Chapter 1
Decomposition of Multiports

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1.1 Introduction

This chapter expects from its reader some basic port-based modeling knowledge and the ability to express these concepts in terms of bond graphs. This means that the reader is supposed to be familiar with the basics of the bond graph notation as explained, for instance, in [13, 16], in particular the nine basic node types C, I, Se, Sf, R, TF, GY, 0, and 1 (cf. Tables 1.1 and 1.2 show the block diagram expansions of their causal instantiations) and their multiport realizations.

As a bond graph is a labeled di-graph, its structure is purely topological. This means that any configuration information of a model has to be added separately. This is the reason that bond graphs are often used for mechanical systems in linear motion and/or with fixed axis rotation, i.e., where the spatial configuration part is fixed, such that scalar variables are sufficient to describe the dynamic behavior. If planar or spatial effects need to be described, then the information about the chosen coordinate frames of the power variables has to be added to the model separately. In that case bond graphs still give an advantage due to the fact that velocity relations found from position relations can be used to find the relations between the conjugate forces without the need to use free-body diagrams, thus reducing the chance of sign errors.

Given that bonds in a bond graph represent bilateral relations between ports that describe specific concepts, there is thus no need for these concepts to be spatially separated. In other words: the interconnection structure of concepts in a model can itself be a mere conceptual structure without any meaning for the actual configuration. From this perspective we will show in the following by means of multiport decomposition [3, 4] how the nine basic concepts that are used in

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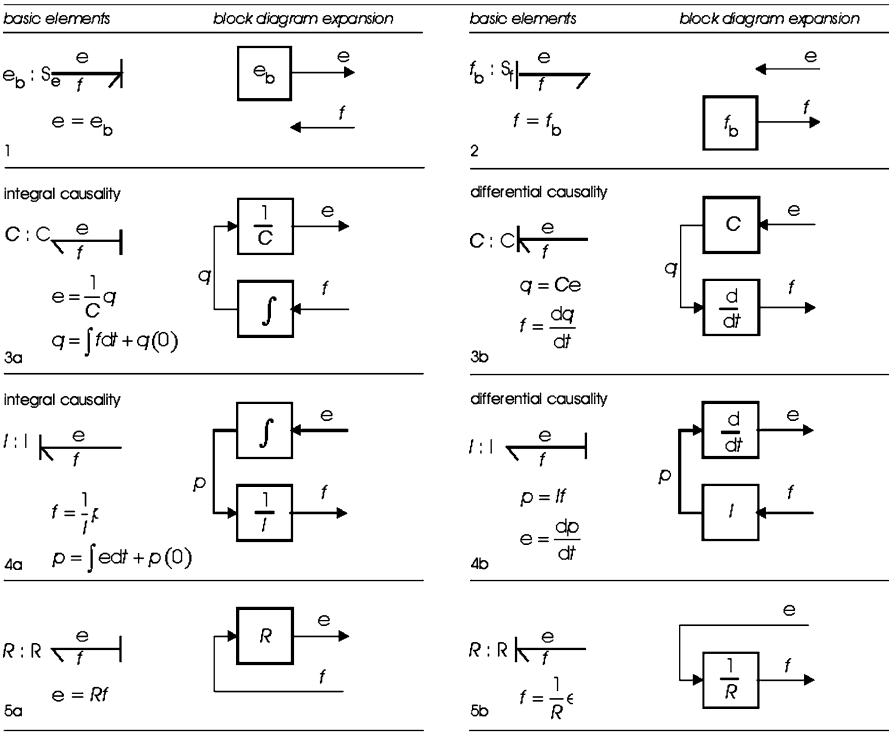
Table 1.1 Basic one-port elements, linear block diagram implementation examples

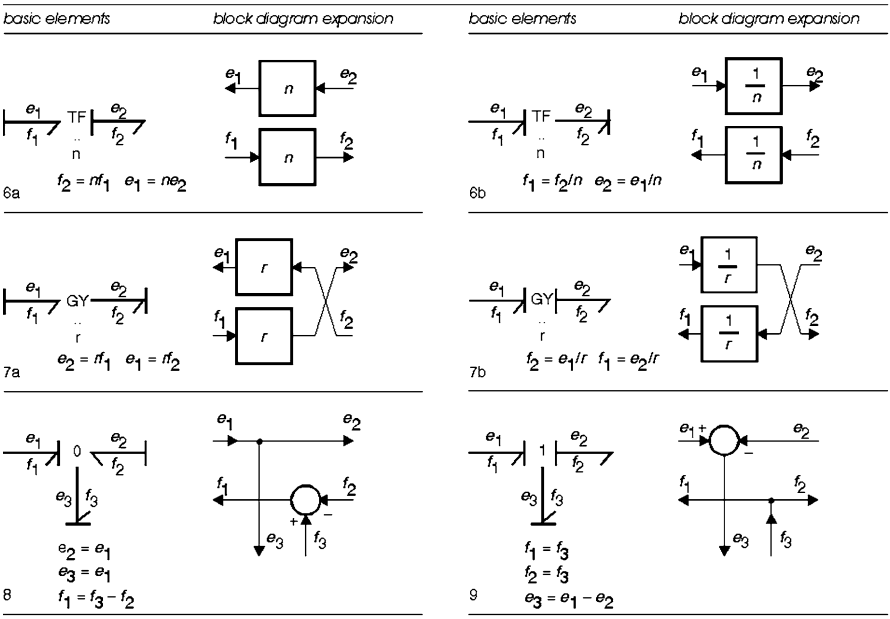


a common bond graph follow from the temporal properties of a physical model, as well as the need to represent the influence of the environment, conceptual interconnection, and basic concepts like reversible and irreversible transduction (link between domains).

A decomposition of a multiport may have multiple purposes:

(1) Getting a better insight in the dynamic properties of a model, as loop gains of causal paths can be easily identified
(2) Getting a better insight in potential simplifications of a multiport due to resulting constitutive parameters that are negligible
(3) Being able to recognize how bond graph fragments can be composed into multiports or other types of basic elements, for instance, to eliminate causal loops
(4) Being able to use techniques for direct linear analysis on a bond graph, like Mason's rule, etc.
(5) Conversion of a bond graph containing multiports into a block diagram or a set of differential equations in state space form in a straightforward manner
(6) Being able to use the graphical input of a simulation package with a bond graph editor that does not support multiport elements

Table 1.2 Basic two- and multiport elements, with linear block diagram implementation examples



First the fundaments of a dynamic model are discussed: energy storage and positive entropy production. Multiport storage is analyzed on the basis of the properties of energy. Next it is discussed that interaction between stored quantities requires the concept of a power continuous multiport and how the relation to the environment can be represented. This approach is able to resolve the analogy paradox and after this is done, irreversible transduction is discussed and its relation to the paradoxal concept of "energy dissipation." Then the power continuous multiport is decomposed and it is motivated that a nonlinear, internally modulated transducer that is able to represent irreversible transduction deserves to be treated as a separate basic concept. Finally the canonical decompositions into basic elements are discussed.

