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Multi-resource emergency supply contracts with asymmetric information in the after-sales services

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ABSTRACT

In this paper, we investigate the contract design in a multi-resource service supply chain between a first line service provider and an emergency supplier under information asymmetry. The service provider is contractually responsible for the timely repair of the assets that fail, under a given service level agreement with the asset owner. To execute a repair, the service provider needs both engineers and spare parts to replace malfunctioning parts. In case of a spare parts stock out, the service provider can either wait for the regular replenishment of parts from the central depot or decide to hand over the entire call to an emergency supplier. For the latter case, a contract between the service provider and the supplier is necessary that specifies how the emergency supplier is compensated by the service provider. Particularly, we investigate what is the best contract the supplier can offer when information on asset reliability only resides with the service provider but remains hidden for the emergency supplier (information asymmetry). In the first type of contracts, the supplier charges the service provider a price, specified in a so-called price-only contract, for each time he takes over a call. As an alternative, we study the so-called revenue-sharing contracts in which the supplier receives a fraction of the service provider's annual revenue and in return agrees to charge a lower price per call. In addition to the standard (single) revenue-sharing contract, we study the implementation of a menu of revenue-sharing contracts. We show that finding a menu of revenue-sharing contracts is not always possible and, if possible, does not necessarily give a higher profit to the supplier than a single revenue-sharing contract. In an extensive numerical experiment, we show that the combination of the single and the menu of revenue-sharing contracts results in, on average, less than 5% loss of the supplier profit under perfect (symmetric) information. Additionally, we find that, while having private information on the assets' failure rates increases the service provider profit, the increase is insignificant, resulting in an additional profit of only 0.06% on average. Finally, we observe that the supplier can increase his profit, on average, up to 14% if he incites the LSP by means of a side-payment mechanism to share his private information.

1. Introduction

The traditional multi-echelon supply chain literature is typically based on the existence of a central planner who has full information on cost factors and (stochastic) demand parameters and is able to establish the central optimal policy in the network (Axsäter and Rosling, 1993). In a multi-echelon supply chain network, however, generally multiple independent decision makers exist (for a review, see Tsay et al., 1999). Such a supply chain differs from a centrally controlled one in two important aspects. First of all, distinct parties in the supply chain generally have different and often conflicting objectives. Second, there may also exist information asymmetries because each party has private information about his or her cost factors or demand, and no party has full information about the entire supply chain. Although decentralized supply chains with asymmetric information have been broadly studied, the literature in after-sales service logistics with asymmetric information is scarce. The after-sales service supply chain differs from a standard supply chain in an important number of aspects. These include the nature of demand, the inventory management aim, the required response, the product portfolio, the performance metric, and

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the inventory turns (Cohen et al., 2006). The after-sales service chain typically involves different parties including the asset owner, service providers, emergency suppliers, and Original Equipment Manufacturers (OEM). The studies in the after-sales literature where contracts between multiple stakeholders are involved, typically concern the interaction between the asset owner (customer) and the service provider, not the interaction between a service provider and his fall-back emergency option, see e.g., Selviaridis and Wynstra (2015) and Bakshi et al. (2015).

In this paper, we study possible contracting scenarios between a local service provider (LSP) and an emergency supplier (she) in the presence of information asymmetry. The LSP (he) is contractually responsible for the upkeep of a group of assets which are subject to random failures. The service level agreement between the asset owner and the service provider, specifying a maximum average repair lead time that the LSP has to satisfy, and a fixed fee (per time unit) which the asset owner transfers to the LSP, is assumed to be given in this study. To fix an asset failure, the LSP sends a service engineer with the appropriate ready-to-use spare part to replace a malfunctioning part that is detected through a failure diagnosis. The LSP holds a number of spare parts in a local stock and employs a group of service engineers in order to meet the service level agreement he promised to his customer, i.e. the asset owner. Depending on the LSP service policy and on various stochastic system characteristics (timing of failures, the repair time and the spare parts replenishment time), he decides on the various spare parts stock levels and the number of service engineers needed in his staff, such that his total service cost is minimized. The LSP may fully rely on himself in providing the requested spare parts and service engineers and follow a "full backlogging policy" in case of a temporary shortage. Another option is to keep a smaller number of spare parts in stock and to employ fewer service engineers, and revert to an emergency supplier (with ample capacity of resources) in the case of a spare part stock out, i.e. follow a "partial backlogging policy". Both policies have been studied by some of the current authors in detail, see Rahimi-Ghahroodi et al. (2017) and Rahimi-Ghahroodi et al. (2019a). These papers emphasize the benefits of joint planning of spare parts and service engineers compared to the common practice of separated planning. In the partial backlogging policy model (Rahimi-Ghahroodi et al., 2017), the emergency shipment cost is given and is fixed per incidental emergency request. Rahimi-Ghahroodi et al. (2019b) extend this model by studying the interaction of the local service provider and the emergency supplier using Stackelberg game theoretical method. In this game, the emergency shipment cost is defined by the supplier such that it maximizes her profit or in a cooperative game such that it maximizes the total supply chain profit in a win–win situation.

In Rahimi-Ghahroodi et al. (2019b), it is shown that the LSP can propose both cost-sharing and revenue-sharing contracts to the supplier as a cooperation tool, leading to better contracts in a win–win situation. In this paper, we study the same problem, but in a more practical scenario, namely, the case of asymmetric information on the asset's reliability. We assume that both players have full information on each other's cost factors, however, the supplier does not have full information on the failure rate of the assets. There are different scenarios that usually lead to information asymmetry in supply chains. For the problem studied in Rahimi-Ghahroodi et al. (2019b), suppose the supplier as the principal, is not interested in any negotiation or cooperation but instead just offers a "take it or leave it" contract. Confronted with this unwillingness, the LSP may decide to keep some of the information on his side hidden for the supplier in the hope to get a more favorable offer from her. In addition, failures of assets generally occur randomly and in order to properly estimate the failure rate, often plenty of analysis and lots of historical data are needed. Hence, sometimes it is the nature of the problem and not the players' decision that creates the information asymmetry. An example is the situation in which the failure rate of the assets is unknown for both players at the time of contract design (cf. Quigley and Walls, 2007). The LSP may be able, before he responds to the supplier's offer, to obtain a better estimate of the failure rate by performing further analysis and investigations. In such a situation, the LSP has better information on the failure behavior which the supplier is lacking. Our model has various characteristic features which are rarely studied in the literature; e.g., for the supplier it is not always better to charge the highest possible price, see Rahimi-Ghahroodi et al. (2019b). Furthermore, the LSP reservation profit is a function of the asset failure rate (type-dependent), which is typically hidden for the emergency supplier. These latter characteristics make the analysis different from the stream of literature on supply chain contracting with asymmetric information.

The contributions in this paper are summarized as follows:

- We study a novel contracting model in the upstream echelon of a service supply chain under asymmetric information.
- We design the optimal single and menu of revenue-sharing contracts, which the emergency supplier offers in case she has no full information on the assets' failure rate.
- We investigate how much of the supplier profit may be lost because of imperfect information and how much of that can be compensated using a single or a menu of revenue-sharing contracts.
- We examine the potential savings the supplier can achieve by using a side-payment mechanism to incite the LSP to share his information on the assets failure behavior.

The remainder of the paper is organized as follows: in Section 2, we review the literature on contracting in after-sales services and supply chains with information asymmetry. We develop a model of asymmetric demand information in an after-sales service supply chain involving a local service provider and an emergency supplier in Section 3. In Section 4, the best price-only contract the supplier can offer in the case of asymmetric information is investigated. Furthermore, we study whether the supplier is able to achieve a higher expected profit by offering a two parameters revenue-sharing contract, in Section 5. In addition, we introduce the menu of contracts with which the supplier can use to screen the LSP type. In Section 6, the value of perfect information is studied in an extensive numerical experiment, and we show how much loss the supplier can compensate by offering each of the proposed contracts. An approach which the supplier can propose to incite the LSP to share his information is discussed in Section 7. Finally, we offer concluding remarks in Section 8.

2. Literature review

This paper is devoted to the design and development of screening contracts with a principal–agent model framework, Stackelberg game theoretical model, for after-sales service logistics. There are two streams of literature that are most relevant for our paper. The first one explores contract design in supply chains with information asymmetry, and the second one concerns after-sales service contracting.

2.1. Contract design in supply chains with asymmetric information

In order to align incentives between different parties in a supply chain, a wide diversity of contracting strategies has been devised and implemented in industry. This, in turn, has generated a significant stream of academic research in supply contracts; see the review articles by Tsay et al. (1999) and Cachon (2003). While most studies have focused on supply contracts for channels where each party in the supply chain has complete knowledge regarding all the parameters across the channel, there is more and more research on designing contracts for supply chains where there is private information held by just one party. Cachon (2003), Chen (2003) and Chan and Chan (2010) provide reviews of this literature with the focus on information sharing and coordination in supply chains. For a more recent review, we refer to Shen et al. (2018). Deng et al. (2013) consider a loss-averse retailer with information asymmetry.

The literature on contracting problems with asymmetric information can in general be classified into two categories: screening problems where the principal is the uninformed party and offers contracts to the agent (the informed party) to induce information revelation, and signaling problems in which the principal is the informed party and offers contracts to the agent (the uninformed party) to signal the true information. The research reported in this paper belongs to the first category, therefore, we limit ourselves to reviewing papers that study principal–agent supply chains with asymmetric information in which the principal is uninformed (screening).

Corbett (2001) generalizes the classic (Q, r) inventory model to the case of a supplier–retailer supply chain with conflicting objectives and asymmetric information. The supplier makes the lot-sizing decision and incurs a fixed cost for each batch produced, while the retailer determines the reorder point r and is responsible for the holding and backorder costs incurred at the retail site. The author derives screening solutions for two scenarios, namely where the supplier only knows the value of the fixed cost, and where the buyer only observes his backorder costs. Under the full information scenario, it is shown that the jointly optimal inventory policy is achievable. However, when the principal cannot observe the agent's internal cost, the centralized optimal solution becomes less favorable for the principal.

Ha (2001) studies a two-echelon supply chain in which a supplier offers several general side-payment contracts in order to incite a retailer to share the information about his cost parameters. The author shows that in a newsvendor environment in which there is asymmetric information about the retailer's costs, it is optimal for the supplier to not service retailers whose costs exceed a cutoff level. Similarly, Corbett et al. (2004) study a setting in which a supplier is uninformed of the buyer's internal cost. The authors assume that the supplier determines the contract parameters and the buyer chooses his order quantity and the retail price which in turn influences the demand of end consumers. They discuss different types of side-payment contracts in which the supplier offers a transfer payment to the buyer for sharing the information regarding his cost structure.

Burnetas et al. (2007) study the application of a menu of quantity discount contracts for a supplier to screen the retailer's demand information. They show that the supplier can earn larger profits with an all-unit discount. Nevertheless, the proposed contracts do not allow the supplier to extract the first-best channel profit. In this context, Mukhopadhyay et al. (2011) find that sharing demand information benefits the supplier on the expense of the retailer and the supply chain. Arcelus et al. (2008) study the case with price-dependent retailer stochastic demand and identify the conditions under which a retailer benefits from passing on to the supplier his private demand information. In contrast, in a manufacturer–retailer system with private retail cost information, Shen and Willems (2012) find that a set of incentive-compatible contracts consisting of wholesale and buyback prices can coordinate the channel (achieve the optimal centralized solution) for any retail cost.

Esmaeili and Zeephongsekul (2010) study the contract design and coordination problem in a one-supplier and one-retailer supply chain with the case that the seller's setup/purchase costs are unknown to the buyer. Moreover, the buyer withholds certain information related to market demand. They propose, a semi-cooperative model, where sharing marketing expenditure is used as an incentive strategy to reveal private information. Çakanyıldırım et al. (2012) study a similar case in which the supplier has private knowledge on her production cost. The supplier demands a reservation profit that depends on her unit production cost (type-dependent reservation profit). A two parameters contract menu of the order quantity and profit percentage is designed by the retailer and is offered to the supplier. It is shown that information asymmetry alone does not necessarily cause channel inefficiency and the menu mechanism can coordinate the supply chain as long as the supplier reserved profit is neither much overvalued nor much undervalued. Babich et al. (2012) study a buyback contract design problem between a supplier and a retailer in which the retailer has private information on demand. They show that offering the optimal buyback contract by the supplier leads to a coordinated solution where the retailer gets only his reservation profit.

