# Comparison of Two Methods for Measurement of Horn Input Impedance

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**Abstract**

*Two methods to measure the acoustic input impedance of a horn are compared. First method measures standing wave patterns in a tube which is loaded by the horn. The input impedance is calculated from the position of the first minimum in the standing wave pattern, and the ratio of maximum and minimum sound pressure level in the tube. Secondly we applied a direct method. A novel flow sensor, the microflown, is used together with a pressure microphone, which are mounted in the throat of the horn. Results from both measurements are compared with simulated models.*

## 0 INTRODUCTION

Horn loudspeakers are widely used in the area where one needs good –frequency independent– directivity of sound, together with high sound pressure levels. Disadvantage of this type of loudspeaker is its production of excessive distortions at high driving levels. In preliminary research concerning compensation of this distortion, it appeared that the transfer characteristics of the horn itself need to be known accurately to yield better performance of distortion reduction techniques [1]. The acoustic input impedance, determined by the geometrical properties of the horn, is considered in the research presented here. Measurement of this acoustic impedance is important for two reasons. In analysis of the behavior of the horn–driver its acoustic load is fully determined by this input impedance. Secondly we can verify wether or not models of a specific horn are accurate enough or if they need refinements.
In this paper we compare two methods to measure input impedance. The first method was proposed by Fahy for measurement of loudspeaker cabinet impedances [2]. We have applied this method towards the horn. Secondly the input impedance is measured directly using a pressure microphone and a novel developed acoustic flow sensor: the microflown [3], both mounted in the throat of the horn.

## 1 THEORY

In modeling of physical systems, it is of significant importance to use as much as possible a priori knowledge. In the case of a horn we use knowledge about the geometrical pattern, i.e. the varying cross–sectional area along axial axis of the horn. Next to this there are the throat and mouth which form the boundary conditions. Basic theory, which uses the loss–free plane wave fundamental horn equation, is well established [4],[5]. The acoustic input impedance, the quotient of the sound pressure and volume velocity in the throat, is found by solving the fundamental horn equation and applying proper boundary conditions.

### 1.1 Geometrical properties

A horn can be considered as an acoustic transformer. It transforms a small area diaphragm into a large area diaphragm, without the difficulties of cone resonances/break–up. The well known family of hyperbolic exponential horns include the conical, exponential and hyperbolic types. The cross–sectional area $S$ as a function of the distance along the axial axis of this family is given by the general horn equation [4]

$$S(x) = S_t\,(\cosh\alpha + T\sinh\alpha)^2 \tag{1}$$

where $T$ is the family parameter, $S_t$ the cross–sectional area of the throat, $\alpha = x/x_0$ with $x$ the axial distance starting from the throat and $x_0$ is the reference axial distance. The family parameter $T$ fully determines the shape of the horn, as $x_0$ depends on $T$, the throat area and mouth area $S_m$.

For $T$ between zero and one we get the hyperbolic cosine horn, and the hyperbolic sine horn for $T$ between one and infinity. For $T \to \infty$ we obtain a conical horn and for $T=1$ we get an exponential horn. In Fig. 1 radius $R$ of a hyperbolic horn as a function of $T$ and $x$ is depicted. Throat and mouth radius and length, were taken from the horn we used in this research: a *FANE acoustics Ltd.* horn. This horn consists of two separate horns, which gives us three distinct cross–sectional areas to determine the family parameter. It is found to be approximately *0.254*, so we are dealing with a hyperbolic cosine horn.

Solving the fundamental horn equation for the hyperbolic horn gives very complex equations for pressure and volume velocity. Avoiding too complex, and thus time consuming, calculations we will approximate the horn by a finite set of concatenated conical horns.

### 1.2 Transmission line approximation

Transmission line modeling of acoustic elements is a good compromise between accuracy and computational complexity [6]. We will use conical horn elements to approximate our horn. This way any horn of arbitrary cross section (even one of which an analytical solution may not exist) can be approximated. The radiation impedance at the mouth of the horn is the important boundary condition for which we can choose various models. We model it by the commonly used circular piston in an infinite baffle. Both acoustic elements will now shortly be reviewed.