1.2 Dynamic System Models of Multidomain Physical Systems: The Key Role of Time

The key concepts used in modeling the dynamic behavior of physical systems are the concepts of *state* and (rate of) *change* (of state). State and change are dialectic concepts: it is impossible to perceive what a state is if it can never change and at the same time it is impossible to understand change without the concept of

state, i.e., some property that may change from one state into the other. In fact, all measurements of time are based on a repetitive change of some state, resulting in a time base.

A dynamic model is not only depending on time, but it should also be "transferable" in time, in other words: it should not lose its competence to describe relevant aspects of reality the moment it is made. This means that some property of the model of a system has to remain constant over time or the variations of this property should be completely determined by a change in the environmental influences on that system, commonly called its boundary conditions. This property that remains constant is called energy. This "transferability" in time of a model is called the *time translation symmetry* requirement and Noether [15] proved that all symmetries result in some form of a conservation principle that should hold for such a model, in this case *energy* conservation, also known as the first principle (or first "law") of thermodynamics. Thermodynamics is often wrongly considered as a mere theory of heat, but its approach of multiport storage of energy [9] can be considered the first systematic approach to multidomain physics, even if no heat or entropy is taken into account. This is why these two principles should not be limited to thermodynamics, but should be the basis of all models of physical systems: without at least one conserved quantity any model would lose its time translation symmetry and which harms its predictive or explaining value.

It is human experience that most (not all!) dynamic processes that we try to describe are characterized by "the arrow of time," i.e., if we would play a movie of the process backwards, it would appear unrealistic. This *time-reflection asymmetry* corresponds to the violation of a conservation principle, viz., that of entropy conservation and results in the *positive entropy production principle*, also called the second principle (or second "law") of thermodynamics. However, in order to properly describe the behavior of entropy, we also need to assign a state to it and consequently the concept of reversible storage. This also corresponds to the remark that not all processes seem irreversible: in some cases friction and other losses can disregarded and the entropy production can be considered to be approximately zero.

1.3 Multiport Storage

All sorts of symmetry principles lead to other conserved quantities, like momentum, angular momentum, electric charge, magnetic flux, and matter (in the incompressible case expressed in its volume or in the flexible case by a strain or displacement, like that of a spring) that can all serve to describe the state of a physical system in which these quantities play a role. When these conserved quantities, represented by a state vector $\mathbf{q}$, can be used to characterize the complete state of a system, there will always be an energy related to them: $E(\mathbf{q})$, which should be a first-degree homogenous function when the extensive states that can also serve as a boundary criterion, viz., amount of matter (moles) and/or available volume, even if they remain constant, are considered as states. This is why all energy density

functions, i.e., energy functions in which the extent defining variable (volume or amount of matter) is set to unity and not considered as a state, are second-degree homogeneous and mostly quadratic. Kinetic energy, for example, is a first-degree homogenous function of momentum p and amount of moles N

$$E(\alpha p, \alpha N) = \frac{\alpha^2 p^2}{2M\alpha N} = \alpha^1 \frac{p^2}{2MN} = \alpha E(p, N) \tag{1.1}$$

where M stands for molar mass and α is an arbitrary parameter to check the degree of the homogenous function. However, the kinetic energy density (per unit mole) is

$$E(p, 1) = \varepsilon(p) \tag{1.2}$$

which is second-degree homogenous (in this case quadratic), because

$$\varepsilon(\alpha p) = \alpha^2 \frac{p^2}{2M} = \alpha^2 \varepsilon(p) \tag{1.3}$$

Obviously, if the amount of moles is assumed to remain constant and not considered a state, then the kinetic energy also appears to be second-degree homogeneous, even quadratic and, as explained later, this leads to a linear constitutive relation that is first-degree homogenous.

If a system is observed in which all these conserved quantities remain constant, i.e., $\mathrm{d}\mathbf{q}/\mathrm{d}t = 0$ and thus $\mathrm{d}\underline{E}/\mathrm{d}t = 0$, the system is said to be in equilibrium (or in static state or in stationary state) and thus not representing any dynamic behavior. In dynamic processes that are not influenced by their environment we commonly observe that they reach an equilibrium state after a certain period of time. The simplest situation of non-equilibrium is if the system can be thought to consist of two parts that are internally in equilibrium such that an energy can be assigned to each of them, while these part are not in equilibrium with respect to each other. After reaching the equilibrium state the conserved states variables of these two parts, including their energies, do not have to be equal in value. A criterion for equilibrium that will be explained later in more detail is that the rates of change of the energy with respect to the conserved state under consideration q_i have to be equal:

$$\frac{\partial E(\mathbf{q})}{\partial q_i^1} = \frac{\partial E(\mathbf{q})}{\partial q_i^2} \tag{1.4}$$

for all states q_i. Obviously, these partial derivatives are related to the total change of the energy in time, the power $P(t)$:

$$P(t) = \frac{\mathrm{d}E(\mathbf{q}(t))}{\mathrm{d}t} = \sum_i \frac{\partial E(\mathbf{q})}{\partial q_i} \frac{\mathrm{d}q_i}{\mathrm{d}t} = \sum_i e_i(\mathbf{q}(t)) f_i(t) = \mathbf{e}^{\mathbf{t}}\mathbf{f} \tag{1.5}$$

where $\frac{\partial E(\mathbf{q})}{\partial q_i} = e_i(\mathbf{q})$ is defined as a generalized effort, $\frac{\mathrm{d}q_i}{\mathrm{d}t} = f_i$ as a generalized flow and $\mathbf{e}$ and $\mathbf{f}$ are the column matrices of these efforts and flows, respectively, such that $\mathbf{e}^t\mathbf{f}$ represents an inner product.

