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A generalised fuzzy cognitive mapping approach for modelling complex systems

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ABSTRACT

Fuzzy Cognitive Maps (FCMs) were developed as a tool for capturing and modelling the behaviour of qualitative system dynamics. However, several drawbacks have been identified that limit FCMs ability in simulating the behaviour of qualitative system. This paper addresses the limitations of FCMs in modelling complex qualitative system dynamics and proposes a generalised Fuzzy Cognitive Mapping (FCM) approach that is able to overcome those limitations. This approach uses fuzzy rules to represent the dynamics of concepts and relations, including time dynamics of relations and introduces a multistep simulation approach that can use several single layer perceptrons to simulate the dynamics of concepts and relations overtime. This approach also incorporates the fuzziness and ambiguity widely associated with expert knowledge when representing and simulating the dynamics of concepts and relations. In this paper, the design of the proposed generalised FCM approach is explained and demonstrated for a real-world case of the consequences of high intensity rainfall in Kampala City, Uganda. This generalised FCM approach creates a new perspective and an alternative approach to model the behaviour of complex qualitative system dynamics using FCMs.

1. Introduction

System Dynamics (SD) is a computer-aided approach to understanding the functioning and behaviour of complex systems such as cities, the climate and ecosystems, for policy analysis and design, initially developed from the work of Jay W. Forrester [1]. According to Jay W. Forrester [2], "System dynamics deals with how things change through time, which covers most of what most people find important. System dynamics involves interpreting real life systems into computer simulation models that allow one to see how the structure and decision-making policies in a system create its behaviour". Complex systems as characterised by Chan [3], is any system featuring a large number of interacting components (agents, processes, etc.) that is often difficult to understand, and hard to solve and requires the development, or the use of, new scientific tools, nonlinear models, out-of equilibrium descriptions, and computer simulations [4]. Complex social systems include human behaviour, and can have concepts interacting in a manner that is quantitative (definitive) and/or qualitative (abstract), with the latter being particularly difficult to include in modelling exercises due to their qualitative nature and the resulting challenges of quantification. The exclusion of such abstract qualitative concepts can bring into question the conclusions arrived at and the models relation to reality. To be able to explain, predict, and understand complexity, it is argued [5] that qualitative phenomena – that can play a substantive role in systems – should be included. Therefore, qualitative systems analysis or qualitative modelling [6] is increasingly being used for analysing the dynamics of complex systems. Kosko [7] introduced fuzzy cognitive maps (FCMs) as a tool for capturing and explaining the behaviour of dynamic qualitative systems [7–9]. FCMs, as explained in [7,10–12] are increasingly been used to model and analyse the behaviour of qualitative systems [13–20]. Over the last 30 years, this fuzzy cognitive mapping (FCM) approach has become increasingly popular due to the ease of design and the low computational requirements for simulating social system dynamics [21,22] , largely using two forms of application, and connected data use and generation: (1) the deductive approach-employing knowledge that is gathered by interviewing experts from the area of application; (2) the inductive approach-an automated and semi-automated approach designed for learning FCM rules based on historical data [13,16,19,20,23–28].

This paper is about designing a generalised fuzzy cognitive mapping approach for capturing, representing, and simulating the behaviour of complex qualitative systems. FCMs in general are seen to have a number of advantages over traditional, quantitative modelling approaches. Advantages of FCMs comprise, e.g., the ability to model data scarce environments with the use of natural language, expressing knowledge, perceptions, experiences or

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beliefs as formulated by the expert or stakeholder, usually characterised by uncertain and vague information [29]. Moreover, FCMs results are easy to interpret by lay people and the public [20]. However, when used to model the behaviour of qualitative SD traditional FCMs also suffer from a number of drawbacks. These drawbacks largely relate to incomplete: (i) consideration of the semantics of causality [30] and hence the limited capture, representation and simulation of causal dynamics; (ii) inclusion of time relations [31–33]; (iii) capture, representation and simulation of fuzziness [34–36]; (iv) simulation of dynamics due to the use of single layer perceptron mechanisms [30]. Several extensions of FCMs have been developed to overcome these drawbacks some of the important ones are discussed in the next section, but most of the developed extensions try to solve specific problems with traditional FCMs and do not try to address the issues for modelling the dynamics of complex qualitative systems.

This paper aims to address several of the limitations in traditional FCMs by introducing a generalised FCM approach that enables the design and simulation of complex qualitative systems. In particular, this approach is the first of a kind that enables capturing, representing and simulating dynamic causal relations including the time dynamics of relations and fuzziness associated with causal reasoning. This is the first contribution that elicits detailed information from experts regarding dynamics of causal interactions including their time relations. Furthermore, this approach is the first of its kind to model the time dynamics of causal relations implicitly.

This paper is divided into six sections. Section 2 of the paper provides an outline of conventional FCMs as well as discusses several advances and a number of limits as regards their use for modelling qualitative systems. Section 3 explains in detail the generalised FCM approach as a tool for modelling complex qualitative systems. This section delves into the framework for capturing, representing, and simulating causal dynamics considering FCMs structure and semantics. This is in series with the presentation of a real world case through simulations in Section 4. In Section 5 and Section 6, the strengths and limitations of the proposed approach in modelling and simulating the behaviour of complex qualitative systems are discussed and future work is suggested.

2. Literature review

This section gives an overview of the traditional FCM approach, and evaluates the strengths and limits of some important advances in FCMs research in modelling system dynamics.

2.1. "Traditional" Fuzzy cognitive maps (FCMs)

Kosko [7] introduced "Fuzzy cognitive maps" in his seminal paper of 1986, and explains the behaviour of qualitative SD by way of causal reasoning using the belief or perception of expert knowledge. Kosko [10] defined FCMs as "Fuzzy signed directed graphs with feedback" and he suggested that they are "analogous to how neural networks learn" [37]. Furthermore, Kosko [7] and his subsequent works [10–12] intended to combine fuzzy logic and neural networks in a way that simulates causal reasoning as determined by linguistic terms.

FCMs as we have come to know them, consist of concepts (linguistic terms) expressed by nodes. Directed arrows with weights explain the relationship between the concepts. These weights describe the strength of the causal relationships with ({−1, 0} and {0, 1}) representing a causal decrease and increase, respectively. Concepts and their interactions are represented by nodes and directed arrows with their weights explain the arrangement of a (given) system. This is represented in the form of an adjacency matrix which, allows for standard algebraic operations for finding relationships between nodes and to automatically learn weights [30].