#### 1.2.1 Conical horn

For $T \to \infty$ in Eq. (1) we find a geometrical pattern known as the conical, of which its cross sectional area along the axial axis is given by

$$S(x) = S_t\,(1 + \alpha)^2 \tag{2}$$

The throat impedance is found from the fundamental horn equation and is found to be [4]

$$z_{A,t} = \frac{\varrho c}{S_t}\left[\frac{j z_{A,m}\frac{\sin k(l-\theta_2)}{\sin k\theta_2} + \frac{\varrho c}{S_m}\sin kl}{z_{A,m}\frac{\sin k(l+\theta_1-\theta_2)}{\sin k\theta_1 \sin k\theta_2} - \frac{j\varrho c}{S_m}\frac{\sin k(l+\theta_1)}{\sin k\theta_1}}\right] \tag{3}$$

where $l$ is the length of the horn, $k\theta_1 = \tan^{-1} kx_1$, $k\theta_2 = \tan^{-1} kx_2$, $x_1$ is the distance from the apex to the throat and $x_2$ is the distance from the apex to the mouth, $\varrho$ being the equilibrium gas density, $k$ the wavenumber, $c$ the speed of sound and $z_{A,m}$ the radiation impedance at the mouth.

Modelling of a horn by a concatenation of equal length conical elements is schematically depicted in Figure 2, where a horn is approximated by *5* conical horns. For each element the radius, $r_m$ at the mouth or $r_t$ at the throat of a conical element, is determined by the local cross sectional area of the horn which is to be approximated. For clarity, $x_1$ and $x_2$ are given in this figure for the final conical section.

Calculation of the horn input impedance starts at the final element where we assume that radiation at the mouth of the horn behaves as a circular piston in an infinite baffle. Input impedance of the final element is now possible which is, on its turn, the mouth impedance for the next element. This recurrent process repeats until we arrive at the throat of the first element from which we obtain the input impedance of the total horn.

### 1.2.2 Radiation impedance

As stated in the previous subsection, we need a model for the radiation impedance at the mouth. Although various possibilities exist, the acoustic impedance of a circular piston set in an infinite baffle is mostly found to be the best approximation particularly when the mouth flares are not too large. This impedance is given by [4]

$$z_A = \frac{\varrho c}{\pi R^2}\left[1 - \frac{J_1(2kR)}{kR}\right] + \frac{j\omega\varrho}{2\pi R^4 k^3} K_1(2kR) \tag{4}$$

with $R$ the radius of the piston and $J_1$ and $K_1$ Besselfunctions defined by the series

$$1 - \frac{J_1(2kR)}{kR} = \frac{k^2R^2}{2} - \frac{k^4R^4}{2^2 3} + \frac{k^6R^6}{2^2 3^2 4} + \dots \tag{5}$$

$$K_1(2kR) = \frac{2}{\pi}\left[\frac{(2kR)^3}{3} - \frac{(2kR)^5}{3^2 5} + \frac{(2kR)^7}{3^2 5^2 7} + \dots\right] \tag{6}$$

Convergence of both series, especially for large piston areas, can be problematic. In our case, however, it appeared that a 100 terms where sufficient.

## 2 MEASUREMENT METHODS

### 2.1 Fahy's method

One method to measure the acoustic impedance is by measuring standing wave patterns inside a tube driven by a loudspeaker which is terminated by a certain acoustic element of which we want to determine the impedance. Fahy used this method to measure loudspeaker cabinet impedance [2], while we will use it to measure the input impedance of a horn.