These generalized efforts are homogenous functions of all states themselves and 1° lower than the degree of the homogeneous energy function:

$$\frac{\partial E(\alpha\mathbf{q})}{\partial \alpha q_i} = \frac{\alpha^n \partial E(\mathbf{q})}{\alpha \partial q_i} = \alpha^{n-1} e_i(\mathbf{q}) \tag{1.6}$$

and form the constitutive relations of a multiport storage element, in a bond graph represented by a multiport C. Some authors use the terminology "C-field" instead of multiport C. As a "field" in physics also has a completely different meaning, it is not used herein to prevent confusion. As the stored energy in a multiport C is a conserved quantity, its mixed second derivatives should be equal

$$\frac{\partial^2 E(\mathbf{q})}{\partial q_i \partial q_j} = \frac{\partial^2 E(\mathbf{q})}{\partial q_j \partial q_i} \tag{1.7}$$

or

$$\frac{\partial e_i(\mathbf{q})}{\partial q_j} = \frac{\partial e_j(\mathbf{q})}{\partial q_i} \tag{1.8}$$

In other words: the Jacobian of $\mathbf{e}(\mathbf{q}(t))$ has to be symmetric according to the principle of energy conservation. This is called Maxwell reciprocity or Maxwell symmetry.

1.4 Resolution of an Analogy Paradox

Most (bond graph) modelers will immediately make the objection that the above definitions of effort and flow are not in line with what they consider to be an effort and a flow: they, together with many other engineers and scientists, consider a force, for example, to be an effort, while a force can indeed be not only the partial derivative of a potential energy with respect to a displacement (deformation of matter or distance in a gravitational field), but also the rate of change of a conserved quantity, viz., the momentum. Others have used the latter argument to consider a force as a flow, leading to an everlasting analogy discussion that highly complicates insight and education [10, 19]. This is why the attribute "generalized" is used: in the generalized bond graphs introduced in [3–5] a force is the generalized effort of the potential domain and the generalized flow of the kinetic domain, while the coupling between these domains is made explicit. In order to come up with one choice for the combination of the kinetic and the potential domains, the so-called mechanical domain, the role of effort and flow in one of these domains has to be inverted, in other words: this domain has to be dualized, i.e., the roles of effort and

flow are interchanged. Obviously, either one of the two domains can be dualized, which causes the just described analogy paradox if the dualization remains implicit. A unit gyrator can be considered an explicit "dualizer" as it equates the flow of each port to the effort of the other port. However, the coupling between the kinetic and the potential domain can only under a certain condition be represented by a dualizing coupling, a unit gyrator called symplectic gyrator [1], viz., the condition that the system is described with respect to an inertial frame, because only then Newton's second law holds. Likewise, a voltage is considered the generalized effort of the electric domain, i.e., the partial derivative of the energy with respect to the electric charge, and the generalized flow of the magnetic domain, i.e., the rate of change of magnetic flux linkage to a number of windings. In this case the additional assumptions that are required to be able to link the electric and the magnetic domains in this way are even more obvious: the rotation operations in Maxwell's equations are only reduced to a simple gyrating relation when the system is assumed quasi-stationary, in other words: when an electric circuit may be assumed not to exchange energy by electromagnetic radiation which would require a third port of this coupling with an irreversible nature. In retrospect, we can also see the quasi-stationary requirement as an additional assumption for the coupling between the potential and the kinetic domain: such a mechanical system also has to be in a quasi-stationary state as it should not radiate acoustic energy. In both cases the radiated power would violate the power continuity of the symplectic gyrator that is assumed to be the interdomain coupling.

1.5 Properties of an Irreversible Transducer

Returning to the system that transcends from non-equilibrium to equilibrium (Fig. 1.1) shows why the generalized effort can be called an *equilibrium-determining variable*: the generalized efforts of the two parts (subsystems) have to be equal when they are in equilibrium. Likewise, the generalized flow can be characterized as an *equilibrium-establishing variable*: there will be a generalized flow between the two subsystems until this flow goes to zero in equilibrium. It is important to note that there is only one flow due to the flow continuity required by the conservation principle that corresponds to a conserved state. Note that this reasoning does not exclude cross-coupling: if the effort difference at one R-type

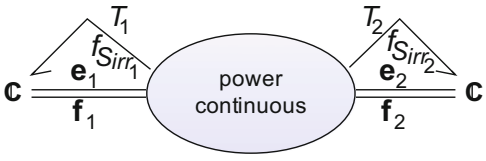


Fig. 1.1 Power continuity of an irreversible transducer due to the energy conservation principle (note that the orientation of the multibonds is not defined yet!)

port of a multiport irreversible transducer is zero, its flow may still be non-zero due to a non-zero effort difference at another R-type port. However, if all effort difference are zero, all flows are zero.

We thus see that the non-equilibrium situation of each of the domains is characterized by one generalized flow, out of one subsystem and into the other, so corresponding with rates of change of the respective conserved states that have an opposite sign (Fig. 1.1), while two generalized efforts with different values are converging to the same value. In other words: there has to be a relation between these efforts and the flow between the subsystems. We also know that during such a process entropy has to be generated in principle and that this entropy is also one of the states. The production can thus be represented by a generalized flow and its conjugate effort is the partial derivative of the energy with respect to the entropy which is known as an absolute temperature $\frac{\partial E}{\partial S} = T$ [9]. Due to the global energy conservation principle and the fact that the concept describing this irreversible process cannot store energy as this would be a "contradictio in terminis," the concept describing the transition from a non-equilibrium into an equilibrium state should be power continuous. If we just consider an irreversible transducer with one entropy producing port which will be labeled as an S-type port, because it act as an entropy source and one R-type port for reasons of simplicity:

$$-e_1 f_1 - e_2 f_2 - T f_{S_{irr}} = 0 \tag{1.9}$$

or, as $f_1 = -f_2$ due to flow continuity:

$$(e_1 - e_2) f_2 = T f_{S_{irr}} \geq 0 \tag{1.10}$$

As the absolute temperature T is positive and the entropy production is zero or positive we can conclude that the generalized effort difference $e_1 - e_2 = \Delta e$ has to have the same sign as the generalized flow f_2, in other words: the generalized flow is always directed from a higher generalized effort to a lower generalized effort, except for cross-coupling effects. At this point it is important to recognize that some efforts, like voltage, have a relative reference, while others, like absolute temperature, pressure, force, or chemical potential, have an absolute reference. In case of a relative reference the flow can only depend on the effort difference and this results in a property called "nodicity" [17]. In non-nodic models like chemical reaction networks the flow can be a function in which the individual values of the efforts are required: this means that such a relation has to be described by an element with at least 2 R-type ports with a nonlinear constitutive relation for which an additional flow continuity constraint holds.