Cao et al. (2013) consider a Stackelberg game between a manufacturer (principal) and a retailer (follower) with asymmetric cost information, where the manufacturer sells the products (to end customers) through the retailer as well as through a direct channel. They assume that the retailer cost is either of type low or high, but that information is not revealed to the manufacturer. The impacts of asymmetric cost information on the equilibrium strategies and profits of different partners have been investigated in detail. Xie et al. (2014) examine a service-oriented manufacturing supply chain with information asymmetry. The manufacturer chooses the product quality and supplies the product to a retailer, who further enhances the product with a particular value-added service before it is sold to the consumers. The costumer's satisfaction depends on the manufacturer's chosen quality and the retailer's service level. They assume that the retailer possesses private demand information. They explore different supply contracts, namely wholesale price, the franchise fee and the retail price maintenance contracts, and identify the one that results in the largest profit for each player and for the supply chain system. Zissis et al. (2015) study a two-node supply chain with one manufacturer producing a single product and one retailer who orders and stores the same product in fixed quantities. They consider the situation where there is some information asymmetry which disturbs the coordination policy. A quantity discount contract is offered by the manufacturer to the retailer in which the manufacturer decides on the discounting and the retailer decides on the quantity. They proved that even with information asymmetry, perfect coordination is feasible under specific conditions.

Kerkkamp et al. (2018) analyze a principal–agent contracting model with asymmetric information between a supplier and a retailer in a classical two-echelon EOQ setting. The retailer has the market power to enforce any order quantity. The supplier wants to minimize her expected costs by offering a menu of contracts with side payments while she has no full information on the retailer's holding cost. They study the situation with two or more types of retailers which have different reserved profits (default option). They propose a sufficient condition which guarantees the existence of unique contracts in the optimal solution for any number of retailer types.

In summary, depending on the dynamics of the system under study, the lack of information on the principal side does not necessarily block the supply chain coordination. The principal can sometimes achieve the centralized optimal solution using a menu-based mechanism. However, she often needs to give the agent more than his reserved profit to enable the coordination.

2.2. After-sales service contracting

In the supply chain contract design literature, after-sales service has received less attention so far. Most studies in after-sales services contracting aim at modeling the interaction of the asset owner (customer) and the service provider, i.e. availability-based contracts, for detailed reviews see Kashani Pour et al. (2016) and Kim et al. (2017). Pricing contracts in the higher echelons of the service supply chains are often needed but are less studied, especially when information is asymmetric. In this paper, we model the contracting in the upstream echelon of after-sales service chains as a game between a service provider and his emergency supplier.

There are a few papers in after-sales service contracting which study the impact of information asymmetry in supply chains. Li et al. (2016) consider performance-based and transaction-based contracts in an after-sales supply chain in which the information about product reliability is only known by the asset owner. The uninformed supplier has to design proper mechanisms by specifying service prices as well as repair capacities so as to overcome the lack of information while maximizing profit. Li and Li (2016) explore service outsourcing with cost information asymmetry. Considering a service seller consigning the service to a service vendor, they examine situations in which the vendor's service cost is either known or unknown to the service seller. They derive optimal contracts for both situations and compare the findings in these two cases to generate insights. A pricing and design problem about after-sales service contracts is investigated by Lan et al. (2017), where a retailer purchases products from a manufacturer and sells these products to consumers. The sales cost is the retailer's private information and can be mined by the manufacturer via techniques from data analytics. They show that the manufacturer can always increase profit if the retailer's cost information is fully learned. Therefore, if the investment cost for data analysis techniques is lower than the expected increase, the investment will be valuable. In addition, they found that, if the expected profit increase is higher than a threshold, the retailer will voluntarily disclose the sales cost information, which leads to a win–win situation.

Product reliability is an important input for after-sales service contracting, so the completeness of product reliability information to different parties is highly important. However, in many cases, knowledge of product reliability is not equally shared throughout the supply chain. This is particularly true when an independent supplier (not an OEM) proposes to provide resources needed for maintaining old equipment that the customer has operated for some time. The supplier does not have access to equipment usage data or failure history, whereas the customer (or the local service provider), i.e., the long-term owner and user of the product, has more accurate information about the product failure rate. The opposite situation is possible as well: if the supplier is also the manufacturer of the product and the product is new, the supplier might have better information about failure rates collected during product development. This opposite setting is considered in Bakshi et al. (2015). They consider an informed principal model and study how reliability can be signaled in after-sales service contracts. See also Liu and Song (2015) in which a screening mechanism is examined in warranty service design when the information on the quality of the product is asymmetric.

One aspect that connects the after-sales service logistics with other supply chain contracting literature is product quality management. There exist studies that investigate contracting in supply chains where the product quality is only known by the manufacturer or supplier. Chao et al. (2009) analyze a situation in which the supplier effort affects the quality level and thus affects the number of recalls after production. They analyze two different types of cost-sharing contracts based on root cause analysis under symmetric and asymmetric information settings. In the asymmetric information scenario, the quality of the supplier's product is not revealed to the manufacturer.

In conclusion, our model differs from the asymmetric information contracting literature in various aspects. The literature on contracting between a service provider and his potential back-up supplier in after-sales service logistics is very limited, especially where the information is asymmetric. Theoretically, our model has some features which are rarely studied in the literature; e.g., for the supplier, it is not necessary optimal to charge the highest possible price because of a sawtooth-like profit function (Rahimi-Ghahroodi et al., 2019b). Furthermore, the LSP reservation profit is a function of the failure rate (type-dependent), therefore it is hidden for the supplier. The inclusion of hidden reservation profit and the sawtooth type supplier profit function makes the analysis different from the stream of literature on supply chain contracting with asymmetric information and increases the structural complexity of optimal contract menus.

3. Model description

We consider a two-echelon after-sales service model with risk-neutral local service provider (LSP) and emergency supplier. The local service provider is contractually responsible for the smooth operation of a group of assets. Under a given service level agreement with the assets owner, he must keep the average repair lead time of failures below a promised level. In return, he receives a fixed fee (per time unit) from the asset owner. For each failed asset, the LSP needs a specific spare part and a service engineer to execute the repair. The LSP keeps these resources locally such that he is able to meet the service level that he promised to the asset owner (maximum average waiting time per repair call). As discussed in Rahimi-Ghahroodi et al. (2017) and Rahimi-Ghahroodi et al. (2019a), the LSP may follow one of two service policy options. He can fully rely on himself in providing the resources and satisfying the repair calls by following a "full backlogging policy". In this policy, spare parts are stocked in sufficient quantities and engineers are employed, while in the case the requested spare parts or a service engineer is not immediately available, the repair call is backlogged until both resources become available again. As an alternative, the service provider can keep less local resources and in the case of spare parts stock out, revert to an emergency supplier with ample capacity of spare parts and service engineers to respond to a repair call. However, the backlogging policy is followed when no service engineer is immediately available upon a request. This policy is studied in Rahimi-Ghahroodi et al. (2017) and is called the "partial backlogging policy". The current paper exploits the results of the model under the partial backlogging policy, but the full backlogging policy is introduced as an alternative for the LSP. In particular, it is known that, if the supplier charges a too high price in case of an emergency shipment, the LSP will use the full backlogging policy instead. Therefore, the emergency supplier needs to carefully evaluate the position of the LSP when she designs a contract. Basic assumptions made in Rahimi-Ghahroodi et al. (2017) and Rahimi-Ghahroodi et al. (2019a), i.e. base stock $(S, S-1)$ spare parts inventory policy, Poisson failure arrival process, and exponential spare parts replenishment time and repair time, also hold in this paper.

The emergency supplier is interested to sign a contract with the LSP to provide him with the requested resources in the case they are not immediately available locally. However, the supplier does not have full information on the failure rate λ of assets as faced by the LSP. The emergency supplier has a probabilistic belief on the failure rate of the assets. This belief is based on historical experience and subjective judgment. To represent information asymmetry in a succinct and analytically tractable way, we assume that all the assets which the LSP is maintaining are either of type l (low failure rate, λ^l) or of type h (high failure rate, λ^h) with $\lambda^h > \lambda^l$. While the LSP observes the assets type perfectly, the supplier believes that the assets the LSP is maintaining are of type l with probability $p \in (0, 1)$.

Given this belief, the supplier needs to decide what is the best contract she can offer to the LSP such that she receives the highest expected profit in the contract. However, the LSP has always the option of following the full backlogging policy (Rahimi-Ghahroodi et al., 2019a). Therefore, the contract that the supplier offers should be at least as profitable for the LSP as in the case of the full backlogging policy. Since the supplier does not know the failure rate and consequently the LSP expected profit in the full backlogging policy, she is facing a more challenging decision to make than in the case of full information (see Rahimi-Ghahroodi et al., 2019b).

In Rahimi-Ghahroodi et al. (2017) and Rahimi-Ghahroodi et al. (2019a), it is assumed that there are multiple types of spare parts and therefore, multiple types of failures. In this paper, for the sake of illustration, we assume that there is just one type of spare part and all failures are of the same type. The assumption of having one type of spare part is relevant in the case where an entire failed system has to be replaced, instead of only some of its constituting parts. Nevertheless, the extension to multiple types of spare parts is rather straightforward.

Table 1
Summary of notations.

S	Spare part stock level
E	Number of service engineers
H	Holding cost per spare part per time unit
O	Hiring cost per service engineer per time unit
U	LSP fixed income per time unit
v	Spare parts regular replenishment rate
v^{em}	Spare parts emergency replenishment rate ($v^{em} > v$)
λ	Failure rate of system (group of assets)
$\lambda_L(S)$	Emergency shipment rate
$W(S,E)$	Expected total waiting time of a repair call (failure) given the stock level and the number of service engineers
W_{max}	Maximum accepted average total waiting time per failure
d	Supplier internal emergency cost per shipment
TP (TP^c)	Total (optimal) expected profit of the supply chain per time unit
FB	Optimal expected profit of the LSP in the full backlogging policy per time unit
SP	Emergency supplier expected profit per time unit
LP	LSP expected profit per time unit
p	Probability that the supplier faces an LSP dealing with assets of type l (low failure rate)
$\mathcal{T}(S,E)$	Transaction cost that the LSP transfers to the supplier in an emergency supply contract per time unit

Some propositions are formulated based on having one type of spare part, and hence they should be modified to handle more spare part types. See Table 1 for a summary of notations.

We investigate a Stackelberg game between the LSP and the emergency supplier in the presence of information asymmetry. The supplier is the principal and she defines the contract terms and offers them to the LSP. The supplier has no perfect information on the assets' failure rate and decides on the best contract terms by considering her probabilistic belief. The LSP can only accept or reject the offer by comparing it with his expected profit when using the full backlogging policy. Regardless of the type of contract between the LSP and the supplier, the supplier incurs an internal cost d for each repair call she satisfies. Note that, in all the following models, we assume that the LSP optimal expected profit using the full backlogging policy (FB), is always less than the total optimal profit under a centralized partial backlogging policy, TP^c (hence with emergency cost per shipment equal to d), otherwise the LSP is always better off by following the full backlogging policy.

The LSP aims to maximize his expected profit while he satisfies the service level (maximum average waiting time) agreed on with the asset owner. The LSP earns a constant income, U, per unit of time. This income is generated from the services offered to his customers and is determined by the market price. Therefore, his optimization problem can be written as

$$(\mathbf{P_{LSP}}) \quad \max_{S,E} LP = \max_{S,E} U - SH - EO - \mathcal{T}(S,E)$$
$$\text{subject to} \quad W(S,E) \le W_{max},$$

where

$$W(S,E) = \left(1 - P^L(S)\right) W_{G/M/E} + \frac{P^L(S)}{v^{em}}, \tag{1}$$

$$P^L(S) = \frac{\rho^S/S!}{\sum_{i=0}^{S} \rho^i/i!}, \tag{2}$$
$$\rho = \frac{\lambda}{v}.$$

The number of spare parts in the replenishment pipeline can be modeled as an $M/M/S/S$ queue. Then, it is easy to show that $P^L(S)$ in (2) equals the loss probability (Erlang B formula) for spare parts because this defines the fraction of repair calls (failures) that is sent to the emergency supplier. Note that the emergency supplier has ample capacity of spare parts and service engineers, therefore there is no queueing for emergency shipment. Eq. (1) gives the total average waiting time of the repair calls. It is the summation of the average waiting time for emergency shipments, which is basically equal to the loss probability times the emergency shipment meantime ($1/v^{em}$) and the average waiting time of repair calls in the service engineers queue. The average waiting time in the service engineers queue is a decreasing function of E, the number of service engineers. The service engineers queue arrival process is in general non-renewal. This correlation between inter-arrival times makes the derivation of the expected waiting time far from straightforward. An accurate method for calculating the expected waiting time in $G/M/E$ queues with general arrival process, exponential service time, and E servers is proposed in Rahimi-Ghahroodi et al. (2017).