The measurement set–up is depicted in Fig. 3. A loudspeaker is attached to one end of a brass tube, while the other end is terminated by the horn. Diameter of the tube is chosen such that there is a smooth transition from tube to horn throat, while the length of the tube has to be at least one–half wavelength of the lowest frequency of interest. A probing microphone is needed for measuring the sound pressure inside the tube. We also want to move it along the axial axis to determine standing wave maxima and minima. Therefore two slots are made where the probing microphone can move along, keeping the slot as tight as possible closed by means of a covering tube. Two slots are necessary because we need a shorter covering tube to measure near the throat of the horn. For the driving loudspeaker a normal full range electrodynamical loudspeaker was chosen. It is not possible to find such a loudspeaker for the range of *300–5000 Hz* with a cone diameter of *22 mm*, and therefore a flange was made to connect loudspeaker to tube. The hereby created volume in front of the loudspeaker has a Helmholtz resonance frequency of *218 Hz*, which is therefore of no concern.

The lowest cutoff frequency of the tube, which has a diameter $d = 22$ *mm*, is given by

$$f_c = \frac{0.5861c}{d} \tag{7}$$

which yields a maximum frequency of *9.2 kHz*. This is sufficiently high as we want to measure in the frequency span of *300–5000 Hz*. The length of the tube is *610 mm* which yields a lower frequency of *283 Hz*.
For determination of the input impedance three values have to be determined. First the position of the sound pressure minimum nearest to the throat, $x_{min}$, and next the maximum and minimum sound pressure of the standing wave pattern in the tube, $p_{max}$ and $p_{min}$, are determined. The complex acoustic impedance is then found from [2]

$$Z = R + jX = \frac{\left(\frac{1.52c}{d^2}\right)(1 - r^2)}{1 + r^2 - 2r\cos(t)} + j\frac{\left(\frac{1.52c}{d^2}\right) 2r\sin(t)}{1 + r^2 - 2r\cos(t)} \tag{8}$$

with $r$ the pressure reflection coefficient and $t$ the reflection phase angle which are given by

$$r = \frac{a - 1}{a + 1} \qquad with \quad a = \frac{p_{\max}}{p_{\min}} \tag{9}$$

and

$$t = \pi\left[\frac{4fx_{\min}}{c} + 1\right] \tag{10}$$

As we only need the ratio of maximum and minimum pressure, the microphone does not need calibrating. This procedure is performed for every frequency in the span of interest. Because it is necessary to seal the tube properly with each measurement, it more straightforward to perform a full frequency sweep of *401* frequency points at every measurement spot in the range of *x=0–393 mm*, with a resolution of *1 mm*. This is a very tedious task which yields an enormous amount of measurement data, especially as we want to measure as accurate as possible, which means *394* separate measurements.

### 2.2 Direct method

Most straightforward method to measure acoustic impedance is of course direct measurement of sound pressure and volume velocity (acoustic flow) inside the throat of the horn. Pressure can be measured using a calibrated microphone, but a flow sensor for midrange frequencies was not available until recently [3]. The microflown, which is depicted in Fig. 4, is based on a mass flow sensor and consists of three heated cantilever shaped resistors located at the end of a silicon diece. The two outermost resistors are used as temperature sensors, while the resistor in the middle is used as heater. Due to an acoustic flow the heat distribution in the sensor will change. A resulting change in the sensing resistors is measured and converted into an electrical output voltage which is proportional to the applied flow. Calibration of the microflown is done for low frequencies in an acoustic shortcut set–up, and for high frequencies by means of a time frame measurement inside a very long tube [7].
Both, microphone and microflown, are mounted in the throat of the horn, thereby firmly sealed to the throat by modeling clay. Influence of the microflown on the sound waves is negligible small as its dimensions are very small ($800 \times 140\ \mu m$ and $2 \mu m$ thick). The microphone has to be placed with more care because its dimensions are such that sound pressure distribution may be disturbed. Best compromise between accurate measurement and least disturbance is found by placing the microphone end flush with the inside of the horn wall. We hereby assume that the pressure in the throat of the horn is equally distributed along the cross sectional area, as in the throat we have plane wave propagation.
Major advantage of this method compared to the previous one, is that the measurement set–up is much simpler and it is performed by one single frequency sweep measurement where an analyzer measures both complex pressure and flow signals.