The above conclusion that the generalized effort difference $e_1 - e_2 = \Delta e$ has to have the same sign as the generalized flow f_2 can also be formulated as follows: the relation between this generalized flow and this generalized effort difference has to lie in the first and third quadrant and thus has to pass through the origin. This explains why the equality of the generalized efforts (the partial derivatives of the energy with respect to their states) is an equilibrium criterion as mentioned before. This means that the slope of this relation, which is commonly called a resistance, is

positive in the origin, but may have any value outside of the origin, which explains why the concept of a negative differential resistance does not violate the principle of positive entropy production.

Relation (1.10) can also be written for all n states:

$$\sum_{i=1}^{n} (e_{1_i} - e_{2_i}) f_{2_i} = \sum_{i=1}^{n} \Delta e_i f_{2_i} = \Delta \mathbf{e}^t \mathbf{f} = T f_{S_{irr}} \geq 0 \tag{1.11}$$

We can also conclude that the power port that represents the irreversibly produced thermal power cannot have a linear constitutive relation, even when all other ports are linear and can be characterized by a resistance matrix R, because

$$f_{S_{irr}} = \frac{\Delta \mathbf{e}^t \mathbf{f}}{T} = \frac{\mathbf{f}^t \mathbf{R}^t \mathbf{f}}{T} \geq 0 \tag{1.12}$$

As only the symmetric part of the resistance matrix contributes to this quadratic form:

$$f_{S_{irr}} = \frac{\mathbf{f}^t \mathbf{R}_s \mathbf{f}}{T} \geq 0 \tag{1.13}$$

and

$$\mathbf{f}^t \mathbf{R}_a \mathbf{f} = 0 \tag{1.14}$$

where $\mathbf{R}_s = \frac{\mathbf{R}+\mathbf{R}^t}{2}$ and $\mathbf{R}_a = \frac{\mathbf{R}-\mathbf{R}^t}{2}$, such that

$$\mathbf{R} = \frac{\mathbf{R} + \mathbf{R}^t}{2} + \frac{\mathbf{R} - \mathbf{R}^t}{2} = \mathbf{R}_s + \mathbf{R}_a \tag{1.15}$$

The symmetry of the part of the relations that contributes to the entropy production is called Onsager symmetry. This symmetry relates the Peltier effect to the Seebeck effect, for example [9]. The other part can be identified as a multiport gyrator that is power continuous as a result of (1.14) and of which we have seen that it contains the dualizing interdomain couplings described earlier.

1.6 Influence of the Environment

It was not discussed yet that energy can also be exchanged with as well as stored in the environment of a system. Typical for the concept of an environment of a system is that there is no information available about its extent: the influence of conserved properties that are exchanged with the environment on the behavior of the system is neglected because no information about these extensive quantities is available: the state of the environment can only be characterized by an intensive state, i.e., a generalized effort that can be considered independent of its conjugate generalized flow. If this is not a valid assumption, the system boundary has to be changed such

that it is valid. The environment can thus be seen as a storage element that is so large with respect to the storage in the system, that it can be considered infinitely large, such that the rate of change of extensive state does not influence the intensive state in a relevant manner. Such an element that represents a boundary condition is a generalized effort source Se. In contrast to a storage element, for which modulation would mean a violation of the energy conservation principle, an effort source can be modulated (MSe). When modulated storage elements are used after all, this implies the implicit assumption that the energy exchange due to modulation is at all times negligible compared to the energy exchange (power) through each of its ports.

If the energy storage of a system is described by a (multiport) storage element and the influence of the environment by effort sources, then all other elements have to be power continuous as a result of the energy conservation principle. The common concept of dissipation, which seems to violate this statement, does not refer to energy, but to one of its Legendre transforms [9, 18], viz., the free energy $F(\tilde{\mathbf{q}}, T) = E(\tilde{\mathbf{q}}, S) - TS$, where $\tilde{\mathbf{q}}$ is the state vector of all states $\mathbf{q}$ from which the entropy S has been eliminated. Furthermore,

$$\mathrm{d}F(\tilde{\mathbf{q}}, T) = \sum_i \frac{\partial E(\tilde{\mathbf{q}}, S)}{\partial q_i} \mathrm{d}q_i - S\mathrm{d}T = \sum_i e_i(\tilde{\mathbf{q}}, T)\,\mathrm{d}q_i - S\mathrm{d}T \tag{1.16}$$

which means that the contribution from the thermal domain to a change in free energy is zero when the temperature is constant. If free energy is assumed to be dissipated in a (multiport) resistor, we thus intrinsically assume that interaction with the thermal domain does not lead to additional dynamic properties of the model, even though temperature variations may influence the system via an activated bond, i.e., modulation by the temperature. In those cases the thermal port is not included in the model. This is the only way to obtain linear models, as the model that describes the entropy production port necessarily has a nonlinear constitutive equation, as we will confirm later. The solvability of linear models explains the "popularity" of models in which the above assumption about the thermal domain is made and where all energy is in fact free energy that can be dissipated.

1.7 Co-Energy

The quadratic representation of the kinetic energy of a constant amount of mass discussed earlier is *equal in value* to the sign inverse of its Legendre transform with respect to the momentum, i.e., $-L[E(p)]_p$, which is a so-called co-energy $E^*(v)$, a function in which the momentum is replaced by the velocity as function argument:

$$-L[E(p)]_p = -\left(\frac{p^2}{2m} - \frac{\mathrm{d}E}{\mathrm{d}p}p\right) = vp - \frac{p^2}{2m} = mv^2 - \frac{1}{2}mv^2 = \frac{1}{2}mv^2 = E^*(v) \tag{1.17}$$

Although the values of kinetic energy and kinetic co-energy are the same, their arguments are not: the kinetic energy is a function of a conserved quantity, the momentum, an extensive variable, while the kinetic co-energy is a function of an equilibrium determining variable (generalized effort), the velocity. This distinction is important when energy functions are used as generating function of constitutive relations of multiports. For example, if we return to the first-degree homogeneous form

$$E^*(v,N) = vp - \frac{p^2}{2MN} = MNv^2 - \frac{1}{2}MNv^2 = \frac{1}{2}MNv^2 = E^*(v,N) \tag{1.18}$$

If this co-energy is used as an energy-like generating function, one obtains for the total material potential

$$\frac{\partial E^*(v,N)}{\partial N} = \frac{\partial}{\partial N}\frac{1}{2}MNv^2 = \frac{1}{2}Mv^2 \neq \mu \tag{1.19}$$

because

$$\mu = \frac{\partial E(p,N)}{\partial N} = \frac{\partial}{\partial N}\frac{p^2}{2MN} = -\frac{p^2}{2MN^2} = -\frac{1}{2}Mv^2 \tag{1.20}$$

In other words: the wrong sign is obtained.