$\mathcal{T}(S,E)$ is the transaction cost that the LSP transfers to the supplier in return to the supplier emergency shipments. The exact form of the transaction cost function depends on the contract between the two and will be discussed in detail later. The emergency supplier is interested in maximizing her own profit which is formulated as:

$$SP = \mathcal{T}(S,E) - d\lambda_L(S), \tag{3}$$

where

$$\lambda_L(S) = \lambda P^L(S).$$

Note that the expected supplier profit depends on the spare parts stock level the LSP chooses in response to the transaction cost $\mathcal{T}(S,E)$ (through emergency shipment rate $\lambda_L(S)$). Hence, by charging a specific transaction cost, the supplier indirectly influences her own income because of the reaction of the LSP in choosing S. Moreover, the supplier profit depends on the failure arrival rate, λ, which is unknown to her. Therefore, the supplier needs to choose her optimal decision based on her probabilistic belief on the failure rate. The profit maximization problem of the emergency supplier then reads:

$$(\mathbf{P_S}) \quad \max_{\mathcal{T}} SP = py^l\left(\mathcal{T}(S^l,E^l) - d\lambda_L^l(S^l)\right)$$
$$+(1-p)y^h\left(\mathcal{T}(S^h,E^h) - d\lambda_L^h(S^h)\right)$$
$$\text{subject to} \quad y^l\left(LP^{l*}(\mathcal{T}) - FB^l\right) \ge 0 \tag{4}$$
$$y^h\left(LP^{h*}(\mathcal{T}) - FB^h\right) \ge 0 \tag{5}$$
$$y^l, y^h \in \{0,1\},$$

where

$$\left(S^j, E^j\right) = \underset{S,E}{\operatorname{argmax}}\left\{U - SH - EO - \mathcal{T}(S,E) \mid W(S,E) \le W_{max}, \lambda = \lambda^j\right\}$$
$$j = l, h,$$
$$LP^{j*}(\mathcal{T}) = U - S^j H - E^j O - \mathcal{T}(S^j, E^j)$$
$$j = l, h.$$

As mentioned earlier, if the LSP optimal profit (given the transaction cost the supplier offers) is below the LSP optimal profit under the full backlogging policy (FB), the LSP declines the supplier offer and switches to the full backlogging policy. The auxiliary variables y^l and y^h determine whether the LSP optimal expected profit in each case, i.e., LP^{l*} and LP^{h*} (under low and high failure rate, respectively) is below his optimal full backlogging profit (FB^l and FB^h). If the latter is true, then constraints (4) or (5) push y^l or y^h (or both) to 0, respectively. Hence, if, given a transaction cost, one of the variables y^l or y^h becomes 0, the supplier will not earn any profit if the LSP has assets with the low or high failure rate respectively. This is because in this case, the LSP will reject the contract since it is below his profit under the full backlogging policy. If both y^l or y^h become zero, the supplier will not make any profit at all.

In the following sections, we study different contracts, namely price-only and revenue-sharing contracts, the supplier can offer to the LSP despite her imperfect information on the assets' failure rate.

4. Price-only contract

In this section, we consider the price-only contract in which there is a single contract parameter, the price per emergency shipment C, that the supplier charges to the LSP. In this case, the LSP and the supplier profits are formulated as follow:

$$LP(S,E,C) = U - SH - EO - C\lambda_L(S),$$
$$SP(S,C) = (C-d)\lambda_L(S).$$

The LSP decides on the best spare part stock level and the number of service engineers given the emergency shipment parameter C, to maximize his expected profit. $LP^*(C)$ gives the LSP optimal expected profit as a function of C. More precisely, $LP^*(C)$ reads:

$$LP^*(C) = \max_{S,E}\left\{U - SH - EO - C\lambda_L(S) \mid W(S,E) \le W_{max}\right\}. \quad (6)$$

Given the optimal policy of the LSP, $SP(C)$ gives the supplier expected profit as a function of C:

$$SP(C) = (C-d)\lambda_L^*(C).$$

where

$$\lambda_L^*(C) = \lambda_L(S^*(C)),$$
$$(S^*(C), E^*(C)) = \underset{S,E}{\operatorname{argmax}}\left\{U - SH - EO - C\lambda_L(S) \mid W(S,E) \le W_{max}\right\}.$$

In the case of full information, this problem is investigated in detail in Rahimi-Ghahroodi et al. (2019b). In the next sections, first, we examine the sensitivity of the problem to the asset's failure rate. Afterward, we investigate the best price-only contract which the supplier can offer to the LSP in the presence of information asymmetry in Section 4.3. Before that, we present some propositions regarding the profit functions in price-only contracts which will be used throughout the paper for solving various proposed problems.

4.1. Drop points

In this section, first, we present Proposition 1 which regards the supplier profit function in a price-only contract. This proposition leads to the definition of *drop points* which is introduced in Proposition 2.

Proposition 1. *In a price-only contract, the optimal emergency shipment rate $\lambda^*(C)$ is a decreasing step function of the emergency shipment cost C.*

The proofs of propositions are given in Appendix. As Proposition 1 states, in a given range, there are a finite number of emergency shipment cost values in which the optimal emergency shipment rate decreases. Let us call these values drop points. There exists an emergency shipment cost threshold C_{th}, above which the LSP prefers to not have a contract with the supplier but instead uses the full backlogging policy, see, e.g., Rahimi-Ghahroodi et al. (2019b). $\Delta(C_{th})$ denotes the set of all drop points in the range of $[0, C_{th}]$, where C_{th} is a positive number. Proposition 2 shows how to find the drop points.

Proposition 2. *Each drop point δ_i ($i = 0,1,\ldots,\ \delta_i < \delta_{i+1}$) has a corresponding spare part stock level S_i (not the other way around), i.e.*

$$\left(S_i, E_i\right) = \underset{S,E}{\operatorname{argmax}}\left\{U - SH - EO - C\lambda_L^s(S) \mid W(S,E) \le W_{max},\ C \in \left(\delta_{i-1}, \delta_i\right]\right\}.$$

Given the stock level value, the drop point can be calculated by the equation below

$$\delta_i = \frac{(S_{i+1} - S_i)H + \left(E_{i+1} - E_i\right)O}{\lambda_L(S_i) - \lambda_L(S_{i+1})},$$

where $\lambda_L(S)$ is the emergency shipment rate given a stock level S.

4.2. Importance of the failure rate

As discussed in Rahimi-Ghahroodi et al. (2019b), in a price-only contract, there exists an emergency shipment cost threshold C_{th}, above which the LSP prefers to not have a contract with the supplier but instead uses the full backlogging policy. It is easy to show that this threshold value is sensitive to the failure rate. Since the supplier does not know the true value of the failure rate, she also does not have full information about this threshold value.

To find the optimal strategy for the emergency supplier in the case of asymmetric information, first, we need to find out how the emergency shipment cost threshold value and the LSP profit change with respect to the failure rate. Let us consider the following example:

Example 1. $\lambda = 1$/day, $\nu = 0.2$/day, $\mu = 0.5$/day, $\nu^{em} = 3$/day, $O = 100$ €/day, $H = 130$ €/day, $W_{max} = 0.05$ day, $d =$ €500/shipment, $U =$ €3500/day

For Example 1, Figs. 1 and 2 show how the LSP optimal expected profit and the emergency shipment cost threshold (C_{th}) change with the failure rate (λ), respectively. Although the LSP expected profit decreases monotonically and rather smoothly by increasing the failure rate, we see an unpredictable and non-monotone behavior of the C_{th} versus the failure rate. One may wonder what causes this unpredictable behavior. For the same example, Figs. 3 and 4 show how the optimal emergency shipment rate (λ_L^*) and the optimal supplier expected profit change as functions of the failure rate (λ) for different emergency shipment cost values (C). Observe that even a small change in the failure rate, can decrease or increase the emergency rate and accordingly, the optimal supplier profit considerably. The LSP optimization problem is a non-linear integer optimization with a (usually) binding single constraint. Therefore, a small change in the failure rate may change the optimal solution (stock levels and the number of service engineers) and accordingly, the optimal emergency shipment rate (λ_L^*) drastically.

The supplier needs to know the value of the failure rate to see what is the threshold value for the LSP. Otherwise, if the supplier offers a price higher than this value, the LSP will reject the offer. As shown in Fig. 2, depending on the parameters of the problem, the emergency threshold value of the LSP with the higher failure rate may be higher or lower than the emergency threshold of the LSP with lower failure rate. Given the failure rate, the supplier is able to calculate the emergency shipment cost threshold value, see Rahimi-Ghahroodi et al. (2019b). Suppose the supplier calculates the emergency shipment cost threshold values based on the two failure rates which she has on her belief set, and calls the LSP with higher threshold value the "soft" type, and the LSP with lower threshold value the "tough" type. It means, she believes that the LSP is of soft type with probability q, which equals either p (if the LSP with low failure rate has the higher threshold value) or $1-p$ (if the LSP with high failure rate has the higher threshold value). Hence

$$C_{th}^s > C_{th}^t,$$

where C_{th}^s and C_{th}^t are the emergency shipment cost threshold values of the soft and tough type LSP, respectively.

4.3. Price-only contract with asymmetric information

In this section, we are interested to see what is the best price-only contract the supplier can offer in the presence of asymmetric information. In contrast to Rahimi-Ghahroodi et al. (2019b), the supplier does not know the true value of the failure rate and she is doubting whether the LSP is of the soft or tough type. The supplier problem is formulated as follows:

$$(\mathbf{P^{PO}}) \quad \max_{C} SP(C) = q(C-d)\lambda_L^{s\,*}(C) + (1-q)y^t(C-d)\lambda_L^{t\,*}(C)$$
$$\text{subject to} \quad C \le y^t C_{th}^t + (1-y^t)C_{th}^s$$
$$C > d$$
$$y^t \in \{0,1\}$$

where

$$\lambda_L^{j\,*}(C) = \lambda_L^j(S_j^*(C))$$

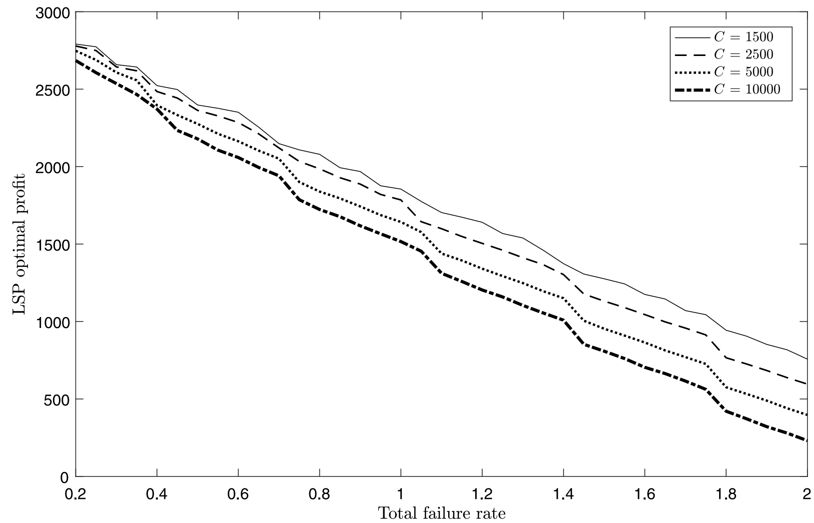

Fig. 1. The LSP optimal expected profit as a function of the failure rate (λ) for different emergency shipment cost (C) in Example 1.

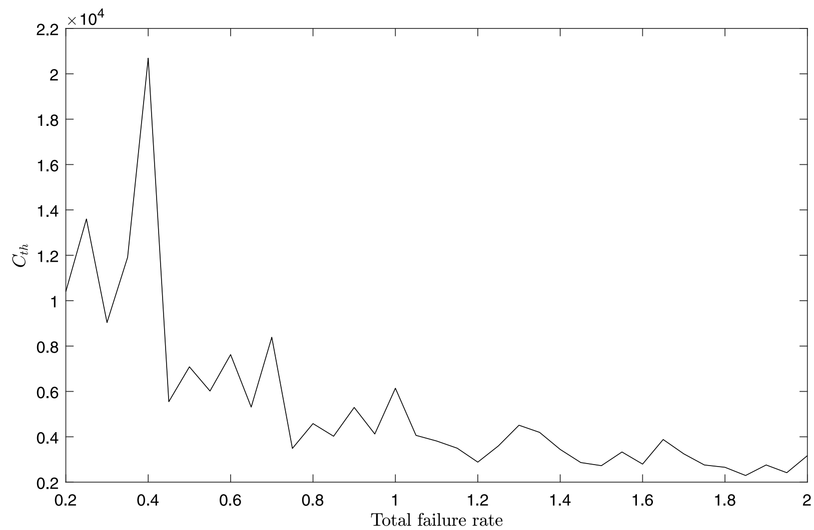

Fig. 2. The emergency shipment cost threshold value as a function of the failure rate (λ) in Example 1.

