

Chapter 3

Energy-Aware Robotics

Stefano Stramigioli

Abstract This chapter has a tutorial nature in introducing a number of useful concepts which resulted by reasoning with power ports rather than with signals, as people usually do in control. Arjan is one of the Godfathers in this way of thinking and he has been a pioneer in bringing these concepts to a new level, introducing proper geometry, a sound system theoretic basis and divulging these issues. This chapter shows how, by using these concepts, it is possible to address or solve certain problems in robotics, control and passivity in a simple and straightforward way. It also presents a formal proof of a claim which is often used as a conjecture and which gives theoretical arguments to counteract the statement which is often used against passivity and saying that passivity is too restrictive and stability is what should be looked for. Many of the concepts reported in the chapter have been the results of discussions with Arjan or are still issues that I am working on with Arjan. It is a great pleasure and honour to have the opportunity to contribute in this way to a recognition of the incredible career of a college and friend for which I have incredible respect from an intellectual and personal point of view.

3.1 Introduction

In many applications of robotics, a controlled robot does interact mechanically with the environment. This interaction means, in system theoretic terms, that the dynamics of the controlled system changes. This change is completely unknown in general and it is, in the opinion of the author, not meaningful in any sense to make hypothesis of linearity, structure or whatsoever of the environment and therefore of this possible change. Furthermore, this change can be discontinuous considering that for example, due to dynamic interaction, bouncing could occur and a consecutive and unpredictable contact/no-contact situation could occur. On the other hand, the robot will physically interact with the environment and the interaction will follow *physical*

S. Stramigioli (✉)
University of Twente, Enschede, The Netherlands
e-mail: S.Stramigioli@utwente.nl

laws, like action and reaction and the first principle of thermodynamics of energy conservation. The first one to specifically address this issue in robotics was Neville Hogan in his famous trilogy [3]. Unfortunately, in the opinion of the author, the core message of Hogan has been often misinterpreted in the robotic literature [14]. From a more systematic and geometrical point of view, the modelling of interaction and behaviour has been presented in [10] and more extensively in [8].

This interaction can be effectively modelled with the concept of a power port known in network theory. The concept of power ports was the basis and essential element used by Paynter in the introduction of Bond Graphs [6]. In Bond Graphs the topology of energetic flows is given the main importance rather than the topology of the physical elements composing the system to be modelled.

A fundamental analysis of methods explaining the basis of bond graphs and their thermodynamical importance has been done by Breedveld [1]. The work of Arjan and Bernhard Maschke on port-Hamiltonian systems together with the deep insight of Peter Breedveld, have started in [5] a new line of research called port-Hamiltonian system theory, which gives a sound system theoretic basis to the use of port concepts in modelling and control. Arjan and Bernhard Maschke have been the pioneers in this line of research and have extended these concepts very elegantly also to distributed parameter systems [13].

The implication of this theory and approach in robotics is unfortunately underestimated, but in the opinion of the author it is the only proper paradigm which can be used to control physical systems which, by their very existence, interact with a physical world where physical energy transfers dictate the way such interaction takes place. The title specifically names “energy awareness” rather than passivity, because the paradigm and ideas presented do not limit in any way the design space of control, but do give methods in order to keep track of the energy flows as a consequence of certain actions in control of robotic systems.

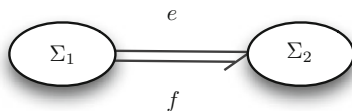
3.2 Why Bother About Power Ports and Energy?

A port models the mean by means of which energy can be exchanged between systems or parts of a system. It can be also used to properly model the interaction between a robot and the environment. Ports can be also used to model the interaction between the actuators of the robot and the robot itself. We can therefore model in this context, any robotic mechanism as a physical system having two multidimensional ports: one modelling the interaction and energy exchange of the robot with the (unknown) environment and one modelling the interaction and energy exchange of the robot with the actuators via which we can modify and shape the robot behaviour via control.

A port is model mathematically with the direct product of a vector space and its dual as

$$\mathcal{P} := \mathcal{V} \times \mathcal{V}^* \tag{3.1}$$

Fig. 3.1 Representation of a power-port to interconnect two systems A and B



in which we can call \mathcal{V} the space of *flows* and \mathcal{V}^* the space of *efforts* or the other way around by dualisation.