This is often not seen, due to the fact that for variable mass with constant density the available volume is used as a state instead of the amount of moles

$$E^*(v,V) = vp - \frac{p^2}{2\rho V} = \rho Vv^2 - \frac{1}{2}\rho Vv^2 = \frac{1}{2}\rho Vv^2 = E^*(v,V) \tag{1.21}$$

$$\frac{\partial E^*(v,V)}{\partial V} = \frac{\partial}{\partial V}\frac{1}{2}\rho Vv^2 = \frac{1}{2}\rho v^2 = p_{\text{dynamic}} \tag{1.22}$$

However, for a pressure, also for a dynamic pressure, an increase in energy corresponds to a decrease in volume, so:

$$p_{\text{dynamic}} = \frac{\partial E(p,V)}{-\partial V} = -\frac{\partial}{\partial V}\frac{p^2}{2\rho V} = \frac{p^2}{2\rho V^2} = \frac{1}{2}\rho v^2 \tag{1.23}$$

In this case the sign error due to the mix up of energy and co-energy is compensated by a second sign error in the partial derivative.

We return to a generic system for which true energy conservation holds and note that all other concepts that are used to describe relations between storage ports have to be multiports that satisfy energy conservation without storage, which means that they have to be power continuous. If we consider an arbitrary power continuous

multiport, it may have mixed causality, in other words its constitutive relation may have the form:

$$\begin{bmatrix} \mathbf{e_1} \\ \mathbf{f_2} \end{bmatrix} = \Psi\left(\begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix}\right) \tag{1.24}$$

where stands for Ψ an arbitrary function. Hogan and Fasse [11] demonstrated by means of scattering variables that such a power continuous constitutive relation can only be of a multiplicative form, i.e.,

$$\begin{bmatrix} \mathbf{e_1} \\ \mathbf{f_2} \end{bmatrix} = \mathbf{J}(.)\begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix} \tag{1.25}$$

where the matrix $\mathbf{J}$ can still depend on any variable.

As a result of power continuity

$$P = \begin{bmatrix} \mathbf{e_1^t} & \mathbf{f_2^t} \end{bmatrix}\begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix} = \begin{bmatrix} \mathbf{f_1^t} & \mathbf{e_2^t} \end{bmatrix}\mathbf{J}^t\begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix} = 0 \tag{1.26}$$

We obtain

$$\mathbf{J} = -\mathbf{J^t} \tag{1.27}$$

At this point we see that we have identified sufficient bond graph concepts to describe dynamic behavior of a system that also interacts with its environment: multiport storages that can interact via a power continuous structure with each other and with the environment that is represented by effort sources (infinite storage) as shown in Fig. 1.2, i.e., three basic concepts. However, given the choice of positive orientation (towards the multiports except for the sources), it turns out that some sign changes would have to be made on the fly that require additional concepts (Fig. 1.3). An experienced bond graph modeler knows that the concept of a junction is needed for this and this is why decomposition of the power continuous connection structure described by (1.25) is discussed next. The need for this additional junction is what is not recognized in the so-called port-Hamiltonian approach, where all

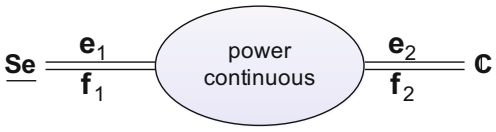


Fig. 1.2 Power continuous interaction between environment and system

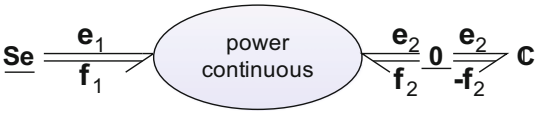


Fig. 1.3 Positive orientation conventions require a 0-junction

orientations to a power continuous structure are chosen positive inward, all positive orientations of multiport storage and dissipation are chosen outward (resulting in an uncommon minus sign in the constitutive relations, but when junction structures have to be combined this results in an anomaly, because junctions have not been defined separately, which may result in error prone sign manipulation at the equation level.

1.8 Decomposition of the Power Continuous Multiport

A quadratic form of a matrix is only zero if it is skew symmetric, cf. (1.27), so it can be decomposed as follows:

$$\begin{bmatrix} \mathbf{e_1} \\ \mathbf{f_2} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix} = \begin{bmatrix} \mathbf{G_e} & \mathbf{T^t} \\ -\mathbf{T} & \mathbf{G_f} \end{bmatrix} \begin{bmatrix} \mathbf{f_1} \\ \mathbf{e_2} \end{bmatrix} \tag{1.28}$$

where

$$\begin{aligned} \mathbf{G_e} &= -\mathbf{G_e^t} \\ \mathbf{G_f} &= -\mathbf{G_f^t} \end{aligned} \tag{1.29}$$

Hogan and Fasse [11] also showed that if we add the property of port-symmetry to a power continuous multiport, there are only two solutions. These solutions appear to be linear without a parameter for all domains where energy is a useful concept: the 0-junction and the 1-junction. Given that these two types of multiports are ideal connectors, these are the only elements that do not need a fixed positive orientation of their ports to guarantee unambiguous use, as a change of orientation is merely a sign change in the summation of either the flows or the efforts, respectively. Using these junctions (in array form [4]) it can be seen (cf. Fig. 1.4) in a straightforward manner that an arbitrary power continuous multiport can be decomposed into two (modulated) multiport gyrators, one with effort-out causality and characterized by the matrix $\mathbf{G_e}$ and one with flow-out causality and characterized by the matrix $\mathbf{G_f}$, as well as a (modulated) multiport transformer characterized by the matrix $\mathbf{T}$. In turn, it can be seen by inspection of the operations of a matrix multiplication, that these multiports can be decomposed into junctions and 2-port (modulated) gyrators and transformers: the multiport gyrator characterized by $\mathbf{G_e}$ by connecting a 1-junction

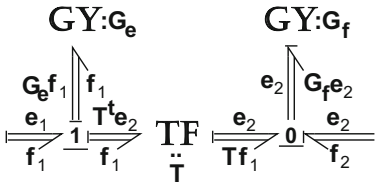


Fig. 1.4 Generic decomposition of a power continuous multiport (**TF** and **GY** may be modulated)

to each port and next connecting each of these ports to all other ports by 2-port gyrators, the multiport gyrator characterized by the matrix $\mathbf{G_f}$ by connecting a 0-junction to each port and next connecting each of these ports to all other ports by 2-port gyrators and finally the multiport transformer characterized by the matrix $\mathbf{T}$ by connecting a 1-junction to each port with effort-out causality and a 0-junction to each port with flow-out causality and next connecting each 1-junction to all 0-junctions by means of 2-port transformers. In this way each 2-port gyrator or 2-port transformer has one of the independent matrix elements of the matrix $\mathbf{J}$ as ratio. As discussed in more detail in [3, 4], these are canonical, immediate decompositions.