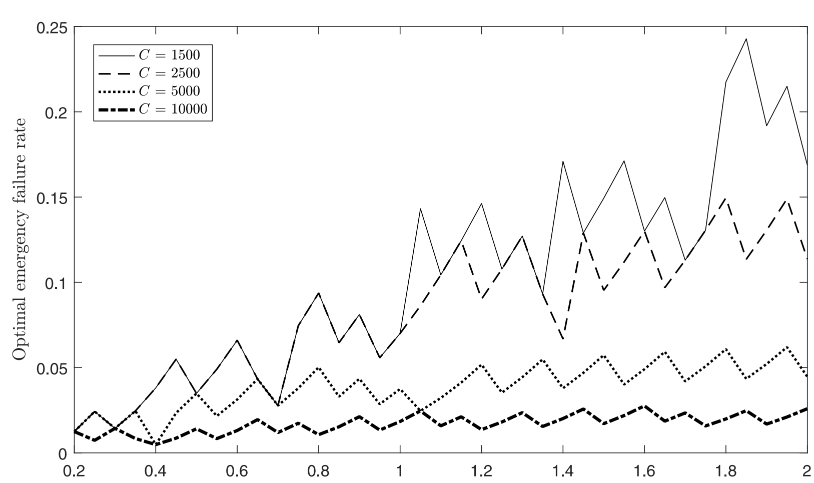

Fig. 3. The optimal emergency shipment rate as a function of the failure rate (λ) for different emergency shipment cost (C) in Example 1.

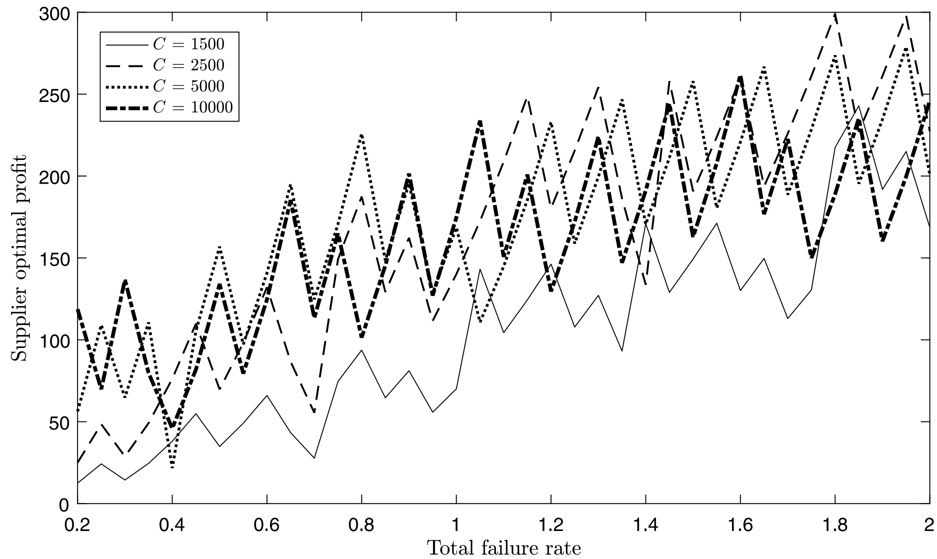

Fig. 4. The supplier optimal expected profit as a function of the failure rate (λ) for different emergency shipment cost (C) in Example 1.

$$j = s, t,$$

$$\left(S_j^*(C), E_j^*(C)\right) = \underset{S,E}{\operatorname{argmax}} \left\{U - S\,H - E\,O - C\lambda_L^j(S) \mid W^j(S,E) \le W_{max}\right\}$$

$$j = s, t.$$

Note that $\lambda_L^{s\,*}(C)(\lambda_L^{t\,*}(C))$ is the optimal emergency shipment rate that the soft (tough) type LSP outsources to the supplier given the emergency shipment cost C. If the supplier offers a price higher than C_{th}^s, then whatever the type of LSP is, he will reject the offer. In case the offer of the supplier is between C_{th}^t and C_{th}^s, then the soft LSP type will accept while the tough LSP will reject the offer. Any emergency shipment price less than and equal C_{th}^t is acceptable for both LSP types. It is obvious that the supplier always offers a price that at least attracts the soft type LSP, therefore the optimal emergency shipment cost is always less than the soft emergency shipment cost threshold value ($C \le C_{th}^s$). The decision variable y^t is equal to 1 if the supplier's offer attracts both types of LSP (i.e., $C \le C_{th}^t$). $W^j(S,E)$ gives the expected total waiting time of the repair calls given that the failure rate is equal to λ^j, $j = s, t$.

In Section 4.1, we introduce the drop points. Proposition 3 suggests how to make use of drop points in order to solve Problem ($\mathbf{P^{PO}}$) efficiently.

Proposition 3. *The optimal solution of Problem* ($\mathbf{P^{PO}}$) *is found in either one of the drop points in the sets $\Delta^s(C_{th}^s)$ (soft LSP) and $\Delta^t(C_{th}^t)$ (tough LSP), or in C_{th}^s or C_{th}^t.*

Proposition 3 shows that to solve Problem ($\mathbf{P^{PO}}$) we only need to search through the drop points and the threshold values. We will use this approach to find the optimal price-only contract in the case of imperfect assets' failure rate information in Section 6. Before that, we investigate other contracts which might enable the supplier to achieve a higher expected profit. In the next section, we introduce the two parameters revenue-sharing contract the supplier can offer to the LSP considering her imperfect information on the failure rate of the assets (LSP type). As a benchmark, we also discuss the coordinated contract in the full information scenario.

5. Revenue-sharing

In this section, we study whether the supplier by offering a revenue-sharing contract is able to achieve a higher expected profit than by offering a price-only contract. In this contract, the supplier receives a fraction of the LSP profit, in return for charging a lower emergency price. This is a two parameters contract (β, C) in which β defines the percentage of the LSP internal revenue (hence excluding emergency shipment cost) that he must transfer to the supplier and C the emergency shipment cost that the LSP needs to pay the supplier per emergency shipment. In this case, the LSP and the supplier profits are formulated as follow:

$$LP(S,E,\beta,C) = (1-\beta)(U - SH - EO) - C\lambda_L(S),$$

$$SP(S,E,\beta,C) = \beta(U - SH - EO) + (C - d)\lambda_L(S).$$

5.1. Benchmark: Coordinated revenue-sharing contract in the full information scenario

The highest total expected profit of the supply chain can be achieved in a contract in which the LSP uses the same number of resources (spare parts and service engineers) as found for the optimal centralized solution. Such a contract is called a *coordinated contract*. In a coordinated contract, if the LSP's share profit is only equal to his reserved profit (his expected profit in the full backlogging policy) out of the total profit, then the supplier will get the highest possible expected profit in this game. In this case, the supplier profit will be

$$SP^* = TP^c - FB,$$

where TP^c is the total optimal centralized profit and FB is the optimal profit of the LSP under the full backlogging policy. As discussed in Rahimi-Ghahroodi et al. (2019b), in the case of full information, this is easily achieved for the supplier. If the supplier offers the following revenue-sharing contract, she will get $TP^c - FB$ profit:

$$\beta = 1 - \frac{FB}{TP^c}, \tag{7}$$

$$C = (1-\beta)d. \tag{8}$$

This contract is not the only revenue-sharing contract that the supplier can offer to get the highest expected profit. Suppose S^c and E^c are the spare part stock level and the number of service engineers in the centralized optimal solution (which maximize the total expected profit):

$$(S^c, E^c) = \underset{S,E}{\operatorname{argmax}} \left\{TP(S,E) \mid W(S,E) \le W_{max}\right\},$$

where

$$TP(S,E) = U - S\,H - E\,O - d\lambda_L(S).$$

This gives

$TP^c = U - S^c H - E^c O - d\lambda_L(S^c).$

In addition, let $(S^*(\beta, C), E^*(\beta, C))$ denote the optimal solution of the LSP expected profit in a revenue-sharing contract with parameters (β, C), i.e.:

$$\left(S^*(\beta,C), E^*(\beta,C)\right) = \underset{S,E}{\operatorname{argmax}} \left\{LP(S,E,\beta,C) \mid W(S,E) \le W_{max}\right\},$$

where

$LP(S, E, \beta, C) = (1-\beta)(U - S H - E O) - C\lambda_L(S).$

The supplier can find all the coordinated revenue-sharing contracts (set of (β, C) values) which gives her $TP^c - FB$ profit by solving the following set of equations:

$$LP(S^c, E^c, \beta, C) = FB, \tag{9}$$
$$\left(S^*(\beta,C), E^*(\beta,C)\right) = (S^c, E^c), \tag{10}$$
$$C \ge 0, \tag{11}$$
$$\beta \in [0,1]. \tag{12}$$

To make sure that the supplier gets the expected profit equal to $TP^c - FB$, the β and C values should be chosen such that the LSP chooses the same stock level (and the number of service engineers accordingly) as S^c (Eq. (10)). This leads to a coordinated solution (with a total profit equal to TP^c). At the same time, the supplier should ensure that the LSP receives just his reserved profit (FB) (Eq. (9)). In this case, the supplier gets all the benefit of coordination. Note that the emergency shipment cost value can be smaller than d, i.e. the supplier will lose money for each emergency shipment. However, she can compensate that by receiving a higher fraction of the LSP profit. Solving the Eqs. (9)–(12) results in the proposition below:

Proposition 4. *Any revenue-sharing contract with parameters that satisfy (13), (14) coordinates the system and gives the supplier the highest possible profit ($TP^c - FB$) and the LSP his expected reserved profit (FB).*

$$\beta \in [\beta_L, \beta_U], \tag{13}$$
$$C = \frac{(1-\beta)(TP^c + d\lambda_L^c) - FB}{\lambda_L^c}, \tag{14}$$

where

$$\beta_L = 1 - \frac{FB\, r^+}{(E^c - E^m(S^c+1))O - H + (TP^c + d\lambda^c)r^+}, \tag{15}$$
$$\beta_U = \min\left(1 - \frac{FB\, r^-}{(E^c - E^m(S^c-1))O + H + (TP^c + d\lambda^c)r^-},\ 1 - \frac{FB}{TP^c + d\lambda_L^c}\right), \tag{16}$$
$$r^+ = 1 - \frac{\lambda_L(S^c+1)}{\lambda_L^c},$$
$$r^- = 1 - \frac{\lambda_L(S^c-1)}{\lambda_L^c}.$$

λ_L^c, S^c, E^c are the emergency rate, spare parts stock level and the number of service engineers in the optimal centralized solution, $\lambda_L(S)$ is the emergency rate given that the stock level is S, and $E^m(S)$ is the minimum number of service engineers that (given the stock level S) satisfies $W(S,E) \le W_{max}$.

It is easy to show that $\beta_L, \beta_U \in [0,1]$. The revenue-sharing fraction value given in (7) is indeed in the range $[\beta_L, \beta_U]$. Inserting this β value in (14) results in $C = (1-\beta)d$(8).

5.2. Emergency shipment cost threshold value in revenue-sharing contracts

In the previous section, we have shown that for any value of $\beta \in [\beta_L, \beta_U]$ (see Eqs. (15) and (16)), if the supplier chooses the emergency shipment cost value given in Eq. (14), she obtains the highest possible expected profit ($TP^c - FB$). This value of emergency shipment cost is also the maximum value that the LSP will accept in a revenue-sharing contract. For a higher C value, the LSP will receive an expected profit lower than FB, hence he will decline the offer and switch to the full backlogging policy. Therefore, for any given sharing fraction value $\beta \in [\beta_L, \beta_U]$, Eq. (17) gives the threshold value for the emergency shipment cost in a revenue-sharing contract above which the LSP will reject the contract:

$$C_{th}(\beta) = \frac{(1-\beta)(TP^c + d\lambda_L^c) - FB}{\lambda_L^c} \tag{17}$$

Since the LSP is transferring a fraction of his internal profit to the supplier, it is obvious that the maximum acceptable emergency shipment cost value (threshold value) for the LSP is lower compared to the price-only contract, i.e.,

$$C_{th}(\beta) \le C_{th}, \quad \beta \in \left[\beta_L, \beta_U\right].$$

For β values outside the range of $[\beta_L, \beta_U]$, the optimal spare part stock level and the optimal number of service engineers are not necessarily the same as for the optimal centralized solution. Therefore, Eq. (17) may not hold. In this case, for a revenue-sharing contract with sharing fraction $\beta \notin [\beta_L, \beta_U]$, the emergency shipment cost threshold value can be found using a bisection search, see Rahimi-Ghahroodi et al. (2019b).