FCMs introduced by Kosko [7] are simulated using the mathematical formulation expressed in Eq. (1).

$$C_j(t+1) = f\left(\sum_{\substack{i=1\\ i\neq j}}^{n} w_{ij} \bullet C_i(t)\right) \tag{1}$$

where n is the number of concepts, $C_j(t+1)$ is the value of concept at the next iteration, $C_i(t)$ is the value of the concept at iteration t and w_{ij} is the weight relation of the interaction between the cause and the effect. This is then mapped to a predefined universe of discourse using transformation functions, the most common being the sigmoid and hyperbolic transformation functions [21]

2.2. Advances in FCMs related to modelling and simulation

In this section some important advancements in FCMs research is analysed to understand their capability in modelling complex qualitative systems. The analysis is presented in Table 1 and for reference a brief review of these advances is presented in Section A of the Appendix.

When modelling complex qualitative SD, ideally, FCMs should be able to capture and model the dynamics of causal relations as perceived by experts. This includes integrating and capturing certain properties of causal dynamics, which can comprise, but are not limited to, the following:

- A cause can take various states or strengths at various instances in time
- A cause cannot have two states or strengths at a given instance in time (two states are only possible in quantum superposition)
- A cause precedes the effect hence temporal dependency is inherent
- The influence of a cause must cause an increase or a decrease only then is the effect felt
- A cause at a particular state can have an effect that is dynamic as a result of a time lag, time delay or time decay
- A cause can have an effect that is dynamic as a result of a change in state or strength (i.e., it can be non-linear, non-monotonic and asymmetric)
- The effect is only felt when there is a change in the state or strength of the affected
- An effect can be a result of conditional causes (co-evolution)

However, conventional FCMs as well as several advances ignore these structural and semantics' particularities when representing causal dynamics. They may therefore produce, at best, too simple representations of a qualitative system.

Furthermore, conventional FCMs and several advances use a single layer perceptron to model and explain the dynamics of qualitative system as a universal property. However, in SD, causal relations can be conditional, probabilistic or possibilistic in nature [5]. Given this knowledge, a single layer perceptron cannot handle simple x-NOR functions; hence, it cannot be considered as a universal approximator [50]. Implying, to be able to explain the dynamics of a system as a universal property multiple single-layer perceptron are needed [30].

Finally, FCMs should ideally also represent uncertainty and vagueness in experts' knowledge. These may be represented and simulated using fuzzy systems and FCMs as envisioned by Kosko [7]; his approach being suggested and intended to be a combination of fuzzy logic and artificial neural networks. The here suggested approach follows that early "tradition".

Table 1
Evaluation of FCMs extensions in modelling complex qualitative system dynamics.

	E-FCMs	FTCMs	RB-FCMs	FGCMs	iFCMs	DCNs	RCMs	BDD-FCMs	tFCMs	T-FCMs	Enhanced-FCMs
Is the study a methodological contribution?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Does methodological contribution demonstrate with simulations the strengths and limits using real-world case studies?	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	Yes
Does the study seek in modelling complex qualitative system dynamics?	Yes	No	Yes	No	No	Yes	Not explicit	No	Not explicit	No	Not explicit
Does the study include the different types of causal relations such as certain, probabilistic, possibilistic and conditional?	No	No	Yes	No	No	No	No	No	No	No	No
Does the study allow the representation of dynamics of causal such as non-linear, non-monotonic and asymmetric causal relations?	Yes	Not explicit	Yes	No	Not explicit	Yes	Yes	No	No	No	Yes
Does the study allow the representation of uncertainty?	No	No	Not explicit	Yes	Yes	No	Yes	No	No	No	Yes
Does the study allow the representation vagueness or hesitancy in expert's reasoning?	No	No	No	Yes	Yes	No	No	Yes	No	No	Yes
Does the study allow the representation of time relations between a cause and an effect, such as time lags and delays?	Yes	Yes	Yes	No	No	Yes	Yes	No	Yes	No	No
Does the study use a multiple, single layer perceptron or a multi-step approach to simulate dynamics?	No	No	Not explicit	No	No	No	Yes	No	No	Yes	Not explicit
Does the study address uncertainty when simulating system dynamics?	No	No	No	Yes	Yes	No	Yes	Yes	No	No	Yes
Does the study explain the evolution of the system through time?	Yes	Yes	Yes	No	No	Yes	Yes	No	Yes	Yes	No
Sources	[31]	[38]	[5,32,34,39–41]	[42,43]	[33,44]	[45]	[46]	[47]	[48]	[49]	[27,35]

3. Methods

In this section, a generalised FCM approach as a tool for simulating complex qualitative systems is discussed. Fig. 1 is an illustration of a generalised FCM framework for designing and simulating complex qualitative system dynamics considering implicit time relations. The most important problem that this FCM approach addresses is the capture, representation and the simulation of the dynamics of causal relations through time discussed in the previous sections.

3.1. A. Knowledge elicitation

Knowledge regarding the system of interest for this study was elicited based on the following questions:

- Has there been a change in the intensity of rainfall in the last five years?
- What are the direct and indirect socio-economic and ecological consequences of this change in intensity rainfall events in Kampala City, Uganda?
- What are the coping strategies in place or deployed for such a rainfall event?

Eliciting information regarding causal interaction between concepts captures the functioning of the system. In the generalised FCM approach causal dynamics were elicited using semi-structured interviews. A combination of fuzzy linguistics and fuzzy numbers were used to elicit information about concepts and their interactions including their time relations. Questions that guided the elicitation of the dynamics of causal relations are outlined in the semi-structured questionnaire provided in the Appendix B.