Note that a gyrators and transformers, except for unit transformers that are in fact equal to bonds, cannot exist inside one domain, as they would violate generalized flow continuity: generalized flows are the rates of change of the conserved quantity that characterizes the domain and neither a transformer nor a gyrator are flow continuous. In other words: they are interdomain couplings. When the objection is made that transformers in mechanical systems seem to violate this reasoning, one should realize that each coordinate in a mechanical model should be considered a separate domain, because a bond graph cannot represent configuration properties: the coordinate frames have to be represented separately and are linked to the scalar values that are used in the topological representation that a bond graph is.

In [3, 4] it was shown that a linear multiport storage element can be decomposed in many ways using a multiport transformer by making use of the congruence properties of its constitutive matrix with a diagonal matrix. However, there is only one congruence decomposition that is canonical, i.e., in which the number of independent parameters of the decomposition is equal to the number of independent parameters of the multiport storage element: the transformation matrix is triangular, such that less 2-port transformers are required. Just like the immediate decompositions of the multiport gyrator and transformer that were just discussed (Fig. 1.4) and that appeared to be canonical, a linear multiport storage element also has such an immediate decomposition: in preferred integral causality all ports are connected to a 1-junction and next each 1-junction is connected to all others via a 0-junction. Finally each junction is connected to a 1-port C. In differential causality the junction types have to be interchanged.

Just like a multiport C, a multiport gyrator also has a congruence decomposition as any skew symmetric matrix is congruent with a block-diagonal matrix with 2×2 symplectic matrices as blocks.

So far we have identified that all models in which energy is a relevant variable can be constructed from 1-port storage elements (C), 1-port effort sources (Se), 2-port (modulated) transformers and gyrators, and 0- and 1-junctions, i.e., 6 basic elements. Given that a unit gyrator can be combined with a 1-port element, dual storage elements (I) and dual sources (Sf) can also be defined, but one should be aware that the asymmetry is then lost between effort and flow as equilibrium-determining and equilibrium-establishing variable, respectively. We then have identified eight basic elements and bond graph modelers will notice that the resistor is still missing: an R-element contradicts energy conservation and can only be used if the influence of the thermal domain on the dynamic properties of a system can be disregarded.

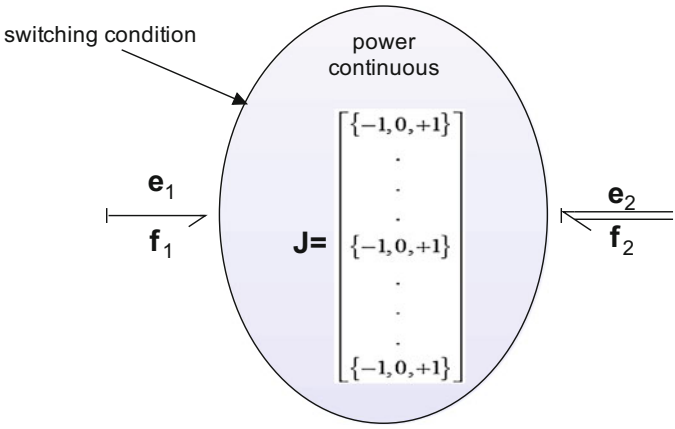


Fig. 1.5 X0-junction as special case of a power continuous multiport (transformer)

Before resolving this paradox two additional basic concepts are added that are in principle not needed, but of such a high conceptual value that they deserve a special representation: the switched junctions. Switched junctions (X0 and X1) represent that the possibility of energy exchange may depend on the state of a system, the easiest example being a bouncing object [7]. As a junction can be considered a power continuous multiport (that is even non-mixing, so in fact a multiport transformer) with a special form of the transformation matrix **J** in (1.25), viz., a column matrix with elements that can only take the values -1 or $+1$, the energy exchange can be switched off by setting all the matrix elements to zero, i.e., by modulating it by a Boolean as a function of some condition that commonly depends on the state of a system (Fig. 1.5). However, since the possibility of modulation of elements is not considered to increase the number of basic elements, we will stick to the common number of nine basic elements.

1.9 Representation of Irreversibility

The elements we have identified so far as needed for a consistent description of energetic behavior (C, I, (M)Se, (M)Sf, (M)TF, (M)GY, (X)0, and (X)1) are all reversible, such that irreversibility due to the second principle of thermodynamics does not seem present, although we have seen that it may be part of a nonlinear power continuous junction structure. This implies the need for a 2-port with a resistive port and an entropy producing port that is power continuous. Such an RS element has indeed been defined, but causes a paradox with Hogan's and Fasse's [11] result that all power continuous 2-ports are either modulated gyrators or modulated transformers. This paradox is resolved by rewriting the constitutive

relations of an RS in such a way that they represent a port-modulated transformer or gyrator, respectively, depending on the causality:

$$\text{RS}: \begin{cases} e = e(f) \\ f_{S_{irr}} = \frac{ef}{T} \end{cases} \tag{1.30}$$

$$\text{MTF}: \begin{cases} f_{S_{irr}} = \left(\frac{e(f)}{T}\right) f \\ e = \left(\frac{e(f)}{T}\right) T \end{cases} \tag{1.31}$$

$$\text{RS}: \begin{cases} f = f(e) \\ f_{S_{irr}} = \frac{ef}{T} \end{cases} \tag{1.32}$$

$$\text{MGY}: \begin{cases} f_{S_{irr}} = \left(\frac{f(e)}{T}\right) e \\ f = \left(\frac{f(e)}{T}\right) T \end{cases} \tag{1.33}$$

In principle, an effort-out causality of the entropy producing port is also possible, but not shown, because this is a rare possibility. As a consequence, a separate RS is in principle not needed, but such a representation would mean that a key property of dynamic system, viz., irreversibility, is not represented by a separate conceptual element symbol, but remains more or less hidden in a specific way to modulate an element that is reversible in its unmodulated form. Therefore, the RS is added as the ninth element and this fundamentally nonlinear element can only be linearized by reducing the influence of the thermal domain to an imposed temperature, such that no dynamic interaction with the thermal domain takes place and entropy does not need to be modeled. Such a change of causality for the multiport storage corresponds to a Legendre transform of its stored energy with respect to the entropy. As we have seen, this Legendre transform $F = E–TS$ is called the free energy, which can be considered to be dissipated in a resistor (R). By doing this the entropy-maximum principle for nonlinear models is transformed into the more common (free!) energy-minimum principle that also applies to linear models.

