# Static deformation of flexure-based mechanisms starting from a kinematic approximation

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## EXTENDED ABSTRACT

## Introduction

Design optimizations in flexure based precision engineering are time consuming. The best mechanical properties of the system in undeformed and deformed configuration have to be found for many designs. An essential step in this analysis involves obtaining the deformed configuration which requires an iterative nonlinear procedure that takes a lot of computation time. Therefore it is advantageous to increase the computational efficiency of this step.

The efficiency can be improved by approximating the deformed configuration by as few parameters as possible, such that it can be obtained fast. This configuration can be used as a starting point to obtain an accurate deformed configuration for the full system. The Global Model Parameterization method [1,2] uses a similar technique in dynamics. This method describes the displacements as a part with large (mainly rigid) displacements, which is described by a few parameters. On this part small (mainly elastic) displacements can be substituted. These small displacements are assumed to be linear such that they can be reduced by a model order reduction technique.

One of the main challenges of this technique is to select appropriate parameters to approximate the configuration. This selection can be made by applying Proper Orthogonal Decomposition [3] on data of previous performed simulations of the full system. This method is appropriate for control purposes, but is not applicable in design optimizations because simulation data of all designs that are analyzed should be available.

This paper shows a kinematic approach to approximate the deformed system that does not require information of the full system, but only requires information of the joints. Flexure mechanism typically contain joints that are standard compositions of flexures, which are designed to allow motion in a few directions and to constrain the other motions. This can be a cross flexure or a more advanced composition [4,5]. These compositions can be seen as building blocks of which information can be saved in a database, or obtained by a parameterization.

## Method

Flexible manipulators are typically kinematically determinate, i.e. if each joint only deforms in its allowed direction, there is one possible configuration for a certain displacement of the end effector. This means that the full configuration of the manipulator can be found by kinematics. To do this, the exact kinematic behavior of each joint, deforming in the allowed direction, should be known, including the parasitic error motion. Because the flexible joints are often standard building blocks, the kinematic behavior of these systems can be saved in a database and reused during simulations. The method is applied on a two-dimensional fourbar mechanism of which the left bar is deformed by 60 degrees. The mechanism contains four cross flexure joints, of which the lower two are fixed to the ground.

The method consist of three steps, schematically visualized in Figure 1. In the first step, a kinematic approach is used to approximate the deformed configuration. In this step each cross flexure is described as one element that only can deform in the allowed direction of the cross flexure. In the figure, this element is visualized by a single bar, but the underlying equations account for nonlinear kinematic effects like pivot shift. In the second step, the deformation of each cross flexure is used to approximate the internal displacements by means of the database. In Figure 1 this is illustrated for one flexure that is modelled by rigid parts and flexible beam elements. The result of this step is the full model, in which the displacements are close to the real deformed configuration. In the third step a few static iterations of the full model are used to obtain the real deformed configuration.

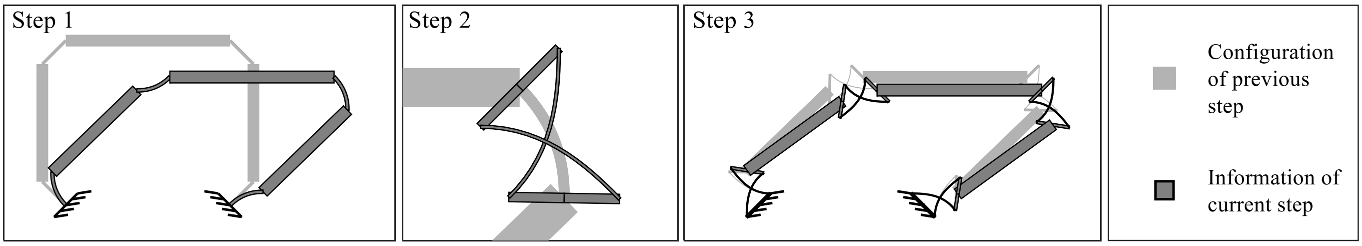


Figure 1. Schematic overview of the three steps in the method for the fourbar mechanism.

**Results**

Results were obtained with the described method ("kinematic approach") and with a full static simulation, i.e. starting from the undeformed configuration static computations were made with the full model ("conventional approach"). Both methods were implemented similarly in Matlab scripts to make simulation times comparable. The accuracy of the solution depends on the number of elements that is used to model each of the two flexures in the cross flexure. Earlier research has shown that for certain applications with large deformations, four elements per flexure resulted in sufficient accuracy [4]. To model more advanced flexure joints, more than 30 elements may be necessary [5].

To compute the deformed configuration of the mechanism with a 60° rotation of the left bar, a stepwise increase of the load is applied to avoid a divergent solution. Figure 2 shows the number of required load iterations. The system is described by independent and dependent coordinates. The independent coordinates are updated each load iteration. For every update of the independent coordinates, another iterative process updates the dependent coordinates. In the conventional approach, the number of load iterations that is required to keep the computation stable increases fast with the number of elements per flexure. In the kinematic approach, the number of iterations is almost constant. In all these kinematic cases, one iteration was sufficient to obtain the approximated configuration (step 1) and the remaining iterations were required to obtain the required accuracy (step 3). Figure 3 shows the average simulation time per iteration. For both methods, the simulation time increases with the used number of elements. The time per iteration in the conventional approach is higher. This is because it takes longer to update the dependent coordinates in the conventional approach, as the load steps in this approach are larger than the load steps in step 3 of the kinematic approach. The combined effect makes that the kinematic approach is 44 times faster than the conventional approach for five elements per flexure.

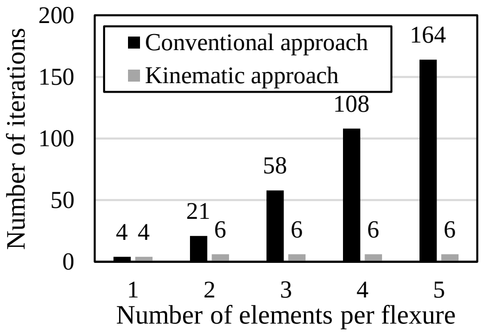


Figure 2. The minimal required number of load iterations to obtain the deformed configuration.

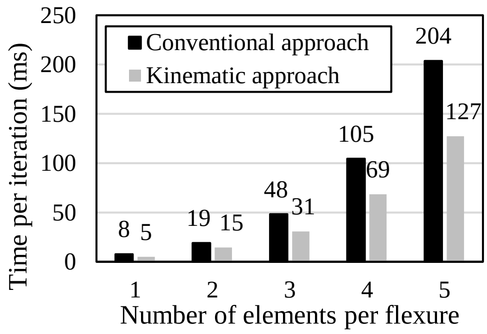


Figure 3. The average time per load iteration to obtain the deformed configuration.

**Conclusions**

Flexure mechanisms are designed to be kinematically determinate, such that the deformed configuration of a system can be approximated by kinematics. This behavior is exploited in the presented "kinematic" approach that requires a characterization of the kinematic behavior of all flexure joints in the system. The characterization of standard flexure joints is saved in a database and used during all simulations that contains these flexure joints. For the fourbar mechanism that was analyzed, the computation time was reduced by a factor of 44 with respect to the "conventional" approach, mainly because fewer iterations were required to solve the system.

In the presented results only a simple cross flexure with fixed dimensions has been considered. The authors are currently working on a method to characterize the kinematics of default joints by parameterization, such that it is independent of its dimensions. This makes the approach more suitable for design optimizations, as the dimensions vary during optimizations.

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