## 3 RESULTS AND DISCUSSION

Measurement results of both methods are depicted in Fig. 5. Comparing them with each other, we notice directly a much noisier result with Fahy's method. This is due to the fact that this result is obtained from a lot of distinct measurements which yields 'position noise', which on its turn affects the calculated impedance from it. This is why measurements are limited to the frequency span of *383–4500 Hz*, outside this span measurements were not useful. Looking at Eq.(8), the sensitivity for an error in $x_{min}$ is quite high especially whenever $t$ is near to $(\pi/2 + n\pi)$ or $(n\pi)$, $n \in \mathbb{N}$. Next to this we observe a difference in the frequencies where the first maximum in the resistance and reactance occurs. This means that we measure a higher cut–off frequency with Fahy's method.

Despite the differences there is, of course, resemblance between both measurements. Most maxima and minima in both resistance as well as reactance have the same quantities and both measurements show a qualitative great resemblance.

A comparison between measurements and the model is of even more interest. In Fig. 6 the results of Fahy's method are compared with the hyperbolic horn model, approximated by a concatenation of *10* conical elements. Increasing the number of elements has no significant influence on the resulting impedance.

Although the results are noisy, it is clear that for frequencies above *2 kHz* we have a fairly great resemblance between measurement and model. Below this frequency however, there is a great error which is clearly caused by a shift of the first maxima in frequency. Possible cause for this discrepancy is the influence of the horn input impedance on the motion of the loudspeaker cone [2]. This is the case when the acoustic reactance of the loudspeaker is equal to or smaller than the horn impedance. For the driving loudspeaker we used ($d=80$ mm, $m=4.5$ g), this yields a frequency of up to approx. *2 kHz* where the influence is significant. Still, the qualitative agreement above this frequency is fairly good, especially for the resistance.

A much greater resemblance, both quantitative and qualitative, is found when comparing the model with the direct measurement results, which are depicted in Fig. 7. Still, we obtain again a difference between the frequencies where the first minima and maxima occur in the lower frequencies, but it is much smaller. We also observe that up to approx. *500 Hz* measurement and model almost coincide (reactance in particular), while between *500* and *600 Hz* both the resistance and reactance become almost constant for about 100 Hz. This yields a frequency–shift between model and measurement which is also observed in the measurement results in Fig. 6, although then between *700* and *900 Hz*.

It is difficult to state what causes these differences between model and measurements. Fact is that the deviation with the direct method is less than with Fahy's method. In the latter we assume that the influence of the microphone probe and slots on the tube do not disturb standing wave patterns. Next to that we have the great sensitivity towards errors in $x_{min}$. We therefore conclude that Fahy's method is less suitable for measuring the input impedance of a horn. Discrepancy between the direct measurement and model are not caused by these interferences. Single assumption made in this measurement is that we do not disturb waves inside the throat of the horn by the placement of the microphone and microflown.

Finally one could state that our model is not accurate enough. The horn used in this research has directing elements inside the mouth of the horn which may affect its impedance. These elements are not taken into account in modeling of the horn. Taking the direct measurement as a reference we adjusted the parameters of the hyperbolic horn model until model and measurement fit on each other. In doing so we need to diminish its length from *31.75 cm* to *27.20 cm* and the family parameter has to be decreased from *0.254* to *0.006*. These are quite large changes which give an improved resemblance between measurement and model, as is seen from Fig. 8. Between *1* and *2 kHz* we still have a rather great discrepancy between model and measurement. For the rest we have indeed obtained a better agreement with measurements.

## 4 CONCLUSIONS

Major conclusion drawn from this research is that direct measurement of (horn input) acoustic impedance is done faster, easier, and more accurate by using a microflown. Using the transmission line approximation for horn modeling together with this measurement we have a valuable tool for modeling horns and other acoustic structures. Any horn can be approximated by this method and its model parameters easily tuned to obtain resemblance between measurement and model.

Also it appeared that Fahy's method is useful for measuring the input impedance of the horn, but is very sensitive to errors in $x_{min}$. Next to that one needs to perform a lot of tedious measurements in which surround noise and system varying conditions (temperature, humidity etc.) affect the measurement. Still, Fahy's method can be seen as a complementary method next to the direct method, especially at higher frequencies.