Obviously, a 1-port R can be generalized into a multiport resistor characterized by a constitutive matrix or a set of interdependent nonlinear constitutive relations with a constitutive Jacobian. In order to decompose it we will have to consider its properties first.

1.10 Decomposition into Basic Elements

The above has made clear that when multiports are assumed to have linear constitutive relations, decomposition in terms of junctions and the remaining seven basic one- and two-elements (cf. Tables 1.1 and 1.2) is possible. As there are many possibilities to do this, the concept of a canonical decomposition was introduced,

which means that the decomposition does require the same number of independent constitutive parameters as the original multiport [3, 4]. Two types of canonical decomposition are possible, the so-called congruence canonical decompositions that are based on the congruence properties of the constitutive matrix and immediate canonical decompositions that follow from direct inspection of the constitutive relations. For 2-ports it has been shown that these two types are exhaustive which strongly suggests that this is also the case for arbitrary multiports [6]. In [3, 4] it was also shown that decomposition of most elements with nonlinear constitutive relations can also be realized by means of internal modulation, with the exception of nonlinear multiport storage elements, as this would introduce multiport artifacts that need to compensate the gyristors that would be caused by the modulation, which would destroy the simplifying nature of a decomposition.

Multiport transformers only have immediate decompositions: their two conjugate constitutive relations consist of a multiplication of a flow vector by a matrix resulting in another flow vector and a multiplication of an effort vector by the transpose of that matrix resulting in another effort vector. A matrix multiplication can be decomposed into multiplications and summations: the multiplications are represented by 2-port transformers with the multiplying matrix element as transformation ratio and the summations are represented by 0-junctions and 1-junctions, respectively, as shown in (Fig. 1.6).

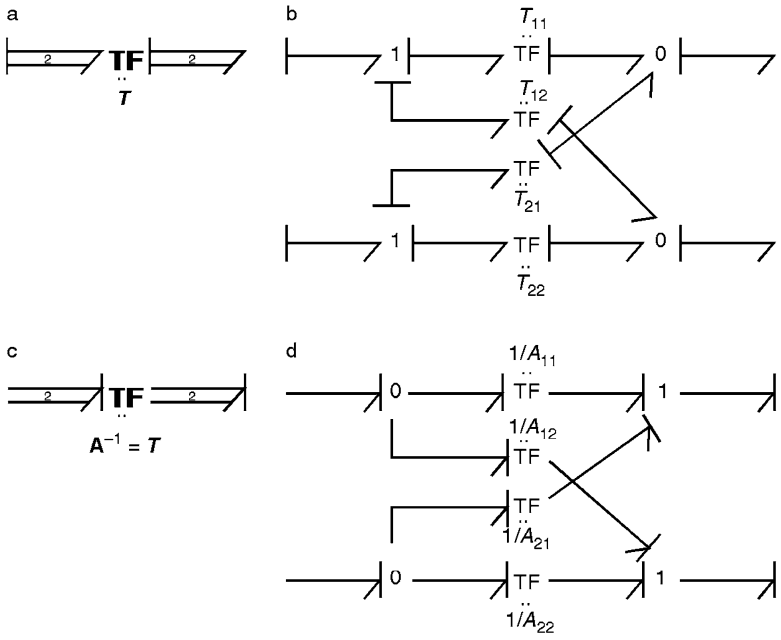


Fig. 1.6 Immediate decomposition of a multiport transformer

A linear multiport resistor or linear multiport storage element is characterized by a symmetric matrix. If such a multiport is connected to a multiport transformer, it can be combined into new multiport element characterized by a matrix $\tilde{\mathbf{C}}$ that equals the inverse original matrix $\mathbf{C}$ pre-multiplied by the transpose of the transformation matrix and post-multiplied by the transformation matrix, resulting again in a symmetric matrix:

$$\begin{aligned}
&\mathbf{T^t C^{-1} T} = \tilde{\mathbf{C}}^{-1} \\
&\tilde{\mathbf{C}} = \left(\mathbf{T^t C^{-1} T}\right)^{-1} = \mathbf{T^{-1} C T^{-t}} \\
&\tilde{\mathbf{C}}^t = \left(\mathbf{T^{-1} C T^{-t}}\right)^{\mathbf{t}} = \mathbf{T^{-1} C^{-1} T^{-t}} = \mathbf{T^{-1} C T^{-t}} = \tilde{\mathbf{C}}
\end{aligned} \tag{1.34}$$

When we combine this result with the property of a symmetric matrix that it is congruent with a diagonal matrix, it is obvious that a combination of a multiport transformers and an array of uncoupled one-ports storage elements or resistive elements can represent a linear multiport storage element or a linear multiport resistive element, respectively (Fig. 1.7).

However, such a linear multiport has $n + \left(n^2 - n\right)/2 = \left(n^2 + n\right)/2$ independent parameters, while a combination of a multiport transformers and an array of one-ports has $n^2 + n$ parameters, such that $\left(n^2 + n\right)/2$ of these parameters are dependent. However, congruence of a symmetric matrix with a unit matrix can be realized by a triangular matrix, which has $\left(n^2 - n\right)/2$ zero elements and all n diagonal elements of a unit matrix are unity, such that $\left(n^2 + n\right)/2$ independent parameters remain.