5.3. Single revenue-sharing contract in the presence of asymmetric information

Proposition 4 specifies the revenue-sharing contracts with which the supplier can get the highest possible profit. However, the supplier is able to offer this revenue-sharing contract only if she knows the true failure rate. In case the supplier is not fully informed, she needs to decide on β and C values that maximize her expected profit given her belief on the type of the LSP. In the asymmetric information scenario in which the supplier is doubting between a low and high failure rate, resulting in a high and low threshold value for the emergency shipment cost (see Section 4.2) and hence a soft or tough LSP, the set of equations below describe the supplier problem to find the optimal revenue-sharing contract:

$$(\mathbf{P^{SRS}})\ \max_{\beta,C} SP(\beta,C) = qSP^s(\beta,C) + (1-q)y^t SP^t(\beta,C) \tag{18}$$

subject to

$$SP^j(\beta,C) = \beta\left(U - S^j(\beta,C)H - E^j(\beta,C)O\right) + (C-d)\lambda_L^j\left(S^j(\beta,C)\right)$$
$$j = s,t,$$
$$C \le C_{th}^s(\beta) + y^t(C_{th}^t(\beta) - C_{th}^s(\beta)),$$
$$(S^j(\beta,C), E^j(\beta,C)) = \underset{S,E}{\operatorname{argmax}}\left\{LP^j(S,E,\beta,C) \mid W^j(S,E) \le W_{max}\right\}$$
$$j = s,t,$$
$$y^t \in \{0,1\},$$
$$C \ge 0,$$
$$\beta \in [0,1],$$

where $LP^j(S, E, \beta, C)$, $j = s,t$ denote the soft and tough LSP expected profit in a revenue-sharing contract with parameters (β, C) and given the spare parts stock level S and the number of service engineers E, i.e.

$$LP^j(S,E,\beta,C) = (1-\beta)(U - S H - E O) - C\lambda_L^j(S), \quad j = s,t.$$

$S^j(\beta,C)$ and $E^j(\beta,C)$, $j = s,t$, are the optimal stock level and the number of service engineers which the LSP of type soft or tough will choose, given the emergency shipment cost C and the sharing fraction value β. Given the values $S^j(\beta,C)$, $\lambda_L^j(S^j)$, $j = s,t$, are the optimal emergency shipment rates for soft and tough LSP type. $C_{th}^j(\beta)$, $j = s,t$ give the maximum emergency shipment cost values that the soft or tough type LSP will accept, given the sharing fraction β (see Section 5.2). Theoretically, we can relax the constraints on the emergency shipment cost to be positive and the β value to be between 0 and 1. However, to keep the model justified in its application, we impose these constraints.

Similar to the price-only contract, Proposition 5 suggests an efficient approach to solve Problem $\mathbf{P^{SRS}}$.

Proposition 5. *Given any β value, the optimal emergency shipment cost in Eq. (18) is either $(1-\beta)\delta_i^j, \delta_i \in \Delta^j(C_{th}^j(\beta))$ or $C_{th}^j(\beta)$, $j=s,t$. Here δ_i^s (δ_i^t)s are the drop points for the LSP of type soft (tough), and $C_{th}^s(\beta)$ and $C_{th}^t(\beta)$ are the maximum emergency shipment costs that, given the sharing fraction β, the soft and tough type LSPs accept in a revenue-sharing contract, respectively.*

As Proposition 5 states, to find the optimal single revenue-sharing contract in the case the supplier doubts whether the LSP is soft or tough, we need to search through different β values, and given each value of β, find the best emergency shipment cost C among the drop points and the emergency shipment threshold values. Proposition 6 shows that there is a maximum β value for a feasible revenue-sharing contract. This enables a faster search for the best β value.

Proposition 6. *The maximum sharing fraction value β_0 that the supplier can use in her revenue-sharing contract which still attracts the LSP is when the supplier charges $C=0$. We have:*

$$\beta_0 = 1 - \frac{FB}{LP^*(0)}, \tag{19}$$

where FB is the expected optimal LSP profit under the full backlogging policy and $LP^*(0)$ is the expected optimal LSP profit under partial backlogging and given an emergency shipment cost value equal to 0.

Note that when $C=0$, the LSP will not necessarily send all the failures to the emergency supplier ($S \neq 0$). Even when the emergency shipment is free, the LSP may still need to keep some resources locally to be able to meet the promised service level (average waiting time constraint).

Proposition 7. *Suppose β_0^s and β_0^t give the maximum sharing fractions that the LSP of type soft and tough accept respectively. The optimal sharing fraction in a single revenue-sharing contract in the case of asymmetric information is in the range of $[0, \max(\beta_0^s, \beta_0^t)]$.*

In the presence of information asymmetry, in contrast to the full information scenario, there is no guarantee that the revenue-sharing contract coordinates the system. Nevertheless, in some instances, the single revenue-sharing contract can coordinate the system and gives the supplier the highest expected profit regardless of the LSP type. Suppose β^g is the solution of the equation below (if there exists a solution):

$$\left\{ \frac{(1-\beta)(TP^c + d\lambda_L^c) - FB}{\lambda_L^c} \mid \lambda = \lambda^s \right\} = \left\{ \frac{(1-\beta)(TP^c + d\lambda_L^c) - FB}{\lambda_L^c} \mid \lambda = \lambda^t \right\}$$

β^g is a sharing fraction value that gives the same value for the emergency shipment cost in a coordinated revenue-sharing contract (Eq. (14)) for both soft and tough LSP type. If β^g is in the range given in Eq. (13) for both types of LSP, then the revenue-sharing contract with parameters $(\beta^g, C_{th}(\beta^g))$ is the optimal contract which coordinates the system and gives the supplier the highest expected profit, and the LSP his reserved profit (regardless of his type).

In an extensive numerical experiment in Section 6, the optimal single revenue-sharing contract that the supplier can offer in the presence of information asymmetry is determined for different instances and is compared with the optimal centralized solution and the price-only contract. In the next section, we investigate whether the supplier is able to achieve more in the case of imperfect information on the failure rate by offering a "menu" of revenue-sharing contracts instead of a single one. We are interested to see how far and under what conditions the supplier can increase her expected profit by using the menu mechanism.

5.4. Menu of revenue-sharing contracts

The supplier is doubting between two failure rate values. To solve the problem, the supplier, instead of a single offer, may offer a menu of revenue-sharing contracts. In each contract, she defines the sharing fraction β and the emergency shipment cost C, specifically for each type of LSP. She needs to design such a menu to maximize her expected profit. According to the revelation principle (Myerson, 1981), the supplier can maximize her profits by establishing no more than one menu option for each LSP type. Thus, when there are two LSP types, the supplier seeks to maximize her own profits by offering two menu options, each one intended to be selected by a different type of LSP.

This objective is constrained by a set of individual rationality (IR) and incentive compatibility (IC) constraints. The IR constraints ensure that each LSP type can benefit from participating (he gets at least a revenue equal to his expected profit under the full backlogging policy). The IC constraints ensure that the LSP type j prefers menu option j over any other one that is offered.

Suppose the supplier designs a menu of two contracts with parameters (β^s, C^s) and (β^t, C^t) where the first one is directed to a soft LSP and the second one to a tough LSP. The supplier can find the optimal menu of contracts by solving the problem below:

$$(\mathbf{P^{MRS}}) \quad \max_{\beta^s,\beta^t,C^s,C^t} SP(\beta^s,\beta^t,C^s,C^t) = qSP^s(\beta^s,C^s) + (1-q)SP^t(\beta^t,C^t),$$

subject to

$$SP^j(\beta^j,C^j) = \beta^j\left(U - S^j(\beta^j,C^j)H - E^j(\beta^j,C^j)O\right) + (C^j-d)\lambda_L^j(S^j(\beta^j,C^j)) \quad j=s,t,$$

$$\left(S^j(\beta^j,C^j), E^j(\beta^j,C^j)\right) = \operatorname*{argmax}_{S,E}\left\{LP^j(S,E,\beta^j,C^j) \mid W^j(S,E) \le W_{max}\right\} \quad j=s,t,$$

$$LP^{j*}(\beta^j,C^j) \ge FB^j \qquad j=s,t, \tag{20}$$

$$LP^{s*}(\beta^s,C^s) \ge LP^{s*}(\beta^t,C^t), \tag{21}$$

$$LP^{t*}(\beta^t,C^t) \ge LP^{t*}(\beta^s,C^s), \tag{22}$$

$$\beta^s,\beta^t \in [0,1], \tag{23}$$

$$C^s,C^t \ge 0, \tag{24}$$

where $LP^j(S,E,\beta,C)$, $j=s,t$ denote the soft and tough LSP expected profit in a revenue-sharing contract with parameters (β,C) and given the spare parts stock level S and the number of service engineers E. $LP^{j*}(\beta,C)$, $j=s,t$, denote the expected optimal profits of the soft and tough LSP, respectively, in a revenue-sharing contract with parameters (β,C).

$$LP^j(S,E,\beta,C) = (1-\beta)(U - SH - EO) - C\lambda_L^j(S) \qquad j=s,t,$$

$$LP^{j*}(\beta,C) = \max_{S,E}\left\{LP^j(S,E,\beta,C) \mid W^j(S,E) \le W_{max}\right\} \qquad j=s,t.$$

Inequalities (20) ensure that each LSP type accepts the contract that is designed for him. Inequalities (21) and (22) are needed to make sure that it is not beneficial for each LSP type to choose the contract that is not designed for him.

Lemma 1. *For any feasible menu of revenue-sharing contracts (which satisfies Constraints (20)–(24)), if $\beta^s > \beta^t$, then $C^s < C^t$, and if $\beta^s < \beta^t$, then $C^s > C^t$.*

Proposition 8. *For the optimal menu of revenue-sharing contracts in the problem $\mathbf{P^{MRS}}$, given β^s and β^t values, the optimal C^s and C^t values are either $(1-\beta^j)\delta_i^j, \delta_i \in \Delta^j(C_{th}^j(\beta^j))$ or $C_{th}^j(\beta^j)$, $j=s,t$.*

In practice, we can determine an optimal menu of revenue-sharing contracts numerically. Based on Proposition 8, similar to single revenue-sharing contract, to find the optimal menu of revenue-sharing contract, we need to search on β^s and β^t values (in the ranges $[0, \beta_0^s]$ and $[0, \beta_0^t]$ respectively), and given certain β values, find the optimal C^s and C^t values among drop points or the emergency shipment threshold values. Nevertheless, to get more insight into the optimal solution, it is worthwhile to further analyze the model theoretically.

Lemma 2. *There does not always exist a feasible menu of revenue-sharing contracts.*

This lemma states that the supplier cannot fully rely on a menu of revenue-sharing contracts and sometimes he only has the option of a single revenue-sharing contract. An example in which there is no feasible menu of revenue-sharing contracts:

$\Lambda = \{0.6, 1.2\}, (\mu, \nu, \nu^{em}, O, H, W_{max}, d) = (0.5, 1.5, 3, 100, 125, 0.1, 0)$. Furthermore, in Section 6, we show that, even if a menu of revenue-sharing contracts exists, the optimal menu does not necessarily yield a higher expected profit to the supplier than the optimal single revenue-sharing contract.

For any λ value, there is a feasible region for C and β in which the LSP accepts the offer. When there are soft and tough LSP types, two scenarios can occur regarding their revenue-sharing contracts feasible regions. In the first scenario, any revenue-sharing contract that is feasible for the tough LSP, is feasible for the soft LSP as well, see Fig. 5a. In the second one, the two feasible regions overlap but part of the solution spaces are only feasible for one of them, see Fig. 5b. The values $\beta_0^j, j = s, t$ are the maximum sharing fraction values (when $C = 0$) that the LSP of each type accepts in the offered revenue-sharing contracts (see Proposition 6). If we know the β_0^s and β_0^t values, we can see which of these two scenarios occur.

It is obvious that if $\beta_0^s > \beta_0^t$, the first scenario occurs and any revenue-sharing contract that is feasible for a tough LSP, is also feasible for a soft LSP, and if $\beta_0^s < \beta_0^t$, the second scenario occurs and soft and tough LSPs' feasible regions are only partially overlapping.