Causal relations are the interaction between concepts, entities or variables. The influence of a cause (at a particular state or strength) must produce an increase or a decrease only then is the effect felt. Causal relations or interactions can be (non-)linear, (non-)monotonic, and (a-)symmetric. Additionally, since a cause precedes an effect, temporal dependency is inherent and it is the effects of time that truly makes a system dynamic. There are two notable causal interactions as a result of temporal dependency: (i) the dynamic influence that one concept has on other concepts due to a change in the state as result of decay or growth; and (ii) the dynamic influence based on the duration of the time elapsed or lagged at a given state of the concept(s). Moreover, effect of a causal relation can be a result of multiple antecedents. These antecedents can have an influence as a result of conditional logic (AND, OR, x-NOR, etc.). Furthermore, several possibilities and the related probabilistic nature of causes are also elicited. All these causal relations are considered and can be included in this generalised FCM approach.

Besides, in SD, it can be observed that an isolated concept in a system can take a particular state or position and can decay or grow over time. This implies that, these states can be absolute (e.g. low or high, big or small, tall or short) at a given instance in time and can decrease or increase over time as consequence of the previous iteration or state. In this generalised FCM approach, during knowledge elicitation the state of the concepts is captured whenever known. In this real-world case of the consequences

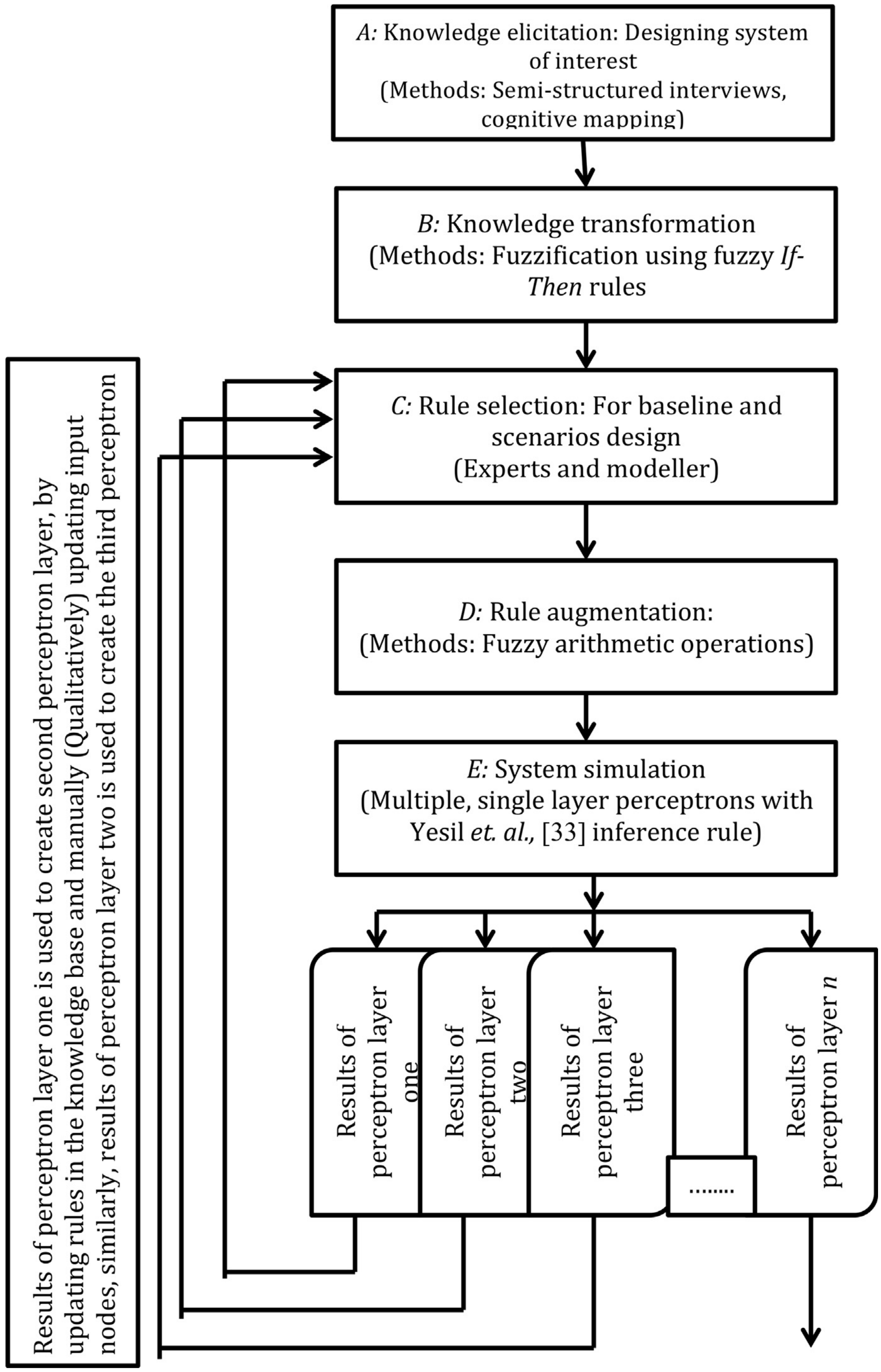

Fig. 1. The generalised FCM approach: A Framework for modelling complex qualitative systems.

high intensity rainfall in Kampala City, several of these (theoretical) causal relations emerge. These causal relations were mapped by experts from various civil society organisations that focus on urban development and environmental planning in Kampala. A sample fuzzy cognitive map elicited from an expert is also provided in the Appendix C.

3.2. B: Knowledge transformation

Concepts and their interactions elicited from experts are stored in a fuzzy rule base or knowledge base. As mentioned before concepts and their interactions were elicited using fuzzy linguistics and fuzzy numbers.[1] Therefore, in the fuzzy rule base, concepts are expressed as fuzzy variables with their states represented using triangular fuzzy numbers. The fuzzy linguistic scale used to interpret the fuzzy number range, of a concept's state/ strength, as provided by the expert's is shown in Figure D1 in Section D of the Appendix. Similarly, the strength of the influence a concept(s) has on another is represented using fuzzy, *If-Then* rules. The fuzzy

[1] Note: For the fuzzy number ranges the attempt was made to elicit the point of the highest belief, to transform the fuzzy number into a triangular fuzzy number. However, some experts provided point of highest belief while other did not, hence, in this study the point of highest belief is assumed to midpoint or modal value of the fuzzy number range.

Table 2
Concept's initial state/activation vector.