Depending on the situation, \mathcal{V} can be a scalar, a finite or an infinite dimensional vector space. In the last case, n-forms and Poincaré duality can be used as introduced in [13]. Considering that a port is *the interface* between two “independent” systems, the mathematical formulation describing the port should not be dependent on the states of the two systems and at the same time should be representable at the “input/output” structure of the two system. In multibody dynamics, this is achieved using the structure of Lie groups, in which the port vector space \mathcal{V} is modeled with a Lie algebra, which is not dependent on any element of the group.

A port should also have an orientation indicating the positive direction of power. In bond-graphs, the direction of a port is indicated with a half arrow as shown in Fig. 3.1. Due to the structure of the port, it is then possible in each instant of time to calculate the power flowing in the positive direction as

$$P = e(f). \quad (3.2)$$

Alternatively, by using scattering, the interaction could be represented by wave variables, also known as scattering variables, which can be geometrically defined for finite [9] and infinite dimensional systems [4] in a geometric way. The difference with this formulation is that the power transfer can be then expressed as an algebraic sum of quantities related to the scattering variables rather than a dual-pairing/product of efforts and flows. This approach has some great advantages in certain situations where the energy transport between the two systems is subjected to physical delay. This has brought to novel insight in geometrical telemanipulation [9].

3.3 The Intrinsically Passive Control (IPC) Framework

In [8], the author has introduced a paradigm called Intrinsically Passive Control. The proposed architecture for a controlled robot interacting with the environment is represented in Fig. 3.2. The basic idea is that, as indicated previously, a robot can be modeled as a physical system having two ports, one with the environment and one with the actuators controlled by the control system. The suggested paradigm is that the control should be conceived as a system which will be coupled using the port structure of the actuators to the control robot. The controller, which is implemented in discrete time, is composed of an Intrinsically Passive Controller (IPC) part and a Supervisor part which can inject energy and control the Robot via the IPC controller.

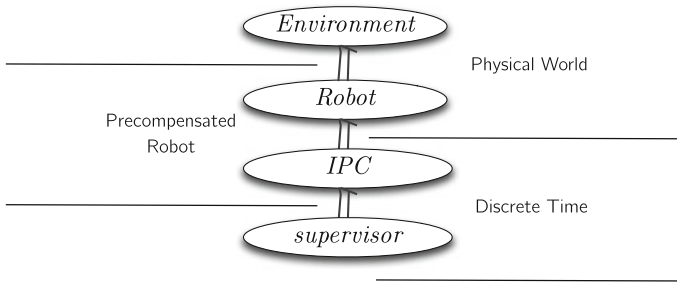


Fig. 3.2 IPC Supervision architecture

This structure has this form because in this way, if the supervisor will not inject energy via the IPC, the energy which can enter the Robot–IPC pair, can only come from the environment. The IPC can be designed on the basis of a model of the Environment, but due to its passive nature, if the Environment will not be as expected, if the supervisor does not inject energy, the interaction will be always passive. This follows the paradigm of what is called *Control by Interconnection* [12].

As it will be formally proved in the next section, if the controlled robot would not be passive seen from the environment side, there exist possible *passive environments* which would destabilise the system when connected to it.

3.4 Passivity as a Must

But why is the concept of port in robotics so important? In robotics the control of robots which interact with an unknown environment should happen stably in interaction with any kind of environment for clear reasons of performance, but more important safety. As said before, once a robot is interconnected with the environment, the stability analysis is only meaningful if the environment is considered as part of the system. Unfortunately, very simplistic and unrealistic models of the environment are used like elastic, purely linear, unilateral or variations of it. The value of such stability proofs is highly discussable considering they only prove stability for a very specific environment. The author argues that in control of systems coupled or interacting with unknown systems, a different paradigm and analysis is necessary as introduced in the previous section.

In this context, the following claims are made:

NP A necessary condition for having stable interaction with an unknown environment is that the controlled robot should result in a passive behaviour seen from the port which interacts with the environment

IPC A necessary condition for achieving the previous point is that, for a physical robot, which is clearly passive as seen composed of a physical system with an interconnection port and a control port, where the controller can supply and drain

energy via actuators, the control should be done via interconnection and should be passive by itself following the IPC paradigm.

The previous claims are at this point conjectures which can be specified more clearly in the following problems statements:

Passivity Control Robot (PCR) If a controlled robot is not passive seen from the environment port, there is always a (passive) environment which can destabilise the interconnected system.

Not Passive State FeedBack (NPSF) For any passive robot, a general control which does not specifically address passivity as a port interconnection (IPC), there is always an environment which could result in an unstable interconnected behaviour as described in PCR.

