INTRODUCTION

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The main subject of this conference was the *chaotic behavior* exhibited by many nonlinear dynamical systems;[1-10] I am referring to the chaotic behavior *inherent* in the solutions of various *deterministic equations* of motion, and not to chaotic behavior obtained by adding external noise sources. This inherent chaotic behavior arises when some orbits (or solutions) have an extremely "sensitive dependence on their initial conditions." Familiar examples are throwing dice and playing roulette and pinball machines. These are deterministic mechanical systems where we literally have the initial conditions in our hands. Yet the results depend so sensitively on the initial conditions that we use them as models of random behavior in conversation and in probability theory. Similar chaotic behavior can arise from even nicer equations of motion,[1-7] i.e., from analytic equations without any of the above singularities (at the edges of the dice, balls, and pins). It also implies that, no matter how much of the past behavior we experimentally observe, we cannot predict much of the future behavior of such orbits.

We discuss both *dissipative* systems, in which the volume of an element of phase space shrinks to zero as time progresses, and *conservative* systems, in which this volume remains bounded forever, away from zero or infinity. The conservative Hamiltonian systems of physics are further subdivided into *integrable* (≃separable) systems, without any chaotic behavior, and *nonintegrable* (≃nonseparable) systems.[1-4] It is among the latter systems that statistical mechanicians search for—and find—*ergodic* systems, i.e., (weakly) chaotic systems in which virtually every orbit covers the surface of constant energy densely and uniformly.[2,3] The general nonintegrable system, however, is not (globally) ergodic, but has chaotic regions throughout its phase space.[1-4] There is evidence for the onset of some diffusive motion, called *Arnol'd diffusion,* in those chaotic regions.[1,2,4] Notwithstanding the abundance of chaotic regions, there is also an abundance of regular regions in which most orbits are confined to invariant tori (inner tubes) in phase space. This follows from the *Kolmogorov–Arnol'd–Moser Theorem,* which shows that a finite fraction of all orbits usually lie on such lower-dimensional tori.[1-4] Hence, ergodicity, and an approach to thermal equilibrium, do not hold for most Hamiltonian systems. Recent developments in these areas are discussed in the papers in the section entitled Ergodic and Integrable Behavior. Applications to physics and chemistry are given under the heading Physics and Chemistry.

One might think that, in a dissipative system, all orbits would eventually be *attracted* to a stationary point; for instance, the point of lowest energy when we add a dissipative perturbation to a Hamiltonian system. This may, indeed, happen. A second possibility is that a simple *periodic attractor*, or *limit cycle*, exists in the familiar van der Pol equation.[4-6] As one changes the value of a certain parameter, μ, which in several systems plays a role similar to that of the Reynolds number in fluid mechanics, one finds that a one loop periodic attractor of period T changes, at some value μ_1, into a double loop attractor of period $2T$. At some μ_2, it acquires four loops and doubles its period again, etc.[6] The μ_k's at which these *period-doubling bifurcations* take place often converge to some finite critical value, μ_∞, producing more and more complicated attractors in the process.[4-6] Beyond μ_∞, the object is no longer an attractor, but other nonperiodic attractors can arise, with complicated shapes in phase space. The motion along some of these attractors can be chaotic, ergodic, and even "mixing."[4,5,7-9] Such *strange attractors* are discussed in the papers grouped under that heading and, together with the above period-doubling attractors, under Chaotic Maps and Flows. Loosely speaking, we can see many periods come and go "in the course of time" in the Fourier spectrum of the motion along a strange attractor. This is reminiscent of the Fourier spectra observed during the onset of *turbulence*[10] and strange attractors are, in fact, found in the (truncated) mode equations for the Navier-Stokes equation of hydrodynamics.[4,5,7-10] These and other recent developments are mainly discussed in the sections entitled Turbulence and Chemical and Fully Developed Turbulence.

The ordering of the sections in this Annal was determined by the order of their presentation at the conference. Many of the following papers will refer you to review articles of the particular field they discuss. I have attached below a list of more introductory and global references that may help bridge the gap between the standard graduate courses and the research discussed in this Annal; I have attempted to order them according to increasing complexity. While our understanding is far from complete, it is exciting to detect progress on these venerable problems of chaotic behavior in deterministic systems and even see some agreement with experiments.

ACKNOWLEDGMENTS

In composing the program, I was actively assisted by our Advisory Committee, whose members are listed on page v. I am very grateful for their willingness to share much personal information and time. In particular, Dr. Joel Lebowitz, the president of The New York Academy of Sciences for 1979, was a constant source of inspiration. Without his aid and comfort this conference would not have taken place. Another essential ingredient was the support the conference received from the agencies listed on page vii. That this conference, which included many theoretical papers, was supported by agencies usually more interested in applications might be seen as a token of confidence in the future applicability of some of the present research, and is greatly appreciated for that reason as well.

Several topics presented at this conference I first learned of at one of its three noteworthy predecessors, namely

1. The 1976 Gordon Conference on Dynamical Instabilites—P. C. Martin and J. P. Gollub, Chairmen
2. The 1977 Como Conference on Stochastic Behavior in Classical and Quantum Hamiltonian Systems—G. Casati and J. Ford, Chairmen
3. The 1977 New York Academy of Sciences Conference on Bifurcation Theory[6]—O. Gurel and O. E. Rössler, Chairmen.

I hope that this conference will, in its turn, inspire someone to organize another conference on Nonlinear Dynamics and recommend to him the excellent staff at The New York Academy of Sciences. I thank Ellen Marks and her staff, Renée Wilkerson and Erna Levine, for their great care and extensive preparations for this conference. It is a pleasure to thank Ann Collins, Bill Boland, India Trinley, and Frederick Bartlett for their cheerful day to day collaboration and equanimous compliance with even the most unreasonable of requests. In short, I thank The New York Academy of Sciences for its very professional support.

In closing, I would like to share the words of Alfred Kahn (the Chairman of the President's Anti-Inflation Task Force) with the as yet unknown chairman mentioned above,

A chairman is to a meeting as a tree is to a dog.

Alfred Kahn
CBS News, 1978.

Introductory References

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10. Swinney, H. L. & J. P. Gollub. 1978. The transition to turbulence. Phys. Today **31:** 41–49; see also their articles in this volume.†

*Conservative systems.
†Dissipative systems.