Concepts	Initial state	Triangular fuzzy number
C1: Rainfall (Intensity)	*More than High*	[0.7000 0.8500 1.0000]
C2: Flooding	*More than High*	[0.7000 0.8500 1.0000]
C3: Electricity shortages & electrocutions	*Moderately High*	[0.6000 0.7000 0.8000]
C4: Housing & property damage	*Lesser than Low*	[0.00 0.2000 0.4000]
C5: Infrastructure damage (incl. transport)	*Moderately Low*	[0.2000 0.4000 0.6000]
C6: Loss of life	*Lesser than Moderate*	[0.2000 0.3000 0.4000]
C7: Displacement	*Moderately High*	[0.6000 0.6500 0.7000]
C8: Traffic jams	*High*	[0.6000 0.7000 0.8000]
C9: Sanitation	*Moderately Low*	[0.3000 0.3500 0.4000]
C10: Economic activity	*Moderately Low*	[0.3000 0.3500 0.4000]
C11: Theft	*Moderately Low*	[0.2000 0.4000 0.6000]
C12: Environmental degradation	*Moderately Low*	[0.1000 0.4000 0.7000]
C13: Absenteeism (work, school & colleges)	*Lesser than Very High*	[0.8000 0.9000 1.000]
C14: Mobility	*Moderately Low*	[0.3000 0.4000 0.5000]
C15: Diseases	*Moderate*	[0.4000 0.5000 0.6000]
C16: Unclog and revive drains	*Moderately Low*	[0.1000 0.3500 0.6000]
C17: Alternative energy: Generators & solar	*Moderately Low*	[0.00 0.3000 0.6000]
C18: Wetlands	*Moderately Low*	[0.3000 0.4500 0.6000]
C19: Early warning systems	*Lower than Low*	[0.1000 0.1500 0.2000]
C20: Relocation & resettlement	*More than High*	[0.7000 0.8000 0.9000]
C21: Rainwater harvesting	*Very Low*	[0.0000 0.1000 0.2000]
C22: Food shortages	*Less than Low*	[0.1000 0.2000 0.300]

rules can also be conditional (And, OR, x-NOR, etc.) Figure D2 illustrated in section D in the appendix shows the fuzzy linguistic scale used to interpret the fuzzy number range of the strength of the causal influence.

3.3. C. Rule selection

The knowledge base stores information regarding the dynamics of concepts and relations as perceived by experts. Fig. 2 shows the concepts and their relations. Fig. 2 is an illustration of a system map (or a fuzzy cognitive map without weights) conceived by experts where time relations between concepts are illustrated. Concepts and arrows in blue express causal interactions that occur both in the immediate, and long term while concepts, and arrows in black, and orange occur or are activated only in the immediate and long term respectively. Expert's knowledge regarding a concept's state and the time taken for the effects to be felt and the influence vary from expert to expert with some overlap. This gives rise to a wealth of knowledge, on the several, probable causations, and possible alternative vis-à-vis the dynamics of the system.

To simulate the system illustrated in Fig. 2 the inputs are defined by the experts, see Table 2. These inputs are activated in fuzzy rule base to extract strength of the causal influence for immediate and the long term. Table 2 illustrates concepts and their states (inputs), as derived from the knowledge of the experts. Concepts in Table 2 at their corresponding state are activated which, in turn activates the fuzzy rules. These fuzzy rules are not defuzzified so as retain fuzziness when simulating the model. Furthermore, rules can overlap due to the diversity in knowledge with expert‘s reasoning. These overlapping rules are augmented in the next section before the model is simulated.

3.4. D. Rule Augmentation

For each concept‘s state the causal relations that are activated, is augmented when there is an overlap in experts‘ reasoning. The rules are augmented based on standard operational laws related to triangular fuzzy numbers (TFNs). Please see section E in the Appendix for a brief overview of the standard operational laws for TFNs. Once the rules are augmented, the strength of the causal influence is represented as *nxn* adjacency matrices. Each *nxn* adjacency matrices is treated as a single perceptron layer. In this study, two perceptron layers are used to model the immediate and long term consequences of high intensity rainfall in Kampala. The weights or the strength of the causal influence in the immediate term after augmentation is illustrated in Table 3. Figure F.1a, in the Appendix illustrates concepts and their relations that are activated for immediate term based on the input.

Table 3
Concepts and the strength of their influence represented as triangular fuzzy numbers.

Concepts and their influence		Strength of influence represented as a triangular fuzzy number
C1 →	*C2*	[0.4167 0.54170.6667]
	C3	[0.5500 0.6125 0.6750]
	C4	[0.4800 0.5300 0.5800]
	C5	[0.1334 0.3167 0.5000]
	C6	[0.4200 0.5300 0.6400]
	C7	[0.2500 0.4750 0.7000]
	C8	[0.3667 0.5667 0.7667]
	C9	[−0.7000 −0.5250 −0.3500]
	C10	[−0.7000 −0.4000 −0.1000]
	C11	[0.4000 0.45000 0.5000]
	C12	[0.2500 0.4250 0.6000]
C2 →	*C4*	[0.6250 0.7125 0.8000]
	C5	[0.5500 0.6500 0.7500]
	C6	[0.1000 0.2500 0.4000]
	C13	[0.5000 0.5500 0.6000]
	C15	[0.5000 0.6000 0.7000]
C3 →	*C6*	[0.1000 0.2000 0.3000]
C5 →	*C3*	[0.1000 0.3500 0.6000]
	C14	[−0.8000 −0.7000 −0.6000]
C12 →	*C18*	[−0.8000 −0.3500 −1.0000]
C15 →	*C6*	[0.5333 0.6000 0.6667]
C16 →	*C2*	[−0.7333 −0.6000 −0.4667]
C17 →	*C3*	[−0.9000 −0.7167 −0.5333]
C19 →	*C2*	[−0.9000 −0.8000 −0.7000]

3.5. E: System simulation

In this generalised FCM approach, concepts and nodes are treated as separate entities. A concept is a complex entity while a node is one representation of a concept. Several FCM extensions [5,28,31,38] use fuzzy rules to try to separate the notion of

