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Quantized Continuous-Time Average Consensus

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1 Introduction

A group of interconnected dynamical systems is said to reach consensus when their internal states converge to a common value. Typically this common value is a function (e.g. the arithmetic mean) of the systems' initial conditions. Communication constraints play a major role in consensus and related problems of distributed computation and control. Such constraints can be represented by a graph of available communication links among agents, together with further restrictions on what information can be exchanged across such links. Over the last few years, the constraint of quantization, that is of communication restricted to a discrete set of symbols, has received significant attention. Although most of to-date works have dealt with discrete-time dynamics (see e.g. [5] and references therein), it is very important to consider the same restrictions in the context of continuous-time dynamics, as recently done in [4]. In this way, it is possible to study the effect of quantization on continuous-time systems without necessarily considering their discretized or sampled-data model. This may be interesting in particular for applications to robotic networks.

2 Quantized consensus and solutions

Besides discussing the general issues of quantization in consensus dynamics, we aim at giving a rigorous treatment of continuous-time average consensus dynamics with uniform quantization in communications. It is well-known that consensus problems can be thought in terms of feedback control systems: As expected, when quantization enters the loop, the stabilization problem becomes more challenging. From the mathematical point of view, a consequence of quantization is that we obtain a system with discontinuous righthand side, whose solutions have to be intended in some generalized sense. In fact we prove by means of an example that classical or Carathéodory solutions actually may not exist. In the literature one can find different approaches to the technical problem of having a system with discontinuous righthand side (see e.g. [3] for a review on these topics). Here we focus on Krasowskii solutions essentially for two reasons. First, there are many handy results concerning existence and continuation of Krasowskii solutions, as well as a complete Lyapunov theory [1, 2]. Second, since the set of Krasowskii solutions includes Filippov and Carathéodory solutions, then results about Krasowskii solutions also hold for Filippov and Carathéodory solutions, in case they exist. On the other hand, the set of Krasowskii solutions may be "too large". In particular, it may contain sliding modes which, from a practical point of view, induce chattering phenomena. To cope with those issues, we propose the use of a quantizer endowed with an hysteretic mechanism, and study the resulting dynamics by a hybrid system approach. Convergence results are given for both the Krasowskii and the hysteretic dynamics: Note that due to the constraint of static uniform quantization we can not precisely obtain consensus. Nevertheless we obtain approximations of the consensus condition we informally refer to as "practical consensus".

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