Proposition 9. *If $\beta_0^s < \beta_0^t$, then there exists at least one feasible menu of revenue-sharing contracts.*

Proposition 9 shows that, in the case the tough LSP feasible region is not fully included in the soft LSP feasible region, then there always exists a feasible menu of revenue-sharing contracts. However, there is still no guarantee that an optimal menu of revenue-sharing contracts results in a better solution (higher expected profit for the supplier) than the optimal single revenue-sharing contract. In an extensive numerical experiment in Section 6, we compare the single and the menu of revenue-sharing contracts and show to what extent each can capture the value of perfect information.

5.5. Other contracts

Our discussion has focused on revenue-sharing contracts. One may think of other contracts with two or more parameters to be used in the case of information asymmetry. We have investigated contracts with a fixed payment which the LSP transfers to the supplier independent of the number of emergency requests in combination with price-only and revenue-sharing contracts. The contract with fixed and per shipment payments appears to have no advantages over revenue-sharing contracts. We analyze the three parameters contract (V, β, C), see the discussion of cost-sharing contracts in Rahimi-Ghahroodi et al. (2019b), (V is fixed payment, β is revenue-sharing fraction and C the emergency shipment cost) to see whether a menu of these three parameters contracts always coordinates the system. Although the supplier is sometimes able to get a higher profit by offering the menu of this three parameters contract compared to a menu of revenue-sharing contracts, still the coordination is not guaranteed. Therefore, we skip the discussion of these contracts in this paper.

Table 2
Parameter values for numerical analysis.

Parameter	Values	Parameter	Values
λ^l	0.5:0.1:1.5	H	50:25:200
λ^h	[1.2:0.2:2]λ^l	W_{max}	[0.01, 0.02:0.02:0.1]
μ	0.5	U	8000
ν	[0.1, 0.25:0.25:1.5]	d	[0, 500, 1000]
ν^{em}	[1.5, 2, 3, 5]ν	p	[0.25, 0.5, 0.75]
O	100		

6. Numerical study

In the previous section, we presented analytical results that provide some insights into the properties of optimal revenue-sharing contracts. In this section, we perform an extensive numerical study, and we investigate the implications of information asymmetry. Specifically, we address the following questions:

How much does the supplier lose by not knowing the information about the assets failure rate (i.e., the LSP type)? How much value is lost in comparison with the case of perfect information when using the best single or the best menu of revenue-sharing contracts, and how often does the use of these contracts result in the coordinated solution? How much does the LSP gain if the supplier is uninformed about the assets failure rate? How often does information asymmetry lead to an offer of the supplier that will be turned down by the LSP?

The parameters setting of the numerical experiment is given in Table 2. The combination of parameter values given in Table 2 results in 582120 instances. We exclude cases in which the emergency shipment cost threshold value (C_{th}) of at least one of the LSP types is smaller than the supplier internal cost per shipment d (in this case there is no way for the supplier to offer an acceptable contract that results in positive profit for her). This reduces the number of instances to be explored to 83570.

To answer the questions proposed above, we take the optimal price-only contract as the basis to represent the case with asymmetric information. This allows us to show the value of perfect information as well as the value of utilizing revenue-sharing contracts. Whenever we talk about the perfect information case, we are referring to a case in which the supplier knows the exact assets' failure rate. We know that, when the supplier has the full information on assets' reliability, she can achieve the highest profit by offering a revenue-sharing contract. In this case, she obtains the whole benefit of coordination and let the LSP only earns his reserved profit. In the next section, we present the value of knowing the asset's reliability information measured by the impact of this information on the supplier expected profit. In Section 6.2, we discuss the value of using revenue-sharing contracts in the case of information asymmetry. A summary of the results of this numerical study is provided in Tables 3 and 4.

6.1. Value of information

To investigate the value of perfect information on the LSP's assets failure rate, for each of the 83570 instances, we compared the supplier expected profit under perfect information with those in the price-only contract under imperfect information through the following formulation:

$$\mathbf{VI} = \frac{SP^* - SP_{PO}}{SP^*} * 100\%,$$

where SP^* is the optimal supplier profit under full information scenario. We define the optimal supplier's profit under perfect information as a benchmark in Section 5.1. We know in that case, the supplier's profit equals $TP^c - FB$, which is obtained through a revenue-sharing

Table 3
Summary of the results of the numerical study based on 83 570 instances.

Number of instances	Price-only	Single revenue-sharing	Menu	Menu+Single revenue-sharing
Coordinated solution	4	46 714 (55.9%)	46 409 (55.5%)	50 607 (60.6%)
Feasible solution	–	–	82 092 (98.2%)	–

contract with the rightly chosen parameters (see (13)–(14)). This contract also coordinates the system. SP_{PO} is the supplier profit using the best price-only contract under asymmetric information on assets' failure rate. Hence, **VI** gives the increase in the supplier expected profit if she can acquire the assets' failure rate information (LSP type), compared to her profit using the best price-only contract (under asymmetric information).

Based on our numerical study, we found that knowing the assets' failure rate information can increase the supplier profit, on average, by 20.50%. We also observed that, in some cases, the value of information can be as high as 82.36%. The value of information increases when the ratio between the high and low failure rates increases, and the value of information has its maximum value when the supplier presumes the same likelihood for the soft and tough LSP (or low and high assets' failure rate), i,e., $p = q = 0.5$ (which represents the maximum variability of the assets' failure rate), see Table 4.

6.2. Value of revenue-sharing contracts

In the previous section, we show the value of perfect information about the asset's reliability and its significant impact on the supplier profit. This presents an opportunity to capture some of the value of information by implementing more complicated contracts. In this section, we investigate how much of the value of perfect information can be captured through the optimal single or through the optimal menu of revenue-sharing contracts. We know that in the case of asymmetric information, a revenue-sharing contract not necessarily coordinates the system, although in general, it will be better than a price-only contract (a price-only contract is a restricted version of revenue-sharing contracts).

As we showed before, for some instances, there does not necessarily exist a feasible menu of revenue-sharing contract (in around 1.7% of the instances), and as expected, if such a menu exists, it is not necessarily better than the best single revenue-sharing contract. Nevertheless, the menu of revenue-sharing contracts can give the supplier up to 44% higher (1.8 times more) profit than the single revenue-sharing contract. Therefore, it is still beneficial for the supplier to use the menu mechanism. The best strategy for the supplier is to use both approaches and for each specific parameter setting, to choose the one (i.e. either the best single or the best menu of revenue-sharing contracts) that gives him the highest expected profit. To measure the value of using revenue-sharing contracts (single or menu), we use the following formulation:

$$\mathbf{VRS} = \frac{SP_{RS} - SP_{PO}}{SP^*} * 100\%,$$

where SP_{RS} is the optimal supplier profit using the best single or menu of revenue-sharing contracts (the one which results in higher profit). The optimal supplier profit using the single or the menu of revenue-sharing contracts can be determined by solving Problems ($\mathbf{P^{SRS}}$) and ($\mathbf{P^{MRS}}$), respectively (see Sections 5.3 and 5.4).

Based on our numerical study, we found that the combination of single and menu of revenue-sharing contracts, compared to price-only contracts, can increase the supplier profit, on average, by 15.96%. We also observed that the maximum value of the revenue-sharing contract (i.e., the maximum **VRS**) was as high as 79.12%. The expected supplier profit under asymmetric information and using the best single or the best menu of revenue-sharing contracts, on average, is only 4.55% lower than the optimal supplier profit under the full information scenario (i.e., the average of **VI** – **VRS**). Moreover, in more than 60% of all instances, the revenue-sharing contracts lead to the optimal coordinated solution. These observations imply that the combination of a single and a menu of revenue-sharing contracts is an efficient way to deal with information asymmetry. We observe that the same conditions that result in the higher value of information also result in a higher value for the revenue-sharing contracts. This is expected since the single and the menu of revenue-sharing contracts capture a large fraction of the value of information. For more details, see Table 4.

Earlier, we mentioned that the LSP might keep the asset failure rate information hidden for the supplier, in this way hoping to receive a better offer. It is obvious that the LSP will not lose anything if he keeps the failure rate information hidden, since he may expect to receive a minimum profit (i.e. his reserved profit) in the full information scenario. However, how much will he gain when the information on the failure rate is hidden for the supplier, and the supplier offers her best single or menu of revenue-sharing contract? Based on our numerical experiment, on average, the LSP receives only a 0.06% higher profit in the asymmetric information case compared to his reserved profit (full information). The maximum difference we found in our instances is 7.3%. If the LSP is of the soft type (i,e. with a high threshold value), he can expect to get a higher profit (on average 0.1% higher than his reserved profit).

The information asymmetry can affect the LSP in another way as well. Although in the menu of revenue-sharing contract there is always a contract that attracts both types of LSP, this is not the case for the single revenue-sharing contract. If the supplier finds the single revenue-sharing contract the best contract that she can offer, there is a chance that the selected contract is not attractive for the tough LSP. In our numerical experiment, in some 12% of all cases, a tough LSP rejects the optimal supplier offer. Although the LSP gets his reserved profit if he rejects the offer, it is typically not preferable for him to reject the contract and switch to the full backlogging policy (since in the partial backlogging policy, he shares the risk with the supplier). In summary, if an LSP is of the soft type, he will always get an attractive offer and on average receive a 0.1% higher profit if he does not share the failure rate information. For the tough LSP type, he may receive unattractive offers in some 12% of the cases (based on our numerical experiment) and on average, gets only 0.007% higher profit (15 times less than the soft type) when not disclosing the failure rate information.

Finally, we note that there was no significant difference observed in the value of information (or the value of revenue-sharing contracts) when changing the various parameters (except those that are already discussed). For more details on the numerical study results, see Tables 3 and 4.

7. Information sharing

There exists a stream of studies in the supply chain contracting literature which investigates the willingness of players for information sharing in a supply chain. As expected, it is not always beneficial for all players to share private information. In this case, the uninformed party may use the side-payment mechanism to acquire the information, see e.g. Yao et al. (2008) and Lan et al. (2017). As we have shown in the previous section, the supplier cannot always achieve the optimal coordinated solution by offering the best single or the best menu of revenue-sharing contracts. When coordination is not possible by means of the proposed contracts, the supplier can go one step further to increase her expected profit. The supplier can offer a side-payment to the LSP as an incentive to reveal his information on the assets reliability.

Table 4
Analysis of the numerical study. SP^*, SP_{SRS}, SP_{MRS} are the supplier profit in the optimal centralized solution, in the best single revenue-sharing contract and in the best menu of revenue-sharing contracts respectively. SP_{RS} is the maximum of SP_{SRS} and SP_{MRS}. LP^s_{RS} and LP^t_{RS} are the optimal soft and tough LSP profit, respectively, given the best revenue-sharing contract.

Condition	VI		VRS	
	Average (%)	Max(%)	Average (%)	Max(%)
all	20.50	82.36	15.96	79.12
$p = 0.25$	18.27	82.36	13.60	79.12
$p = 0.50$	23.33	70.41	18.21	63.50
$p = 0.75$	20.04	65.61	16.20	65.61
$\lambda^h/\lambda^l = 1.2$	19.53	62.18	13.93	62.18
$\lambda^h/\lambda^l = 1.4$	20.03	63.55	15.16	61.24
$\lambda^h/\lambda^l = 1.6$	20.41	65.42	15.92	63.95
$\lambda^h/\lambda^l = 1.8$	20.88	82.36	16.68	63.5
$\lambda^h/\lambda^l = 2.0$	21.72	79.12	18.18	79.12

Indicator	Condition	Average (%)	Max(%)
$\frac{SP^* - SP_{RS}}{SP^*}$ [a]	all	4.55	45.65
$\frac{SP_{MRS}}{SP_{SRS}}$	all	87.70	180.56
$\frac{LP^s_{RS} - FB^s}{FB^s}$	all	0.107	7.29
$\frac{LP^t_{RS} - FB^t}{FB^t}$	all	0.007	2.39

[a] Equal to **VI** – **VRS**.