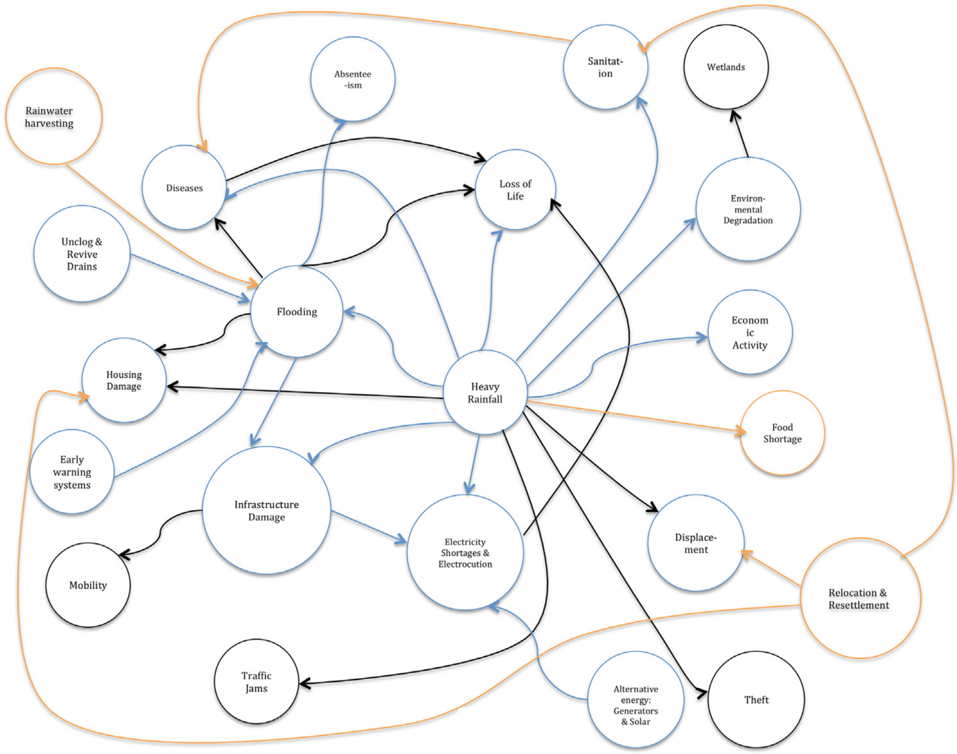

Fig. 2. System map of the consequences of a high intensity rainfall with time relations.

a concept from a node. Most studies that include fuzzy rules for the representation of concepts and their interaction within the system invariably de-fuzzify Eq. (2) their results because classical fuzzy arithmetic are not considered in the conventional FCMs approaches (see Eq. (1)).

$$C = \frac{\int xg(x)dx}{\int g(x)dx} \tag{2}$$

In this generalised FCM approach fuzzy rules are explained using fuzzy numbers—specifically TFNs to help represent the fuzziness and ambiguity typically associated with linguistic terms. The results of the fuzzy rules are not de-fuzzified so as to retain the fuzziness and ambiguity associated with concepts [36]. The fuzzy output from the fuzzy rule base is inferred using an alternative method [35] that includes fuzzy numbers and fuzzy arithmetic, as expressed in Eq. (3).

$$C_j(t+1) = \sum_{i=1}^{n} \tilde{w}_{ij} \otimes C_i(t) \tag{3}$$

Since TFNs are used to represent concepts and relations, simulations run by applying Eq. (3), however results provide only standard approximations. The real result is found either by using the α-cut method or the extension principle method [51–55]. In the generalised FCM approach the extension principle (Equation E.7) is used to map the results of to the pre-defined universe of discourse using the hyperbolic tangent function Eq. (5).

$$C_j(t+1) = f(\tilde{C}_j(t+1)) \tag{4}$$

where $f(x)$ is

$$f(x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \tag{5}$$

In this generalised FCM approach the results produced are TFNs. However, for ease in interpreting the results the centre of gravity of the triangle is identified using Eq. (2). The results presented in Section 4, retain both the TFN as well as the de-fuzzified values. The simulation results explain the relative change that a concept has undergone which is then translated into the absolute change that the concept undergoes. A qualitative translation scale helps convert the relative change into the absolute scale (see section G in the Appendix for the scale). Finally, the output or the new concept states are used as input to activate the long term influences. This is modelled to understand the long term effects of high intensity rainfall if it continues.

4. Results

In this section the simulations of the system shown in Fig. 2 is discussed. Section 4.1 explains the immediate term consequences of a high intensity rainfall event (the baseline story). Section 4.2 explains the long term consequences of a high intensity rainfall event modelled as a result of the baseline story. This simulation approach models two discrete events in succession.

4.1. Modelling the behaviour of the immediate consequence of high intensity rainfall in Kampala, Uganda

The system map of the immediate term consequences is simulated at the unit-time of six hours. The unit-time of six hours is extracted from the expert‘s reasoning. Most experts felt that the immediate term consequences takes place within a day, with most interactions playing out within six hours. Table 3, illustrates the initial state of each concept and Table 4, illustrates the strength of each causal relation for immediate term as perceived by the experts. Concepts *C19*, *C17*, *C16* and *C1* are controlled through each step of the iteration as they are input nodes. Fig. 3 presents the simulation results of the immediate term consequences of high intensity rainfall.

Each graph in Fig. 3 shows the TFN range (bands in black and red), which represents the relative increase or a decrease,

Fig. 3. The evolution of the immediate term consequences of high intensity rainfall.

each concept experiences during iterations. The asterisks (*) show the centre of gravity of the TFNs, the black band illustrates the uncertainty range and the red band indicates the iteration when the system starts to stabilise. Table 4 illustrates how each concept behaves/ unfolds. The second column in Table 4 represents the relative change (increase/decrease) that a concept has undergone in fuzzy linguistics. The TFN range is translated using the fuzzy linguistic scale (see section G in the Appendix). Column three represents the number of iterations taken before each concept stabilises. Each iteration is considered as one unit-time and column four in Table 4 represents the time taken before each concept stabilises in hours and column five in days. Column six is the translation of the relative change (column three) to the absolute change that a concept has undergone.

For example the results illustrated in Fig. 3 and Table 4 suggests that concepts *C2: Flooding*, *C13: Absenteeism (work, school & colleges)*, *C15: Diseases*, does not show any increase or decrease (relative change) after stabilisation, i.e. after, 3, 2.25 and 3.75 days, respectively. Since, these concepts do not experience any increase or decrease, the state of these concept do not change thus no absolute change from their initial state (presented in Table 2) is experienced. Similarly, concepts *C11: Theft*, and *C18:*

Table 4
The evolution of the immediate term consequences of high intensity rainfall event.