Discrepancies between model and measurement are caused by elements which were not taken into account, like the directing elements in the mouth of the horn. Also, we have assumed that the radiation impedance at the mouth of the horn behaves as a vibrating circular piston inside an infinite baffle. Clearly this is an approximation which highly affects calculation of input impedance, and as stated by other authors better approximations need to be developed for it [6].

Additional applications of the microflown used in this research, are in linearizing methods where a prediction of the acoustic flow in the throat of the horn is needed to compensate nonlinear propagation of sound waves inside a horn [8]. We could use the flow signal as a feedback signal to avoid modeling or equalization of the horn driver transfer function.

## 5 ACKNOWLEDGEMENT

The help of H. Kuipers with building the measurement set–up for Fahy's method is gratefully acknowledged.

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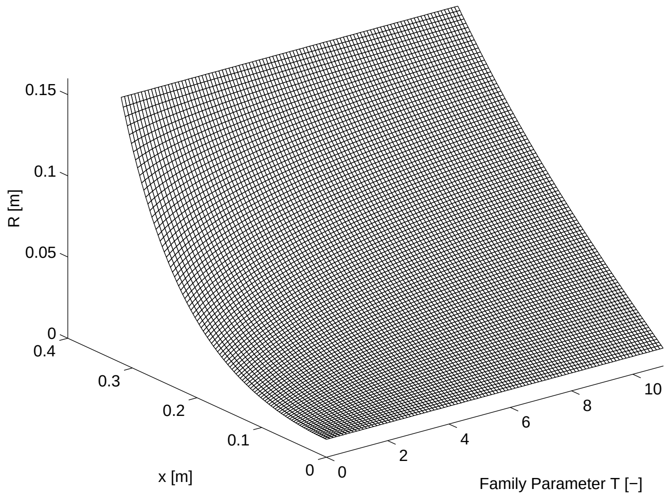


*Fig. 1: Radius R of a hyperbolic horn as function of family parameter T and axial distance x.*

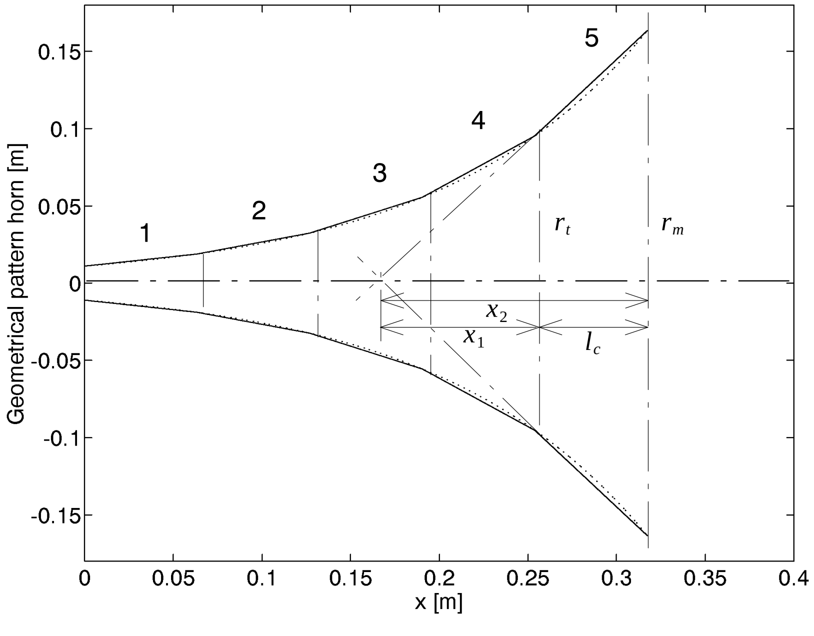


*Fig. 2: Modeling of a horn by 5 equal length ($l_c$) conical elements. Distances to apex for final element are shown.*