Characterisation of Stable Active Environment (CSAE) Given a Robot controlled passively via interconnection (IPC), we can characterise the active environments which would result in a stable interconnected behaviour.

The argument PCR is important because it proves NP. The argument NPSF could formally prove that the only proper and safe way to control interactive systems should use the IPC methodology for robustness and that any other state feedback cannot ensure stable behaviour under uncertainty of the plant. Last but not least, CSAE would give a method to characterise and relax hypothesis on the passivity of the environment or humans, as often criticised in the haptic literature. In this work PCR will be formally proven. NPSF and CSAE are conjectures at this stage and work is in process to see if they can be formally proved, maybe with extra conditions.

3.4.1 The PCR Problem

The following theorem is a formal proof of PCR.

Theorem 3.1 *Given a non-passive system Σ with input output pair (u, y) , there always exist a passive system $\bar{\Sigma}$ which connected to Σ will give rise to an unstable behaviour of the interconnection of Σ and $\bar{\Sigma}$.*

Proof Non-passiveness of Σ implies that $\exists \bar{u}(t)$ such that the integral of minus the supply rate is unbounded, which means we can extract infinite energy from the system. Indicate with $\bar{y}(t)$ the output corresponding to the input $\bar{u}(t)$. This means that we can define the extracted energy function $H_o(t)$ as

$$H_o(t) = \int_0^t \langle \bar{u}(s) | \bar{y}(s) \rangle ds \quad (3.3)$$

By construction $\lim_{t \rightarrow \infty} H_o(t) = \infty$. This implies that due to the continuity of $H_o(t)$, \exists a bounded $H_{\min} := \min_t H_o(t)$.

We will now constructively define a passive system $\bar{\Sigma}$ which will generate the input $\bar{u}(t)$.

$$\dot{x} = n(t)\bar{y} \quad (3.4)$$

$$\bar{u} = n(t)\frac{\partial H}{\partial x} \quad (3.5)$$

with $H(x) = \frac{1}{2}x^2$ and $n(t) = \frac{\bar{u}(t)}{\frac{\partial H}{\partial x}}$. It is easy to see that the previous system is passive (even conservative) with storage function $H(x)$. By initialising $x(0) = \sqrt{2H_{\min} + \Delta}$ for any $\Delta > 0$, it can be seen that by construction $\frac{\partial H}{\partial x}(t) > 0 \quad \forall t > 0$ and it is therefore always possible to calculate $\bar{u}(t)$. By setting as interconnection $\bar{u} = \bar{u}$ and $\bar{y} = \bar{y}$, we by construction have that

$$\lim_{t \rightarrow \infty} H_0(x) = \lim_{t \rightarrow \infty} H(x) = \infty \Rightarrow x \rightarrow \infty$$

which proves instability of the coupled system having a state diverging.

The previous proof is simple and reasonably straightforward, but the theorem's implications are far reaching. First of all, the theorem is general and nonlinear. This means that, if a controlled robot is not passive, it is possible to construct an environment, maybe by a second controlled robot, which would be passive and if connected to the original robot would result in an unstable system. This clearly gives a strong reason to create a passive behaviour for any robot which would potentially interact with an unknown environment in order to ensure stable and safe behaviour.

3.5 Connecting to the Discrete World

Everything done so far is treated in continuous time. One important issue in practical applications is that clearly, the controller will be implemented digitally. In order for this framework to be solid, we therefore need a way to couple the continuous and discrete world which will not violate the energy balance and therefore which will not create or destroy energy in the coupling between the continuous and discrete world. This has been introduced in [11] and will be recalled hereafter.

Consider the port interconnection of a continuous time Hamiltonian system H_C and a discrete Hamiltonian system H_D through a sampler and zero-order hold. Suppose that H_C has an admittance causality (effort in/flow out) and therefore H_D has an impedance causality (flow in/effort out).

During the dynamic evolution of the two systems between time kT and $(k+1)T$, where T is the sampling time and k is a positive integer, the effort supplied to H_C by H_D will be constant due to the zero-order hold assumption. We will indicate this value as $e_d(k)$. If we indicate the power port at the continuous side with $(e(t), f(t))$, we clearly have

$$e(t) = e_d(k) \quad t \in [kT, (k+1)T]$$

By looking at the energy flow towards the continuous system, we can see that if we indicate with $\Delta H_C^{\text{in}}(k)$ the energy which flows through the input power port from time kT up to time $(k+1)T$, we obtain

$$\begin{aligned} \Delta H_C^{\text{in}}(k) &= \int_{kT}^{(k+1)T} e_d^T(k) f(s) ds \\ &= e_d^T(k) \int_{kT}^{(k+1)T} f(s) ds \\ &= e_d^T(k) (x((k+1)T) - x(kT)) \end{aligned} \quad (3.6)$$

where we indicated with $x()$ the integral of the continuous time flow $f(t)$.