Concept	Relative change	Total No. of iterations	Total time taken at unit-time (six hours)	Time in days	Absolute change in state
C1: Rainfall (Intensity)	Controlled/ Input variable	N.A.	N.A.	N.A.	*More than High*
C2: Flooding	No change	12	72	3	*More than High*
C3: Electricity shortages & electrocutions	Increases less than a little	14	84	3.5	*Lesser than Very High*
C4: Housing & property damage	Increases little	12	72	3	*Moderately Low*
C5: Infrastructure damage (incl. transport)	Increases little	14	84	3.5	*Moderately High*
C6: Loss of life	Increases little	16	96	4	*Moderately Low*
C7: Displacement	Increases little	12	72	3	*Less than Very High*
C8: Traffic jams	Increases little	13	78	3.25	*Less than Very High*
C9: Sanitation	Decreases little	13	78	3.25	*More than Very Low*
C10: Economic activity	Decreases little	12	72	3	*More than Very Low*
C11: Theft	Increases less than very little	13	78	3.25	*Moderately Low*
C12: Environmental degradation	Increases less than a little	9	54	2.25	*Moderate*
C13: Absenteeism (work, school & colleges)	No change	15	90	3.75	*Lesser than Very High*
C14: Mobility	Decreases more than very little	15	90	3.75	*Moderately Low*
C15: Diseases	No change	15	90	3.75	*Moderate*
C16: Unclog & revive drains	Controlled/ Input variable	15	90	N.A.	*Moderately Low*
C17: Alternative energy: Generators & solar	Controlled/ Input variable	N.A.	N.A.	N.A.	*Moderately Low*
C18: Wetlands	Decreases less than very little	14	84	3.5	*Moderately Low*
C19: Early warning systems	Controlled/ Input variable	N.A.	N.A.	N.A.	*Lower than Low*

N.A. — Not Applicable

Wetlands experience some small relative change. The results suggest that *C11: Theft* increases after 3.25 days while *C18: Wetlands* decreases after 3.5 days, respectively. However, the increase and decrease is so small that the absolute change in state is not discernible. Furthermore, *C3: Electricity shortage and electrocutions*, *C12: Environmental degradation* experience a discernible increase in 3.5 and 2.25 days, respectively. The translation of this relative increase is evident as these concepts experiences an absolute change from its initial state. Concept *C3: Electricity shortage and electrocutions* increases from Moderately *High* to *Lesser than Very High* and concept *C12: Environmental degradation* increases from *Moderately Low* to *Moderate*. The remaining concepts (except for the controlled concepts) demonstrate a large discernible change overtime as illustrated in Fig. 3 and Table 4.

4.2. Modelling the behaviour of the long term consequence of high intensity rainfall in Kampala, Uganda

In this section the absolute change experienced by each concept in the immediate term (Table 4) is used as feedback to activate the rules of the long term consequences as perceived by the experts. From Fig. 2 it is evident that some concepts and causal connections in the long term emerge while others disappear. This is also evident when comparing Table 5 with Tables 3 and 4 (or Figure F1a and Figure F1b) some concepts, and causal relations emerge while others disappear and the influence of concepts change based on the time taken for the effect to be felt. For example, *C21: Rehabilitation and resettlement*, *C22: Rainwater harvesting*, and *C22: Food shortages* are concepts that emerge in the long term. *C22: Rainwater harvesting* influences *C2: Flooding*, while *C21: Rehabilitation and resettlement* influences *C15: Displacement*, *C4: Housing & property damage* and *C9: Sanitation*. Other causal relations that emerge in the long term is the influence of *C1: Rainfall (Intensity)* on *C22: Food shortages*, and *C5: Infrastructure damage (incl. transport)* on *C6: Loss of life* and *C13: Absenteeism (work, school & colleges)*. Similarly, concepts that disappear in the long term for example are *C8: Traffic jams*, *C14: Mobility* and *C18: Wetlands*. Some causal connections that disappear for example is the influence of *C1: Rainfall (Intensity)* on *C4: Housing & property damage*, *C6: Loss of Life*, and *C7: Displacement*. The graphical representation of the system (long term consequences of high intensity rainfall) is illustrated in Figure F1b. Column two in Table 5, shows the updated state of each concept as TFNs and column four shows the strength of the influence of the causal connection.

The simulation results of the system illustrated in Figure F1b and Table 5 are presented in Fig. 4 and Table 6. Note: The unit-time the system is modelled at is 3 days for the long term consequences because experts perceive that most long term causal interactions take place within 3 days. The results present the evolution of the system given the changes in the immediate term. Hence, to trace the absolute change each concept has undergone (presented in Table 6) it must be compared against its updated initial state (presented in Table 5 as TFNs. For example, *C6: Loss of life* and *C22: Food shortages* experience a discernible increase. This relative increase suggests an absolute change in concept *C6: Loss of life* and *C22: Food shortages* from it initial state. Concept *C6: Loss of life*, increases from *Moderately Low* to *Moderately High* and *C22: Food shortages*, increases from *Lesser than Low* to *Low* in 33 days. Furthermore, *C5: Infrastructure damage (incl. transport)* and *C11: Environmental degradation* experiences a large discernible increase. The translation of this relative increase suggests that absolute state changes from *Moderately High* to *Very High* for *C5: Infrastructure damage (incl. transport)* and from *Moderate* to *Lesser than High* for *C11: Environmental damage* in 27 and 24 days, respectively. The results for the remaining concepts are presented in Table 6.

5. Discussions

This paper tries to address the issue of modelling the dynamics of qualitative systems using FCMs. This was the intended goal of FCMs as envisioned by Kosko [7]. In this paper, a new generalised FCM approach was introduced that tries to address the issues with modelling qualitative SD using FCMs. Eliciting and modelling the dynamics associated to a cause and an effect and by demonstrating it using a real world case of the socio-economic consequences of high intensity rainfall in Kampala, addresses most issues with using FCMs in simulating qualitative SD. To elaborate knowledge regarding the various states of a concept, the dynamic influence that a concept at a given state has on another based on the time taken for the effect to be felt is elicited. The authors demonstrate the means of eliciting causal dynamics from experts (see questionnaire presented in the Appendix) and advocate that it is necessary to elicit detailed information regarding the dynamics of a system from experts to

Table 5
Updated concept states based on immediate term outputs and their corresponding causal influence.