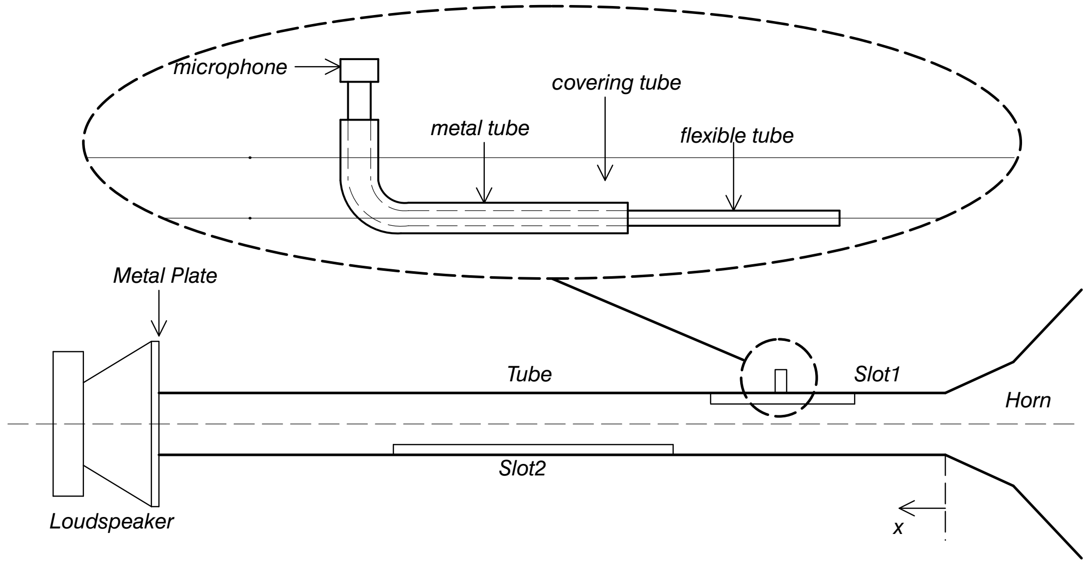


*Fig. 3: Measurement set–up according to Fahy's method, with magnified microphone–probe set–up. Measuring microphone can be inserted in one of both slots.*

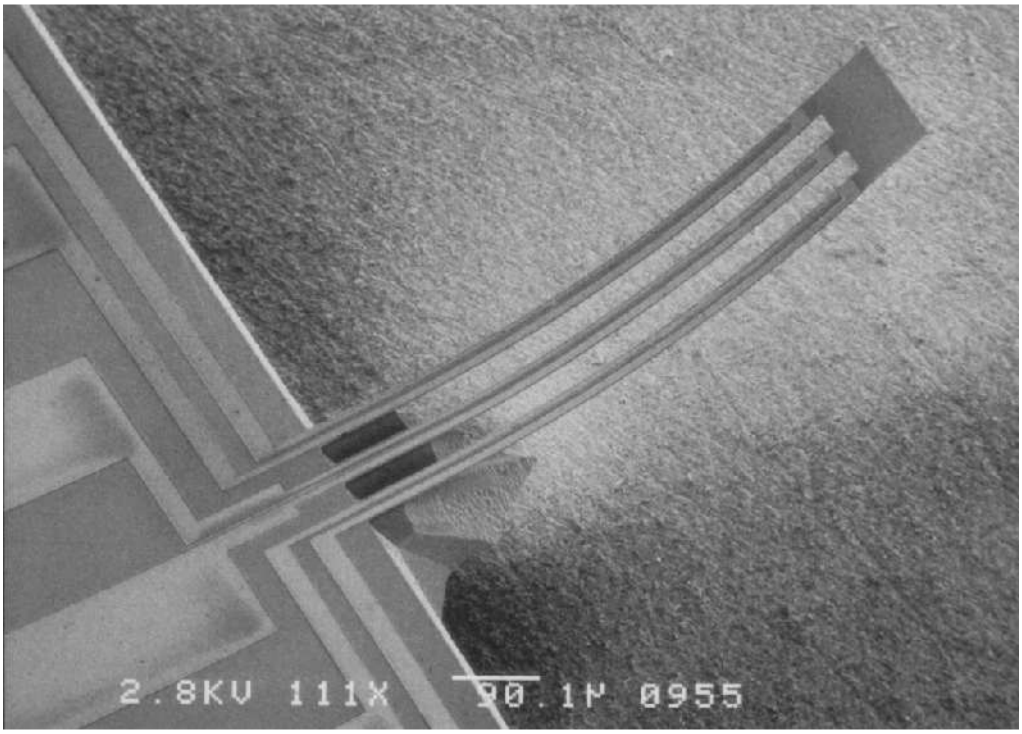


*Fig. 4: The μ–flown, showing the three cantilever shaped resistors of which the outer two are used as sensing resistors while the middle resistor is used as heater.*