If the diagonal elements of such a triangular matrix are set to unity and the unit matrix is replaced by an arbitrary diagonal matrix, a combination of n one-port elements with a multiport transformer with $\left(n^2 + n\right)/2$ independent parameters is obtained, which is decomposable by immediate decomposition (Fig. 1.7), hence a

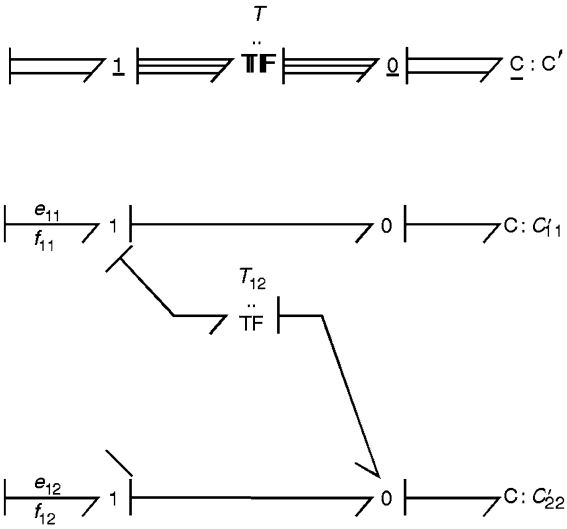


Fig. 1.7 (De)composition of a multiport, C respectively, into a multiport C (2-port shown for simplicity)

canonical decomposition. However, one should keep in mind that the decomposition of a multiport transformer depends on its causality: causal inversion of the decomposed form leads to algebraic loops.

Immediate decompositions of multiport storage or resistive elements are also possible by rewriting the matrix relations. For example, for two-ports this results in the so-called Π- and T-type decompositions, depending on causality. For storage elements the disadvantage of this canonical decomposition is that the number of stored quantities and initial conditions (three in case of a two-port) becomes larger than the number of independent states and initial conditions of the multiport (two in case of a two-port). More details can be found in [3, 4], but what is important here is that all linear multiports are decomposable into the basic 1-ports and 2-ports in a junction structure of bonds and 0- and 1-junctions.

1.11 Conclusion

The nine common basic elements of bond graphs were shown to result from the fundamental principles of physics and many implicit assumptions that are often not considered. It will improve the modeling process, when a modeler is more aware that these assumptions may not hold. The use of the bond graph notation already helps a modeler when the grammar rules and semantics of this graphical language are respected. The additional awareness that a bond graph stems from multiport storage, effort sources to represent the environment and a power continuous multiport that may also contain irreversible transduction, may improve the modeling process even further. For example, using the concept of the generalized domain in a generalized bond graph notation makes clear that the required flow continuity only allows the use of C, (M)Se, (X)0, (X)1, and thermal (M)RS elements inside the domain, while (M)TF, M(GY), and other (M)RS elements are interdomain couplings. If the thermal domain is excluded and power is considered a flow of free energy, then C, (M)Se, (X)0, (X)1, and R are intradomain elements. Obviously, multiport C and (M)R can serve as interdomain couplings too.

A central role plays the resistive multiport: if seen as a generalization of a simple 1-port R we have seen that the generic relation between an effort and a flow vector also generates a part that does not contribute to the entropy production, viz., the skew-symmetric part of its Jacobian, which can be represented by a multiport gyrator. By contrast, if we start from the power continuity that also should hold for irreversible transducers, we saw that an RS can in principle be replaced by a modulated transformer or gyrator. One could say that the first approach is kind of bottom up from the common approach that uses free energy that can be dissipated, while the second approach starts from the energy conservation principle. The first approach has the advantage that we can recognize that close to equilibrium of domains with an absolute reference no gyrator can exist, such that processes only have the nature of relaxation, while for from equilibrium the Jacobian of nonlinear

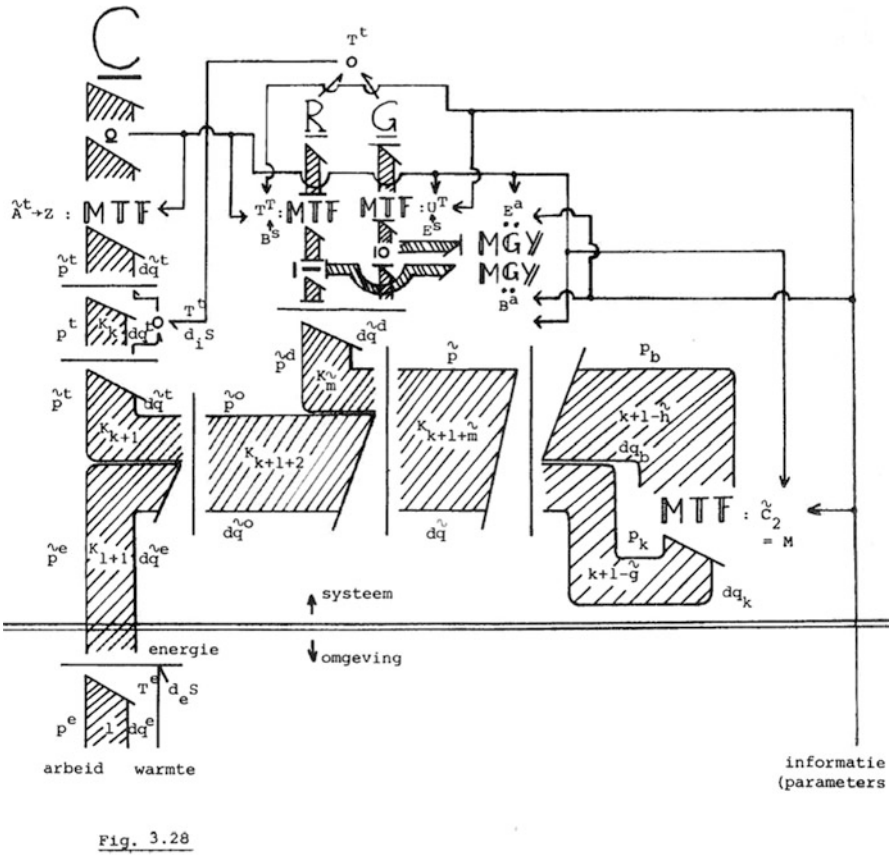


Fig. 1.8 Historic [1] multibond decomposition (except MTF and MGY)

relation may result in gyrating couplings that, for instance, can explain chemical oscillations or the use of a transistor to create an oscillator while biased into a non-equilibrium operating point.

Finally, a picture from the author's master thesis [1] (Fig. 1.8: ignore the Dutch and the uncommon choice of some symbols, like the G referring to a resistive port in conductive causality, no addition of an S to the symbol of an irreversible transducer and a direct sum definition that allowed a sign change) is shown to demonstrate that a multibond representation of a generic decomposition (except for the MTF's and MGY's of which immediate decomposition is straightforward) has been available for over 35 years. The rather cryptic symbols for effort and flow variables, bond dimensions, and constitutive matrices are the result of the fact that this multibond graph tried to represent and extend an analysis by [12] of the power continuous conceptual structure in thermodynamic systems. Obviously, it was not recognized at that time that the RS is just a part of the power continuous interconnection structure and, as such, a nonlinear "constraint."

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