Concept or cause	Updated concept state/strength	Concept affected	Strength of the causal influence/unit-time days
C1: Rainfall (Intensity)	[0.7000 0.8500 1.0000]	*C2* *C3* *C5* *C6* *C9* *C11* *C12* *C22*	[0.5000 0.6000 0.7000] [0.1000 0.1500 0.2000] [0.5000 0.5500 0.6000] [0.000 0.1000 0.2000] [−0.5000 −0.2500 0.0000] [0.000 0.15000 0.3000] [0.8000 0.8500 0.9000] [0.4000 0.4750 0.5500]
C2: Flooding	[0.7000 0.8500 1.0000]	*C13* *C15*	[0.7000 0.8000 0.9000] [0.3500 0.4750 0.6000]
C3: Electricity shortages & electrocutions	[0.7500 0.8500 0.9500]		
C4: Housing & property damage	[0.2000 0.4000 0.6000]		
C5: Infrastructure damage (incl. transport)	[0.4000 0.6000 0.8000]	*C3* *C6* *C13*	[0.0000 0.2000 0.4000] [0.5000 0.5500 0.6000] [0.2000 0.2500 0.3000]
C6: Loss of life	[0.3500 0.4500 0.5500]	*N.A.*	
C7: Displacement	[0.8000 0.8500 0.9000]	*N.A.*	
*C8: Traffic jams**	[0.8000 0.9000 1.0000]		
C9: Sanitation	[0.1000 01500 0.2000]	*N.A.*	
*C10: Economic activity**	[0.1000 01500 0.2000]		
C11: Theft	[0.2000 0.4000 0.6000]	*N.A.*	
C12: Environmental degradation	[0.2000 0.5000 0.8000]	*N.A.*	
C13: Absenteeism (work, school & colleges)	[0.8000 0.9000 1.000]	*N.A.*	
*C14: Mobility**	[0.3000 0.4000 0.5000]		
C15: Diseases	[0.4000 0.5000 0.6000]	*N.A.*	
C16: Unclog & revive drains	[0.7000 0.8500 1.0000]	*C2* *C9*	[−0.5000 −0.4000 −0.3000] [0.4000 0.5000 0.6000]
C17: Alternative energy: Generators & solar	[0.00 0.3000 0.6000]	*C3*	[−0.6500 −0.5500 −0.4500]
*C18: Wetlands**	[0.3000 0.4500 0.6000]	*N.A.*	
C19: Early warning systems	[0.1000 0.1500 0.2000]	*C2*	[−0.5000 −0.4000 −0.3000]
*C20: Relocation & resettlement***	[0.7000 0.8000 0.9000]	*C4* *C7* *C9*	[−0.3000 −0.2000 −0.1000] [−0.3000 −0.2000 −0.1000] [−0.3000 −0.2000 −0.1000]
*C21: Rainwater Harvesting***	[0.0000 0.1000 0.2000]	*C3*	[−0.7500 −0.6500 −0.5500]
*C22: Food shortages***	[0.1000 0.2000 0.3000]	*N.A.*	

* Concepts deactivated for unit-time days
**New concepts that are activated for unit-time days
N.A. — Not Applicable

be able to use FCMs as a tool for dynamic simulations. These causal dynamics elicited is represented as fuzzy rules to enhance flexibility and modelled using, several, single layer perceptrons to explain the behaviour of the system. The issue regarding uncertainty is also addressed when simulating the system. Accordingly, the results presented in Section 4.1 explains the immediate term consequence of 'high intensity rainfall', Section 4.2 explains the long term consequence of 'high intensity rainfall' if it were to continue. These two sections explain the socio-economic dynamics of high intensity rainfall event. The results describe the increase or decrease (relative change) as well as the change in state (absolute change) experienced by each concept over time. The results also explain the uncertainty range of the relative change that a concept experiences.

Unlike conventional FCMs, and most advances in FCMs discussed, the advantages of this approach are the following (i) it can model complex qualitative systems while explaining the evolution of a system not only as some relative change but also as the absolute change a concept undergoes (which is considered more important than the former [30]) (ii) it considers and incorporates the dynamics associated with the inclusion of time when modelling causal connections while explaining the evolution of a system through time and (iii) addresses the issue of uncertainty or fuzziness by eliciting fuzzy number ranges from experts, representing causal relation as fuzzy rules and by simulating the model using an inference method similar to that of Kosko's [7] which, allows fuzzy arithmetics'. Furthermore, the GFCM approach provides flexibility in capturing, representing and simulating the dynamics of causes and effects including their time relations. The use of multiple layers of perceptrons enhances the simulation of the time dynamics of the system. Thus the generalised FCM approach explains the dynamics of a system to greater detail in comparison to traditional FCMs and their advances and the results produced are robust, enabling better decision-making. However, the generalised FCM approach has a few drawbacks, the elicitation of causal dynamics is intensive and more time consuming. The point of highest belief is not acquired, thus, limiting the representation of asymmetric causal relation and each unit-time of causal connection is modelled separately.

6. Conclusion

This paper has explored the feasibility of the generalised FCM approach or generalised FCMs (GFCMs) in modelling complex qualitative system dynamics. Conventional FCMs cannot model the dynamics of complex qualitative systems. GFCMs proposes eliciting a greater detail of knowledge regarding the dynamics between concepts and their interaction with others to be able to