Remark 3.2 It is important to realise that, in most useful mechanical applications like haptics, $e_d(k)$ will correspond to forces/moments that a controller would apply to an inertial element. In this case, $x()$ would be nothing else than a position measurement of the masses the controller pushes on.

It is now straightforward to state the following theorem:

Theorem 3.3 (Sample Data passivity) *If in the situation sketched before, we define for the interconnection port of H_D*

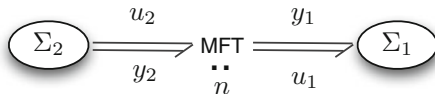
$$f_d(k) := \frac{x(kT) - x((k+1)T)}{T}, \quad (3.7)$$

we obtain an equivalence between the continuous time and discrete time energy flow in the sense that for each n :

$$\sum_{i=1}^n e_d^T(i) f_d(i) = - \int_0^{nT} e^T(s) f(s) ds \quad (3.8)$$

Remark 3.4 It is important to notice that the exact equivalence is achieved only by the definition of Eq. 3.7 in which $x()$ is usually the easiest variable to measure in real applications. The negative sign appearing in Eq. 3.8 is consistent with the fact that the power flowing into the continuous system is minus the power flowing into the discrete side.

Fig. 3.3 Representation of a power-port to interconnect two systems Σ_1 and Σ_2



3.6 Energy Routing

An important technique which has been originally introduced by Duidam and Stramigioli in [2] and called by Ortega DSER in [7] (Duindam Stramigioli Energy Router) allows to direct energy flows without compromising passivity.

3.6.1 Controlling the Energy Directions and Magnitude Among (Sub)systems

To introduce this, with reference to Fig. 3.3 let us start from the situation in which only two ports are considered connecting two systems Σ_1 and Σ_2 and indicated with $(u_1, y_1) \in \mathcal{V}_1 \times \mathcal{V}_1^*$ and $(u_2, y_2) \in \mathcal{V}_2 \times \mathcal{V}_2^*$ and for which we indicated inputs and outputs of the two systems with u_i and y_i , respectively, for $i = 1, 2$. For simplicity of exposition, let us consider $\mathcal{V}_1 = R^n$ and $\mathcal{V}_2 = R^n$. A power continuous interconnection of the two systems is implemented by using the following relations which correspond in bond graphs to a multidimensional transformer or gyrator

$$u_1 = n \ y_2 \quad (3.9)$$

$$u_2 = n^T \ y_1 \quad (3.10)$$

where n and n^T is any linear map and its dual. Clearly, we have that

$$u_1^T \ y_1 = y_2^T \ n^T \ y_1 = y_2^T \ u_2 \quad (3.11)$$

which proves energy continuity. In the previous relation, n can be changed continuously or discontinuously and independently of its value the power continuity will hold by construction. We can therefore vary n also as function of the port variables, creating effectively a system which allows energy flow only in a specific direction. Suppose for example we want to force energy flowing from Σ_2 to Σ_1 . This can be achieved simply by enforcing the direction of the power. Considering the positive power of Fig. 3.3 goes from Σ_2 to Σ_1 , we want to achieve $y_1^T \ u_1 > 0$ indicating positive power flow toward Σ_1 . Using Eqs.(3.9) and (3.10) this can be done by choosing

$$n = \alpha y_1 y_2^T \quad (3.12)$$

for a positive α . It is easy to see that by this construction a negative α will force a flow of energy in the opposite direction and its magnitude will control the amount of energy transfer. At all effects, α can be used to control the amount and direction of energy flow. It is also important to notice that energy will follow in the direction controlled iff energy is available which will result in values of $y_i \neq 0$.