Suppose the LSP demands a reveal fee of $\mathcal{F}$ to reveal the assets' failure rate. After the supplier acquires the failure rate information, she will be able to offer the coordinated revenue-sharing contract, which results in the highest profit for her and the reserved profit for the LSP (excluding the reveal fee). Therefore, if both supplier and LSP accept this side-payment and information sharing mechanism, their expected profit will be

$$SP_s = -\mathcal{F} + q(TP^c_s - FB^s) + (1-q)(TP^c_t - FB^t) = -\mathcal{F} + SP^*,$$
$$LP^j_s = \mathcal{F} + FB^j \qquad j = s,t,$$

where SP_s, LP^j_s are the expected supplier and LSP profits, for $j = s,t$ respectively, after the LSP reveals the assets' failure rate and receives the reveal fee $\mathcal{F}$. SP^* gives the expected supplier profit if she has the full information on the assets' failure rate. TP^c_j, $j = s,t$ is the optimal centralized expected profit in the case the LSP is of type soft or tough. The LSP is willing to share his private information and the supplier is willing to pay the reveal fee if their expected profit becomes higher than their profits under asymmetric information (and using the best revenue-sharing contract), i.e.

$$SP_s > SP_{RS}, \tag{25}$$
$$LP^j_s > LP^j_{RS} \qquad j = s,t, \tag{26}$$

where SP_{RS} and LP^j_{RS} are the expected profits of the supplier and LSP (of type soft or tough), respectively, under asymmetric information and given the best single or the best menu of revenue-sharing contracts. This gives us the feasible range of the reveal fee $\mathcal{F}$ such that both players are willing to participate in this side-payment and information sharing mechanism:

$$LP^j_{RS} - FB^j < \mathcal{F} < q(TP^c_s - FB^s) + (1-q)(TP^c_t - FB^t) - SP_{RS} \qquad j = s,t.$$

If the supplier has the right to choose the reveal fee amount freely, he might offer to pay just a bit more than $\min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t)$. But why should the minimum of these two amounts be sufficient? Suppose the LSP type k has a lower threshold value for the reveal fee than the LSP type k' (k can be soft or tough), i.e.

$$LP^k_{RS} - FB^k < LP^{k'}_{RS} - FB^{k'}.$$

If the supplier offers a reveal fee amount that is just a bit higher than $LP^k_{RS} - FB^k$, two scenarios can happen. If the LSP is of type k, he will accept this offer and reveal his private information. In this case, he expects a higher profit than in the case of asymmetric information. If the LSP is type k', then this offer makes his expected profit less than his expected profit under asymmetric information. However, he knows that if he rejects this information sharing offer, the supplier will immediately know his type, and accordingly the asset's failure rate (because a type k LSP would not reject the offer). In that case, the supplier will offer the LSP the optimal single revenue-sharing contract knowing that the LSP is of type k'. This makes the LSP expected profit equal to his reserved profit $FB^{k'}$ and he will not receive any reveal fee. Therefore, even for the LSP of type k' which has a higher threshold value for the reveal fee than the offered one, it is better to accept the information sharing offer and reveal his information on the assets' reliability by receiving the reveal fee.

Suppose the reveal fee can be determined in a cooperative environment. In this case, the players make an agreement in which the supplier transfers a fraction of her expected potential saving to the LSP in order to acquire his private information. Note that the supplier needs to pay the LSP at least $\min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t)$ amount, otherwise the LSP, whatever his type might be, will not share his information on the assets' failure rate. Therefore, the equation below gives the maximum saving $\mathcal{Z}$ that the supplier can achieve if the LSP reveals his information on the failure rate:

$$\mathcal{Z} = SP^* - SP_{RS} - \min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t).$$

Suppose they agree on a sharing fraction θ which gives the fraction of the total supplier's benefit of information sharing, $\mathcal{Z}$, which the supplier transfers to the LSP (via the reveal fee). Given the sharing fraction θ, the reveal fee is equal to

$$\mathcal{F} = \theta\mathcal{Z} + \min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t)$$
$$= \theta\left(SP^* - SP_{RS}\right) + (1-\theta)\min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t). \tag{27}$$

If the supplier transfers the reveal fee in Eq. (27) and the LSP reveals his information, the supplier and the LSP expected profits are as follows:

$$SP_s = \theta SP_{RS} + (1-\theta)SP^* - (1-\theta)\min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t), \tag{28}$$

$$LP^j_s = \theta\left(SP^* - SP_{RS}\right) + (1-\theta)\min(LP^s_{RS} - FB^s, LP^t_{RS} - FB^t) + FB^j \quad j = s,t, \tag{29}$$

It is easy to show that for any $\theta \in (0,1)$, the Eq. (28) and (29) satisfies the conditions on (25) and (26).

We examine the information sharing mechanism for our numerical test example (Section 6) with 83570 instances. We calculate how much the supplier and the LSP can increase their expected profit if they use this information sharing mechanism for different benefit-sharing fraction (θ) values. Based on our numerical experiment, we observe that the supplier, on average, can increase her expected profit up to 14.23% compared to her profit when she is not fully informed on the asset reliability. The increase in her profit can get as a high as 83.14%. Similarly, we look for the average and maximum benefit the LSP can achieve. Compared to his reserved profit (FB), the soft LSP type, on average, does not benefit from this information sharing mechanism. He expects to lose, on average, up to 0.27% of his profit. In some instances, the soft LSP can save up to 4.17% in his profit, but he also risks a loss of up to 7.27%. For the tough LSP, the situation is more promising on average and more controlled on extreme cases. The tough LSP, on average, can increase his expected profit up to 0.16%. The maximum saving and loss we observe for the tough LSP in our numerical experiment are 1.53% and 2.38% respectively. For more information about the numerical result, see Fig. 6.

In summary, for the cases where coordination is not achievable by using the best single or the best menu of revenue-sharing contracts, the

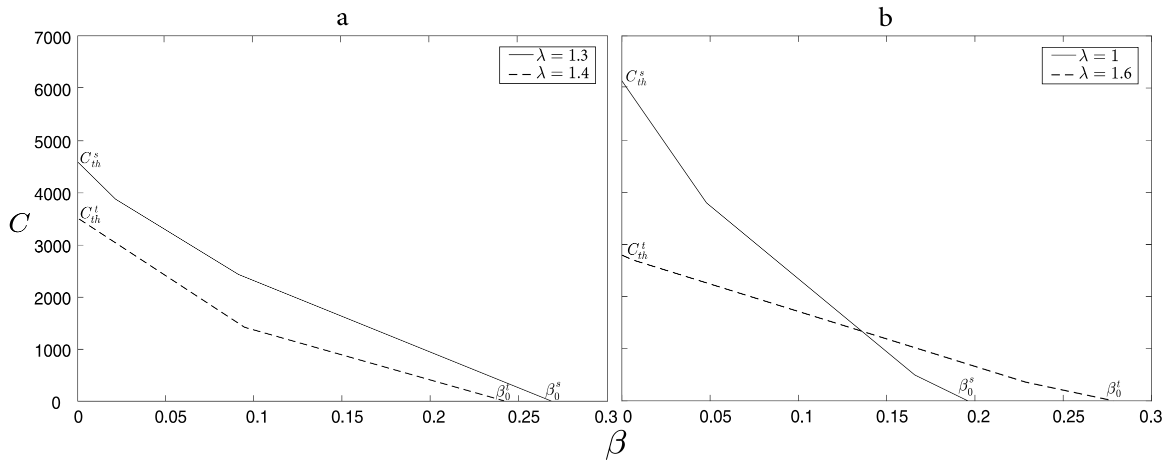

Fig. 5. The (β, C) values where the LSP of soft and tough type will accept the revenue-sharing contract (of Example 1). The failure rate of each LSP type is given. β_0^s and β_0^t are the maximum sharing fraction values that the supplier can use in her revenue-sharing contracts offers if the LSP is of soft and tough type respectively.

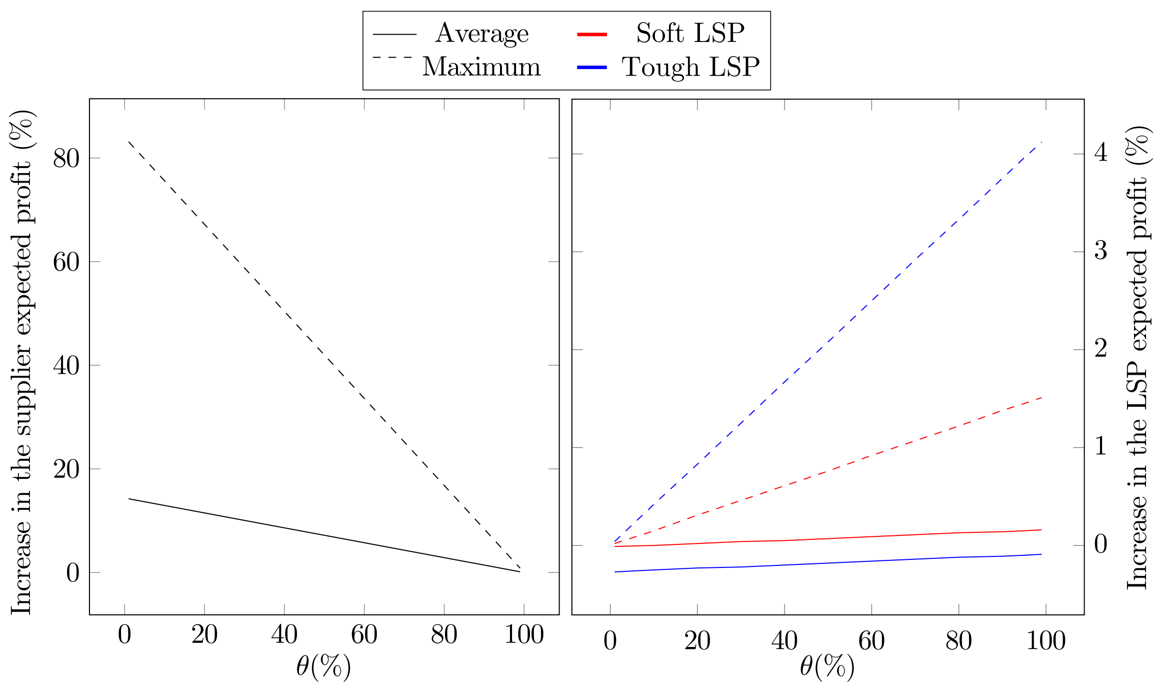

Fig. 6. Average and maximum increase (or decrease) in the supplier and the LSP expected profit (compared to the asymmetric information case) using the information sharing mechanism and as a function of the benefit-sharing fraction θ. Values are based on a numerical experiment with 83570 number of instances (Table 2).

supplier can increase his expected profit considerably, using this side-payment and information sharing mechanism. For the LSP the situation depends on his type. If the LSP is of soft type, it is better for him to not participate in this information sharing mechanism from the beginning. Although, he can save up to 4.17% in some cases, on average he will lose even for θ values close to 1. For the tough LSP, this mechanism is slightly more attractive. Although the tough LSP can also lose up to 2.38% of his profit, on average we found a modest 0.16% potential saving in his expected profit.

8. Conclusion

In this paper, we study a Stackelberg game between a local service provider (LSP) and an emergency supplier in which the supplier is the principal and offers the contracts. The LSP has limited local resources and if a repair call cannot be satisfied due to a stock out of the relevant spare part, he relies on an emergency supplier to take over this call completely. We considered a situation in which the emergency supplier who serves the LSP, has less information about the failure of the assets than the local service provider. We explored three different ways in which the supplier might offer a contract to the LSP in the case of asymmetric information. In the price-only contract, the supplier charges the LSP a fixed price per emergency shipment request. The supplier needs to choose a price that maximizes her expected profit considering that the LSP may face either low or high asset failure rates. The higher the price, the lesser the number of emergency requests the supplier may expect, and in case the price exceeds a threshold value, the LSP will reject the offer. As a second option, the supplier can use a two parameters revenue-sharing contract which we show to always give a higher profit to the supplier. We also study the use of a menu of revenue-sharing contracts with which the supplier can screen the LSP type (asset's failure rate) by offering two different revenue-sharing contract terms.

As expected, the presence of information asymmetry causes the supplier to lose some profit. If she relies on a price-only contract, she will lose more than 20% of her profit and rarely (in less than 0.01% of all cases) is able to achieve the optimal coordinated solution. With the help of a single and a menu of revenue-sharing contracts, the supplier loses less than 5% of her profit compared to the perfect information scenario. We propose an incentive for information sharing which the supplier can use when the single or the menu of revenue-sharing contracts cannot capture the value of information entirely. With this approach, the supplier is able to increase his expected profit, on average, up to 14%.

The LSP never gets lower expected profit when the information is hidden for the supplier compared to the full information scenario, however his gain will be negligible. Given the supplier's optimal strategy in offering a single or a menu of revenue-sharing contracts, a soft LSP on average gets 0.1% more profit when the information is asymmetric. For the tough LSP, the additional profit is negligible. In addition, in some 12% of the cases, he gets offers that he needs to reject (it gives him less than his reserved profit). However, if the tough LSP is willing to share the asset reliability information he can improve his revenue, on average, up to 0.16% (of his reserved profit).

For the sake of illustration, we assume that there are only two types of LSP. However, an extension to the case of more than two types of LSP is straightforward. Note that for a higher number of LSP types, finding the optimal menu of contracts becomes computationally more expensive. The game theory model that is studied in this paper is based on the model on Rahimi-Ghahroodi et al. (2017). To keep the model tractable, we simplify the problem to the case that there is one type of spare part. The extension to multiple types of spare parts is possible and rather straightforward. Some propositions are currently formulated for the case of only one type of spare part. Their extension to multiple part types requires some modifications and slightly more involved proofs.