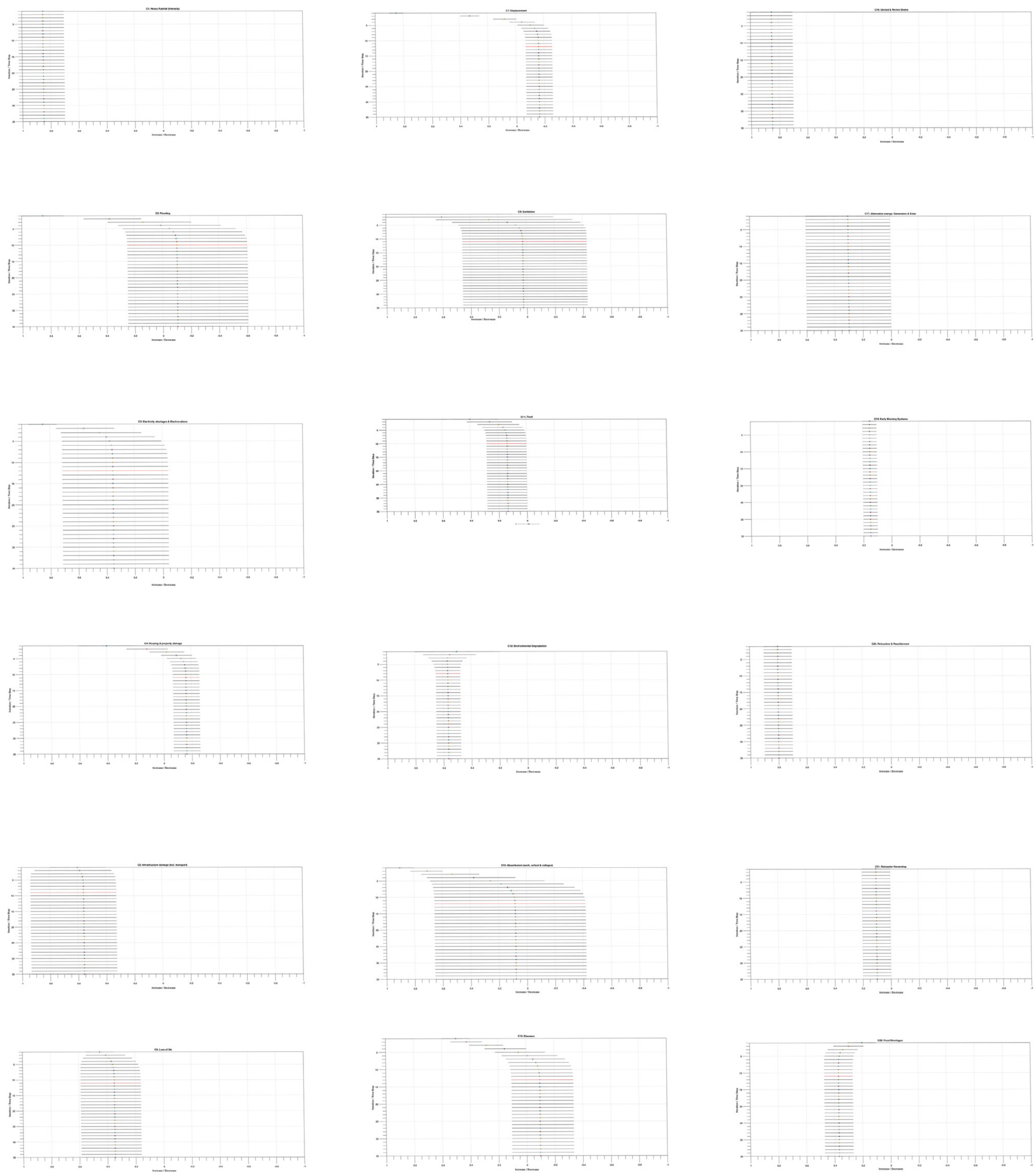

Fig. 4. The evolution of the long term consequences of high intensity rainfall.

robustly simulate SD. GFCMs uses fuzzy rules to represent these dynamics of concepts and relations, including the time dynamics of relations, and introduces a multistep simulation method that incorporates several, single-layer perceptrons to simulate the dynamics, i.e. temporally explicit development of concepts and relations. Furthermore, this approach can explain both the relative and absolute change to a given system while retaining fuzziness and ambiguity typically associated with expert knowledge when representing and simulating the dynamics of concepts and relations. GFCMs create a new perspective and alternative option to modelling the behaviour of complex qualitative system dynamics with FCMs. Despite the possible increase in knowledge required, this approach is more versatile in its use and a possible improvement over other extensions of FCMs when trying to model the dynamics of real world complex qualitative systems.

Future work will be carried to (i) understand the role of explicit time relations in modelling and simulating GFCMs; and (ii) design and test qualitative methods that can best elicit the highest point of belief when obtaining fuzzy numbers from experts and stakeholders.

Table 6
The evolution of the long term consequences of high intensity, rainfall event.

Concept	Relative change	Total No. of iterations	Total time taken at unit-time (three days)	Absolute change in state
C1: Rainfall (Intensity)	Controlled/Input variable	N.A.	N.A.	*Moderately High*
C2: Flooding	Decreases less than very little	10	30	*More than High*
C3: Electricity shortages	No change	12	36	*Lesser than Very High*
C4: Housing & property damage	Decrease very little	11	33	*Moderately Low*
C5: Infrastructure damage (incl. transport)	Increase Moderately	9	27	*Very High*
C6: Loss of life	Increases little	11	33	*Moderately High*
C7: Displacement	Decreases less than very little	12	36	*Lesser than Very High*
C8: Traffic Jams	Deactivated	N.A.	N.A.	N.A.
C9: Sanitation	No change	11	33	*Very Low*
C10: Economic activity	Deactivated	N.A.	N.A.	N.A.
C11: Theft	Increase less than very little	10	30	*Moderately Low*
C12: Environmental degradation	Increase moderately	8	24	*Less than High*
C13: Absenteeism (work, school & colleges)	Increases less than very little	12	36	*Lesser than Very High*
C14: Mobility	Deactivated	N.A.	N.A.	N.A.
C15: Diseases	No change	13	39	*Moderate*
C16: Unclog & revive drains	Controlled/Input variable	N.A.	N.A.	*Moderately Low*
C17: Alternative energy: Generators & solar	Controlled/Input variable	N.A.	N.A.	*Moderately Low*
C18: Early warning systems	Controlled/Input variable	N.A.	N.A.	*Lower than Low*
C19: Wetlands	Deactivated	N.A.		N.A.
C20: Relocation & Resettlement	Controlled/Input variable	N.A.	N.A.	*High*
C21: Rainwater Harvesting	Controlled/Input variable	N.A.	NA	*Very Low*
C22: Food shortages	Increases little	11	33	*Low*

N.A. — Not Applicable

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to https://doi.org/10.1016/j.asoc.2019.105754.

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Appendix A. Supplementary data

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