all of the many simulation examples.

Even better convergence is expected for identifiable process parameters by using the first-order regulator or preferably controller which also allows disturbance rejection. It remains to be investigated what are the implications for higher-order and/or non-minimum phase processes.

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