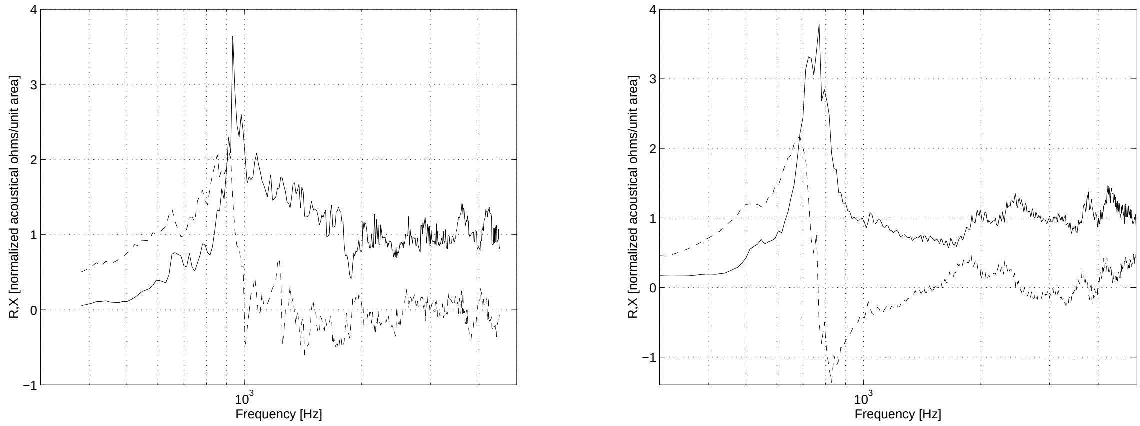


*Fig. 5: Measurement results of Fahy's (left hand–side) and direct (right hand–side) method. Solid lines depict normalized throat resistance R and dashed lines normalized throat reactance X.*

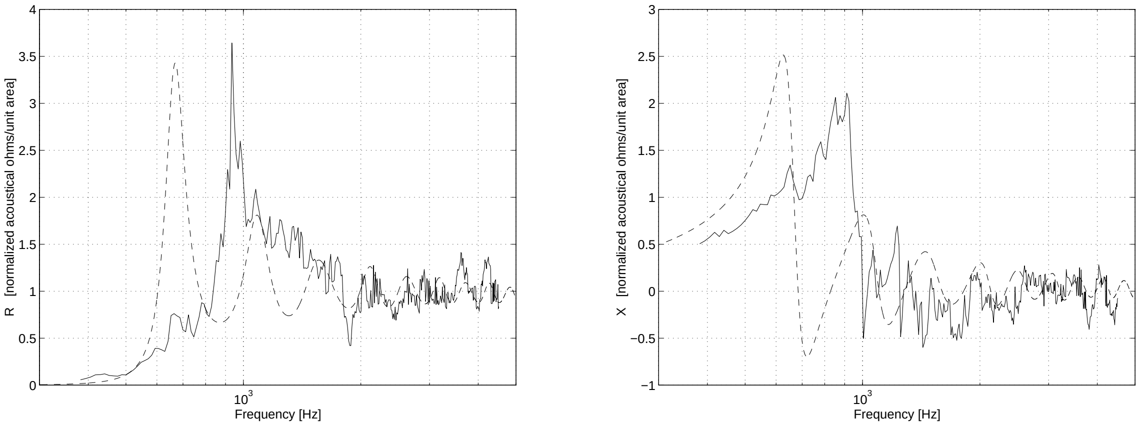


*Fig. 6: Measurement results of Fahy's method (solid–lines) compared with simulation results of hyperbolic horn approximation with 10 conical elements (dashed lines). Left hand–side shows normalized throat resistance R and right hand–side normalized throat reactance X.*

