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Identification of compartmental models of diffusive systems by linear regression

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Abstract. A linear regressive model structure and output predictor, both in algebraic form, are deduced from an LTI state space system with symmetric tridiagonal system matrix structure. In practice, systems with such a system matrix arise in modeling of distributed system configurations, and are known as *compartmental* systems. Two diffusion example cases are approximated by compartmental models and simulations of one case are worked out. Simulation results indicate that the method may be attractive for estimation, prediction and insight in compartmental systems, especially when physical knowledge is to be preserved.

1 Introduction

In input-output relations of (compartmental) diffusive systems, physical parameters appear non-linearly, resulting in the use of (constrained) non-linear parameter estimation techniques with its short-comings regarding global optimality and computational effort. Our approach is to handle the parameter estimation and prediction problem, initially for an LTI infinite-dimensional system, via discretization and a linear regressive parametric realization of the system in order to obtain unique estimates. We are motivated to pursue this path instead of following a 'black box' data-based approach—like subspace identification, because we can keep track of the original (physical) parameters.

In practice, compartmental modeling frequently arises in distributed system configurations. The interested reader is referred to [2] for identifiability results of these systems by an algebraic approach. In the specific case that the matrices A, B, C and D of a compartmental LTI system, say Σ^d, is linear in the parameters ϑ, we are able to find another realization of Σ^d which is suited for linear regressive estimation and prediction. After some integral transform of Σ^d, we obtain a set of linear equations of the form $\varphi^T M^{-1}\psi = b$, with $M = qI - A(\vartheta)$, A the system matrix and q the time backshift operator. In the specific case that A is a symmetric tridiagonal matrix, explicit solutions for M^{-1} are known, *e.g.* [1].

Hence, the *key* is to find M^{-1}, such that we may rewrite Σ^d as a linear regressive set of equations in a newly defined parameter vector θ and in past and current input-output data. From hereon, it is rather straightforward to arrive at an estimate $\hat{\theta}$ using existing estimation techniques (Total Least Squares, IV, etc.).

2 Case studies

Consider the nonsingular tridiagonal matrices $M_\mathrm{A} \in \mathbb{R}^{n\times n}$ and $M_\mathrm{B} \in \mathbb{R}^{n\times n}$. Note that we slightly abuse our notation by the subscripts A and B which correspond to our example cases A and B. We define,

$$M_\mathrm{A} = \begin{pmatrix} a & b & 0 & \cdot & 0 \\ b & a & b & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & b & a & b \\ 0 & \cdot & 0 & b & a \end{pmatrix}, \quad M_\mathrm{B} = \begin{pmatrix} c & b & 0 & \cdot & 0 \\ b & a & b & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & b & a & b \\ 0 & \cdot & 0 & b & a \end{pmatrix}, \tag{1}$$

where a, b and c are scalars corresponding to physical and possibly, discretization parameters. They are assumed constant and $c \neq a \neq 0 \neq b$. Here, matrix M corresponds to the resolvent of the system matrix A of Case A and B.

- *Case A:* a compartmental approximation of a boundary control system with Dirichlet boundary conditions;
- *Case B:* a compartmental approximation of a boundary control system with one Neumann boundary condition.

We will study the inverse of M_A and M_B and the use of $M_\bullet^{-1}$ for estimation of θ, prediction of the observation y at time instant t_k and parametric output sensitivities of the compartmental system. Some results are reported in [3].

References

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