This construction can be easily generalised to the situation in which instead of a two port, we consider a multidimensional Dirac structure connecting n systems Σ_i for $i = 1, \dots, n$. Suppose that by convention, all positive orientations are chosen towards the systems that the Dirac structure connects. In this case, using the same kind of notation, we would have by power continuity that

$$y_1^T u_1 + \dots + y_n^T u_n = 0 \quad (3.13)$$

and this will have to be realised by a relation of the form

$$\begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} = S \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad (3.14)$$

where S can be a skew symmetric matrix of proper dimension: $S^T = -S$. Suppose it is the goal to control the flow direction and magnitude of energy to the first system $y_1^T u_1$. We have that

$$y_1^T u_1 = y_1^T S_1 \begin{pmatrix} y_2 \\ \vdots \\ y_n \end{pmatrix} \quad (3.15)$$

where $(0 \ S_1)$ is the first row of the skew symmetric matrix S . By clearly choosing

$$S_1 := \alpha_1 y_1 (y_2 \cdots y_n) \quad (3.16)$$

we can by choosing α_1 choose the direction and magnitude of power flow towards system Σ_1 and this will fix the first row, and for the skew symmetric constraint column, of the matrix S . By proceeding in a similar way, it would be possible to use the extra degrees of freedom still available in the choice of the matrix S in order to select other energy flows to the remaining systems. A similar analysis could also be carried out by using scattering which would directly represent positive and negative energy flows towards the systems and from the systems attached to the Dirac structure.

3.6.2 Energy Tanks and Tracking

Another way to use energy routing is to keep track of the energy which is used to perform a certain operation. This can be used to prevent instability of certain control actions. Suppose for example to control a robot which interacts with an unknown environment, a general control law which would not specifically monitor the amount of energy injected to the system, could potentially destabilise an interaction with an unknown environment as proven previously, if the energy would not be bounded by a passive behaviour. It is therefore useful to have a strategy which is able to allocate a certain *energy budget* to perform a specific operation and take proper actions if this amount of energy has been used. This action may be to adapt the control to prevent instability, or to analyse the situation and possibly adapt the control strategy providing extra energy. This is why the author talks about energy awareness rather than passivity which could seem restricting the applicability of the paradigm.

Consider the energy tank to have an associated positive definite energy function $H(s) = 1/2s^2$ with s a scalar. The energy budget can be initialised by a proper initial value of s . Assume to have n subsystems as in the previous section which need to be controlled and assume to have a control law $u = f(x, y)$ where u represents the column vector of all inputs, x the vector of states of the systems and y the dual outputs of u . To have power continuity we can consider the following interconnection between the energy tank and the systems:

$$\begin{pmatrix} \dot{s} \\ u_1 \\ \vdots \\ u_n \end{pmatrix} = S \begin{pmatrix} \frac{\partial H}{\partial s} \\ y_1 \\ \vdots \\ y_n \end{pmatrix} \quad (3.17)$$

again with a skew symmetric matrix S and also considering that $\dot{H} = \frac{\partial H}{\partial s} \dot{s}$.

It is possible to show that, under the condition that $\frac{\partial H}{\partial s} \neq 0$, it is possible to calculate a skew symmetric matrix S which satisfies the control relation $u = f(x, y)$ and at the same time monitors the energy necessary for that action by the value of the energy function H . By only monitoring the scalar s is therefore possible to see when the available budget of energy has expired. This simple idea, paradoxically can be used to “passivise” any control law, but building a safety mechanism which would prevent to inject indiscriminate energy into a control system leading to instability. In other words, we can implement the control law $u = f(x, y)$ until the energy set in the beginning is finished and then switch to a different control action to prevent loss of passivity and ensure stable interaction with any passive environment as proved in Theorem 3.1.

3.6.3 Projections

In many robotic applications it is useful to use projectors operators in order to implement certain control strategies. For example, in case of under-actuation, it is not possible to servo a complete force which would be desired, but that should be first projected on the subspace of forces which are implementable. If this projection is done naively, it can result in loss of passivity. By using the method shown in the previous section it is possible to monitor the energy consequence of such actions, but for the sake of clarity and with a didactical goal, hereafter, the example of projections will be further constructed.