CRediT authorship contribution statement

S. Rahimi-Ghahroodi: Conceptualization, Methodology, Software, Data curation, Writing - original draft, Data curation, Visualization. **A. Al Hanbali:** Conceptualization, Investigation, Supervision, Writing - review & editing, Project administration, Funding acquisition, Resources. **W.H.M. Zijm:** Investigation, Supervision, Writing - review & editing, Funding acquisition. **J.B. Timmer:** Formal analysis, Writing - review & editing.

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Appendix. Proofs of propositions

Proposition 1.

Proof. For any given emergency shipment cost C, the LSP chooses the spare part stock level and the number of service engineers such that his expected profit (6) is maximized while the average waiting time constraint is satisfied. By increasing the emergency shipment cost by a very small number ϵ, the LSP decides to add more spare parts to counter act the increase in the emergency shipment cost only if the new internal cost, consisting of spare parts inventory and service engineers hiring cost, is less than the change in emergency shipment cost, i.e.,

$$\left[S^*(C+\epsilon)-S^*(C)\right]H+\left[E^*(C+\epsilon)-E^*(C)\right]O < C\lambda_L(S^*(C))-(C+\epsilon)\lambda_L(S^*(C+\epsilon)),$$

where $\lambda_L(S^*(C)) = \lambda_L^*(C)$, and $S^*(C)H$ is the optimal inventory holding cost and $E^*(C)O$ is the optimal hiring cost of the service engineers given the emergency shipment cost C. In this point, the optimal LSP solution changes and the emergency rate decreases. Due to the fact that S and E are positive integers, this solution change happens for a finite number of emergency shipment cost values. The solution mainly changes due to the increase in the spare parts inventory and not necessarily a change in the number of service engineers. The reason for that is, according to the partial backlogging policy the addition of an extra spare part by the service provider will decrease the emergency shipment rate see (2)–(3). Note that, by increasing the number of service engineers, the LSP can reduce the expected total waiting time, however, this does change the emergency shipment rate. This shows that the optimal emergency failure rate, $\lambda_L^*(C)$, is a decreasing step function in the emergency shipment cost. Finally, we note that the set of drop points is discrete due to the fact that the S and E are positive integers. □

Proposition 2.

Proof. By definition, a drop point is an emergency shipment cost value which increasing it with a small amount, changes the optimal emergency shipment rate. This corresponds to a change of optimal spare part stock level. Suppose the optimal stock level given the emergency shipment cost δ_i equals S_i and it changes to S_{i+1} if the emergency shipment cost goes a bit higher than that. Note, the optimal emergency shipment rate $\lambda_L(S)$ is a step function in S, however, the optimal LSP expected profit is a differentiable function in S. Due to this property, it is easy to show that the LSP profit given the emergency shipment cost δ_i is the same for the stock levels S_i and S_{i+1}, i.e.

$$U - S_iH - E^m(S_i)O - \delta_i\lambda_L(S_i) = U - S_{i+1}H - E^m(S_{i+1})O - \delta_i\lambda_L(S_{i+1})$$

which results in the equation of δ_i in Proposition 2. □

Proposition 3.

Proof. It is shown in Proposition 1 that the optimal emergency shipment rate is a decreasing step function in the emergency shipment cost. Base on that, it is easy to show that the supplier profit function (3) in a price-only contract, given a shipment rate value, is a piecewise linear function in the emergency shipment cost. Therefore, we can reformulate the supplier profit function (in Eq. (6)) as follows

$$SP = \begin{cases} (C-d)l_1 & 0 \le C \le \delta_1, \\ (C-d)l_2 & \delta_1 < C \le \delta_2, \\ \dots & \\ (C-d)l_n & \delta_{n-1} < C \le C_{th}, \end{cases} \tag{30}$$

where δ_is are the drop points which are introduced in Section 4 and $l_i, i = 1,\dots,n$ is the optimal emergency shipment rate given that the emergency shipment cost is between δ_{i-1} and δ_i. From this formulation, it is obvious that in a price-only contract under full information scenario, the optimal emergency shipment cost is either one of the δ_i values (drop points) or C_{th} value. We can have the same argument for our problem ($\mathbf{P^{PO}}$). If y^t equal 0, then we have the same formulation as (30). For the case y^t equals 1, we have the following formulation for the objective function of Problem ($\mathbf{P^{PO}}$):

$$SP = \begin{cases} (C-d)l'_1 & 0 \le C \le \delta'_1, \\ (C-d)l'_2 & \delta'_1 < C \le \delta'_2, \\ \dots & \\ (C-d)l'_{n^s+n^t} & \delta'_{n^s+n^t-1} < C \le C^t_{th}, \end{cases} \tag{31}$$

where

$$\delta'_i \in \Delta',$$
$$\Delta' = \{\delta^t_1,\dots,\delta^t_{n_t},\delta^s_1,\dots,\delta^s_j \mid \delta^s_j \le C^t_{th}, \delta^s_{j+1} > C^t_{th}\},$$

and, n^s and n^t are the number of drop points in the range $[0, C^s_{th}]$ and $[0, C^t_{th}]$ in the case the LSP is of the soft or tough type, respectively. Given the formulation in (31), it is easy to see that the optimal solution can be found in either one of the drop points of both LSP types, or C^s_{th} and C^t_{th} values. Therefore, either y^t equal 0 or 1, the optimal solution of Problem ($\mathbf{P^{PO}}$) is either one of the drop points or one of the threshold values associated with both LSP types. □

Proposition 4.

Proof. Eq. (9) gives

$$C = \frac{(1-\beta)(TP^c + d\lambda^c_L) - FB}{\lambda^c_L}$$

By definition $C \geq 0$. Therefore

$$0 \leq \frac{(1-\beta)(TP^c + d\lambda^c_L) - FB}{\lambda^c_L},$$

$$\beta \leq 1 - \frac{FB}{TP^c + d\lambda^c_L}.$$

Eq. (10) suggests that the stock level S^c is the maximizer of the LSP profit function. Hence, the LSP profit given the revenue-sharing contract (β, C) should be higher in S^c than in $S^c - 1$ and $S^c + 1$, i.e.,

$$(1-\beta)(U - S^c H - E\,O) - C\lambda^c_L \geq (1-\beta)(U - (S^c+1)H - E^m(S^c+1)O) - C\lambda_L(S^c+1),$$

$$(1-\beta)(U - S^c H - E\,O) - C\lambda^c_L \geq (1-\beta)(U - (S^c-1)H - E^m(S^c-1)O) - C\lambda_L(S^c-1).$$

These two inequalities give

$$\beta \geq 1 - \frac{FB\,r^+}{(E^c - E^*(S^c+1))O - H + (TP^c + d\lambda^c)r^+},$$

$$\beta \leq 1 - \frac{FB\,r^-}{(E^c - E^*(S^c-1))O + H + (TP^c + d\lambda^c)r^-}.$$

We assume that $FB < TP^c$ (Note, if $FB > TP^c$, then the LSP is always better off with the full backlogging policy and any type of contract with the supplier will not be beneficial for him). Moreover, since $\lambda_L(S)$ is decreasing in S, $r^- < 0 < r^+$. Therefore, it is easy to show that $\beta_U > \beta_L$. □

Proposition 5.

Proof. First, we show that the drop points of a revenue-sharing contract with parameters (β, C) is given by $(1-\beta)\delta, \delta \in \Delta(C_{th})$ in which $\Delta(C_{th})$ is the set of drop points below C_{th} in the price-only contract. For any revenue-sharing contract, the drop points are the emergency shipment values that by increasing them by a small value, the optimal stock levels will change. Similar to Proposition 2, suppose S_i and S_{i+1} are the optimal stock levels of the LSP in the revenue-sharing contract (β, C) given the emergency shipment cost δ^β_i and $\delta^\beta_i + \epsilon$ (a small positive number), respectively. Base on the continuity of the LSP profit function, It is easy to show that the LSP profit is the same for the stock levels S_i and S_{i+1} given the emergency shipment cost δ^β_i, i.e.

$$(1-\beta)(U - S_i H - E^m(S_i)O) - \delta^\beta_i \lambda_L(S_i)$$
$$= (1-\beta)(U - S_{i+1}H - E^m(S_{i+1}))\delta^\beta_i \lambda_L(S_{i+1}),$$

where $E^m(S)$ is the minimum number of service engineers such that given the stock level S, $W(S, E) \leq W_{max}$, and $\lambda_L(S)$ is the emergency shipment rate given the stock level S. The equation above gives

$$\delta^\beta_i = (1-\beta)\frac{(S_{i+1} - S_i)H + \left(E^m(S_{i+1}) - E^m(S_i)\right)O}{\lambda_L(S_i) - \lambda_L(S_{i+1})},$$

which gives

$$\delta^\beta_i = (1-\beta)\delta_i$$

The rest of the proof is similar to the proof of Proposition 3. □

Proposition 6.

Proof. It is easy to show that the LSP expected profit in the revenue-sharing contract is decreasing in both β and C values (see Eq. (7)). Therefore, the maximum β value that the LSP accepts is when the $C = 0$. LSP accepts a contract offer if it gives him at least as much as his expected profit using the full backlogging policy, FB, i.e.

$$(1-\beta_0)(U - S^*H - E^*O) = FB, \tag{32}$$

where (S^*, E^*) is the optimal solution of the LSP in a revenue-sharing contract with parameters (β_0, C). It is obvious that (S^*, E^*) is the optimal solution of the LSP in a price-only contract with $C = 0$, i.e.,

$$LP^*(0) = U - S^*H - E^*O. \tag{33}$$

Therefore, Eq. (32) and (33) result in Eq. (19) □

Proposition 7.

Proof. The proof is straightforward based on Proposition 6. □

Lemma 1.

Proof. It is easy to show that the LSP profit is decreasing in C and β. If $\beta^s > \beta^t$ and $C^s > C^t$, then the soft LSP gets more profit if he chooses the contract of tough LSP than his dedicated contract. Therefore, the constraint (21) will be violated. The same happens for the constraint (22) if $\beta^s < \beta^t$ and $C^s < C^t$. □

Proposition 8.

Proof. Similar to Proposition 5, it is easy to show that $(1-\beta^j)\delta^j_i, \delta_i \in \Delta^j(C^j_{th}), j = s, t$ are the drop points of the emergency shipment rate in the case of soft or tough LSP type. To show that the optimal emergency shipment costs of the optimal menu of contracts are among these drop points or the threshold values, we can prove by contradiction.

Suppose the optimal emergency shipment costs C^s and C^t are not one of the drop points or the threshold values. By definition, by increasing each of the emergency shipment cost up to the next drop point (or the threshold value) the optimal emergency shipment rate (of both soft and tough LSP type) will not change. Moreover, it is easy to show that all the constraint stay feasible. Therefore, the supplier can get a higher profit by increasing the emergency shipment costs up to the next drop point (or the threshold value). Hence, given fraction values β^s and β^t, the optimal emergency shipment costs in the menu of revenue-sharing contracts are always among the drop pints or the threshold values. □

Proposition 9.

Proof. By definition

$$C^s_{th}(0) > C^t_{th}(0).$$

We know that for any LSP type,

$$C_{th}(\beta_0) = 0,$$
$$\beta_0 = 1 - \frac{FB}{LP^*(0)}$$

It is easy to show that $C_{th}(\beta)$ is decreasing in β. Therefore, if $\beta^s_0 < \beta^t_0$, there is a β^g where

$$C^s_{th}(\beta^g) = C^t_{th}(\beta^g),$$
$$C^s_{th}(\beta) > C^t_{th}(\beta), \quad \beta < \beta^g,$$
$$C^s_{th}(\beta) < C^t_{th}(\beta), \quad \beta > \beta^g.$$

Hence, the revenue-sharing contracts with parameters of the set

$$\mathcal{R}^s = \left\{(\beta^s, C^s) \mid \beta = [0, \beta^g), \quad C^s = \left(C^t_{th}(\beta), C^s_{th}(\beta)\right]\right\}$$

is only acceptable by the soft LSP and the revenue-sharing contracts with parameters of the set

$$\mathcal{R}^t = \left\{(\beta^t, C^t) \mid \beta = (\beta^g, \beta^t_0),\quad C^t = \left(C^s_{th}(\beta), C^t_{th}(\beta)\right]\right\}$$

is only acceptable by the tough LSP. Therefore, any menu of two contracts each from one the sets $\mathcal{R}^s$ and $\mathcal{R}^t$ is a feasible menu. □

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