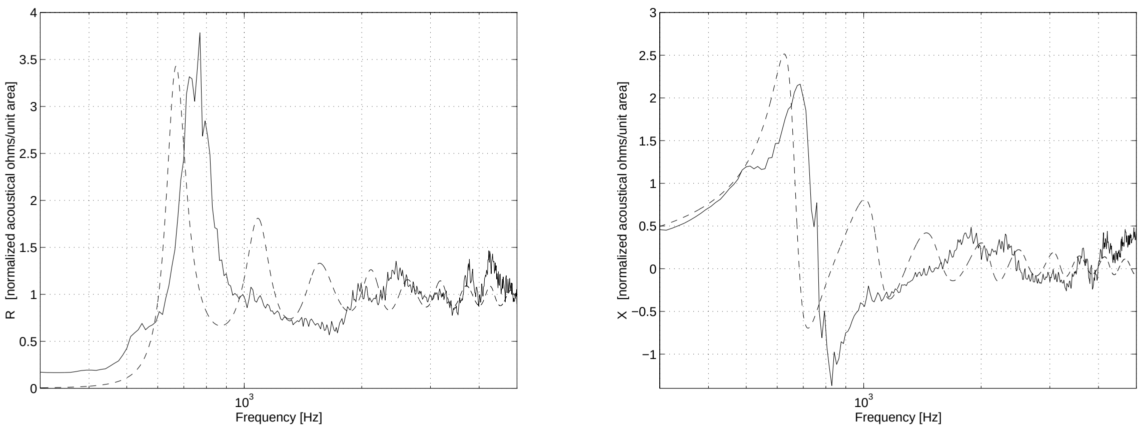


*Fig. 7: Measurement results of direct method (solid–lines) compared with simulation results of the hyperbolic horn approximation with 10 conical elements (dashed lines). Left hand–side shows normalized throat resistance R and right hand–side normalized throat reactance X.*

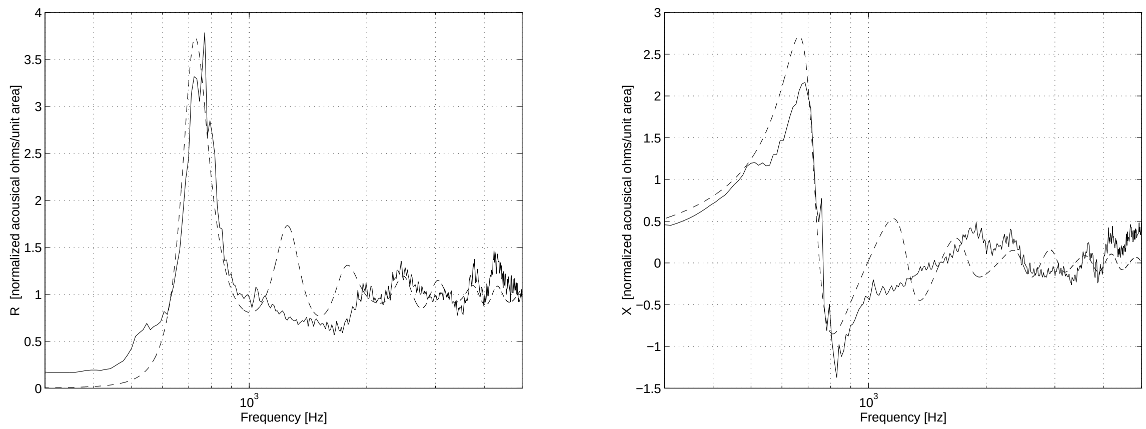


*Fig. 8: Optimized simulation results of the hyperbolic horn approximation with 10 conical elements (dashed lines) on measurement results of direct method (solid–lines). Left hand–side shows normalized throat resistance R and right hand–side normalized throat reactance X.*