Consider a control law which would calculate the applied force F to an under-actuated robot. Assume the motion of the robot can be measured and let us indicate with \dot{x} its velocity. From a port point of view, the controlling port of this robot would then be (\dot{x}, F) and $F^T \dot{x}$ would be the power supplied to the robot. Suppose that, for reasons which are not going to be discussed here, we want to apply to the robot an elastic force with some specific geometrical properties. We can create consistently an elastic force by defining an elastic energy function $H(x)$ which, after integrating the velocity of the robot \dot{x} could calculate the force to be applied as $F = \frac{\partial H}{\partial x}$. Unfortunately, due to the under-actuation of the robot, we first need to project the gradient of H to the subspace of applicable forces. If we indicate with P such a projection, we could indicate the control law with:

$$F = P \frac{\partial H}{\partial x}. \quad (3.18)$$

Unfortunately, such an operation alone would break passivity considering that this operator is acting only on the force and not dually on the velocity. The passivity could be recovered by integrating for the state of the spring $P^T \dot{x}$ rather than \dot{x} , but this would drastically change the control law because the state of the spring would not be anymore representing the configuration of the robot. This paradox is showing that such a projection on the force only, will inject or extract energy from the system and if we are able to exactly monitor this, we can prevent that the projection action would result in loss of passivity and possibly an unstable behaviour. What we can do is therefore specifically to model the energy which is necessary to recover this passive behaviour. This can be clearly done by framing the control operation of the projection as a general control law $u = f(x, y)$ as explained in Sect. 3.6.2 but we will do it constructively hereafter, in order to give better insight.

Let us indicate with $v = \dot{x}$ the real velocity of the robot and with $\bar{v} := P^T v$. If we want to conserve the integration of v rather than \bar{v} for the state of the controller, we can model this by adding a new power port and using what in bond graphs is called a 0-junction (representing one of Kirchhoff's laws), which is an element whose connected bonds all have the same effort F and for which the flows sum algebraically: $v = \Delta v + \bar{v}$. We can now model the energy used for "balancing" the projection, with a new storage element (energy tank) which we can represent with an energy function $\bar{H}(s) = \frac{1}{2} s^2$. By then setting

$$\Delta v = C \frac{\partial \bar{H}}{s} \quad (3.19)$$

$$\dot{s} = C^T F \quad (3.20)$$

and choosing

$$C = (v - \bar{v}) / \frac{\partial \bar{H}}{\partial s} \quad (3.21)$$

we can easily check that, as long as $s > 0$, and there is energy available in the tank, the projection operator will achieve the original goal without the shortcoming of losing information about the pose of the robot in the elastic control and by having an exact quantification of the energy which such an action requires. If we furthermore slightly modify Eq.(3.21) to be

$$C = \begin{cases} (v - \bar{v}) / \frac{\partial \bar{H}}{\partial s} & s > \varepsilon \text{ or } \Delta v F \geq 0 \\ 0 & s \leq \varepsilon \text{ and } \Delta v F < 0 \end{cases} \quad (3.22)$$

where $\Delta v F > 0$ indicates power flow towards the storage tank $\bar{H}(s)$, we can also handle the singularity. This system will implement the desired compensation as long as energy will be available. Further modifications could for example inject energy to the tank $\bar{H}(s)$ by redirecting energy from possible damping actions as presented in the next section.

3.6.4 What About Damping?

Very often, especially for the control of mechanical systems, damping plays an important role. The effect of damping is clearly to irreversibly extract energy from the system. On the other hand, it may be useful to extract energy without necessarily getting rid of it, but rather store it somewhere else, very much in a similar fashion as introduced in the previous section. From a thermodynamical point of view, dissipation is an irreversible transformation of energy from any domain to the thermal domain leading to an increase of entropy. We can use this metaphor, but from a control point of view, we can buffer this energy and use it for other possible means. This operation does not create energy and it is therefore passive and perfectly consistent with the framework. A small modification of what is presented in Sect. 3.6.2 allows to implement this. If for example we increase the dimension of u and y of 1, we can add a relation:

$$u_{n+1} = B y_{n+1} \quad (3.23)$$

where B could be a varying, but positive damping coefficient. The effect of such an action is that any energy which would be extracted via the port (u_{n+1}, y_{n+1}) would automatically be used to increase the energy buffer $H(s)$ to be used as previously described. The possibility of time varying the damping B allows, from the point of view of control, to shape the dynamics of the system in a desirable way and the presented framework will ‘automatically’ take care that the energy balance will be accounted for.

3.7 Conclusions

In this chapter some basic concepts of what the author calls energy-aware robotics have been presented. It has been shown that the passive behaviour of a robot which can interact with an unknown environment is a must to ensure stable interaction with any passive environment. Different methodologies have been presented which give an idea on how, thanks to the use of port concepts, it is possible to structure control loops in such a way that all energy flows can be made explicit and passivity ensured. These techniques can also be used in telemanipulation and many other fields of robotics successfully to ensure a stable behaviour.

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